Realization of stochastic phase-space quantization on a collective field theory

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We consider the collective-field-theoretical description of a many-body quantum system (one dimensional for simplicity). We show how the stochastic phase-space quantization prescriptions can be applied to such a system. We define integration measures for the inner product and prove, constructively, that it is not necessary to employ effective quantities (e.g., effective action, Hamiltonian, etc.) in this scheme, unlike the conventional formulation of collective field theory. We offer intuitive interpretations of these results in terms of particle spreading and minimum uncertainty. A specific, simple example is explicitly worked out.

I. INTRODUCTION

The stochastic approach to quantization advocated by Prugovečki and collaborators[1,2] has a number of advantages stemming from the fact that it is formulated directly in phase space. Essential to the realization of the stochastic scheme is the existence of a family of functions $\{\xi_{p,q}(x)\}$, which resolve (continuously) the identity in $L^2(\mathbb{R}^n)$, where $\mathbb{R}^n$ stands for configuration space. We write, formally,

$$\int_\Gamma |\xi_{q,p}\rangle dq\, dp \langle \xi_{q,p}| = 1 , \tag{1}$$

where Γ denotes the phase space of the system. In terms of the functions $\xi_{q,p}(x)$, (1) reads:

$$\int_\Gamma \xi^*_{q,p}(x)\xi_{q,p}(x')dq\, dp = \delta(x-x') . \tag{1'}$$

Through the family $\{\xi_{q,p}\}$ one is able to achieve an irreducible representation for the canonical quantization mappings pertinent to the stochastic scheme and whose most popular form is[1-4]

$$\hat{Q} = q + i\hbar\frac{\partial}{\partial p} , \tag{2a}$$

$$\hat{P} = -i\hbar\frac{\partial}{\partial q} . \tag{2b}$$

Henceforth we shall set $\hbar = 1$. A slight modification of (2a) and (2b) adhering to a requirement of a more specific nature has been discussed in Ref. 5.

Our own viewpoint of stochastic quantization, as emerges through our work in Refs. 5–7, is that it generates a formalism which has a natural way of describing collective, or coherent, behavior on quantum systems. This point is actually underlined by the relation that connects the customary wave function $\phi(x)$ in configuration space with the phase-space wave function relevant to the stochastic scheme:[1,2]

$$\psi(q,p) = \int \xi_{q,p}(x)\phi(x)dx . \tag{3}$$

As clearly seen, the phase-space wave function $\psi(q,p)$ constitutes a superposition of ordinary wave functions $\phi(x)$.

In this paper we shall pursue our interpretation of stochastic quantization further by directing our attention to collective field theory. To the extent that a field theory constitutes an extreme instance of a many-body system, we anticipate that the methodology of phase-space quantization, in its stochastic formulation, will find a natural realization within a field-theoretical context.

There are two basic reasons behind our choice of collective field theory as a system with a continuous infinity of degrees of freedom on which to explicitly test our interpretation of Prugovečki's stochastic quantization. First, collective field theory is nonrelativistic, hence, easier to deal with. Second, it is a field theory which is directly borne out of a first quantized (N-body) system, where the stochastic description is well understood.

The story of collective field theory dates back to almost four decades ago. Our analysis, on the other hand, will rely exclusively on the more recent work of Jevicki and Sakita[8,9] who revisited the problem in connection with the $1/N$ expansion in field theory. Our present objectives are, of course, quite different.

To further motivate our thinking, let us now direct our attention to phase-space matters, whose role in our scheme is quintessential. A number of authors[10] have already emphasized the decided advantages stemming from the formulation of history over paths directly in phase space. This point has been made irrespective of the stochastic approach to quantization. The phase-space history over paths leads to the following expression for the transition amplitude $\langle q',t'|q,t\rangle$:

$$\int \frac{\mathcal{D}q(\tau)\mathcal{D}p(\tau)}{2\pi\hbar}\exp\left[i\int_t^{t'}[p\dot{q}-H(p,q)]d\tau\right]\Bigg|_{\substack{q(t)=q\\ q(t')=q'}} . \tag{4}$$

Now if the Hamiltonian has the customary structure

$$H=\frac{p^2}{2m}+V(q)\ , \tag{5}$$

then the momentum integration can be easily performed (it is of a Gaussian nature) leading to the standard, configuration-space, path-integral formula. There are instances, on the other hand, where the momentum integration leads to an effective action which replaces the original one. Generally speaking, such a situation arises with the quantization of constrained systems.

Interestingly enough, collective field theory, as developed through conventional quantization prescriptions in coordinate representation, is eventually formulated in terms of an effective action. The explicit content of this statement will be given in Sec. II where the results of Jevicki and Sakita[8,9] will be briefly reviewed. The general idea is that a nontrivial Jacobian factor enters the path-integral formula and the same Jacobian is involved in the inner product between state functionals. In the present work we shall define a collective-field-theoretical scheme within the framework of a stochastic phase-space approach to quantization.

Given the phase-space content of our collective-field-theoretical description we expect that it will be possible to dispose with the aforementioned Jacobian factor. We shall concentrate on this particular aspect of our construction, irrespective of whether it simplifies the Hamiltonian descripton of the system. Our main concern is to understand the subtleties involved in the stochastic approach to collective field theory rather than to explore its possible practical advantages. We shall, nevertheless, discuss a specific example within the stochastic collective field theoretical context advocated in our present work.

Our paper is organized as follows. In Sec. II we present our main construction: a collective field theory adhering to the premises of stochastic, phase-space quantization. The focal point of our efforts will be the specification of integration measures which define corresponding inner products. In Sec. III we determine conditions under which no extra Jacobian factor enters the stochastic collective-field-theoretical scheme. We discuss a simple example to which we apply the new formalism. We also offer some comments concerning the meaning of not having the extra Jacobian factor. In Sec. IV we present some general thoughts regarding the extension of our work to relativistic field theories.

II. PHASE-SPACE COLLECTIVE FIELD

A. Conventional collective field theory: brief summary

Our considerations will be conducted, for simplicty, on a one-dimensional many-body system. Let there be a collection of spinless particles dispersed within the interval (a,b). The total number of particles in our collection is N. Let $S[x]$, $x=(x_1,x_2,\ldots,x_N)$, denote the action functional which results after a Wick rotation ($t\to -it$) has been performed. In that case we may talk of a partition function for the system which is given by the functional integral

$$Z_N=\int\cdots\int\prod_{i=1}^{N}\mathcal{D}x_i e^{-S[x]}\ . \tag{6}$$

The first step toward formulating a collective field theory for the system is to introduce the density operator

$$\phi(q)=\frac{1}{N}\sum_{i=1}^{N}\delta(q-x_i) \tag{7}$$

obeying the (obvious) constraint

$$\int\phi(q)dq=1\ . \tag{8}$$

The operator $\phi(q)$ serves as a collective field variable. In terms of this variable the partition function (6) is given by the formal expression

$$Z_N=\int\mathcal{D}\phi(q)e^{-S[\phi]}J[\phi]\ , \tag{9}$$

where the Jacobian factor $J[\phi]$ enters on account of the change of variables. It will be given an explicit form, once the integration measure in (9) is defined.

At the same time the wave functions $\psi(x_1,x_2,\ldots,x_n)$ in the original description of the N-body system become functionals of the form $\Phi[\phi]$.

We now give concrete meaning to the integration measure $\mathcal{D}\phi(q)$ through the Fourier expansion

$$\phi_k=\frac{1}{L}\sum_{i=1}^{N}e^{-ikx_i},\quad k=\frac{2\pi n}{L},\quad n=\pm1,\pm2,\ldots\ , \tag{10}$$

where L is the length of the interval $[a,b]$, i.e., the "volume" of our system.

In terms of the Fourier components ϕ_k, the Jacobian $J[\phi]$ is given by

$$J[\phi]=\int\prod_{i=1}^{N}\mathcal{D}x_i\prod_{k\,(\neq0)}\delta\left[\phi_k-\frac{1}{L}\sum_{i=1}^{N}e^{-ikx_i}\right]\ , \tag{11}$$

whereas (9) takes the form

$$Z_N=\int\prod_k d\phi_k J[\phi]e^{-S[\phi]}\ . \tag{12}$$

We next focus our attention on the action functional $S[\phi]$. It turns out that all nontrivial information comes from the kinetic part which reads $\frac{1}{2}\sum_{i=1}^{N}\hat{p}_i^{\,2}$ in the original N-body description. Denoting the corresponding functional operator by $T[\phi]$, we have[8]

$$T[\phi]=\frac{i}{2}\sum_k(-k^2\phi_k)\left[-i\frac{\partial}{\partial\phi_k}\right]+\frac{1}{2}\sum_{k,k'}\frac{kk'}{L}\phi_{k-k'}\left[-i\frac{\partial}{\partial\phi_k}\right]\left[-i\frac{\partial}{\partial\phi_{-k}}\right]\ . \tag{13}$$

As part of the Hamiltonian functional operator the above expression assumes the form

$$T[\phi,\pi]=-\frac{i}{2}\sum_k k^2\phi_k\pi_{-k}+\frac{1}{2}\sum_{kk'}\frac{kk'}{L}\phi_{k-k'}\pi_{-k}\pi_{k'}\ , \tag{14}$$

where we have made the (obvious) definition $\pi_{-k}\equiv -i(\partial/\partial\phi_k)$. From (10) one surmises that Hermitian conjugation amounts to

$$\phi_k^\dagger = \phi_{-k} \;, \tag{15a}$$

$$\pi_k^\dagger = \pi_{-k} \;. \tag{15b}$$

It turns out that the Hamiltonian is not Hermitian under the above conjugation procedure unless the Hilbert space is furnished with the inner product[8,9]

$$(\psi_1,\psi_2) = \int \prod_{k\,(\neq 0)} d\phi_k J[\phi]\Phi_1^\dagger[\phi]\Phi_2[\phi] \;. \tag{16}$$

We observe the omnipresence of the Jacobian factor $J[\phi]$ throughout this construction. In particular, the effective action functional mentioned in the Introduction reads

$$S_{\text{eff}}[\phi] = S[\phi] - \ln J[\phi] \;, \tag{17}$$

where $S[\phi]$ is the expression resulting from the straight substitution of (7) into the original action $S[x]$.

We close our survey with the important remark that the Jacobian entering the inner product (16) can be traded with a redefinition of the canonical momenta. Explicitly, the similarity transformation $\Phi[\phi] \to J^{-1/2}[\phi]\Phi[\phi]$, which eliminates $J[\phi]$ from (16), implies the following readjustment for the canonical momentum operator π_k:

$$\pi_k \to J^{1/2}[\phi]\pi_k J^{-1/2}[\phi] = \pi_k + J^{1/2}[\pi_k, J^{-1/2}[\phi]]$$

$$= \pi_k + \tfrac{1}{2} i \frac{\partial \ln J[\phi]}{\partial \phi_k} \;. \tag{18}$$

By substituting the kinetic energy term, one obtains an effective Hamiltonian H_{eff} which, by construction, is Hermitian with respect to an inner product not involving a Jacobian factor.

B. Preparatory remarks on the stochastic scheme

Our objective, as stated in the Introduction, is to set up a collective field theory within the framework of the stochastic phase-space approach to quantization. From the outset we agree to adopt the standard practice of elevating wave functions, associated with first quantization, to field operators, associated with second quantization. In particular, Eq. (3) will be carried over as a relation among field operators.

There are now certain pertinent aspects of the stochastic scheme which we would like to review before undertaking our explicit analytical considerations. The first remark pertains to the fact that in the stochastic framework neither position nor momentum eigenstates are, in general, available. The modifications of the canonical mapping (2a) and (2b) introduced in Ref. 5 for the explicit purpose of realizing such eigenstates leads to a Hilbert space inner product with respect to which neither $\hat{Q}$ nor $\hat{P}$ are Hermitian operators. At first sight, it appears that this occurrence will present a severe handicap in our effort to define a functional integration measure for the sought-after collective field theory. After all, the integration measure defined through the Fourier decomposition (10) for the conventional collective field theory relies on the Hermiticity of the momentum field operator.

We do know, on the other hand, that the stochastic quantization scheme is unitarily equivalent to the conventional one.[2] Furthermore, the work of Berezin and Šubin[11] establishes the existence of kernels $K(p,q;x,y)$ which connect phase-space quantization conditions of the form (2a) and (2b) to corresponding ones realized on pair of dual variables (x,y) with the conventional representation $x \to \hat{X} = x$, $y \to \hat{Y} = -i(\partial/\partial x)$ (see also Ref. 12 for an application of the Berezin-Šubin work to stochastic phase-space quantization). We, therefore, anticipate an eventual success in constructing a complete set of vectors within the context of our approach.

Our final comment pertains to our own work on stochastic phase-space quantization. The point has been abundantly made[5–7] that this approach is a natural one for accommodating states of minimum uncertainty. As already implied above, the simultaneous presence of p and q variables in the wave function leads to particle descriptions which do not support absolute localization either in coordinate or in momentum. Consider, in this connection, the interpretation given to the generating functions $\xi(x-q)$ which give rise to the resolution family of functions according to

$$\xi_{q,p}(x) = \exp(ipx)\xi(x-q) \;. \tag{19}$$

These generating functions define,[1,2] through their modulus squared, confidence functions

$$h(x-q) = |\xi(x-q)|^2 \;, \tag{20}$$

which furnish a measure of the "spreading" of a quantum-mechanical particle. (Corresponding arguments can be made in momentum space.)

We have speculated[5,6] that the stochastic phase-space quantization scheme accommodates, in a natural manner, the well-known coherent states of minimum uncertainty. The latter comprise a complete (but not orthonormal) set of functions. In our following work we shall find it convenient to rely on a similar set of functions which is complete but not orthonormal.

C. Construction of a phase-space collective field theory

We consider an N-body setup similar to the one in Sec. II A. The first step, in our attempt to construct a stochastic collective field theory, is to transcribe the density operator given by (7) into a corresponding phase-space quantity. The bridging relation is given by (3). Substituting (7) into (3) and using (19), we obtain an initial expression for the collective field which is relevant to our new description. It reads

$$\psi(q,p) = \frac{1}{N}\sum_{i=1}^{N} e^{ipx_i}\xi(x_i-q) \;. \tag{21}$$

For reasons of emphasis, related to the fact that we have generated the resolution family of functions $\{\xi_{q,p}\}$ from functions defined on coordinate spaces, c.f. (19), we shall write $\psi_p(q)$, instead of $\psi(q,p)$ from here on.

In Ref. 6 we have identified the above expression with the so-called Bloch functions, widely employed in solid-state physics[13] in connection with electronic lattices. According to that interpretation, the generating functions $\xi(x_i-q)$ correspond to wave functions pertaining to the particle located at the lattice site i. The relevant

Schrödinger equation contains a Hamiltonian with potential energy determined by the interactions of said particle with the other particles as well as external forces.

We observe that our transcription into the phase-space language has amended the idealized description of the collection of particles in terms of δ functions by a more realistic one in terms of Bloch functions.

The lattice arrangement, though not essential from a physical point of view, presents a concrete advantage in that it allows us to employ Bloch functions. Given that we are actually considering systems with a sizeable number of particles, the collective-field-theoretical approach is meaningless; otherwise, the lattice setup is equivalent to assuming a uniform density distribution. We shall adopt this arrangement from now on. (Alternatively, we can adopt a more cautious point of view and simply state that our subsequent construction pertains exclusively to lattice systems.)

Our pressing task at this point is the search for completeness. To this end, we shall trade the lattice periodic Bloch functions with the, unitarily equivalent, Wannier functions $\psi_i(q)$ defined by

$$\psi_i(q)=\int_{\rm BZ}\frac{dp}{L}e^{-ipx_i}\psi_p(q) , \qquad (22)$$

where the integral goes over the first Brillouin zone (BZ) with volume L in the reciprocal lattice.

The Wannier functions obey the completeness relation

$$\sum_{i=1}^{N}\psi_i(q)\psi_i(q')=\delta(q-q') . \qquad (23)$$

They are not, however, orthonormal.

By trading the p variable (Bloch functions) with the i variable (Wannier functions) we are now in possession of a localized description of the system with a basis labeled by the particle number. To make further progress we must become more specific about the generating functions $\xi(q-x_i)$. According to our interpretation, c.f. discussion following Eq. (21), we can think of the $\xi(q-x_i)$ as wave functions pertaining to each individual particle. They can be expanded in a basis of functions which describe the corresponding individual particle. One such expansion is

$$\xi(q-x_i)=\sum_k c_k^i e^{i(q-x_i)k}, \quad k=\frac{2\pi n}{L}, \quad n=\pm1,\pm2,\dots . \qquad (24)$$

In other words, $\xi(q-x_i)$ is conceived of as a distribution around the particle position x_i which replaces the idealized description, in the conventional collective field case, of the same particle through $\delta(q-x_i)$.

Substituting (21) in (22), we obtain

$$\psi_j(q)=\frac{1}{N}\sum_k\int_{\rm BZ}\sum_i c_k^i e^{ip(x_i-x_j)}e^{-ikx_i}e^{-iqk}$$
$$\equiv\frac{1}{N}\sum_k\psi_{j,k}e^{-iqk} . \qquad (25)$$

We now introduce our collective field through its expansion in the Wannier basis

$$\varphi(q)=\sum_i a_j\psi_j(q) . \qquad (26)$$

Substituting (25) into the above expansion we obtain an alternative expansion of the collective field in a Fourier basis which we exhibit as follows:

$$\varphi(q)=\sum_k\varphi_k e^{iqk} . \qquad (27)$$

Appealing to (25) and (26), we determine that φ_k is of the general form

$$\varphi_k=\frac{1}{N}\sum_j b_j(k)e^{-ikx_j}, \quad k=\frac{2\pi n}{L}, \quad n=\pm1,\pm2,\dots . \qquad (28)$$

In sharp contrast to (10), the Fourier expansion of φ_k has nonunity coefficients.

A particularly simple situation arises when we divide phase space into cells of volume 2π and employ for the $\xi(q-x_i)$ corresponding step functions per lattice site.

The confidence function $h(q,x_i)$ now reads (the c_i are complex parameters)

$$h(q-x_i)=\begin{cases}|c_i|^2 & \text{if } |x_i-q|<\epsilon\\ 0 & \text{if } \epsilon<|x_i-q|<a ,\end{cases} \qquad (29)$$

where ϵ denotes the spreading of the wave (step) function and a the lattice spacing in configuration space. In this case the c_k^i entering (24) do not depend on k and (28) assumes the simpler form

$$\varphi_k=\frac{1}{N}\sum_j b_j e^{ikx_j} , \qquad (28')$$

where the b_j are independent of k.

Whether in the form of (28) or that of (28′), our integration measure emerges as $\prod_{k\,(k\neq0)}d\varphi_k$. An alternative choice stems from (26), namely, $\prod_j da_j$. For practical reasons, however, we shall stick with our first choice throughout.

III. DISPENSING WITH THE JACOBIAN FACTOR

In Sec. II we successfully conducted a search for completeness which produced an assortment of integration measures, appropriate to the collective-field-theoretical scheme. We shall now proceed to examine whether the Hermiticity of the Hamiltonian can be effected in a natural way, without the necessity of introducing a Jacobian factor into the inner product. The strategy we shall employ will utilize work already carried out within the framework of the conventional collective field theory.[8,9]

To set ourselves up for achieving such a strategy we shall make the simplifying assumption that the coefficients $b_j(k)$ entering (28) [or b_j entering (28′)] are the same for each particle: $b_1(k)=b_2(k)=\cdots=b_N(k)\equiv b(k)$. Such an assumption is entirely within the premises of the collective approach according to which the original variables $x=(x_1,\dots,x_N)$ are to be replaced with ones that are *symmetric* with respect to particle exchange.

Under this working assumption (28) becomes

$$\varphi_k=\frac{b(k)}{N}\sum_{j=1}^{N}e^{-ikx_j},\quad k=\frac{2\pi n}{L},\quad n=\pm1,\pm2,\ldots \quad (30)$$

Upon consulting (10), the above relation tells us that

$$\varphi_k=\frac{Lb(k)}{N}\phi_k \quad (31)$$

as well as that

$$\frac{\partial}{\partial\phi_k}=\frac{Lb(k)}{N}\frac{\partial}{\partial\varphi_k} \quad (32)$$

[with $b(k)=b$ for the simpler description according to (28′)].

Introducing the conjugate momentum $\tilde{\omega}_{-k}=-i(\partial/\partial\varphi_k)$, relevant to the stochastic scheme, we determine

$$\pi_k=\frac{Lb(k)}{N}\tilde{\omega}_k . \quad (33)$$

Referring to the conventional collective scheme, we now recall the redefinition (18) through which the Jacobian is removed from the inner product at the expense of replacing the Hamiltonian with H_{eff}. Restricting ourselves to the kinetic term, the latter reads

$$T_{\text{eff}}=\tfrac{1}{2}\sum_{k,k'}\pi_{-k}\Omega_{kk'}[\phi]\pi_{k'}+\tfrac{1}{8}\sum_{k,k'}k^2k'^2\phi_{-k}\Omega^{-1}_{kk'}[\phi]\phi_{k'} , \quad (34)$$

where we have ignored a term that is cancelled once the level of the vacuum state is adjusted to zero. In the above relation $\Omega_{kk'}[\phi]$ is given by

$$\Omega_{kk'}[\phi]=\frac{kk'}{L}\phi_{k-k'} . \quad (35)$$

Substituting (31) and (33) in (34), we obtain

$$T_{\text{stoch}}=\tfrac{1}{2}\sum_{k,k'}\tilde{\omega}_{-k}\mathcal{W}_{kk'}[\varphi]\tilde{\omega}_{k'} +\tfrac{1}{2}\sum_{k,k'}k^2k'^2\varphi_{-k}\mathcal{W}^{-1}_{kk'}[\varphi]\varphi_{k'} , \quad (36)$$

where

$$\mathcal{W}_{kk'}=\frac{kk'}{N}b(-k)b(k')\varphi_{k-k'} . \quad (37)$$

Let us now determine circumstances under which (37) is consistent with $J[\varphi]$ being unity, i.e., no Jacobian factor is to enter the stochastic collective field description. From (28) we easily surmise that a necessary condition for this to happen is the following:

$$\int\prod_{j=1}^{N}dx_j\prod_{k(k\neq0)}\delta\left[\varphi_k-\frac{b(k)}{N}\sum_i e^{-ikx_i}\right]=1 \quad (38)$$

or

$$\int\prod_{j=1}^{N}dx_j\prod_{k(k\neq0)}\delta\left[\frac{N\varphi_k}{b(k)}-\sum_i e^{-ikx_i}\right] =\prod_{k(k\neq0)}\frac{|b(k)|}{N} . \quad (39)$$

Multiplying and dividing the left side by L, as well as using (32), we obtain

$$\int\prod_{j=1}^{N}dx_j\prod_{k(k\neq0)}\delta\left[\phi_k-\frac{1}{L}\sum_i e^{-ikx_i}\right] =\prod_{k(k\neq0)}\frac{L|b(k)|}{N} . \quad (40)$$

The expression on the left is none other than the Jacobian $J[\phi]$ entering the conventional collective field theory [c.f. Eq. (11)]. We conclude that if $b(k)$ is chosen according to

$$J[\varphi]=\prod_{k(\neq0)}\frac{L}{N}|b(k)| , \quad (41)$$

then $J[\varphi]=1$.

For the simpler case described by (28′) the above condition reads

$$J[\varphi]=\prod_{k(\neq0)}\frac{L}{N}|b| . \quad (41')$$

In conclusion, we have determined explicit conditions under which a collective field theory constructed in accordance with the prescriptions of phase-space stochastic quantization does not involve a Jacobian in the sense that the latter is unity. Moreover, we have found that the Hermitian Hamiltonian corresponding to this approach has the general form (kinetic energy part):

$$T_{\text{stoch}}=\tfrac{1}{2}\sum_{k,k'}\tilde{\omega}_{-k}\frac{kk'}{N}|b|^2\varphi_{k-k'}\tilde{\omega}_{k'} +\tfrac{1}{8}\sum_{k,k'}\varphi_{-k'}\frac{Nkk'}{|b|^2}\varphi^{-1}_{k-k'}\varphi_{k'} . \quad (42)$$

The above expression, of course, adheres to the stochastic description given by (28′). For the more general case described by (28) we must replace $|b|^2$ by $|b(k)|\,|b(k')|$.

Following Ref. 8, let us pursue, within the context of the present approach, an application pertaining to the Bohm-Pines electron plasma. Besides the kinetic term (42), we must also include the Coulomb interaction between electron pairs. In the Fourier transform language we are using, the aforementioned potential-energy term reads (we are now working in three dimensions)

$$U_{\mathbf{kk}'}=\frac{4\pi e^2}{|\mathbf{k}|^2}V\delta_{\mathbf{kk}'} , \quad (43)$$

where V is the volume of the system.

Taking into account the interpretation of $|b|^2$ as describing a spreading for each particle, we surmise the following behavior for large N:

$$|b|^2\varphi_{k-k'}\sim\frac{N^2}{V}\delta_{\mathbf{kk}'} . \quad (44)$$

Substituting the above in (42) and adding the potential-energy term (43), we write

$$H_{\text{stoch}}=\frac{1}{2}\sum_k \left[\frac{k^2N}{V^2}\tilde{\omega}_{-k}\tilde{\omega}_k + \left[\frac{4\pi e^2 V}{k^2}+\frac{1}{4}\frac{k^2V^2}{N}\right]\varphi_{-k}\varphi_k \right], \qquad (45)$$

which identifies the plasma frequency as[8]

$$\omega_k^2=\frac{4\pi e^2 N}{V}+\frac{1}{4}k^4 .$$

We close this section with the following remark. In earlier work[5,6] on stochastic quantization we arrived at an inner product pertaining to "phase-space" wave functions of the form

$$(\psi_1,\psi_2)=\int dp\, dq\, \psi_1^*(p,q)\psi_2(q,p)e^{-(1/2)(p^2+q^2)} .$$

In other words, the stochastic scheme at first quantization level introduces a density function, equivalently a Jacobian factor, which corresponds to a particle spreading that adheres to minimal uncertainty. The same exact density function also appears in connection with particle descriptions through coherent states.[14] In quite a contrasting manner, the collective-field-theoretical description according to the stochastic scheme utilizes the presence of the particle spreading to eliminate a Jacobian factor, whose presence in the conventional description is unavoidable. In fact, as (41) and/or (41′) suggest, *the aforementioned Jacobian factor can be interpreted as a measure of the particle spreading.* Our final intepretation, then, is that the stochastic description trades the Jacobian of the conventional formulation with the particle spreading, which it inherently contains.

IV. OUTLOOK

In this concluding section we exclusively offer some general speculations. We have already remarked that collective field theory provides the simplest possible framework within which we were able to study the application of phase-space stochastic quantization prescriptions to a system with a continuous infinity of degrees of freedom. The fact that we have worked in a nonrelativistic context has greatly facilitated our discussion.

The application of stochastic phase-space prescriptions to relativistic field theories calls for extra considerations. In this connection we may recall that the many-particle content of a relativistic field theory is attained through the second quantization procedure and its accompanying Fock space construction for the Hilbert space of states. This whole edifice is based on a formulation which relies exclusively on a Heisenberg picture. It is difficult to envision a stochastic phase-space description of field theory along these lines.

Consider, on the other hand, the possibility of casting a field theory into a Schrödinger-like picture.[15] The conventional quantization rules now read

$$\hat{\phi}(x)=\phi(x), \quad \hat{\pi}(x)=-i\frac{\delta}{\delta\phi(x)} .$$

Generalizing these quantization prescriptions to the following ones:

$$\hat{\Phi}(x)=\phi(x)+i\frac{\delta}{\delta\pi(x)}, \quad \hat{\Pi}(x)=-i\frac{\delta}{\delta\phi(x)}$$

would appear to be the first step towards a "stochastic field theory." The corresponding state functionals will now be of the form $\Psi[\phi(x),\pi(x)]$. Any substantial progress along such lines is heavily dependent upon a fuller understanding of a Schrödinger field theory and its particle content.

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