# Positronium formation in the ground and $n=2$ levels in an $e^+$-H collision

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The positronium formation processes in the ground and excited $n=2$ levels have been investigated in $e^+$-H atom collision using the Glauber eikonal approximation. Both the differential and total formation cross sections have been studied in the intermediate- and high-energy regime. Present eikonal results are found to differ appreciably from the corresponding first-order Born values even at very high energies. The total-cross-section results have been compared with other existing theoretical values as well as with the corresponding Born cross sections.

## I. INTRODUCTION

The study of positron collisions with atoms and molecules is currently receiving increasing attention both from experimental and theoretical investigators. Because of the availability of more intense positron beams and sophisticated detectors, direct measurements of positronium (Ps) formation in positron-atom collisions have now become feasible. This, in turn, has stimulated many theoretical workers to study the Ps formation due to capture of an atomic electron where a positron beam is incident on a neutral atomic target. Positronium formation cross sections have been recently measured by Charlton *et al.*,[1] Diana and co-workers,[2,3] and Fromme *et al.*[4] over a wide range of incident positron energies for different target systems, but unfortunately a significant discrepancy is noted between the results of the two experimental groups (Charlton *et al.* and Fornari *et al.*), which becomes more pronounced with increasing positron energy. This gives added incentive to the theoretical workers for further investigations of Ps formation. The theoretical and experimental situations have recently been reviewed by Humberston[5] and by Griffith,[6] respectively.

Since the experimental results for Ps formation include the contribution from all energetically allowed states, theoretical cross-section data for capture to excited states are highly needed in order to make a meaningful comparison with the experiments. Unfortunately, most of the theoretical works in the literature refer to the formation of Ps in its ground state and very few attempts (Sil *et al.*,[7] Khan *et al.*,[8] and other references cited therein) have been made to calculate the excited-state Ps-formation cross sections.

We present in this paper the differential as well as the total cross sections for Ps formation to ground and excited states $(2s,2p)$ in $e^+$-H collision, in the framework of the Glauber eikonal approximation, taking a consistent account of the projectile-nucleus potential term (both in the phase and in the interaction) in order to satisfy the boundary condition properly. Though no experimental results are yet available in the literature for an atomic hydrogen target, serious attempts have already been initiated[9,10] in this direction and the results are expected to be reported in the near future. Our major interest concerns intermediate- and high-energy regions.

It is now well known that the first-order Born approximation (FBA) is not adequate to predict a reliable result for a rearrangement process, and the higher-order effects should also be taken into account, especially at very high energies. The eikonal approximation which takes account of some higher-order effects has been successfully applied to predict the total cross sections by several authors in charge transfer processes by heavy particle impact (Sinha *et al.*,[11] and other references cited therein).

In a very recent work Tripathi *et al.*[12] have reported preliminary calculation of Ps formation to the ground state for $e^+$-H collisions in the Glauber eikonal approximation following the technique developed by Sinha *et al.*[11] The salient feature of this new technique is the reduction of the six-dimensional, exact boundary-corrected eikonal amplitude to a single-dimensional one, without resorting to any peaking-type approximation. Previously, this reduction was possible only up to a two-dimensional integral. The resulting one-dimensional integral in the new approach[11] has been evaluated numerically by a simple quadrature method. Although, in the present work, the main emphasis is given on the calculation of Ps formation to the $n=2$ level, some of the results for the ground-state capture also are being discussed here for the sake of completeness. Further, expecting that the major contribution to excited-state capture comes from the $n=2$ level and that the formation cross section falls off as $n^{-3}$,[5] the present calculation also allows us to predict the total Ps-formation cross sections at intermediate and high energies.

Only the post form of the Glauber eikonal amplitude has been employed in the present work. We would like to point out here that in the case of electron–hydrogen-atom elastic scattering, no post prior discrepancy has been noted in the exchange term by the present authors.[13]

## II. THEORY

The expression for the Glauber eikonal amplitude (post form) for Ps formation in the $e^+$-H collision, taking account of both the interaction terms, is given as[10]

 

$$A_{if}=-\frac{\mu}{2\pi}\int\int e^{-i\mathbf{k}_f\cdot\mathbf{s}}\Phi_f^*(\mathbf{r}_{12})\left[\frac{1}{r_1}-\frac{1}{r_2}\right]\Phi_i(\mathbf{r}_2)\times e^{i\mathbf{k}_i\cdot\mathbf{r}_1}\left[\frac{r_{12}-z_{12}}{r_1-z_1}\right]^{-i\eta}d\mathbf{r}_1 d\mathbf{r}_2\ , \tag{1}$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are the position vectors of the incident positron and the bound electron from the target nucleus which is taken to be infinitely heavy and at rest. $\mathbf{r}_{12}$ denotes the relative coordinate of the electron and positron; $\mathbf{s}$ is the position vector of the center of mass of the positronium atom with $\mathbf{s}=\frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)$ and $\eta=1/v$, $\mathbf{v}$ being the velocity of the incident positron. $\mathbf{k}_i,\mathbf{k}_f$ are the initial and final momenta; $\Phi_i$ and $\Phi_f$ refer to the initial $1s$ and final ($1s$, $2s$, or $2p$) bound-state wave functions of the H and Ps atoms, respectively. We have used atomic units throughout the work in which $\mu$, the reduced mass of the final system, comes out to be 2.

As in our previous work,[11,13] we now use the following contour-integral representation for the eikonal phase term occurring in Eq. (1):

$$y^{-i\eta}=-\frac{1}{2i\sin(\pi i\eta)\Gamma(i\eta)}\int_c(-\lambda)^{i\eta-1}e^{-\lambda y}d\lambda\ . \tag{2}$$

Thus, in view of (1) and (2), the expression for the amplitude can now be written as

$$A_{if}\approx\int_{\Gamma_1}\int_{\Gamma_2}\left[\int\int e^{-i\mathbf{k}_f\cdot\mathbf{s}}\Phi_f^*(\mathbf{r}_{12})\left[\frac{1}{r_1}-\frac{1}{r_2}\right]\Phi_i(\mathbf{r}_2)e^{i\mathbf{k}_i\cdot\mathbf{r}_1}p_1(\lambda_1,\mathbf{r}_1)p_2(\lambda_2,\mathbf{r}_{12})d\mathbf{r}_1 d\mathbf{r}_2\right]d\lambda_1 d\lambda_2\ , \tag{3}$$

where

$$p_1(\lambda_1,\mathbf{r}_1)=e^{-\lambda_1(r_1-z_1)}(-\lambda_1)^{-i\eta-1}$$

and

$$p_2(\lambda_2,\mathbf{r}_{12})=e^{-\lambda_2(r_{12}-z_{12})}(-\lambda_2)^{i\eta-1}\ ;$$

$z_1$ and $z_{12}$ are the $z$ components of $\mathbf{r}_1$ and $\mathbf{r}_{12}$, respectively, the $z$ axis being taken along $\mathbf{k}_i$.

Using Fourier-transform techniques, the space integrations (over $\mathbf{r}_1$ and $\mathbf{r}_2$) in (3) can be performed analytically. The actual space integral occurring in (3) can be generated by parametric differentiations of the function $J_0$:

$$J_0=\frac{\pi^2}{(\beta^2-\alpha\gamma)^{1/2}}\ln\left[\frac{\beta+(\beta^2-\alpha\gamma)^{1/2}}{\beta-(\beta^2-\alpha\gamma)^{1/2}}\right]\ , \tag{4}$$

where $\alpha$, $\beta$, and $\gamma$ are functions[11] of $\mathbf{k}_i$, $\mathbf{k}_f$, $\lambda_i$, and $\lambda_f$.

Instead of using this analytic form of $J_0$, which contains a logarithmic branch cut we use the following contour-integral representation:

$$J_0=2\pi^2\int_0^\infty\frac{1}{\alpha x^2+2\beta x+\gamma}dx\ , \tag{5}$$

where the product $\alpha\gamma$ has been split as first suggested by Sinha and Sil[14] in such a manner that both the functions $\alpha$ and $\gamma$ are individually linear functions of the integration variables $\lambda_1$ and $\lambda_2$. By virtue of this choice we can perform the $\lambda_1$ and $\lambda_2$ integrations analytically.

We now proceed to carry out the $\lambda_1$ and $\lambda_2$ integrations analytically following the method developed by Sinha *et al.*,[11] leaving behind the $x$ integration occurring in (5). The result of integration ($I_0$) with respect to $\lambda_1$ and $\lambda_2$ can be expressed in terms of the Gauss hypergeometric function ($_2F_1$)

$$I_0=\frac{-4\pi^2}{a}\left[\frac{b}{a}\right]^{i\eta}\left[\frac{c}{a}\right]^{-i\eta}\left[\frac{ad}{bc}\right]^{i\eta}{}_2F_1(1-i\eta,-i\eta,1,z)\ , \tag{6}$$

with $z=1-bc/ad$; $a$, $b$, $c$, and $d$ are functions of the variable $x$ and the parameters $\mathbf{k}_i$, $\mathbf{k}_f$, $\lambda_i$, and $\lambda_f$.

Thus we are finally left with a one-dimensional real integral over the variable $x$. We have made a general computer code to calculate the $_2F_1$ function over the entire complex plane of the argument making use of its proper analytic continuations. The functions containing branch cuts in (6) are also computed with proper care. The convergence of the final one-dimensional integral has been tested by increasing the number of Gaussian-quadrature points.

## III. RESULTS AND DISCUSSIONS

The positronium-formation cross-section results have been computed for the process $e^+ + \mathrm{H}(1s) \rightarrow (e^+e)(1s,2s,2p)+\mathrm{H}^+$ in the framework of the Glauber eikonal approximation. As a check of our program we have reproduced the corresponding Born results for different energies, putting $\eta=0$ in the same computer program.

Table I records the present eikonal cross-section results for Ps formation to ground state together with the corre-

TABLE I. Ps-formation cross sections for the process $e^+ + H(1s) \rightarrow (e^+e)(1s) + H^+$ (in units of $\pi a_0^2$). The numbers in square brackets indicate the power of 10 by which the entry is to be multiplied.

| $E$ (Ry) | This work | Drachman *et al.*[b] | DWPO[a] Model $A$[c] | DWPO[a] Model $B$[d] | FBA |
|---|---|---|---|---|---|
| 3 | 8.29[−1] | 7.09[−1] | 5.76[−1] | 6.14[−1] | 7.85[−1] |
| 4 | 3.63[−1] | 3.24[−1] | 2.66[−1] | 2.87[−1] | 3.51[−1] |
| 5 | 1.65[−1] | 1.62[−1] | 1.39[−1] | 1.50[−1] | 1.73[−1] |
| 6 | 8.54[−2] | 8.70[−2] | 7.71[−2] | 8.38[−2] | 9.21[−2] |
| 8 | 2.77[−2] | 2.99[−2] | | | 3.13[−2] |
| 10 | 1.08[−2] | 1.21[−2] | 1.22[−2] | 1.34[−2] | 1.26[−2] |

[a] Reference 15.
[b] Reference 16.
[c] Model $A$: potential of Temkin and Lamkin (Ref. 21).
[d] Model $B$: potential of Callaway *et al.* (Ref. 22)

sponding FBA and distorted-wave polarized-orbital (DWPO) results of Khan and Ghosh[15] and the coupled static results of Drachman *et al.*[16] for comparison. It is evident from the table that the eikonal results give good agreement with those of Drachman *et al.*[16] and the results due to Khan and Ghosh[15] (model $B$ in Table I) at intermediate energies. At lower energies (below 4 Ry), however, the present results tend to overestimate the other theoretical values. This may be due to the fact that the DWPO (Ref. 15) and the coupled static approximations[16] are mainly low-energy approximations, whereas the eikonal approximation is supposed to be valid at intermediate and high energies only. At high energies ($\sim > 5$ Ry) the Born cross sections are always higher than all the other theoretical values quoted in Table I. In fact, we have noted[12] that at still higher energies ($\sim$2000 eV) the Born-approximation results for ground-state Ps formation lie much above the present eikonal values.

Table II displays the present cross-section values for $2s$ and $2p$ states for various energies along with the corresponding Born and DWPO results of Khan *et al.*[8] where available. The ground-state (Born-approximation and present) results for these energies are also included for proper comparison.

It is apparent from Table II that, at energies below 50 eV, the present $2s$ and $2p$ results always overestimate the Born-approximation as well as the DWPO (Ref. 8) values. But above 50 eV, the eikonal results approach the corresponding FBA values, coincide at a particular energy depending on the state to which capture is taking place, and then continue to lie much below the Born-approximation results throughout the energy region. Further, for the $2p$ state the discrepancy between the FBA and the eikonal results is much higher than for the $1s$ and $2s$ states for en-

TABLE II. Ps-formation cross sections for the process $e^+ + H(1s) \rightarrow (e^+e)(1s,2s,2p) + H^+$ (in units of $\pi a_0^2$). The numbers in square brackets indicate the power of 10 by which the entry is to be multiplied.

| | ($1s$-$1s$) | | ($1s$-$2s$) | | | ($1s$-$2p$) | | |
|---|---|---|---|---|---|---|---|---|
| eV | FBA | This work | FBA | DWPO[a] | This work | FBA | DWPO[a] | This work |
| 30 | 1.65 | 1.82 | 1.85[−1] | 1.58[−1] | 2.20[−1] | 1.13[−1] | 8.5[−2] | 1.45[−1] |
| 40 | 8.47[−1] | 8.75[−1] | 1.11[−1] | | 1.24[−1] | 5.70[−2] | 4.1[−2] | 5.75[−2] |
| 50 | 4.60[−1] | 4.55[−1] | 6.56[−2] | 6.19[−2] | 6.88[−2] | 2.90[−2] | 1.5[−2] | 2.53[−2] |
| 60 | 2.69[−1] | 2.53[−1] | 3.93[−2] | | 3.94[−2] | 1.56[−2] | | 1.21[−2] |
| 70 | 1.63[−1] | 1.49[−1] | 2.43[−2] | | 2.34[−2] | 8.65[−3] | | 6.24[−3] |
| 80 | 1.03[−1] | 9.19[−2] | 1.55[−2] | 1.86[−2] | 1.44[−2] | 5.00[−3] | 2.6[−3] | 3.41[−3] |
| 100 | 4.50[−2] | 3.89[−2] | 6.88[−3] | 9.32[−3] | 6.08[−3] | 1.90[−3] | 1.2[−3] | 1.17[−3] |
| 125 | 1.91[−2] | 1.56[−2] | 2.83[−3] | | 2.39[−3] | 6.33[−4] | | 3.77[−4] |
| 150 | 8.92[−3] | 7.07[−3] | 1.31[−3] | | 1.07[−3] | 2.49[−4] | | 1.43[−4] |
| 175 | 4.56[−3] | 3.53[−3] | 6.64[−4] | | 5.26[−4] | 1.09[−4] | | 6.11[−5] |
| 200 | 2.40[−3] | 1.90[−3] | 3.61[−4] | | 2.79[−4] | 5.25[−5] | | 2.87[−5] |
| 300 | 3.67[−4] | 2.66[−4] | 5.12[−5] | | 3.75[−5] | 5.05[−6] | | 2.63[−6] |
| 400 | 8.72[−5] | 6.13[−5] | 1.19[−5] | | 8.44[−6] | 8.82[−7] | | 4.49[−7] |
| 500 | 2.74[−5] | 1.90[−5] | 3.70[−6] | | 2.57[−6] | 2.19[−7] | | 1.10[−7] |
| 700 | 4.60[−6] | 3.10[−6] | 6.08[−7] | | 4.12[−7] | 2.57[−8] | | 1.27[−8] |
| 1000 | 6.45[−7] | 4.34[−7] | 8.53[−8] | | 5.66[−8] | 2.50[−9] | | 1.23[−9] |
| 2000 | 1.29[−8] | 8.55[−9] | 1.69[−9] | | 1.09[−9] | 2.45[−11] | | 1.20[−11] |
| 3000 | 1.17[−9] | 8.24[−10] | 1.63[−10] | | 1.04[−10] | 1.57[−12] | | 7.75[−13] |
| 4000 | 2.20[−10] | 1.55[−10] | 3.06[−11] | | 1.95[−11] | 2.20[−13] | | 1.09[−13] |
| 8000 | 4.19[−12] | 2.66[−12] | 5.26[−13] | | 3.32[−13] | 1.87[−15] | | 9.74[−16] |

[a] Reference 8.

ergies above 50 eV. In fact, the discrepancy is almost 40–50 % for all the energies greater than 50 eV.

Regarding the comparison with the DWPO (Ref. 8) results, it is evident from Table II that for the $2s$ state the DWPO (Ref. 8) results are higher than the eikonal as well as the FBA values for higher energies (e.g., 100 eV), while for energies lower than 60 eV, the situation is the reverse, i.e., for these energies, the DWPO (Ref. 8) values lie always below the eikonal and Born results.

From the $2p$ state, on the other hand, the DWPO (Ref. 8) cross sections are always below the FBA and eikonal results for the energy range 30–80 eV, while at 100 eV, the eikonal $2p$ results are somewhat smaller than the DWPO (Ref. 8) values (see Table II). Comparing the results of 80 and 100 eV for the $2p$ state, it seems that the eikonal and the DWPO (Ref. 8) cross-section curves cross each other in the above energy range (i.e., 80–100 eV).

It may be inferred from a comparison of the results in Table II that, unlike the $e^{\pm}$-H elastic scattering cross sections, the Ps-formation cross sections ($1s$, $2s$, and $2p$) in the eikonal approximation do not converge to the corresponding first-order Born results, even at an energy as high as 1000 eV. This feature is in conformity with the earlier theoretical findings in the distorted-wave approximations[17,18] for ground-state Ps formation. This may be due to the fact that at very high incident energies, the first-order Born approximation is not adequate for a rearrangement collisional process; the higher-order effects should also be taken into account.

From Table II, it is also apparent that the cross sections $\sigma_{2p} < \sigma_{2s} < \sigma_{1s}$, and at higher energies the ratio of $\sigma_2$ ($\sigma_2 = \sigma_{2s} + \sigma_{2p}$) and $\sigma_1$ (i.e., $\sigma_{1s}$) approaches the value 0.125, satisfying the $n^{-3}$ rule ($n$ being the principal quantum number). For example, the ratio $\sigma_2/\sigma_1$ at energy 2000 eV is 0.1287, while at 8000 eV it is 0.1252. It is expected that the contributions from the higher states ($n > 2$) will decrease gradually and hence it would be quite reasonable to predict from the cross sections $\sigma_1$ and $\sigma_2$ an estimate for the total Ps-formation cross section to all possible states with the help of the expression $\sigma_c = \sigma_1(1 + 1.616\sigma_2/\sigma_1)$, as done by Jackson and Schiff.[19]

Table III displays the present $n=2$ level capture cross sections ($\sigma_{2s} + \sigma_{2p}$) and total Ps-formation cross sections ($\sigma_{1s} + \sigma_{2s} + \sigma_{2p}$) along with the corresponding FBA and DWPO (Ref. 8) results where available. Below 50 eV the present ($n=2$)-level capture cross sections are always higher than the corresponding FBA results, while from 50 eV and onwards, they always lie below the first-order Born values. This feature is also noted in the behavior of the total ($\sigma_{1s} + \sigma_{2s} + \sigma_{2p}$) Ps formation cross sections in the present eikonal approximation.

The DWPO (Ref. 8) results, on the other hand, are always smaller than the FBA and present eikonal results in the lower-energy region (see Table III), while at high energies they give higher estimates of the cross sections.

Figures 1–4 exhibit the present differential cross sections in the ground and $2s, 2p$ states at different incident energies. The differential cross section at energy 50 eV shows a sharp minimum at the angle $\sim 40°$ for the $1s$ and $2s$ states, while for the $2p$ states the curve falls almost monotonically (see Fig. 1) as the angle increases, without showing any structure. With increasing energy the minima for the $1s$ and $2s$ curves shift towards smaller angles and become sharper. For energies 100 and 200 eV the

TABLE III. Ps-formation cross sections in the $n=2$ level and total Ps-formation cross sections in $e^+$-H collisions (in units of $\pi a_0^2$). The numbers in square brackets indicate the power of 10 by which the entry is to be multiplied.

| | $n=2$ level ($2s+2p$) | | | Total ($1s+2s+2p$) | |
|---|---|---|---|---|---|
| eV | FBA | DWPO[a] | This work | FBA | This work |
| 30 | 2.98[−1] | 2.43[−1] | 3.65[−1] | 1.95 | 2.18 |
| 40 | 1.68[−1] | | 1.81[−1] | 1.01 | 1.06 |
| 50 | 9.46[−2] | 7.69[−2] | 9.41[−2] | 5.55[−1] | 5.49[−1] |
| 60 | 5.49[−2] | | 5.15[−2] | 3.24[−1] | 3.04[−1] |
| 70 | 3.29[−2] | | 3.00[−2] | 1.96[−1] | 1.79[−1] |
| 80 | 2.05[−2] | 2.12[−2] | 1.78[−2] | 1.23[−1] | 1.10[−1] |
| 100 | 8.78[−3] | 1.05[−2] | 7.25[−3] | 5.38[−2] | 4.61[−2] |
| 125 | 3.46[−3] | | 2.77[−3] | 2.26[−2] | 1.84[−2] |
| 150 | 1.56[−3] | | 1.21[−3] | 1.05[−2] | 8.28[−3] |
| 175 | 7.73[−4] | | 5.87[−4] | 5.33[−3] | 4.12[−3] |
| 200 | 4.13[−4] | | 3.08[−4] | 2.81[−3] | 2.21[−3] |
| 300 | 5.62[−5] | | 4.01[−5] | 4.23[−4] | 3.06[−4] |
| 400 | 1.28[−5] | | 8.89[−6] | 1.00[−4] | 7.02[−5] |
| 500 | 3.92[−6] | | 2.68[−6] | 3.13[−5] | 2.17[−5] |
| 700 | 6.34[−7] | | 4.25[−7] | 5.23[−6] | 3.52[−6] |
| 1000 | 8.78[−8] | | 5.78[−8] | 7.33[−7] | 4.92[−7] |
| 2000 | 1.71[−9] | | 1.10[−9] | 1.46[−8] | 9.65[−9] |
| 3000 | 1.64[−10] | | 1.05[−10] | 1.33[−9] | 9.29[−10] |
| 4000 | 3.08[−11] | | 1.96[−11] | 2.52[−10] | 1.75[−10] |
| 8000 | 5.28[−13] | | 3.33[−13] | 4.72[−12] | 2.99[−12] |

[a] Reference 8.

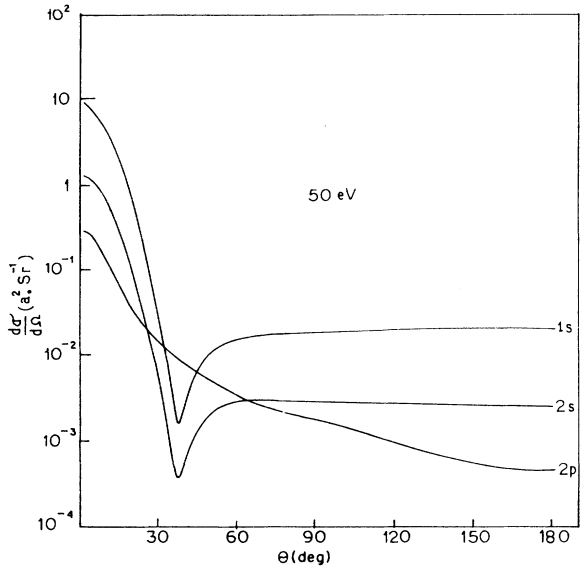


FIG. 1. Differential cross sections for positronium formation in positron-hydrogen atom collisions.

minima for the 1*s* and 2*s* states occur at almost the same angle, ~30° (see Figs. 2 and 3). But as we go on increasing the energy (e.g., at 500 eV) a secondary minimum appears (see Fig. 4) for both the 1*s* and 2*s* differential curves, which gets more and more pronounced with increasing energy (see Fig. 5 for 10 000 eV). Figure 5 shows the prominent occurrence of these two minima for both 1*s* and 2*s* states: a sharp one at around ~30° and the other comparatively less sharp, at about 80°.

For the 2*p* state, on the other hand, from 200 eV onwards, a single minimum appears (see Fig. 4) which again becomes more and more prominent with increasing energy (see Figs. 3–5). The angle at which the minimum occurs is nearly fixed (~73°) for all energies (200–10 000 eV).

The occurrence of the first minimum in the 1*s* and 2*s* differential curves may be ascribed to the fact that the

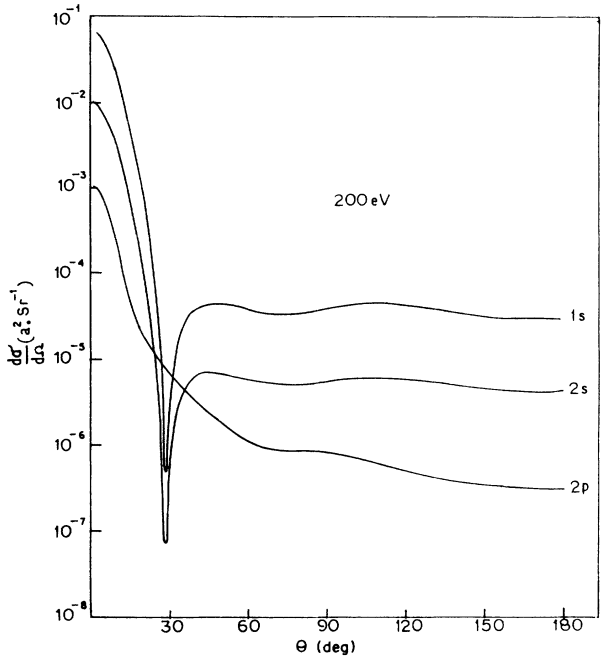


FIG. 3. Differential cross sections for positronium formation in positron-hydrogen atom collisions.

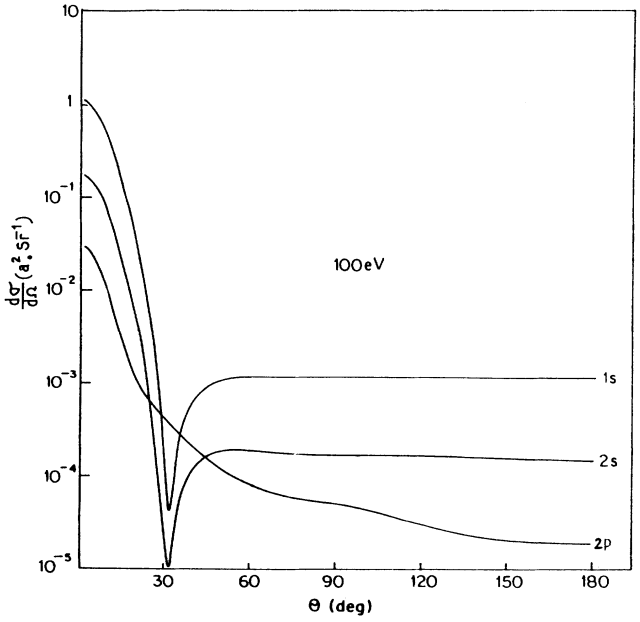


FIG. 2. Differential cross sections for positronium formation in positron-hydrogen atom collisions.

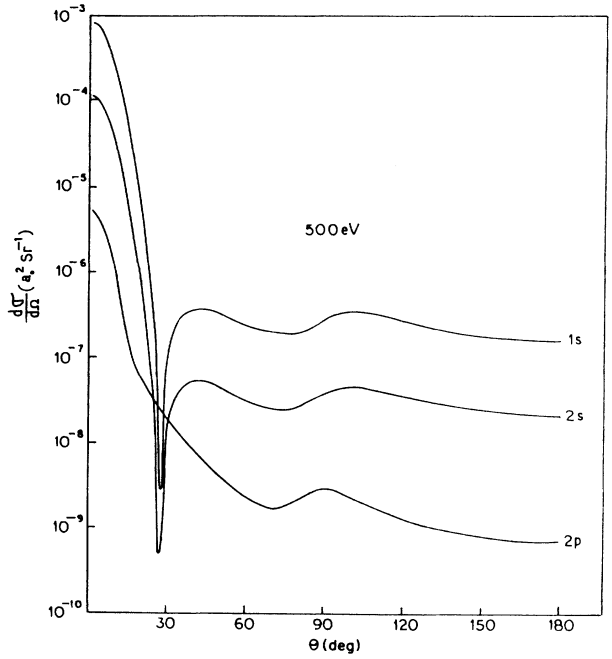


FIG. 4. Differential cross sections for positronium formation in positron-hydrogen atom collisions.

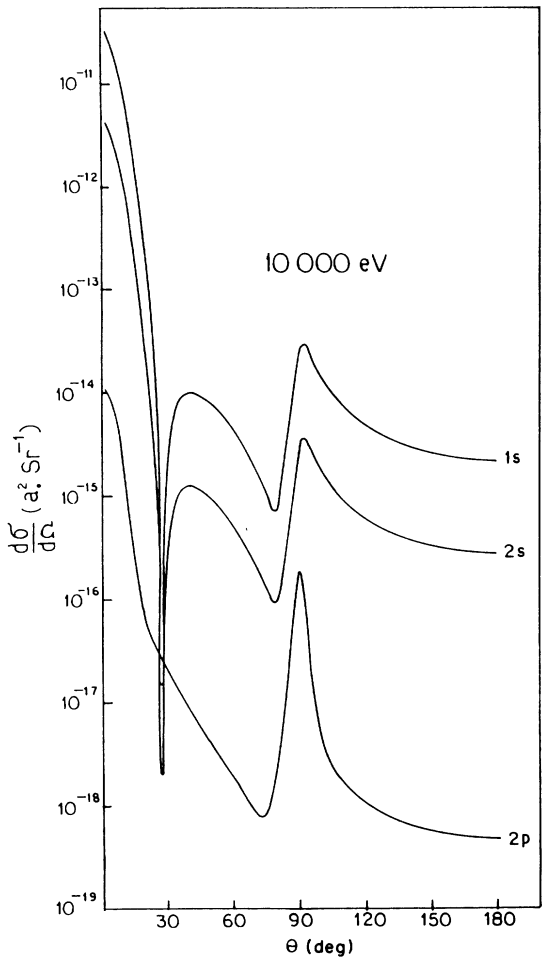


FIG. 5. Differential cross sections for positronium formation in positron-hydrogen atom collisions.

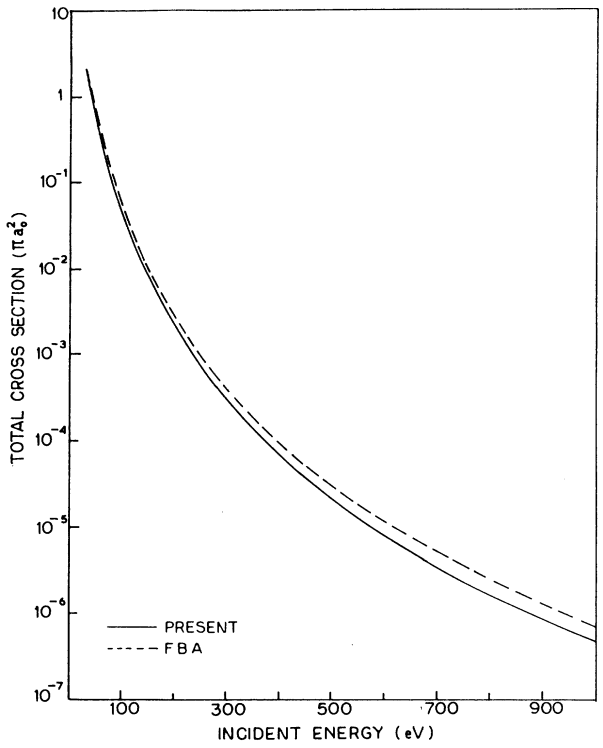


FIG. 6. Comparison of the total ($\sigma_{1s}+\sigma_{2s}+\sigma_{2p}$) cross sections for Ps formation in positron-hydrogen atom collisions obtained from eikonal calculations and FBA calculations.

contributions from the attractive and the repulsive parts of the interaction potential to the scattering amplitude interfere destructively at this angle. The same feature has also been noted by the previous authors in their positronium-formation calculations.[7,16,17,20] Since with increasing energy both the amplitudes for attractive and repulsive parts become more and more peaked in the forward direction, the position of the minimum (due to the destructive interference between the two) shifts towards smaller angles as energy increases. The minima for the $m$-degenerate states occur at different scattering angles and, as a result, the total differential cross sections for the $2p$ state do not exhibit such a minimum.[7] The appearance of the secondary minima at higher energies for the $1s$ and $2s$ differential curves and the single minimum for the $2p$ differential curve may be due to the second-order effect. Here, we would like to point out the fact that no such secondary minimum arises for the FBA calculation, even at very high energy ($\sim 10$ keV). We have also computed some extreme-high-energy ($\sim 50$ keV) eikonal differential cross sections (not given here) and have noted that no other minimum appears in the differential curves.

Figure 6 compares the present total formation cross sections ($\sigma_{1s}+\sigma_{2s}+\sigma_{2p}$) with the corresponding results of FBA calculations. The present curve lies always below the FBA curve (as evident from the figure) except for very low energies (30–40 eV). Below 30 eV, the comparison is not meaningful since both the eikonal and the Born approximation are not supposed to be valid in this energy region.

The literature lacks any experimental data for the positronium formation in $e^+$-H collision and thus we are unable to compare our findings with experiment. But since the eikonal approximation has been quite successful in predicting the charge transfer cross sections at intermediate and high energies in ion-atom collisions, we feel that the present results are likely to give a reasonable estimate of the Ps-formation cross sections at intermediate and high energies.

Further, since the present evaluation technique of the eikonal Ps-formation amplitude requires only a simple one-dimensional numerical integration, it offers a much easier and more economical treatment for studying the Ps-formation process involving multielectron targets. However, for a proper judgement of different theoretical results we have to await an accurate experiment.

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