Testing of the line element of special relativity with rotating systems

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Experiments with rotating systems are analyzed from the perspective of a test theory of the Lorentz transformations which permits, in principle, the verification of the latter's simultaneity relation.

I. INTRODUCTION

The testing of the Lorentz transformations (LT's) is often referred to as the testing of the line element of special relativity (SR). The significance of the experiments involved in this task is determined with the help of Robertson's test theory[1] (RTT). A more recent and widely referenced test theory of SR by Mansouri and Sexl[2–4] is equivalent to RTT, as explicitly shown by MacArthur.[5]

Rotating systems have played a prominent role in the testing of the LT's. Mansouri and Sexl[3] have discussed these systems in connection with the time dilation effect. In the recent literature, this role has become even larger as they have additionally been used in connection with the discrimination of transformations such as

$$x'=\gamma(x-Vt),\quad y'=y,\quad t'=\gamma^{-1}t\ , \tag{1}$$

from the LT's.[6,7] RTT is not sufficiently comprehensive for this discrimination, as a result of an assumption it contains relative to the relation of simultaneity. This assumption has been removed from a revised version of Robertson's test theory,[8,9] to be denoted as RRTT. The present paper is concerned with analyses of experiments with rotating systems from a RRTT perspective.

We shall concentrate on the resonant absorption of photons in centrifuges and on the coordinated universal time system (UTC). The *Photon and centrifuge experiment* can be analyzed by a variety of methods. The first of these methods is due to Møller,[10] who used it in connection with discriminating between the Galilean and Lorentz transformations. Mansouri and Sexl[3] later adapted it to RTT, but only in a relatively low order of approximation. The Mansouri and Sexl result was eventually used by Spavieri[7] for distinguishing the LT's from transformations (1). This use is inconsistent with the assumptions made in the derivation of that result. It is pertinent, therefore, to produce analyses of the photon and centrifuge experiment in the appropriate context, i.e., RRTT. This still can be done by several methods and, in retrospect, the analysis should be carried out in a higher order of approximation.

The Møller method of analysis is relatively cumbersome in orders higher than that of Mansouri and Sexl. Maciel and Tiomno[6] (MT) have produced a higher-order analysis by a different, very elegant method. Although MT do not mention RRTT, they work in the same spirit of RRTT, since they do not impose *ab initio* any particular synchronization procedure. MT conclude that indeed the photon and centrifuge experiment can be used to determine whether the LT's or transformations (1) can be said to be more fundamental. Their work requires some qualifications. It also requires corroboration by other methods of analysis, given the potential importance of their result. For this purpose we have used in a RRTT context the same elegant method of analysis that Misner, Thorne, and Wheeler[11] (MTW) use in a relativistic context. We have found exactly the same result as MT. A qualification to the MT work is, however, necessary. The LT's and transformations (1) differ in their respective concepts of simultaneity. As explicitly stated by Grøn, the relativity of simultaneity plays an essential role in the kinematical resolution[12] of the Ehrenfest paradox. It follows that, while there is no Ehrenfest paradox in SR, the absolute simultaneity relation appears to preclude the existence of this solution in the context of transformations (1). Distortions of the disk—in response to the inconsistency of the longitudinal and transverse contractions—may follow. Therefore, if one uses the photon and centrifuge experiment to distinguish between the LT's and transformations (1), one has to include in the analysis of experimental results the possibility of the existence of such distortions.

Let us next consider the UTC system. This system is related, by its very nature, to different important topics in SR. In addition to the Ehrenfest paradox, these are the Sagnac effect, the comparison of slow-clock transport and Einstein's synchronization procedures, and the difference in rate between clocks which move with the same speed in different directions in the inertial frames to which the coordinates (x',y',t') of Eqs. (1) apply. These topics have to be included in any meaningful analysis of the UTC system, when it is used for discriminating between the LT's and transformations (1).

The contents of the paper are organized as follows. In Sec. II we briefly summarize RTT and the kinematical aspects of RRTT. We show in Sec. III that requiring a kinematical solution to the Ehrenfest paradox already discriminates in principle the LT's from transformations (1). In Sec. IV we consider in RRTT the Møller method of analysis of the photon and centrifuge experiment in the same order as in the Mansouri and Sexl derivation and the Spavieri discussion. The higher-order, MT analysis is discussed in Sec. V. The result of the MT calculation is corroborated in Sec. VI by the MTW analysis. UTC is considered in Sec. VII. Results are summarized in Sec. VIII.

II. REVISED ROBERTSON'S TEST THEORY OF SPECIAL RELATIVITY

In Robertson's test theory, inertial frames are materialized by small free-falling elevators. This permits the introduction of Cartesian coordinates. Isotropy is then postulated in at least one local frame S. In conjunction with appropriate choices of space-time origins and orientations of spatial axes, the postulate of isotropy enables Robertson to considerably reduce the number of non-null independent coefficients in the coordinate transformations from the frame S to another inertial frame S' of velocity V. Robertson additionally assumes (or chooses, or imposes) the equality of the to and from speeds of light and derives a set of coordinate transformations which, when inverted, can be written as

$$x'=a(x-Vt),\quad y'=ey,\quad t'=j(t-Vx)\ . \tag{2}$$

[Notice that these transformations do not contain those of Eqs. (1) as a particular case.] The coefficients a, e, and j, which depend on the velocity V of the transformation, have to be determined by experiment. Using the Minkowski metric in S, $dt^2-dx^2-dy^2$ ($c=1$), and the transformations inverse to (2), the metric takes in S' the form

$$ds^2=g'_{00}dt'^2+g'_{11}dx'^2+g'_{22}dy'^2\ . \tag{3}$$

Robertson uses ideaized experimental results for the Michelson-Morley (MM), Kennedy-Thorndike (KT), and Ives-Stilwell (IS) experiments. He thus obtains $g_{00}=-g_{11}=-g_{22}=1$, i.e., that the Minkowski metric is also valid in the inertial frames S'. This, of course, is equivalent to determining that the coefficients a, e, and j in Eqs. (2) are the same ones that appear in the Lorentz transformations. The Robertson proof has eventually been used by experimentalists as a test theory of the line element of SR. Neither Robertson nor Mansouri and Sexl have constructed the dynamics pertaining to the different coordinate transformations among which the test theory intends to discriminate.

In 1984, one of us advocated that test theories should also include dynamical considerations[8] rather than being restricted to the kinematics. It was also proposed[8] that the testing of the line element should be based on the more general transformations

$$x'=a(x-Vt),\quad y'=ey,\quad t'=hx+jt\ , \tag{4}$$

a point that also Spavieri[7] has explicitly made (it is also implicit in the MT work). These transformations are consistent with all the aforementioned Robertson postulates, but do not assume the equality of the to and from speeds of light. This equality implies that[9]

$$h+Vj=0\quad (\mathrm{RE}) \tag{5}$$

which, upon substitution in (4), yields (2). The last equation will be denoted as the Robertson-Einstein (RE) relation. The three great optical experiments that Robertson considers are not now sufficient to determine the four coefficients a, e, h, and j in Eqs. (4). These three experiments, respectively, imply that[8,9,13]

$$a=\gamma e\quad (\mathrm{MM})\ , \tag{6a}$$

$$j+Vh=a\gamma^{-2}\quad (\mathrm{KT})\ , \tag{6b}$$

$$j+Vh=\gamma^{-1}\quad (\mathrm{IS})\ . \tag{6c}$$

When substituted in Eqs. (4), these three experiments only permit us to reach the following one-function family of coordinate transformations:

$$x'=\gamma(x-Vt)\ , \tag{7a}$$

$$y'=y\ , \tag{7b}$$

$$t'=hx+(\gamma^{-1}-hV)t\ . \tag{7c}$$

We shall denote Eqs. (7) as the h family. It contains both the LT's and transformations (1) as special cases.

The four relations (MM), (KT), (IS), and (RE), together with transformations (4), yield the LT's. The time dilation factor with respect to the frame S is given by $j+Vh$, as can be seen from the time transformation in the equations inverse to (4):

$$x=Ax'+BVt'\ , \tag{8a}$$

$$y=Ey',\quad z=Ez'\ , \tag{8b}$$

$$t=Cx'+Bt'\ , \tag{8c}$$

where

$$A=a^{-1}(1+Vhj^{-1})^{-1}\ , \tag{9a}$$

$$B=(j+Vh)^{-1}\ , \tag{9b}$$

$$C=a^{-1}h(j+Vh)^{-1}\ , \tag{9c}$$

$$E=e^{-1}\ . \tag{9d}$$

Hence

$$\Delta t'(\Delta x'=0)=B^{-1}\Delta t=(j+Vh)\Delta t\ . \tag{10}$$

Equation (6c) thus implies that the IS experiment determines the time dilation factor. As first stated by Robertson,[3] the KT experiment means that the two-way speed of light is independent of the velocity of the laboratory. It translates into the relation (6b) between the time dilation and longitudinal contraction factors,[8,13] respectively, $j+hV$ and a. Similarly, the MM experiment means that the two-way speed of light is independent of the path of the ray.[3] It translates into the relation (6a) between the longitudinal and transverse contraction factors,[8,13] respectively given by the coefficients a and e.

III. EHRENFEST PARADOX AND THE TESTING OF THE LORENTZ TRANSFORMATIONS

We now deal with the Ehrenfest paradox as a basis for the distinguishability of transformations (1) from the LT's. This paradox formulates the apparent inconsistencies present in the kinematics of the rotating disk.[14] The premises of the paradox are that the periphery of a rotating disk is contracted by the factor $(1-\omega^2R^2)^{1/2}$, whereas the radius, which is transverse to the motion, is not contracted. The ratio of periphery to radius is no longer 2π but $2\pi(1-\omega^2R^2)^{1/2}$. Notice that we are not concerned here with the internal geometry of the rotating disk, but rather with the shape of the disk relative to an inertial frame comoving with the center of the disk. It is worth pointing out that in the derivation, say, by Møller,[15] of that geometry the implicit assumption is made without explanation that there is no paradox or at least that the disk in rotation remains a disk. This assumption is unwarranted in a RRTT context.

The varied solutions to the Ehrenfest paradox span several decades (see, for instance, Refs. 12 and 16–18). These solutions leave something to be desired, but further consideration of these approaches is beyond our scope. Suffice it to mention that they are inconsistent with the widely accepted assumption that measuring rods in these systems have exactly the same length as measuring rods in the inertial frames in which they are instantaneously at rest. In other words, the lengths of the rods are independent of the acceleration.

We have only encountered one solution to the Ehrenfest paradox that is consistent with this assumption. It is due to Grøn,[12] and states that the motion that would realize the aforementioned contraction of the periphery is inconsistent with SR kinematics. Hence, in SR, there is no paradox since its premises are not valid.[19] Since the relativity of simultaneity is essential to the resolution of the paradox, a nonconventionalist framework is presupposed. It is not clear what kinematical solution could be provided to the Ehrenfest paradox by those who maintain that the relativity of simultaneity is a matter of convention.

Let us now consider the same paradox in the context of transformations (1) and under the assumption that these transformations may in principle be distinguishable from the LT's. In Eqs. (1), simultaneity is absolute. Grøn's stated solution no longer applies. Let us first assume for simplicity in the argument that the center of the rotating disk is at rest in the preferred frame S. Then the contractions or lack of contractions relative to the frame S are the same as in SR. As a result of the inconsistent longitudinal and transverse requirements to which the paradox refers, a rotating disk will be subjected to stresses that will result in strains. Since S is a frame of isotropy, the disk will be strained (to different degrees at different points), but will remain a disk if the velocities are not excessively high. Let us now assume that the center of the disk is at rest in an inertial frame S' different from the preferred frame. Such frames are not in general frames of isotropy.[20] Hence the deformations caused by the longitudinal-versus-transverse inconsistency may, in this case, not even permit the disk to maintain the shape of a disk in the rest frame of its center. In the context of transformations (1), the specific degree of elasticity or plasticity of the disk has to be considered in the calculation of the shape of the rotating disk.

We have just shown that the LT's and transformations (1) already produce different physics in the case of a rotating disk: in the context of Eqs. (1), a rotating disk (even) in the preferred frame is not equivalent to a disk that follows the laws of relativity. The relativistic disk is only stressed by the forces that give centripetal acceleration to its different points. Disks in a world governed by Eqs. (1) experience the aforementioned additional stresses and strains.

A complete analysis of rotating disk experiments for the testing of SR should include the finding of the shape of the disk in the context of the test theory. Even if the disk is elastic and velocities are small, this is not just a problem of elasticity in the way we presently know it. Under the contractions that are involved in the theory of elasticity, the atoms and molecules become more (or less) closely packed. By contrast, a change of the scale of space itself is the case in the contractions of the LT's and pertinent alternatives. This is an enormously involved problem which in addition requires the development of a dynamics for the test theory. Moreover, the complications increase in the context of testing the LT's and not just of comparing them with transformations (1). Indeed, Eqs. (1) and the LT's are just two members in the h family, the coefficient h indicating the amount of relativity of simultaneity present in each member of the family.

In the rest of the paper we shall make the assumption of the *well-behaved disk*. We mean by this that the disk does not experience deformations and that, therefore, the speed of all points in its periphery is the same. As we have argued, this is inconsistent in sufficiently high approximation orders (for further details, see Sec. V). However, since this is the assumption that everybody has made so far (by not contemplating the possibility of distortions of the disk), we shall also make it for sorting out a contradiction between the MT and Spavieri results. Furthermore, since a dynamics consistent with Eqs. (1) is not known, the predictions of the test theory for the well-behaved disk constitute a reference point against which any non-null experimental results might be compared, if necessary.

We may summarize this section as follows. Grøn's kinematic solution to the Ehrenfest paradox shows that rotating disks argue in favor of the distinguishability in principle of the LT's from transformations (1). It also indicates that the testing of SR with rotating systems is, at some order of approximation, intertwined with the solution given to this paradox.

IV. MØLLER METHOD OF ANALYSIS

Next we present a self-contained analysis of the photon and centrifuge experiment by the Møller method of analysis. The experiment is described in Refs. 10 and 11. Experimental results can be found in Ref. 3. The present argument differs from a comparable argument by Mansouri and Sexl[3] in that we do not assume any unwarranted simultaneity relation disguised in the form of a synchronization procedure (see Appendix A for details). Also, the analysis of Mansouri and Sexl is not completely self-contained, as it is a modification of a similar proof by Møller for the Galilean case.[10]

A plane wave travels in a straight line from emitter to absorber. The characteristics of this wave in the frame of isotropy S will be denoted as $(\omega,\mathbf{k})$. In the rest frames of absorber and emitter, the frequency will be called ω_α and ω_e. It is a simple matter to relate ω_e and ω_α to $(\omega,\mathbf{k})$. One readily finds[21]

$$\omega_{\alpha,e}=B_{\alpha,e}(\omega-\mathbf{k}\cdot\mathbf{u}_{\alpha,e}) , \tag{11}$$

where $B_{\alpha,e}$ denotes the result of evaluating the function B of Eq. (9b) for the velocities $\mathbf{u}_{\alpha,e}$ of absorber and emitter in the rest frame of the center of the disk. From (11), one gets

$$\omega_\alpha=\omega_e\frac{B_\alpha}{B_e}\frac{1-(\mathbf{k}/\omega)\cdot\mathbf{u}_\alpha}{1-(\mathbf{k}/\omega)\cdot\mathbf{u}_e} . \tag{12}$$

The emitted frequency is ω_0. Furthermore, in the preferred frame and in a vacuum situation, the ray velocity of light $\mathbf{U}$ is simply given by

$$\mathbf{U}=\frac{\omega}{k}\frac{\mathbf{k}}{k}=\frac{\mathbf{k}}{\omega} ,$$

since we have set $c=1$ (hence ω equals k). Thus

$$\omega_\alpha=\omega_0\frac{B_\alpha}{B_e}\frac{1-\mathbf{U}\cdot\mathbf{u}_\alpha}{1-\mathbf{U}\cdot\mathbf{u}_e} . \tag{13}$$

Let us denote by $\mathbf{V}$ the velocity of the center of the centrifuge, at rest on the earth, with respect to the *ab initio* rest frame S. As already argued by Spavieri,[7] the candidate value for this velocity is $10.^{-3}$ On the other hand, typical velocities $\mathbf{u}_{\alpha,e}$ of points of the periphery of centrifuges relative to its center have a magnitude of about 10^{-6}. We shall define as δ the quantity 10^{-3} and expand in powers of it. The velocities $\mathbf{V}$ and $\mathbf{u}_{\alpha,e}$, respectively, represent one and two powers of δ. We calculate in third order of δ, the dominant order in which the calculation yields nontrivial results. In this way we obtain

$$\frac{B_\alpha}{B_e}=\frac{1+(J+H)u_e^2}{1+(J+H)u_\alpha^2}=1+2(J+H)[\mathbf{V}\cdot(\mathbf{u}'_e-\mathbf{u}'_\alpha)] , \tag{14}$$

where the parities of j and h have been taken into account in defining $j(V)=1+JV^2+\cdots$ and $h(V)=HV+\cdots$. We similarly have

$$\frac{1-\mathbf{U}\cdot\mathbf{u}_\alpha}{1-\mathbf{U}\cdot\mathbf{u}_e}=\frac{1-\mathbf{U}\cdot\mathbf{u}'_\alpha-\mathbf{U}\cdot\mathbf{V}}{1-\mathbf{U}\cdot\mathbf{u}'_e-\mathbf{U}\cdot\mathbf{V}}$$
$$=1+\mathbf{U}\cdot(\mathbf{u}'_e-\mathbf{u}'_\alpha)+(\mathbf{U}\cdot\mathbf{V})[\mathbf{U}\cdot(\mathbf{u}'_e-\mathbf{u}'_\alpha)] . \tag{15}$$

In deriving (14) and (15) we have used an expression for the addition law of velocities that can be found in Ref. 21. The same expression can be used to substitute for $\mathbf{U}$ in terms of $\mathbf{U}'$. We need to do so only in first order since $\mathbf{U}$ multiplies factors which are of the second or third order. Thus

$$\mathbf{U}=\frac{\mathbf{U}'+\mathbf{V}}{1-H(\mathbf{U}'\cdot\mathbf{V})}=\mathbf{U}'+\mathbf{V}+H\mathbf{U}'(\mathbf{U}'\cdot\mathbf{V}) . \tag{16}$$

Under the assumption of the well-behaved disk, $\mathbf{u}'_e-\mathbf{u}'_\alpha$ is perpendicular to $\mathbf{U}'$. Equations (15) and (16) yield

$$\frac{1-\mathbf{U}\cdot\mathbf{u}_\alpha}{1-\mathbf{U}\cdot\mathbf{u}_e}=1+\mathbf{V}\cdot(\mathbf{u}'_e-\mathbf{u}'_\alpha) . \tag{17}$$

Hence

$$\omega_\alpha=\omega_0\{1+[1+2(J+H)][\mathbf{V}\cdot(\mathbf{u}'_e-\mathbf{u}'_\alpha)]\} . \tag{18}$$

The idealized null result for this experiment, $\omega_\alpha=\omega_0$, thus implies

$$J+H=-\tfrac{1}{2} , \tag{19}$$

which is the second-order version of the (IS) relation, Eq. (6c).

The result just obtained has several implications. Contrary to Spavieri's claim, this experiment cannot differentiate in third order of δ between the LT's and transformations (1), since both sets of transformations satisfy Eq. (19). Similarly, this experiment does not yield the relativistic one-way speed of light from a RRTT perspective, as substitution of (19) in (16) only replaces J for H (see also Appendix A). Finally, contrary to Mansouri and Sexl's presentation of results,[3] this is not a first-order experiment in any meaningful way. Indeed, Eq. (18) shows that this is a second-order experiment in the velocities. The same comment applies by reference to Eq. (19), since J is the coefficient of the second-order expansion of the function j.

The present authors have carried this method of analysis to the δ^5 order, but only for the configuration in which emitter and absorber are on opposite ends of a diameter. The calculation is extremely cumbersome. [A null result is obtained *vis-à-vis* the discrimination of transformations (1) from the LT's. The more general result obtained by MT (Ref. 6) and by the present authors in Sec. VI also becomes null for this particular configuration.]

V. ANALYSIS OF MACIEL AND TIOMNO

In their analysis of the photon and centrifuge experiment, MT neglect possible effects related to the Ehrenfest paradox as being, they contend, of higher order than those they consider. In other words, they are assuming that the disk is well-behaved in their approximation. As justification, they refer to a paper by Ives,[17] which does not actually prove the MT statement. We now show that the terms that may arise from not having a kinematical solution to the Ehrenfest paradox in the context of absolute simultaneity are indeed of the fifth order in δ, contrary to the MT contention. We then review their analysis.

The geometry of the well-behaved disk enters the derivation of the result (19) when we use that $\mathbf{U}'\cdot(\mathbf{u}'_e-\mathbf{u}'_\alpha)$ is zero. Because of the distortions of the disk, this term may differ from zero by quantities like $\mathbf{U}'\cdot(\Delta\mathbf{u}'_e-\Delta\mathbf{u}'_\alpha)$. Hence the order of the fractional frequency shifts is the order of $|\Delta\mathbf{u}'|$, which is the same as that of $\omega|\Delta\mathbf{r}'|$ and of $\mathbf{u}'|\Delta\mathbf{r}'|/\mathbf{r}'$ where $\Delta\mathbf{r}'$ represents the strains in the disk. These strains are due to the different Lorentz contractions relative to the preferred frame or frame of isotropy, contractions that now have an absolute character and are proportional to $(1-\mathbf{u}^2)^{-1/2}$. Since $\mathbf{u}$ is approximately equal to $\mathbf{V}+\mathbf{u}'$ and $\mathbf{V}$ is the same for all points of the disk, $|\Delta\mathbf{r}'|/\mathbf{r}'$ may be of the order of $\mathbf{V}\cdot\mathbf{u}'$. We can thus expect distortions of the disk (associated with the Ehrenfest paradox) of the order of $\mathbf{u}'(V\cdot\mathbf{u}')$, i.e., of the fifth order in δ. In spite of this, and due to the reasons and purposes stated in Sec. III, we shall continue the discussion of the MT derivation as if the assumption of the well-behaved disk were justified. This assumption is also implicitly present in the work by Spavieri and by Mansouri and Sexl. As a conclusion to one of their papers,[3] the last two authors state that the photon and centrifuge experiment, among others, cannot distinguish between SR and preferred frame theories. The expectation would thus be that there is an error in the MT derivation and corresponding result. We deal with this aspect of the derivation next.

Close scrutiny of the MT derivation shows that their equation

$$\Phi(t)=\Phi(0)+\omega_0 t+V\nu_0\cos[\Phi(0)+\omega_0 t]+O(\nu_0^2V^2) \quad (20)$$

is incorrect, as substituting $t=0$ into it shows. One also observes that in the process of obtaining it, the authors first differentiate an equation and the result is then integrated. Such a procedure cannot give anything new but an integration constant. We have removed this unnecessary complication from the derivation. What remains is just a substitution of Ives coordinates [to follow the name given by MT to the coordinates on the left-hand side of Eqs. (1)] in terms of Lorentz coordinates in the expression for the time-dependent angular coordinates of emitter and absorber. If one does this, one readily obtains their Eq. (2) and the remainder of the proof continues without flaws. Before we proceed to reproduce the MT result by an alternative calculation, let us elaborate on the role of rigidity in the MT derivation and in the contents of the paper itself.

One readily notices that the role of rotating disks in the MT calculation is that of providing, at least in principle, both emitter and receiver with the same Ives speed [speed defined in terms of the coordinates that enter Eqs. (1)] in a circular trajectory. The same effect should be obtained if one just assumed that emitter and absorber are two beads moving with constant Ives speed over a nonrotating ring. Rigidity is not of the essence of the calculation itself; the equality of the Ives velocities is. We can obtain the MT effect, even in a relativistic world, by providing emitter and absorber with the program of motion briefly described as "constant Ives speed over a ring." However, we do not want to introduce explicitly a particular program of motion since the value of the final result, the frequency shift, will depend on what program of motion we introduce. We do want nature itself to decide whether the program of motion will be a particular one, if simultaneity is absolute, or a different one, if simultaneity is relative. For this purpose we should choose a physical problem with a *neutrally specified* program. It is then the inner workings of nature, i.e., the *dynamics*, that will decide whether the resulting program of motion is one or another (say, constant Ives speed versus constant relativistic speed). The particular dynamics that the world follows will thus manifest itself in the form of different Doppler results for the neutrally specified problem. Since one does not know the *Ives dynamics*, MT bypassed this difficulty by choosing "rigid rotating disks" for generating the neutral program of motion. The introduction of Ives rigidity therefore represents an assumption of a practical nature rather than a fundamental, structural postulate of a theory. It is not a perfect solution to the problem of the photon and the centrifuge, but it is a reasonable hypothesis in the context of the testing of the LT's against transformations (1).

VI. MISNER-THORNE-WHEELER METHOD OF ANALYSIS

The following derivation of the non-null result by Mansouri and Sexl (MS) is based on a similar relativistic derivation by MTW.[11] The two basic physical magnitudes that enter this analysis are the four velocities of source and absorber and the four momenta of the photons. They are given by

$$u^{\mu'}=\frac{dt',d\mathbf{r}'}{ds'(\mathbf{V})}=\frac{1,\mathbf{u}'_{e,\alpha}}{ds'(\mathbf{V})/dt'}\ , \quad (21)$$

$$p'_\mu=(E',-\mathbf{p}')=(\omega',-\mathbf{k}')\ , \quad (22)$$

where $\hbar=1$, and where $\mathbf{u}'_{e,\alpha}$ represents the velocities in the $S'(V)$ frame of emitter and absorber. All these equations apply equally well to all velocities $\mathbf{V}$ such that $V<c$ and, at least, to all possible functions a, e, h, and j which permit a one-to-one correspondence of the coordinates on the left of Eqs. (4) to the Cartesian coordinates of the pseudo-orthonormal frames of Minkowski space-time. Contraction of (21) and (22) yields a scalar. The meaning of the scalar can be obtained by going to the rest frame of the emitter (absorber), which is at rest in its proper reference frame S'' of velocity $\mathbf{u}$. Thus

$$u^{\mu'}=\frac{1,0}{ds''(\mathbf{u})/dt''}\ . \quad (23)$$

The contraction (p,u) of the four-vectors p and u is a scalar. The form of this scalar in terms of physical magnitudes will depend on the coefficients a, e, h, and j. By calculating in the rest frame of the emitter (absorber) we have

$$(p,u)=\frac{\omega''}{ds''(\mathbf{u})/dt''}=\omega''[g_{00}(u)]^{-1/2}\ , \quad (24)$$

where ω'' is the energy of the emitted (absorbed) photon. We notice that (p,u) only depends on the coefficient g_{00}, when referred to the rest frame of emitter (absorber). Such is not the case in general. We thus have

$$\omega''=(p,u)[g_{00}(u)]^{1/2}\ . \quad (25)$$

Next we calculate in the comoving frame the scalar (p,u) for both the emission and absorption of a photon. From (21) and (22) one gets

$$(p,u)=\frac{\omega'-\mathbf{k}'\cdot\mathbf{u}'}{[ds'(\mathbf{V})/dt']_{e,\alpha}}\ . \quad (26)$$

At this point we abandon the generality of the calculation of Sec. IV and specialize (26) to a world governed by Eqs. (1). This is automatically achieved by using in the denominator the metric that corresponds to Ives coordinates:

$$\frac{ds(\mathbf{V})}{dt'}=[(1-\mathbf{V}\cdot\mathbf{u}')^2-u'^2]^{1/2} . \quad (27)$$

[Notice the appearance of other coefficients of the metric now that we are referring (p,u) to magnitudes of inertial frames other than the rest frame of the emitter or the absorber.] We thus have

$$\frac{\omega''_\alpha}{\omega''_e}=\frac{(\omega'-\mathbf{k}'\cdot\mathbf{u}'_\alpha)}{(\omega'-\mathbf{k}'\cdot\mathbf{u}'_e)}\frac{[(1-\mathbf{V}\cdot\mathbf{u}'_e)^2-u_e'^2]^{1/2}}{[(1-\mathbf{V}\cdot\mathbf{u}'_\alpha)^2-u_\alpha'^2]^{1/2}} , \quad (28)$$

where we have used the fact that g_{00} is unity for the metric that corresponds to the Ives coordinates. At this point it is convenient to use relativistic quantities for the next step in the calculation. The wave vector $\mathbf{k}'$ does not go in the direction of emitter to absorber. We relate it to its relativistic homologue, $\mathbf{k}'_r$. Thus we have

$$\mathbf{k}'=\mathbf{k}'_r+\omega'\mathbf{V}=\omega'(\mathbf{c}+\mathbf{V}) , \quad (29)$$

with $|\mathbf{c}|=1$. Hence, in fifth order of δ, we have that the first factor in the right-hand side of (28) can be written as

$$\frac{\omega'-\mathbf{k}'\cdot\mathbf{u}'_\alpha}{\omega'-\mathbf{k}'\cdot\mathbf{u}'_e}=\frac{1-\mathbf{c}\cdot\mathbf{u}'_\alpha-\mathbf{V}\cdot\mathbf{u}'_\alpha}{1-\mathbf{c}\cdot\mathbf{u}'_e-\mathbf{V}\cdot\mathbf{u}'_e}$$

$$=\frac{1-W-V_\alpha}{1-W-V_e}=1+(V_e-V_\alpha)(1+W) , \quad (30)$$

and the second factor as

$$\frac{[(1-\mathbf{V}\cdot\mathbf{u}'_e)^2-u_e'^2]^{1/2}}{[(1-\mathbf{V}\cdot\mathbf{u}'_\alpha)^2-u_\alpha'^2]^{1/2}}=[1-V_e-u_e'^2][1+V_\alpha+u_\alpha'^2]$$

$$=1-(V_e-V_\alpha) , \quad (31)$$

where

$$V_{e,\alpha}\equiv\mathbf{V}\cdot\mathbf{u}'_{e,\alpha},\quad W\equiv\mathbf{c}\cdot\mathbf{u}'_\alpha=\mathbf{c}\cdot\mathbf{u}'_e . \quad (32)$$

Thus

$$\omega''_\alpha/\omega''_e-1=(V_e-V_\alpha)W . \quad (33)$$

The dominant parts of $V_{e,\alpha}$ and W are, respectively, the third- and second-order terms so that, in order to get (33) in fifth order, it suffices to calculate the angles in zeroth order. Hence

$$W=u'\cos\Phi_0 ,$$

$$V_e-V_\alpha=2uV\cos(\omega t+\Phi_0)\sin\Phi_0 ,$$

where Φ_0 is half the angle between emitter and absorber. We finally have

$$\omega''_\alpha/\omega''_e-1=u'^2V\sin(2\Phi_0)\cos(\omega t+\Phi_0) , \quad (34)$$

which is the same result that MT obtained by a different method.

VII. COORDINATED UNIVERSAL TIME

Several authors[7,22,23] have discussed the detection with stable clocks of variations in the speed of light due to the motion of the earth. In particular, Spavieri has claimed that the comparison of readings in the UTC system amounts to the performing of the extra type of experiment required by RRTT. We shall now contest this claim, for it has been made with neglect of several important aspects of this problem: the effect of slow-clock transport, Sagnac effect, and the different time dilations at different points on the periphery of a rotating disk.

A. Slow-clock transport versus Einstein's synchronization in the inertial frames of RRTT

A familiar attempt at measuring the one-way speed of light by time-of-flight measurement involves the task of first synchronizing clocks by slow-clock transport. After calculating the time of flight of light for different members of the Robertson family, Spavieri[7] states without proof that "the time delay measured with the help of clock transport is equivalent to that given by Einstein's synchronization, i.e., $t_E=L_0/c$." By comparison of these two times, Spavieri concludes that this time-of-flight method can be used to distinguish between the LT's and transformations (1). Notice that the argument, if correct, would apply even in inertial frames. In the quoted statement, Spavieri appears to be misinterpreting a correct result by Mansouri and Sexl,[2] who obtained that the slow-clock transport and Einstein's synchronization procedures "agree if and only if the time dilation factor is given by the special relativistic value. . . ." In other words, this time-of-flight measurement is equivalent to the IS experiment, a result which is corroborated by our analysis in Appendix B.

Mansouri and Sexl did not calculate either the times of fight or the delays by slow-clock transport for the different members of the Robertson family. We have calculated them in Appendix B. Both the time of flight and the delay are found to depend on the particular member of the family. The difference between the times of flight and the time delays is, however, a constant for all members of the Robertson family that satisfy the IS relation. The value of the constant is the time of flight of special relativity, which is consistent with the Mansouri and Sexl result. On the other hand, transformations (1) cannot be distinguished from the LT's since both of them satisfy the IS relation. Spavieri's result is incorrect because of his incorrect time delay for slow-clock transport. Notice, by the way, that in SR this time delay is zero and not L_0/c.

Coincidentally, one of us (J.G.V.) had previously committed a similar mistake[13,24] to that made by Spavieri. It was stated that slow-clock transport cannot give a relation among the coefficients of the transformations. This was based on the argument[13,25] that, if the transformations are given by differentiable functions, any effect caused by slow-clock transport with velocity $\mathbf{u}$ will go to zero as $\mathbf{u}$ itself goes to zero. This is certainly correct if, in the expression for the corresponding effect, $\mathbf{u}$ multiplies a term that does not grow too fast as **u** goes to zero. Unfortunately, the verification of this condition was overlooked.

B. Measuring the one-way speed of light in the periphery of a rotating disk using slow-clock transport

This measurement has not been analyzed in a RRTT context. We shall not do it here, for the results will depend on the solution given to the Ehrenfest paradox in the context of transformations (1). However, it is necessary to point out that the Sagnac effect is being overlooked in the Spavieri argument. For completeness, we discuss this briefly below.

In a rotating disk, the speed of light is not a constant in general.[15,25] This is why naive Einstein's synchronization, $\Delta t = L/c$, does not work and gives rise to the Sagnac effect, which is the correction to the incorrect synchronization in a relativistic context. This effect is detectable by sending a light signal a whole turn around the earth. It has been experimentally confirmed by means of the global positioning system of satellites.[26,27] It has a trivial explanation from the perspective of an inertial frame comoving with the center of the earth, which we briefly describe.

A light ray travels along the periphery of a rotating disk until it returns to the same point P' fixed to this periphery. With respect to the inertial frame S in which the center of the disk is assumed to be at rest, the point P' has moved a distance $r\omega t$, where ω is the angular velocity of the disk and t is the time elapsed. Relative to the frame S, light has covered a distance ct. This distance is given by $2\pi r + r\omega t$. Thus

$$ct = 2\pi r + r\omega t \ . \tag{35}$$

From the perspective of observers on the disk, light takes a time t' to return to P' which, in the first order, is equal to t. If these observers use Einstein's synchronization on the rotating disk (i.e., if they set $ct = 2\pi r$), they commit an error (Sagnac effect) given by

$$\delta t \simeq \frac{r\omega t}{c} \simeq \frac{r\omega}{c}\frac{2\pi r}{c} \ . \tag{36}$$

Rather than "synchronizing a clock with itself" after a complete turn around the periphery of the disk, we might wish to synchronize clocks at P' and P'', separated by an angle ϕ. Instead of (36), we would now have

$$\delta t \simeq \frac{r\omega}{c}\frac{\phi r}{c} = \frac{VL}{c^2} \ , \tag{37}$$

where L is the length of the arc between P' and P'' and V is the tangential velocity at the periphery of the disk. Equation (37) thus gives the correction in rotating disks for the Einstein synchronization correction of the inertial case.

Although this effect is very real, it cannot be detected by using slowly transported clocks on the periphery of the disk with velocity v. These clocks move with velocity $V \pm v$ with respect to an inertial frame comoving with the center of the disk. The time dilation factor is now given by

$$[1-(V\pm v)^2/c^2]^{1/2}$$

which, in the pertinent order, differs from $[1-V^2/c^2]^{1/2}$ by the quantity Vv/c^2. Multiplication by the time of travel of the transported clock yields again VL/c^2, which represents a kind of Sagnac effect for slow-clock transport also. It follows that if we use slow-clock transport on the periphery of the disk and, as is also the case in inertial frames, we do not correct the readings of the transported clocks, the neglected correction will cause the velocity of light in rotating disks to appear with the value c. A similar analysis of these two Sagnac effects would also have to be done in RRTT before one can draw any implications from the data of the UTC. Hence Spavieri's claims in this respect have to be disregarded.

C. Measuring the one-way speed of light in the periphery of a rotating disk without slow-clock transport

There is a different time-of-flight experiment that does not involve a previous synchronization by slow-clock transport. Let two clocks be located at points P' and P'' on the periphery of a rotating disk. Their synchronization is ignored. One studies the evolution, as a function of orientation, of the arrival times at P'' of pulses that leave P' at regularly spaced proper time intervals. This time-of-flight experiment seems to be what Spavieri had in mind when he stated that "Actually the clock transport can be provided by the earth's rotation. . . and no diurnal changes are observed. . . ." (Notice that the velocity provided by the earth rotation is not the relative velocity between two clocks that enters the comparison of the slow clock transport and Einstein's synchronization procedures.) In making his argument, Spavieri fails to consider that, from the perspective of RRTT, clocks at different points on the periphery of the disk tick with different rates. As Grøn has shown,[23] this has to be considered even in SR (when the behavior of the clocks is analyzed from the perspective of an inertial frame that is not comoving with the center of the rotating disk). Before any conclusions can be drawn as to whether UTC can be used in this way to differentiate between Eqs. (1) and the LT's, this difference in clock rates has to be included in the analysis in RRTT context.

Let us finally observe that this experiment is similar in nature to the centrifuge experiment, which has already been analyzed (for well-behaved disks) in fifth order.

VIII. SUMMARY OF RESULTS

In recent years, contradictory results on rotating disk experiments have been published in this journal.[6,7] We have shown that the analyses by Spavieri[7] were completely inappropriate and we have reproduced, by an alternative calculation, the main result in the work by MT.[6] The conflict has been resolved.

Another result exhibited in this paper is the realization that the Ehrenfest paradox constitutes in principle a powerful argument in favor of the distinguishability of the LT"s from transformations (1). Unfortunately, it has limited practical value unless a consensus should develop as to how one should deal with it in the context of theories with an absolute simultaneity relation. It interferes with the analysis of the resonant absorption of photons in centrifuges and other similar experiments.

The relation of different experiments to the determination of the speed of light has been illustrated. Slow-clock transport and Einstein's synchronizations have also been compared in RRTT.

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APPENDIX A: DISGUISING THE IVES-STILWELL EXPERIMENT AS A MEASUREMENT OF THE ONE-WAY VELOCITY OF LIGHT

In Sec. IV we have corroborated the Mansouri and Sexl result that, in a low order of approximation, the photon and centrifuge experiment is equivalent to the IS experiment. We now illustrate a few issues that are often confused in the literature, namely, (a) that IS is completely unrelated to the speed of light, (b) that a measurement of the two-way speed of light can be disguised as a measurement of the one-way speed of light, and (c) that, similarly, an IS experiment can be disguised as an experiment measuring the one-way speed of light. Finally, (d) we specialize these considerations to the Mansouri and Sexl analysis of the resonant absorption of photons in centrifuges in low order of the velocities.

(a) The relations (MM), (KT), and (RE) represent independent statements about the speed of light. Thus, at least three relations between the coefficients of transformations (4) are necessary to completely determine the one-way speed of light. The sufficiency of these three relations for the same purpose is shown as follows. Consider the Robertson transformations (RT's) which satisfy (MM), (KT), and (RE). They can be written as

$$x'=j(x-Vt),\quad y'=j\gamma^{-1}y,\quad t'=j(t-Vx)\ . \tag{A1}$$

The relation between the coordinates on the left of Eqs. (A1) and Lorentz-Einstein coordinates, $\{t_L, r_L\}$, is[9]

$$\{t',r'\}=j\gamma^{-1}\{t_L,r_L\}\ . \tag{A2}$$

This subfamily will be written as RT's+(MM)+(KT)+(RE). Its members do not satisfy, in general, the (IS) relation. As Eq. (A2) shows, the velocity of light for this subfamily is a constant, as in SR. The (IS) relation can thus be said to be independent of the one-way speed of light.

(b) The (RE) relation amounts to the equality of the to and from speeds of light and, as a consequence, to the equality of the one-way and two-way speeds. This relation has to do only with reversing the direction of the speed of light, not with changing the orientation of the path. The (KT) relation, on the other hand, means that the two-way speed of light is independent of the direction of the path of light. If the two-way speed of light depends on the direction of the path, so does necessarily the one-way speed, regardless of whether the (RE) relation is satisfied or not. Suppose now that we were in a hypothetical world in which the (MM) and (RE) relations were known to be satisfied, for one reason or another, but the (KT) experiment had not yet been performed. In such a world the one- and two-way speeds would coincide but would be direction dependent. If one then performed a (KT) experiment and a null result were obtained, said experiment might be misinterpreted to determine the constancy of the one-way speed of light although it clearly is a two-way experiment.

(c) Let us denote as (XX) those relations that satisfy

$$\text{RT's}+(\text{MM})+(\text{KT})+(\text{IS})+(XX)=\text{LT's}\ . \tag{A3}$$

[A particular case of (XX) is, of course, (RE).] If

$$\text{RT's}+(\text{MM})+(\text{KT})+(XX)=\text{RT's}+(\text{MM})+(\text{KT})+(\text{RE})\ ,$$

the velocity of light of the subfamily RT's+(MM)+(KT)+(XX) is the relativistic one and the (IS) experiment does not appear to determine this velocity. But if these two subfamilies do not coincide, the (IS) experiment appears to determine this velocity. [The reason is that, in the process of requiring compliance with (IS), one is also achieving compliance with (RE).] The proper way of stating this is to say that the four relations (MM), (KT), (IS), and (XX) together determine the one-way speed of light, but no subset of these relations does.

(d) We finally consider the family that Mansouri and Sexl use in their analysis of the photon and centrifuge experiment.[3] In the order in which Mansouri and Sexl work out this problem, two of their undetermined coefficients, b and d, become unity. In addition, those authors impose the synchronization relation $\epsilon=a'/b$, which derives from slow-clock transport in inertial frames.[2] So, their subfamily is

$$\text{RT's}+(b=1)+(d=1)+(\epsilon=a'/b)\ . \tag{A4}$$

The transformations of this subfamily are

$$x=X-VT,\quad y=Y,\quad t=T+2\alpha Vx\ , \tag{A5}$$

where α is a coefficient in the expansion $a=1+\alpha V^2+\cdots$. For comparative purposes, the subfamily RT's+(MM)+(KT)+(RE) can be written in the same order as

$$x=X-VT,\quad y=Y,\quad t=T-Vx\ . \tag{A6}$$

Transformations (A5) and (A6) do not coincide (except for a particular value of α), which means that the velocity of light of the subfamily (A4) is not the relativistic one. Transformations (A5), however, become the LT's when the (IS) relation is imposed on them. This means that the (IS) experiment will appear to determine the one-way speed of light (of course, it does not, as we already know). The relations $(b=1)$, $(d=1)$, $(\epsilon=a'/b)$, and (IS) together determine the one-way speed of light, but no subset of these four relations does. From the perspective of RRTT, the difficult part in the testing of the LT's is in finding experiments that will yield a relation like $\epsilon = a'/b$, and which should not be imposed by convention.

APPENDIX B: SLOW-CLOCK TRANSPORT AND EINSTEIN'S SYNCHRONIZATION FROM THE PERSPECTIVE OF THE REVISED ROBERTSON'S TEST THEORY OF SPECIAL RELATIVITY

In this appendix, we shall calculate the correction that has to be introduced in the procedure for synchronizing clocks by slow-clock transport. This correction is then taken into account when performing a time-of-flight measurement of the one-way speed of light.

Let two identical clocks at the same point P_1 read the same time. One of them is slowly moved from P_1 to another point P_2 of an inertial frame. Upon arrival at P_2, we would set a clock at P_2 to read what the moving clock reads, after applying a Robertson-slow-transport correction, which we now calculate.

The line element for the Robertson family is given in Eq. (40) of Ref. 9. For a slowly moving clock it yields, in first order of $\mathbf{u}'$,

$$ds = \{B\gamma^{-1} + \gamma[(C/V) - A](\mathbf{V}\cdot\mathbf{u}')\}dt' , \tag{B1}$$

where all the functions are evaluated at V. For $\mathbf{u}'=0$, the relation between line element and proper time is obtained, namely,

$$ds = B_u \gamma_u^{-1} d\tau , \tag{B2}$$

where the subscript u denotes the evaluation of the respective functions at u. Eliminating ds between (B1) and (B2) and solving for $d\tau$, we get the expression for the proper time in terms of the differentials of the coordinates in arbitrary inertial frames. Thus

$$d\tau = BB_u^{-1}\gamma^{-1}\gamma_u dt' + \gamma\gamma_u B_u^{-1}[(C/V) - A](\mathbf{V}\cdot\mathbf{u}')dt' . \tag{B3}$$

Let $\Delta t'$ be the arbitrarily large time (of S') that the moving clock takes to go from P_1 to P_2. Let $\Delta\tau$ be the corresponding time elapsed in the rest frame of the moving clock. We use Eq. (B3) to obtain the correction, $\delta t'$,

$$\begin{aligned}\delta t' &= \Delta t' - \Delta\tau \\ &= -\frac{1}{B^{-1}\gamma}\frac{\partial(B^{-1}\gamma)}{\gamma V}\overline{P_1P_2} \\ &\quad -\gamma^2 B^{-1}[(C/V) - A](\mathbf{V}\cdot\mathbf{n}')\overline{P_1P_2} ,\end{aligned} \tag{B4}$$

where we have replaced $\overline{P_1P_2}$ for $\mathbf{u}'\Delta t'$ and where $\mathbf{n}'$ is the unit vector in the direction of $\mathbf{u}'$. Notice the disappearance of the arbitrarily large time $\Delta t'$ through multiplication by the arbitrarily small quantity $\mathbf{u}'$. We now expand $\delta t'$ in first oder in V and obtain

$$\delta t' = -2V(1/2 + J + H)\overline{P_1P_2} + (1+H)(\mathbf{V}\cdot\mathbf{n}')\overline{P_1P_2} . \tag{B5}$$

In SR, $J+H = -\frac{1}{2}$ and $H = -1$; thus $\delta t'$ equals zero in the rest frame of P_1 and P_2.

We next calculate the velocity of light in the S' frame under the usual assumption that it is a constant in Robertson's preferred frame. When we express the velocity $\mathbf{U}'$ of light in S' as a function of the velocity $\mathbf{V}$ of the laboratory, we obtain an expression that contains the functions a, e, h, and j evaluated at V, namely,[21]

$$\mathbf{U}' = \frac{e\mathbf{U} + \mathbf{V}[(a-e)V^{-2}(\mathbf{U}\cdot\mathbf{V}) - a]}{hV^{-1}(\mathbf{U}\cdot\mathbf{V}) + j} , \tag{B6}$$

where $\mathbf{U}$ is the velocity of light in the preferred frame. The functions a, e, and j are even. Furthermore, $a(0)=e(0)=j(0)=1$. Thus, in first order,

$$\mathbf{U}' = \mathbf{U}[1 - hV^{-1}(\mathbf{U}\cdot\mathbf{V})] - \mathbf{V} , \tag{B7}$$

and, therefore, the magnitude U' of $\mathbf{U}'$ is given by

$$U' = 1 - (1+H)(\mathbf{e}'\cdot\mathbf{V}) , \tag{B8}$$

where h has been defined as in Appendix A and where $\mathbf{e}'$ is the unit vector in the direction of $\mathbf{U}'$. The time of flight in the Robertson family is in first order given by

$$t' = \overline{P_1P_2}[1 + (1+H)(\mathbf{e}'\cdot\mathbf{V})] . \tag{B9}$$

If the clock at P_2 has been appropriately synchronized, a light ray that leaves P_1 at $t=0$ arrives in P_2 at t given by Eq. (B9). Through slow-clock transport, the clock at P_2 is reading a time τ given by $t' - \delta t'$. So, although the speed of light is given in first order by the right-hand side of (B9) for all members of the Robertson family, we actually measure an apparent time of flight $t' - \delta t'$ given by

$$t' - \delta t' = \overline{P_1P_2} + 2V(\tfrac{1}{2} + J + H)\overline{P_1P_2} , \tag{B10}$$

where we have used the fact that $\mathbf{n}=\mathbf{e}'$. If the apparent time of flight takes the relativistic value $\overline{P_1P_2}$ (over c), we obtain $J+H = -\frac{1}{2}$, which is the second-order version of the (IS) relation. We again find that this experiment does not measure the speed of light or a relation of the fourth type in RRTT. It is really an IS-type experiment.

Finally, by setting $J+H = -\frac{1}{2}$ in Eq. (B5), we notice that the slow-clock-transport correction still depends on H. It is not a constant as Spavieri incorrectly claims. The constant comes only from the combination of (B5) and (B9), once the (IS) relation has been assumed.

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