Quantum electrodynamics based on self-fields, without second quantization: Apparatus dependent contributions to $g-2$

A. O. Barut and Jonathan P. Dowling
Department of Physics, Campus Box 390, University of Colorado, Boulder, Colorado 80309
(Received 22 April 1988)

Using a formulation of quantum electrodynamics which is not second quantized, but rather based on self-fields, we calculate the energy shifts of an electron bound by a magnetic field in the vicinity of an infinite-plane conductor. We confirm the recent result of Kreuzer that the energy shift arising from the plate-induced change in the magnetic moment, $\Delta\mu/\mu=-\alpha/4Rm$, is exactly canceled by a similar change $\Delta m/m=-\alpha/4Rm$ in the mass. Thus no change occurs in the spin-precession frequency to order α/Rm, in agreement with Brown *et al.* This cancellation of the two effects resolves an apparent controversy in recent literature over whether such a shift to the spin-precession frequency ω_s occurs. There is, however, a boundary-induced change in the cyclotron frequency ω_c which we calculate in the quantum result as $\Delta\omega_c/\omega_c=\alpha/8Rm$ to order α. Our method of approach is novel in that it uses only the self-field to compute radiative corrections; there are no vacuum fluctuations.

I. INTRODUCTION

It has long been known that radiative energies are shifted in the presence of boundaries. Perhaps the most famous example of this phenomenon is the Casimir effect, whereby two parallel-plane conductors experience a force of attraction, usually explained as due to the boundary condition demanded upon the surrounding electromagnetic vacuum fluctuations.[1] However, the concept of zero-point fluctuations is not the only way to explain this effect; precisely the same attraction between the plates can be derived in the absence of hypothetical field fluctuations if one includes instead self-field effects emanating from the plates themselves.[2] Similarly, the long-range Casimir-Polder van der Waals force between an atom and a conducting surface can be computed by coupling the atom to the surface through the intermediary of the zero-point fluctuations,[3] or by considering the effect of the boundary on the self-field of the electron in the absence of any of the vacuum fields.[4] Finally, these two rather different points of view, vacuum fluctuations[5] (VF) versus self-fields[6] (SF), lead to the same result when one computes the effect of a boundary upon the rate of spontaneous emission or upon the Lamb shift of an atom.

While the SF approach is classically grounded in the theory of radiation reaction—the VF method has no classical analogue, since the classical solution to the homogeneous Maxwell equations is usually taken as $A_\mu\equiv 0$, stochastic electrodynamics notwithstanding. For a survey of the curious duality between the VF and SF approaches to QED, we refer the reader to Milonni.[7] It shall suffice to say here that the VF picture is not the only one, as is commonly believed, and that there are those who believe that the vacuum field is more of a formal contrivance or artiface than a "real" physical thing. Perhaps self-field phenomena are the same in certain cases *as if* a fluctuating zero-point field were present.

The most accurate measurement of the electron g factor, $a_e:=(g-2)/2$ comes from an experiment conducted by Dehmelt and co-workers.[8] (The notation $A:=B$ means that A is being defined as equal to B.) A single electron is suspended in a Penning trap under the influence of a uniform magnetic field, and the experimental result

$$a_e=1\,159\,652\,193(4)\times 10^{-12} \tag{1}$$

is obtained. For a typical trap the distance R from the bound electron to the trap wall is on the order of $R\approx\frac{1}{3}$ cm. Thus it is of interest to approximate the effect of the boundary upon the QED free space shift a_e, to see at what point we would expect to see a change in the experimental outcome.

In the Penning-trap experiments the electron is bound by a strong magnetic field into Landau orbitals. The cyclotron and spin-precession frequencies, ω_c and ω_s are measured, and the value of $g-2$ is related to these two quantities by

$$a_e=\frac{\omega_s-\omega_c}{\omega_c}=\frac{(g/2)-1}{1}=\frac{g-2}{2}\ . \tag{2}$$

Thus we see that boundary-induced shifts to ω_s and to ω_c must both be reckoned with.

Over the years calculations have been performed for electrons in various states of motion near conducting boundaries of many different geometries. For simplicity, we shall restrict our discussion to an electron in a strongly magnetic field, executing cyclotron motion in a plane parallel to and a distance R from a perfect plane conductor. From dimensional arguments one can see that changes in either ω_s or ω_c will have the form $(h/2\pi=c=1)$

$$\frac{\Delta\omega}{\omega}\propto\frac{\alpha}{mR} \tag{3}$$

to first order in the fine-structure constant α.

There has been a controversy in recent literature over whether there is indeed any correction to ω_s to first order in the parameter α/Rm. Brown and his co-workers find no correction to ω_s in this order of approximation, whereas others seem to obtain such a shift.[9] The discrepancy had been blamed on lack of gauge invariance or on the inappropriate use of the image method. However, Kreuzer and Svozil,[10] in a series of elegant papers, have used the image method and a manifestly gauge-invariant approach to show that, to order α/Rm, magnetic moment corrections are exactly canceled by mass corrections, leaving no overall correction to ω_s for an electron bound in a Gaussian wave packet between parallel-plate conductors. Thus if one computes $\Delta\mu$ without including the effect of Δm, one finds a correction to ω_s, whereas if both effects are taken together there is no correction.

There is, however, a plate-induced correction to ω_c to this order, as pointed out by Brown *et al.*[11] The change $\Delta\omega_c$ is essentially classical in origin, and forms the dominant contribution to δa_e.

Hitherto, all calculations of ω_s have been carried out using either the full apparatus of standard QED, or by coupling zero-point fluctuations to the electron in a nonrelativistic (NR) approach.

In the present paper we shall use a formulation of QED which is not second quantized and which contains no vacuum fluctuations but rather is based on self-fields. We compute all corrections to the energy of an electron executing cyclotron motion in a stationary plane parallel to a perfectly conducting planar surface to first order in the parameter α/Rm. We find, in exact agreement with the full QED calculation of Kreuzer,[10] no shift to ω_s in this order due to cancellation of the effect of magnetic moment and mass shifts:

$$\frac{\Delta\mu}{\mu}=-\frac{\alpha}{4Rm},\quad \frac{\Delta m}{m}=-\frac{\alpha}{4Rm}\ . \tag{4}$$

However, we do find two corrections to ω_c. There is a direct shift in the orbital motion, $\Delta\omega_{\mathrm{OS}}$ (where OS refers to orbital shift), due to the electromagnetic interaction of the charge with its image in the wall, and a second correction, $\Delta\omega_{\mathrm{MS}}$ (where MS refers to mass shift), due to the change of the electron mass in the the presence of the conductor. The sum of the two shifts yields

$$\frac{\Delta\omega_c}{\omega_c}=\frac{\Delta\omega_{\mathrm{OS}}+\Delta\omega_{\mathrm{MS}}}{\omega_c}=\frac{\alpha}{8Rm} \tag{5}$$

in the quantum regime, as opposed to the classical limit of large quantum numbers n.

It is interesting that the cavity corrections to QED, which we calculate here via the (relativistic) Green's functions appropriate for the cavity, are the same whether the matter part (4) is treated relativistically or nonrelativistically. Thus the physics of the problem is already contained in a NR theory which includes the self-field of the electron from the beginning. It is therefore not necessary to introduce a second quantized radiation field with its resultant vacuum fluctuations to understand the mechanism responsible for boundary-induced radiative corrections to the magnetic moment and mass of the electron.

II. SELF-FIELD APPROACH TO QED

In standard QED one deals with a bare electron and a second quantized radiation field; the self-field is added back on to the electron one photon at a time through an expansion in Feynman diagrams. In the self-field approach, as developed in the relativistic version by Barut and Kraus,[2] and in the NR version by Barut and van Huele,[13] the self-field of the electron is included from the beginning and the radiation field is classical. Vacuum fluctuations are conspicuously absent.

The standard QED results for the Lamb shift and spontaneous-emission rates are derived from the self-field theory in Refs. 12 and 13. The NR version of the theory was adapted by Barut and Dowling to account for boundary-induced changes in spontaneous emission[4] and Lamb shifts, as well as to describe long-range Casimir-Polder van der Waals forces between an atom and a conducting surface.[6] Recently Barut, Dowling, and van Huele[14] have arrived at a cutoff-dependent value for the free-space value of $g-2$ which is correct in sign for all values of cutoff and correct in magnitude for the reasonable cutoff choice of $\Lambda/m=\frac{3}{4}$.

In the present paper we adapt this previous NR free-space calculation of $g-2$ to include boundary effects. The results are finite and cutoff independent and agree exactly with the fully relativistic, standard QED calculation of Kreuzer.[10]

An electron is assumed to be surrounded by an electromagnetic field $A_\mu(x)$ which can be separated conceptually into the sum of an external field A^e_μ and a self-field A^s_μ. A^e_μ obeys the homogeneous Maxwell's equations while the field tensor $F^s_{\mu\nu}$ constructed from A^s_μ

$$F_{\mu\nu}:=\partial_\mu A_\nu-\partial_\nu A_\mu$$

obeys the inhomogeneous equation

$$(F^{\mu\nu}_s)_{,\mu}=ej^\nu\ , \tag{6}$$

where $j^\mu(x)$ is the electron's own four current ($e>0$ throughout). Equation (6) may be solved for A^s_μ by use of an electromagnetic Green's function $D_{\mu\nu}$ to yield

$$A^s_\mu(x)=e\int dy\, D_{\mu\nu}(x-y)j^\nu(y)\ , \tag{7}$$

where the choice of the gauge is to be specified.

All calculations of energies will proceed from an action formulation with $w(x)$, an action density, and

$$W=\int dx\, w(x;\varphi;A) \tag{8}$$

the total action with $x=x_\mu$, $A=A^e_\mu+A^s_\mu$, and φ the matter field. For bound states the action W is related to the total energy E of the system by[12,13]

$$W_{fi}=(2\pi)\delta(E_f-E_i)E\ , \tag{9}$$

where the subscripts i and f indicate initial and final energies.

For the problem at hand we use the Pauli Hamiltonian

$$H=\frac{1}{2m}[\boldsymbol{\sigma}\cdot(\mathbf{p}-e\,\mathbf{A})]^2+eA_0 \tag{10}$$

to construct the symmetrized action density ($\boldsymbol{\sigma}$ is the usual Pauli spin vector operator):

$$w=\varphi^*\left[\frac{1}{2m}[(\overleftarrow{\nabla}+ie\,\mathbf{A})\cdot\boldsymbol{\sigma}][\boldsymbol{\sigma}\cdot(\overrightarrow{\nabla}-ie\,\mathbf{A})]+eA_0-i\partial_t\right]\varphi + \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}\,, \tag{11}$$

where we have now included $F^s_{\mu\nu}F^{\mu\nu}_s$, and the Hamiltonian H has been made symmetric by integration by parts (IP). Here $\varphi^*\overleftarrow{\nabla}\varphi:=(\nabla\varphi^*)\varphi$. [Throughout, equality of action densities shall mean equality with respect to IP and surface terms which, along with $A^s_\mu(x)$, are presupposed to vanish at infinity.] The $\varphi(x)$ form a field of two-component Pauli spinors. Notice we have not included any contributions from image charges *ab initio*. These effects arise naturally through the boundary conditions to be imposed on A^s_μ through the Green's function $D_{\mu\nu}$.

The current may be computed from Eq. (11) via the Euler-Lagrange equations. Variation of W with respect to φ^*

$$\frac{\delta W}{\delta\phi^*}-\partial_\mu\frac{\delta W}{\delta\varphi^*{}_{,\mu}}=\left[-\frac{\nabla^2}{2m}+\frac{ie}{m}\mathbf{A}\cdot\nabla+\frac{ie}{2m}\nabla\cdot\mathbf{A}-\frac{e}{2m}\boldsymbol{\sigma}\cdot\mathbf{B}+\frac{e^2}{2m}A^2\right]\varphi=0$$

yields the *Pauli equation of motion*. Variation of W with respect to A_μ yields Maxwell's equations,

$$\frac{\delta W}{\delta A^s_\nu}-\partial_\mu\frac{\delta W}{\delta(A^s_\nu)_{,\mu}}=-ej^\nu+(F^{\mu\nu}_s)_{,\mu}=0$$

only if one takes j^μ as

$$\frac{\delta W}{\delta A^s_\mu}=:-ej^\mu\,, \tag{12}$$

which can be written in the form

$$j^\mu=\varphi^*\left[1,\frac{1}{2mi}\overleftrightarrow{\nabla}+\frac{1}{2m}(\overleftarrow{\nabla}\times\boldsymbol{\sigma}-\boldsymbol{\sigma}\times\overrightarrow{\nabla})-\frac{e}{m}\mathbf{A}\right]\varphi =[\rho,\mathbf{j}_M+\mathbf{j}_{\rm SM}+\mathbf{j}_F]\,, \tag{13}$$

with

$$\mathbf{j}_M=\varphi^*\frac{\overleftrightarrow{\nabla}}{2mi}\varphi\quad\text{(momentum)}\,,$$
$$j_{\rm SM}=-\frac{1}{2m}[\varphi^*(\boldsymbol{\sigma}\times\nabla\varphi)-(\nabla\varphi^*\times\boldsymbol{\sigma})\varphi]\quad\text{(spin momentum)}\,,$$
$$\mathbf{j}_F=-\frac{e}{m}\mathbf{A}\quad\text{(field)}\,, \tag{14}$$

where $\varphi^*\overleftrightarrow{\nabla}\phi:=\varphi^*\nabla\varphi-(\nabla\varphi^*)\varphi$ and $\rho=\varphi^*\varphi$ as usual. Equation (14) is the the NR analogue of the Gordon decomposition of the Dirac current $e\Psi\gamma_\mu\Psi$.

III. CALCULATION OF THE TOTAL ACTION

To determine the contribution of the contraction $F_{\mu\nu}F^{\mu\nu}$ to the action density in Eq. (11) we write, for the self-field A^s_μ alone,

$$\begin{aligned}\tfrac{1}{4}F^s_{\mu\nu}F^{\mu\nu}_s&=\tfrac{1}{4}A^s_{[\mu,\nu]}F^{\mu\nu}_s\\ &=\tfrac{1}{4}(A^s_{[\nu}F^{\mu\nu}_s)_{,\mu]}-\tfrac{1}{4}A^s_{[\nu}(F^{\mu\nu}_s)_{,\mu]}\\ &=-\frac{e}{2}A^s_\nu j^\nu\,,\end{aligned} \tag{15}$$

where $[\cdot,\cdot]$ implies antisymmetrization in the indices, and IP has been employed. Keeping in mind that j_μ is a function of the total field $A=A^s+A^e$ we insert the expression (14) for j into (15) to get

$$\begin{aligned}\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}&=-\tfrac{1}{2}eA^s_\mu j^\mu\\ &=\varphi^*\left[-\frac{e}{2}A^s_0-\frac{ie}{2m}\mathbf{A}^s\cdot\nabla-\frac{ie}{4m}\nabla\cdot\mathbf{A}^s+\frac{e}{4m}\boldsymbol{\sigma}\cdot\mathbf{B}^s-\frac{e^2}{2m}(\mathbf{A}^e\cdot\mathbf{A}^s+A_s^2)\right]\varphi\,,\end{aligned} \tag{16}$$

where IP has been used to sandwich all the ∇'s between φ^* and φ. The interaction terms of Eq. (11) for w may be written

$$\begin{aligned}&\varphi^*\left[\frac{1}{2m}[(\overleftarrow{\nabla}+ie\,\mathbf{A})\cdot\boldsymbol{\sigma}][\boldsymbol{\sigma}\cdot(\overrightarrow{\nabla}-ie\,\mathbf{A})]+eA_0\right]\varphi\\ &\quad=\varphi^*\left[-\frac{1}{2m}\nabla^2+\frac{ie}{m}\mathbf{A}\cdot\nabla+\frac{ie}{2m}\nabla\cdot\mathbf{A}-\frac{e}{2m}\boldsymbol{\sigma}\cdot\mathbf{B}+\frac{e^2}{2m}A^2\right]\varphi\,.\end{aligned} \tag{17}$$

If we call $\mathbf{B}=\mathbf{B}^e+\mathbf{B}^s=\nabla\times\mathbf{A}^e+\nabla\times\mathbf{A}^s$, then the sum of (16) and (17) gives for the total action density w

$$w=\varphi^*\left[-\frac{\nabla^2}{2m}+eA^e_0+\frac{e}{2}A^s_0-i\partial_t+\frac{ie}{m}\mathbf{A}^e\cdot\nabla+\frac{ie}{2m}\mathbf{A}^s\cdot\nabla-\frac{e}{2m}\boldsymbol{\sigma}\cdot\mathbf{B}^e-\frac{e}{4m}\boldsymbol{\sigma}\cdot\mathbf{B}^s+\frac{e^2}{2m}A_e^2+\frac{e^2}{2m}\mathbf{A}^e\cdot\mathbf{A}^s+\frac{ie}{2m}\nabla\cdot\mathbf{A}^e+\frac{ie}{2m}\nabla\cdot\mathbf{A}^e+\frac{ie}{4m}\nabla\cdot\mathbf{A}^s\right]\varphi\,. \tag{18}$$

The terms involving A^e_μ alone yield the standard quantum-mechanical (QM) motion of the charge and those containing A^s_μ give rise to radiative corrections in analogy to classical radiation reaction-type effects. In particular, the $\boldsymbol{\sigma}\cdot\mathbf{B}^s$ term yields magnetic moment corrections, and from $\mathbf{A}^s\cdot\nabla$ we get orbital energy shifts and mass renormalization.

IV. CHOICE OF GAUGE AND BOUNDARY CONDITIONS

Following Kreuzer and Svozil,[10] we choose to work in the axial gauge. (Of course, physical results will be independent of the choice of gauge, but use of the axial gauge facilitates the calculations when boundaries are present.) The condition that $E_\parallel$ and $B_\perp$ vanish on the surface S of a conductor can be written covariantly as

$$F_{\mu\nu}n_\alpha\epsilon^{\mu\nu\alpha\beta}|_S \equiv 0, \quad \forall\beta \tag{19}$$

where $n_\alpha=[0,\mathbf{n}]$, with $\mathbf{n}$ normal to S in the rest frame of the conductor. If we now restrict ourselves to the situation of a perfect plane conductor located at $z=-R$, then Eq. (19) is implied by the choice of the axial gauge condition

$$0=n_\mu A^\mu = A_3 \ , \tag{20}$$

where $n_\mu=[0,\mathbf{z}]=[0,0,0,1]$. With this choice of gauge the Green's function $D_{\mu\nu}$ of Eq. (7) can be written as[10]

$$D_{\mu\nu}(x-y)=-\frac{1}{(2\pi)^4}\int\frac{dk\, e^{ik\cdot(x-y)}}{k^2+i\epsilon}\left[g_{\mu\nu}-\frac{n_\mu k_\nu+n_\nu k_\mu}{n\cdot k}+\frac{n^2k_\mu k_\nu}{(n\cdot k)^2}\right], \tag{21}$$

where the $+i\epsilon, \epsilon>0$ provides the correct causal behavior. If we define

$$D(k;x-y):=\frac{1}{(2\pi)^4}\frac{e^{ik\cdot(x-y)}}{k^2+i\epsilon} \tag{22}$$

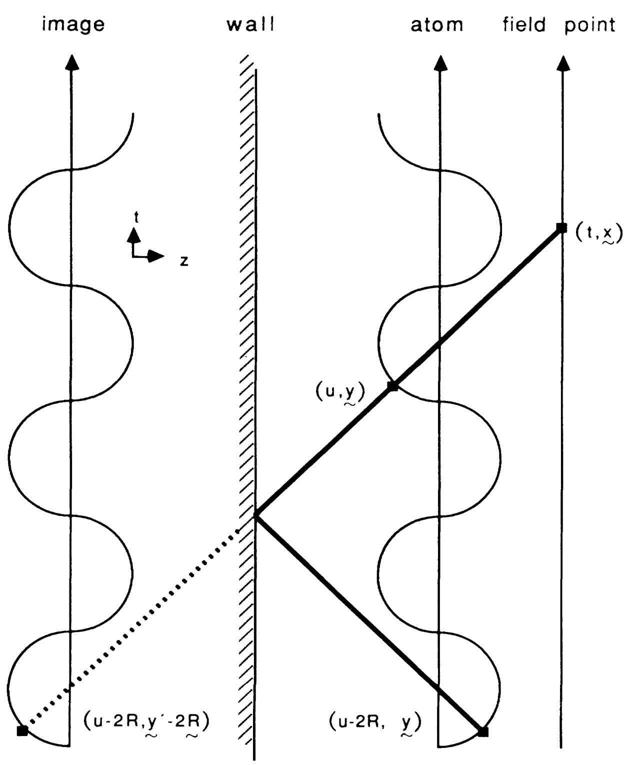

FIG. 1. The value of the electromagnetic Green's function at the field point $[t,\mathbf{x}]$ is determined by the current at the source point at $[u,\mathbf{y}]$ and by the image current at the image source point $[u-2R,\mathbf{y}-2\mathbf{R}]$ where we define $\mathbf{y}':=(y_1,y_2,y_3)$ and $\mathbf{R}:=(0,0,R)$. Notice that the image source point is retarded in both space and time.

then the Green's function solution for A^s_μ can be written

$$-\mathbf{A}^s(x)=e\int\int dy\, dk D(k;x-y) \times\left[\mathbf{j}(y)-\frac{(\mathbf{k}\cdot\mathbf{j})\hat{\mathbf{z}}+(\hat{\mathbf{z}}\cdot\mathbf{j})\mathbf{k}}{\hat{\mathbf{z}}\cdot\mathbf{k}}+\frac{(\mathbf{k}\cdot\mathbf{j})\mathbf{k}}{(\hat{\mathbf{z}}\cdot\mathbf{k})^2}\right] \tag{23a}$$

and also

$$A^s_0(x)=-e\int\int dy\, dk D(k;x-y)\left[1-\frac{\omega^2}{(\hat{\mathbf{z}}\cdot\mathbf{k})^2}\right]\rho(y), \tag{23b}$$

where $\mathbf{j}=\mathbf{j}(x)$, $\rho(y)=j_0(y)$, and $k=k^\mu=[\omega,\mathbf{k}]$.

To effect the correct boundary condition, we use the method of images. Consider the world path of an electron executing cyclotron orbits near our plane conductor as shown in Fig. 1. Clearly the value of $A^s_\mu(x)$ at the field point $x^\mu=[t,\mathbf{x}]$ is the sum of the contributions from the current ej^ν at source point $y^\mu=[s,\mathbf{y}]$ and the contribution of the image charge current $-ej'\nu$ at the retarded source point $y'\mu-2R^\mu=[s-2R,\mathbf{y}'-2\mathbf{R}]$ where $\mathbf{y}'=(y_1,y_2,-y_3)$. (Henceforth a prime indicates a change of sign of the 3 coordinate). Here $R=R^\mu=[R,\mathbf{R}]$ $=[R,0,0,R]$ in a notation clear by context. In the Green's-function solution (7) for $A^s_\mu(x)$ we now insert the total current

$$j^\nu(y)=j^\nu_0(y)+\Delta j^\nu_0=j^\nu_0(y)-j_0\nu(y'-2R)$$

to get, regardless of gauge,

$$\begin{aligned}A^s_\mu(x)&=e\int dy\, D_{\mu\nu}(x-y)[j^\nu(y)-j^{\nu'}(y'_r)]\\&=e\int dy[D_{\mu\nu}(x-y)-D_{\mu\nu}(x-y'_r)]j^\nu(y)\\&=:e\int dy[D^0_{\mu\nu}(x-y)+\Delta D_{\mu\nu}(x-y)]j^\nu_0(y)\\&=:A^0_\mu(x)+\Delta A_\mu(x)\ ,\end{aligned} \tag{24}$$

where the self-field only is being considered. The $A_\mu^0(x)$ will give the free-space radiative corrections with $D_{\mu\nu}^0$ in the Coulomb gauge; these corrections have been computed elsewhere.[14]

For the remainder of the paper, we shall be concerned only with the boundary-induced shifts which arise from $\Delta A_\mu(x)$ (we drop the superscript 0), with $\Delta D_{\mu\nu}$ in the axial gauge.

V. MAGNETIC MOMENT

Since the axial gauge demands $A_3=0$ we may take $\mathbf{A}=\mathbf{A}_\parallel=(\frac{1}{2})\mathbf{B}_\perp\times\mathbf{x}$ with $\mathbf{B}=\mathbf{B}_\perp=B\mathbf{z}$, $\mathbf{B}=\mathbf{B}^e+\mathbf{B}^s$; $\mathbf{B}^e$ is constant and uniform and $\mathbf{B}^s$ is yet to be determined.

Referring back to the action density of Eq. (18) the term with $\nabla\cdot\mathbf{A}^e$ vanishes, and one can show $\nabla\cdot\mathbf{A}^s=0$ also. We assume only an external magnetic field $\mathbf{B}^e$ and thus $A_0^e\equiv0$ as well. As usual for weak fields, the term containing $|\mathbf{A}^e|^2$ is neglected, and that proportional to $\mathbf{A}^e\cdot\mathbf{A}^s$ is proportional also to $|\mathbf{A}^e|^2$, and so we drop it too. The total action can now be written

$$w=\varphi^*\left[-\frac{\nabla^2}{2m}+\frac{ie}{m}\mathbf{A}^e\cdot\nabla-i\partial_t-\frac{e}{2m}\boldsymbol{\sigma}\cdot\mathbf{B}^e-\frac{e}{4m}\boldsymbol{\sigma}\cdot\mathbf{B}^s+\frac{ie}{2m}\mathbf{A}^s\cdot\nabla+\frac{e}{2}A_0^s\right]\varphi=:\sum_{k=1}^{7}w_i\ . \tag{25}$$

We shall assume that the zeroth-order energy is contained in $W_0=W_1+W_2+W_3$. This is just the energy of a spinless charge executing Landau orbitals in a plane parallel to the conductor. The spin and radiative corrections are included in $W':=W_4+W_5+W_6+W_7$.

To illustrate how energies are extracted, we compute first the contribution from $W_4=(-e/2m)\boldsymbol{\sigma}\cdot\mathbf{B}^e$, the energy due to the *normal* magnetic moment.

We begin by making the Fourier expansion

$$\varphi(x)=\sum\!\!\!\!\!\!\int_n \varphi_n(\mathbf{x})e^{-iE_nt}\ , \tag{26}$$

where the $\varphi_n(\mathbf{x})$ are initially the exact discrete and continuous solutions to the total equation of motion contained in Eq. (25) with E_n as corresponding eigenvalues (n stands for all eigenvalues). Hence

$$\begin{aligned}W_4&=\int dx\,\varphi^*\left[-\frac{e}{2m}\boldsymbol{\sigma}\cdot\mathbf{B}^e\right]\varphi\\&=-\frac{e}{2m}\sum_{n,m}\int\int d^3x\,dt\,\varphi_n^*(\boldsymbol{\sigma}\cdot\mathbf{B}^e)\varphi_m e^{i\omega_{nm}t}\\&=-\frac{e}{2m}(2\pi)\sum_{n,m}\int d^3x\,\varphi_n^*(\boldsymbol{\sigma}\cdot\mathbf{B}^e)\varphi_m\delta(\omega_{nm})\\&=-\frac{e}{2m}(2\pi)\sum_n\langle n|\boldsymbol{\sigma}\cdot\mathbf{B}^e|n\rangle\ ,\end{aligned} \tag{27}$$

where $\omega_{nm}:=E_n-E_m$, and t integration yields a δ function. If we assume, to first order in the iteration, that the φ_n are solutions to the unperturbed motion in W_0, we may extract the perturbation to the nth level and, as per Eq. (9), division by 2π yields the energy

$$E_r^{(n)}=\frac{W_4^{(n)}}{2\pi}=-\frac{e}{2m}\langle n|\boldsymbol{\sigma}\cdot\mathbf{B}^e|n\rangle\ ,$$

which is the standard QM result for the interaction energy of the normal magnetic moment with an external magnetic field $\mathbf{B}^e$.

We now turn our attention to the anomalous magnetic moment correction contained in $W_5=(-e/4m_0)\boldsymbol{\sigma}\cdot\mathbf{B}^s$. Recall we are considering here only the plate corrections induced by $\Delta A_\mu^s(x)$ in the axial gauge. Now since $\Delta A_\mu^s(x)$ is a function of $j^\nu(y)$, inspection of Eq. (14) shows that to $O(\alpha)$ only $\mathbf{j}_F$ will contribute to Δa_e, Δa_e being the plate correction to the free-space anomalous magnetic moment a_e. Using j'_μ and $e\to-e$ in Eq. (7) with the Green's function $\Delta D_{\mu\nu}$ of expression (24) in the axial gauge, as in Eq. (23), we obtain

$$\Delta\,\mathbf{A}^s(x)=e\int\int dy\,dk\,D(k;x-y)\times\left[j'(y)-\frac{(\mathbf{k}\cdot\mathbf{j}')\hat{\mathbf{z}}+(\hat{\mathbf{z}}\cdot\mathbf{j}')\mathbf{k}}{\hat{\mathbf{z}}\cdot\mathbf{k}}+\frac{(\mathbf{k}\cdot\mathbf{j}')\mathbf{k}}{(\hat{\mathbf{z}}\cdot\mathbf{k})^2}\right]\ , \tag{28}$$

where

$$\mathbf{j}'=\mathbf{j}'_F(y)=\frac{e}{m}\mathbf{A}'\rho(y)\ .$$

If one now considers $\nabla\times\mathbf{A}^s$, then the operation $\nabla\times$ becomes $+i\mathbf{k}\times$ and terms proportional to $\mathbf{k}$ vanish, so we drop them. Now, to $O(\alpha)$, $\mathbf{A}=\mathbf{A}^e$. Further, we anticipate the dipole approximation (DA) in which $\mathbf{x}=\mathbf{y}$ and so saying we take $\mathbf{A}^e(\mathbf{y})\approx\mathbf{A}^e(\mathbf{x})$. Applying the DA, $e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\approx1$, we get

$$\nabla\times\Delta\,\mathbf{A}^s(x)=\frac{e^2}{m}\int\int dy\,dk\,D(k;t-s+2R)\times\left[\nabla\times\mathbf{A}'_e-\frac{1}{u}(\nabla\times\hat{\mathbf{z}})(\mathbf{k}\cdot\mathbf{A}'_e)\right]\rho(y)\ . \tag{29}$$

The second term in square brackets vanishes upon integration over the solid angle $d\Omega_k$ and so we eliminate it. Remaining is $\nabla\cdot\mathbf{A}^e(\mathbf{x})=:\mathbf{B}^e$. Writing in the DA

$$k\cdot(x-y+2R)\approx\omega(t-s+2R)-2\mathbf{k}\cdot\mathbf{R}$$

in D, we have

$$\Delta W_5^F=\int dx\,\varphi\left[-\frac{e}{4m}\sigma\cdot\mathbf{B}^s\right]\varphi$$
$$=-\frac{e^3}{4m^2}\int\int\int dx\,dy\,dk\,D(k;t-s)e^{i2k\cdot R}\varphi^*(\sigma\cdot\mathbf{B}^e)\varphi\rho(y)$$
$$=-\frac{e^3}{4m^2}\sum_{n,m,p,q}\int\int\int dx\,dy\,dk\,D(k;t-s)e^{i2k\cdot R}\varphi_n^*(\mathbf{x})(\sigma\cdot\mathbf{B}^e)\varphi_m(\mathbf{x})\varphi_p^*(\mathbf{y})\varphi_q(\mathbf{y})e^{i(\omega_{nm}t+\omega_{pq}s)}$$
$$=-\frac{e^3}{4m^2}\frac{1}{(2\pi)^2}\sum_{n,m,p,q}\int\frac{d^3k}{\omega_{pq}^2-\lambda^2}e^{i2\omega_{pq}R}e^{-i2\mathbf{k}\cdot\mathbf{R}}\langle n|\sigma\cdot\mathbf{B}^e|m\rangle\langle p|q\rangle\delta(\omega_{nm}+\omega_{pq})\ ,$$

where the δ function comes from t and s integrations. (The Δ indicates a plate correction and the superscript F shows that only the field term of the current is under consideration.) The δ function is satisfied by $\omega_{nm}+\omega_{pq}=0$. For this condition we choose the self-energy solution $n=q$ and $m=p$, yielding upon integration over $d\omega$ ($|k|=:(\lambda,2\lambda R=:\xi)$

$$\Delta W_5^F=\int dx\,\varphi\left[-\frac{e}{4m}\sigma\cdot\mathbf{B}^s\right]\varphi$$
$$=-\frac{e^3}{4m^2}\int\int\int dx\,dy\,dk\,D(k;t-s)\times e^{i2k\cdot R}\varphi^*(\sigma\cdot\mathbf{B}^e)\varphi\rho(y)$$
$$=\frac{e}{2m}\frac{\alpha 2m}{4Rm}\sum_n\langle n|\sigma\cdot\mathbf{B}^e|n\rangle\ , \tag{30}$$

with $\alpha=e^2/4\pi$ in our units. Extracting the contribution to level n and converting to energy, we get

$$\Delta E_5^{(n)}=\frac{\Delta W_5^{(n)F}}{2\pi}=\frac{e}{2m}\frac{\alpha}{4Rm}\langle n|\sigma\cdot\mathbf{B}^e|n\rangle\ . \tag{31}$$

Thus the total energy proportional to $\sigma\cdot\mathbf{B}^e$ is

$$E_4+\Delta E_5=-\frac{e}{2m}\langle\sigma\cdot\mathbf{B}^e\rangle\left[1-\frac{\alpha}{4Rm}\right] \tag{32}$$

or

$$\frac{\Delta\mu}{\mu}=-\frac{\alpha}{4Rm}\quad\text{(plate shift)}\ . \tag{33}$$

We compare this to be the free-space anomalous magnetic moment, $\delta\mu$, also calculated in the present NR theory[14]

$$\frac{\delta\mu}{\mu}=-\frac{\alpha}{2\pi}\frac{4\Lambda}{3m}$$

(free-space shift, before mass renormalization), where Λ is a cutoff in the photon momentum $|\mathbf{k}|$. (We use δ for a free-space effect and Δ for a boundary effect.)

The interpretation within the present theory is that in free space the self-field acts back on the particle so as to decrease the spin—in a drag or spin retardation effect—leading to a decrease of magnetic moment given by Eq. (34). In the presence of a conducting boundary, the interaction of the change with the field of the image charge further decreases the rate of spin.

VI. MASS RENORMALIZATION

If we take the mass which appears in the kinetic energy $W_1=-\nabla^2/2m$, to the coefficient of inertia for the free-space mass, then any radiative corrections arising from ΔA_μ^s which give rise to another term proportional to ∇^2 must be used to renormalize this mass. The term $W_6=(ie/2m)\mathbf{A}^s\cdot\nabla$ gives just such a contribution if one isolates $\mathbf{j}\rightarrow\mathbf{j}_M=\varphi^*\nabla\varphi/2mi$ in the reckoning of ΔA^s. Inserting $\mathbf{j}_M$ from Eq. (14) into the expression (28) for $\Delta\mathbf{A}^s$ we obtain

$$\Delta\mathbf{A}^s=-\frac{ie\pi}{Rm}\int dy\int_0^\infty d\xi\,D(k;t-s+2R)\times\frac{\sin\xi}{\xi}\varphi^*(y)\nabla_y^{\parallel}\varphi(y)\ , \tag{35}$$

where we have used the DA, integrated over $d\Omega_k$, and dropped a contact term singular in the variable $\cos\theta_k$. We define $\lambda:=|\mathbf{k}|$ and $\xi:=2\lambda R$. Notice how the component of the momentum perpendicular to the wall, proportional to $\nabla_\perp$, has canceled out of the expression; this is a consequence of the axial gauge formulation of the problem.[15] Thus we see that the mass shift Δm is actually a tensor quantity with a component $\Delta m_\parallel$ arising from $\nabla_\parallel$ and another component $\Delta m_\perp$ which is zero. The action ΔW_6^M is then

$$\Delta W_6^M=\frac{\alpha}{2Rm^2}\sum_{n,m}\int_0^\infty d\xi\frac{\sin\xi}{\xi}\times\left[-\langle n|\nabla|n\rangle\cdot\langle m|\nabla^{\parallel}|m\rangle+e^{ia_{nm}}\frac{\xi^2}{a_{nm}^2-\xi^2}\times\langle n|\nabla|m\rangle\cdot\langle m|\nabla^{\parallel}|n\rangle\right]\ , \tag{36}$$

where $a:=2R\omega$ and $a_{nm}:=2R\omega_{nm}$, and $\xi=2|\mathbf{k}|R$. The superscript M notates that only the momentum term of the current has been used, while Δ indicates that only the plate corrections to the free-space term W_6 are under consideration. With respect to the symmetry in n and m in $\sum_{n,m}$, we may write a partial fraction expansion[13]

$$\sum_{n,m}\frac{\xi^2}{a_{nm}^2-\xi^2}=\frac{1}{2}\sum_{n,m}\left[\frac{a_{nm}}{a_{nm}-\xi}-\frac{a_{nm}}{a_{nm}+\xi}-2\right]$$
$$\to\sum_{n,m}\left[\frac{a_{nm}}{a_{nm}+\xi}-1\right],$$

which is then inserted into the above expression to obtain

$$\Delta W_6^M=-\frac{\alpha}{2Rm^2}\sum_{n,m}\left[\frac{\pi}{2}\langle n|\nabla|n\rangle\cdot\langle m|\nabla^{\parallel}|m\rangle+\left[\frac{\pi}{2}-\int_0^\infty d\xi\frac{a_{nm}}{a_{nm}-\xi}\frac{\sin\xi}{\xi}\right]e^{ia_{nm}}\times\langle n|\nabla|m\rangle\cdot\langle m|\nabla^{\parallel}|n\rangle\right]. \quad (37)$$

Barut and van Huele show that the first term in the square brackets vanishes if one does not use the DA, so we drop it. The last term in square brackets has been shown by Barut and Dowling[4,6] to give rise to boundary-induced changes in the Lamb shift and spontaneous-emission rates. Thus we carry only the middle term which yields a mass renormalization (MR). Since we seek a self-energy contribution of an electron in level n upon itself, we may set $n=m$ in $\exp(ia_{nm})$. Hence, converting to energy in extracting the nth energy level,

$$\Delta E_{\text{MR}}^{(n)}=\frac{\Delta W_{\text{MR}}^{(n)}}{2\pi}=-\frac{\alpha}{8Rm^2}\sum_m\langle n|\nabla|m\rangle\cdot\langle m|\nabla^{\parallel}|n\rangle$$
$$=-\frac{\alpha}{8Rm^2}\langle n|\nabla_{\parallel}^2|n\rangle . \quad (38)$$

Calling m_0 the observed, free-space mass and m the plate-renormalized value, we have

$$-\frac{\nabla^2}{2m}=-\frac{\nabla^2}{2m_0}-\frac{\alpha}{4m_0R}\left[\frac{\nabla_{\parallel}^2}{2m_0}\right],$$

which then implies

$$m_{\parallel}=m_0\left[1-\frac{\alpha}{4mR}\right],$$
$$m_{\perp}=m_0 .$$

These two expressions, when taken together, give

$$m=m_0\left[1-\frac{\alpha}{4mR}\right]$$
$$=:m_0+\Delta m \quad (39)$$

or

$$\frac{\Delta m}{m}=-\frac{\alpha}{4mR}\quad\text{(plate shift)} . \quad (40)$$

This is then the plate-induced shift in the dynamic mass. For comparison, the free-space mass shift as given in the present NR self-field theory[14] is

$$\frac{\delta m}{m}=\frac{8\Lambda}{3m}\left[\frac{\alpha}{2\pi}\right]\quad\text{(free-space shift)} , \quad (41)$$

where, unlike in the magnetic moment case, the signs of the two shifts δm and Δm are different. In free space the mass of the particle increases due to the inclusion of the mass energy of the self-field, in analogy to mass renormalization as used in classical radiation reaction theory. Near a conducting boundary the self-field of the charge, which contributes to the electromagnetic mass, becomes decreased due to the boundary condition; hence the observed free-space mass is decreased by Δm as given in Eq. (40).

We notice that Δm and $\Delta\mu$ are equal magnitude and sign. Thus if we express the total energy shift proportional to $\boldsymbol{\sigma}\cdot\mathbf{B}^e$ in terms of the free-space mass m_0 we see

$$E_4+\Delta E_5=-\frac{e}{2m}\boldsymbol{\sigma}\cdot\mathbf{B}^e\left[1+\frac{\Delta\mu}{\mu}\right]$$
$$=-\frac{e}{2m_0}\boldsymbol{\sigma}\cdot\mathbf{B}^e\left[1-\frac{\Delta m}{m}+\frac{\Delta\mu}{\mu}\right]$$
$$=-\frac{e}{2m_0}\boldsymbol{\sigma}\cdot\mathbf{B}^e , \quad (42)$$

where the Δm and $\Delta\mu$ effects have cancelled out to $O(\alpha)$, leaving the original free-space spin-precession energy.

Thus we see, in agreement with the observation of Kreuzer,[10] the equality of $\Delta\mu/\mu$ with $\Delta m/m$ effects a cancellation of any contribution from the conductor to the spin-precession frequency ω_S to within order α/Rm. It is clear that a calculation of $\Delta\mu$ which does not also include the effect of Δm will lead to an erroneous prediction of a non-null value for $\Delta\omega_s$; thus resolving the discrepancy between the works of Ref. 9 and the conclusions of Brown *et al.*[12]

It is rewarding to note that although our self-field calculation is nonrelativistic, uses the dipole approximation, contains no second quantization of the electromagnetic field and requires no zero-point radiation vacuum fluctuations; it is nevertheless able to reproduce precisely the same magnitudes for the boundary shifts $\Delta\mu$ and Δm as those obtained by Kreuzer[10] in his fully relativistic, standard QED calculation. This illustrates the power of the self-field approach to QED.

VII. SHIFT IN CYCLOTRON FREQUENCY

A. Direct electromagnetic shift

In the present theory, one contribution to the shift in the cyclotron frequency $\omega_c=eB^e/m$ comes from the term $W_6=(ie/2m)\mathbf{A}^s\cdot\nabla$, which gives rise to changes in the Landau levels. The analysis, which by now is standard, of W_2 for the free-space Landau solutions gives for the nth energy level

$$\Delta E_2^{(n)}=\left\langle n\left|\frac{ie}{m}\mathbf{A}^e\cdot\nabla\right|n\right\rangle=-\frac{e}{2m}\langle n|\mathbf{B}^e\cdot\mathbf{L}|n\rangle , \quad (43)$$

where $\mathbf{L}$ is the angular momentum operator. We have used the relation,

$$\mathbf{A}\cdot\mathbf{p}=\tfrac{1}{2}(\mathbf{B}\times\mathbf{r})\cdot\mathbf{p}=\tfrac{1}{2}\mathbf{B}\cdot(\mathbf{r}\times\mathbf{p})=:\tfrac{1}{2}\mathbf{B}\cdot\mathbf{L}\ .$$

To see how the presence of the conductor effects the motion, let us consider ΔA_μ^s as given by Eq. (28) with

$$\mathbf{j}'(y)\to\mathbf{j}'_F(y)=(e/m)\,\mathbf{A}'(y)\rho(y)=(e/m)\,\mathbf{A}_\parallel(y)\rho(y)\cong(e/m)\,\mathbf{A}_\parallel(\mathbf{x})\rho(y)$$

as found in expression (14). We have again anticipated the use of the dipole approximation. The action ΔW_6^F now becomes

$$\Delta W_6^F=\frac{ie^3}{8\pi Rm^2}\sum_{n,m}\int_0^\infty d\xi\frac{\sin\xi}{\xi}\frac{\xi^2}{a_{nm}^2-\xi^2}e^{ia_{nm}}\times\langle n\,|\,\mathbf{A}_\parallel^e\cdot\nabla|m\,\rangle\delta_{nm}\ , \qquad (44)$$

where $a=2R\omega$, $a_{nm}=2R\omega_{nm}$, and $\xi=2|\mathbf{k}|R$ as before. The superscript F notates that only the field term of the current is being used. The δ_{nm} in Eq. (44) will act to eliminate the retardation factor a_{nm} from this piece of the action. In the limit of large quantum numbers, the spread of each energy level will prevent an exact cancellation and a retardation factor of $\cos(2R\omega_{nm})$ will emerge. This the factor which appears in the result of Brown *et al.*[11] Extracting the nth-level contribution, converting to energy, and recalling that $\mathbf{A}^e\cdot\nabla=(i/2)\mathbf{B}^e\cdot\mathbf{L}$, we have

$$\Delta E_6^{(n)F}=\frac{\Delta W_6^{(n)F}}{2\pi}=\frac{e}{2m}\frac{\alpha}{8Rm}\langle n\,|\mathbf{B}^e\cdot\mathbf{L}|n\,\rangle\ . \qquad (45)$$

Expansion of Eqs. (45) and (43) gives for the direct orbital shift (denoted by the os subscript)

$$\frac{\Delta\omega_{\rm os}}{\omega_c}=-\frac{\alpha}{8Rm}\ . \qquad (46)$$

This is the direct orbital shift due to the electromagnetic interaction of the orbiting electron with the wall. The indirect shift due to the plate-induced mass change is computed in Sec. VII B.

B. Indirect shift due to the mass change

In addition to the direct shift, $\Delta\omega_{\rm os}$, computed above, there is an indirect shift in ω_c due to the fact that the mass m which appears in the free-space formula $\omega_c=eB/m$ of Eq. (43), is no longer the free-space mass m_0 but rather the plate-shifted mass $m_0+\Delta m$, where Δm is given by the relation (40). This shift is also classical in origin. Classically, the observed free-space mass m_0 is the sum of the bare mass m_b and the electromagnetic mass δm; δm is smaller in the presence of a conducting boundary, a result which can be derived totally classically. In any case, one must use the plate-shifted mass in expression (43) for the normal cyclotron motion. Let us recall expression (40) for the plate-induced shift of the electron mass:

$$\frac{\Delta m}{m}=-\frac{\alpha}{4mR}\ . \qquad (47)$$

Including Eq. (47) in the expression (43) yields

$$\Delta E_2=-\frac{e}{2m}\langle n\,|\mathbf{B}^e\cdot\mathbf{L}|n\,\rangle=-\frac{e}{2m_0}\langle n\,|\mathbf{B}^e\cdot\mathbf{L}|n\,\rangle\left[1+\frac{\alpha}{4mR}\right] \qquad (48)$$

and hence an additional change in frequency, $\Delta\omega_{\rm ms}$, due to the boundary-induced mass shift (ms)

$$\frac{\Delta\omega_{ms}}{\omega_c}=\frac{\alpha}{4mR}\ , \qquad (49)$$

which, when combined with the orbital result, $\Delta\omega_{\rm os}$, yields a total shift in the cyclotron frequency

$$\frac{\Delta\omega_c}{\omega_c}=\frac{\Delta\omega_{\rm os}}{\omega_c}+\frac{\Delta\omega_{\rm ms}}{\omega_c}=\frac{\alpha}{8Rm}\ . \qquad (50)$$

Previous calculations of $\Delta\omega_c$, such as those of Brown *et al.*,[9] seem to include both the direct orbital shift, $\Delta\omega_{\rm os}$, and more subtle mass change effect, $\Delta\omega_{\rm ms}$, together in one calculation. Their result is valid for the classical limit of large quantum numbers, however, and is not directly comparable with our calculation here. Barton and Fawcett[16] obtain our Eq. (49) using standard QED and also a classical argument. We get, in addition, the contribution from Eq. (48), which, together with Eq. (49), leads to the overall result of expression (50). The self-field formalism used in this paper is very different from the formalism used in the standard theory, and hence it is extremely difficult to make a step-by-step comparison of the two calculations. We note that the method used by Barton and Fawcett is not the fully relativistic QED theory, but rather an approximation whereby the electromagnetic vacuum field is taken as a perturbation upon nonrelativistic electron equation of motion. This is similar to the nonrelativistic approach used here, with the exception that the vacuum field is replaced with the self-field of the particle. Hence the two approaches are only approximations to a fully relativistic approach, which could explain the discrepancy between our result and that of Barton and Fawcett. In any case, the primary result of this paper is that of Eq. (42) which shows that there is no plate correction to the spin-precession frequency to first order, resolving the apparent dispute in the literature. We leave the detailed analysis of the plate-induced cyclotron frequency shift under various limits—quantum, classical, retarded regime, nonretarded regime, etc.—to future work.

We see then that there is a danger of not considering *all* possible boundary-induced effects together in calculations of this type. The interplay of various contributions in the total energy can lead to cancellations or modifications one might not expect by concentrating on only one piece of the total action.

VIII. SELF-ENERGY SHIFT

To make a more complete analysis of the action W we lastly turn our attention to $W_7=(e/2)A_0^s$, the self-energy shift. Taking ΔA_0^s from Eqs. (23), we have

$$\Delta A_0^s(x)=e\int\int dy\,dk\,D(k;x-y+2R)\left[1-\frac{\omega^2}{(\hat{\mathbf{z}}\cdot\mathbf{k})^2}\right]\rho(y)\ , \tag{51}$$

leading to

$$\Delta W_7=\frac{e^2}{2}\frac{1}{(2\pi)^4}\int\int dx\,dy\int\int d\lambda\,\lambda^2 d\Omega_k\frac{e^{i\omega(t-s)}}{\omega^2-\lambda^2}\times\cos(2\lambda Ru)e^{-i\omega 2R}\rho(x)\rho(y)$$
$$=-\frac{e^2}{2\pi}\int\frac{d\xi}{2R}\frac{\sin\xi}{\xi}\sum_n\delta_{nn}$$
$$=-\frac{\alpha\pi}{2R}\sum_n\delta_{nn}\ , \tag{52}$$

where $u=\cos\theta_k,\lambda=|\mathbf{k}|$, etc. Converting to the energy of level n, we obtain

$$\Delta E_7^{(n)}=\frac{W_7^{(n)}}{2\pi}=-\frac{\alpha}{4R}\ , \tag{53}$$

which is simply the electrostatic energy of interaction between the charge and its image in the wall. Notice how in our formulation, energies such as this are not put *ab initio* into the Hamiltonian, as is done in the nonrelativistic, vacuum fluctuation approach, but rather arise naturally through the self-field's reflecting of a change in the boundary conditions.

The energies in Eq. (53) also may be interpreted as a change in the free-space electromagnetic mass; $\Delta m=-\alpha/4R$, or $\Delta m/m=-\alpha/4Rm$, consistent with the result found previously in the relation (40). It is easy to see that this mass shift is classical in origin. Equation (52) may be analyzed classically by simply inserting the classical charge density, $\rho(x)=\delta(x)$; the result is precisely the same, namely, Eq. (53). Hence the mass shift near a conducting boundary is indeed a classical phenomenon.

IX. CONCLUSIONS

We hope that in the course of this paper we have illustrated the power of the self-field approach to QED. From a single action we were able to extract all manner of boundary-induced, QED radiative energy shifts, in a theory which is not second quantized. The action W here contains more information still: boundary effects on spontaneous emission and on the Lamb sift, with accompanying long-range Casimir-Polder van der Waals forces (as discussed in Refs. 4 and 6), and more. All this is obtained by simply including the self-field of the particle to begin with, and seeing to it that this self-field obeys the boundary conditions of the surrounding space. It is a remarkably economical approach to the theory of radiative corrections, with the intuitive and appealing classical limit of radiation reaction theory. Nowhere have we used vacuum fluctuations.

The controversy over whether the spin-precession frequency ω_s shifts to $O(\alpha/Rm)$ near a plane conductor appears to have been resolved in the negative due to the cancellations of the effects of $\Delta\mu$ and Δm. The shift in ω_c to this order, however, will have an impact upon future measurements of $g-2$ in Penning tapes, a fact which Brown and his co-workers have analyzed extensively.[9,17]

The self-field approach has been used to give a fully relativistic account of spontaneous emission,[18] and work is in progress to apply it to the Lamb shift[19] and $g-2$. Further research is also forseen in using the self-field method in the areas of Casimir-Polder forces, atoms in blackbody radiation, and phenomenon such as the Unruh effect and Hawking radiation.

ACKNOWLEDGMENTS

One of us (J.P.D.) would like to acknowledge partial support from the Italian Ministry of Foreign Affairs during a stay as a visiting scientist at the International Centre for Theoretical Physics in Trieste, where a portion of this work was carried out.

[1]H. G. B. Casimir, Proc. K. Ned. Akad. Wet., Ser. B **51**, 793 (1948); G. Plunien, B. Müller, and W. Greiner, Phys. Rep. **134**, 87 (1986).

[2]L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon, Oxford, 1960), pp. 368–376; J. Schwinger, L. L. DeRoad, Jr., and K. A. Milton, Ann. Phys. (N.Y.) **115**, 1 (1978); P. W. Milonni, Phys. Rev. A **25**, 1315 (1982).

[3]H. G. B. Casimir and D. Polder, Phys. Rev. **73**, 360 (1948); G. Barton, J. Phys. B **7**, 2134 (1974).

[4]A. O. Barut and J. P. Dowling, Phys. Rev. A **36**, 2550 (1987).

[5]G. Barton, Proc. R. Soc. London, Ser. A **320**, 251 (1970); P. W. Milonni and P. L. Knight, Opt. Commun. **9**, 119 (1973); E. A. Power and T. Thirunamachandran, Phys. Rev. A **25**, 2473 (1982).

[6]A. O. Barut and J. P. Dowling, Phys. Rev. A **36**, 649 (1987).

[7]P. W. Milonni, *Foundations of Radiation Theory and Quantum Electrodynamics*, edited by A. O. Barut (Plenum, New York, 1980), pp. 1 and 22.

[8]H. Dehmelt, Phys. Rev. D **34**, 722 (1986).

[9]G. Barton, Proc. R. Soc. London, Ser. A **320**, 251 (1970); M. Babiker and G. Barton, *ibid.* **326**, 255 (1972); **326**, 277 (1972); G. Barton and H. Grotch, J. Phys. A **10**, 1201 (1977); E. Fischbach and N. Nakagawa, Phys. Rev. D **30**, 2356 (1984).

[10]K. Svozil, Phys. Rev. Lett. **54**, 742 (1985); M. Kreuzer and K. Svozil, Phys. Rev. D **34**, 1429 (1986); M. Kreuzer, J. Phys. A (to be published); see also A. C. Tang, Phys. Rev. D **36**, 2181 (1987) for a discussion of mathematical procedures.

[11]L. S. Brown and G. Gabrielse, Rev. Mod. Phys. **58**, 233 (1986); L. S. Brown *et al.*, Phys. Rev. Lett. **55**, 44 (1985); D. G. Boulware, L. S. Brown, and T. Lee, Phys. Rev. D **32**, 729 (1985).

[12]A. O. Barut and J. Kraus, Found. Phys. **13**, 189 (1983).

[13]A. O. Barut and J. F. van Huele, Phys. Rev. A **32**, 3187 (1985).

[14]A. O. Barut, J. P. Dowling, and J. F. van Huele, Phys. Rev. A **38**, 4405 (1988).

[15]H. Grotch and E. Kazes, Phys. Rev. D **13**, 2851 (1976); Am. J. Phys. **45**, 618 (1977).
[16]G. Barton and N. S. J. Fawcett, Phys. Rep. **170**, 1 (1988).
[17]L. S. Brown, G. Gabrielse, K. Helmerson, and J. Tan, Phys. Rev. A **32**, 3204 (1985); L. S. Brown, K. Helmerson, and J. Tan, *ibid.* **34**, 2638 (1986).
[18]A. O. Barut and Y. I. Salamin, Phys. Rev. A **37**, 2284 (1988).
[19]A. O. Barut and Y. I. Salamin (unpublished).