Super-regenerative laser receiver: Transient dynamics of a laser with an external signal

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In super-regenerative detection, a weak external signal is amplified in an oscillator that is periodically switched on and then quenched. It is shown that very weak optical signals, slightly above the noise level, can be detected effectively by a laser used as a super-regenerative receiver. The detector output for different signal levels and its operating characteristics are determined by numerical integration of stochastic differential equations that represent the growth of laser radiation from a background consisting of signal and noise.

I. INTRODUCTION

The detection of low-level signals is of importance in the theory and practice of optical communications and has motivated much research in recent times. Many of the techniques that were used originally in radio and microwave engineering have proved to be applicable in the optical regime as well. In this paper we examine the method of super-regenerative detection that was used with great success in radar receivers,[1] and its application to optical frequencies. The characteristic feature of a super-regenerative receiver is an oscillator that is periodically switched on and off.

The operation of a super-regenerative laser receiver is easily explained with reference to the schematic arrangement shown in Fig. 1. A ring laser is operated with a Faraday rotator and Q switch in the cavity. The Faraday rotator ensures unidirectional operation, while the Q switch is used to turn the laser on and off periodically in response to an electronic driving signal. In the absence of an external signal, the laser intensity grows from a background of spontaneous emission noise after it is abruptly Q switched at time $t=0$. The solid curve in Fig. 2(a)

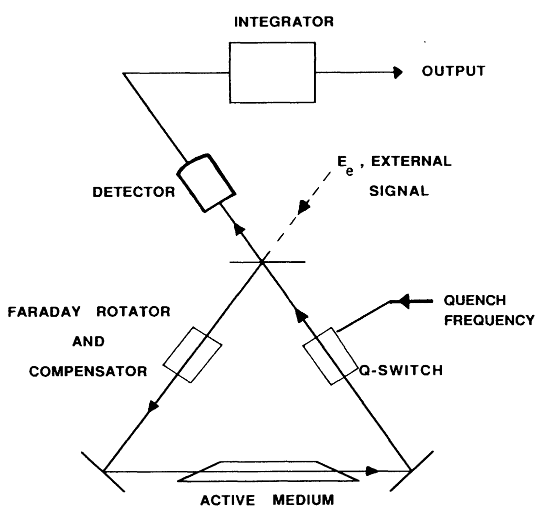

FIG. 1. Schematic arrangement for super-regenerative laser receiver. The integrator determines the area under the time trace of the intensity of the laser as it is periodically switched on and off.

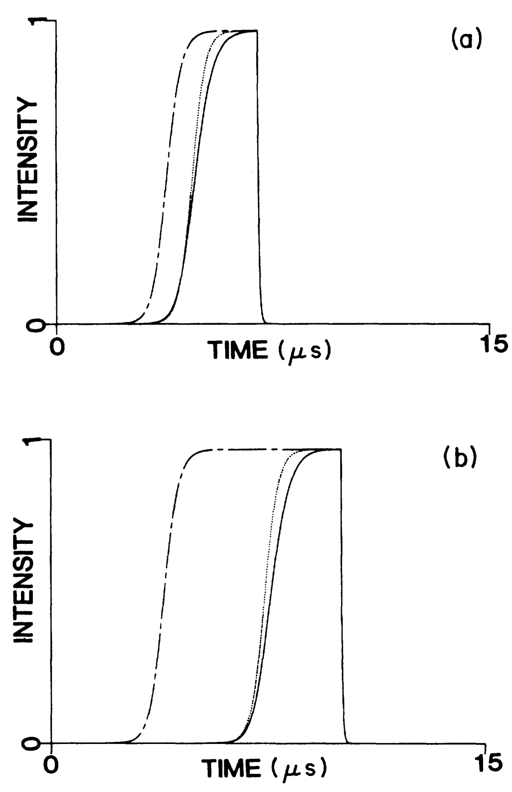

FIG. 2. Growth of the mean intensity with time. Ten thousand trajectories were averaged to obtain this curve. The net gain is $a_0=2.16\times10^6$ s^{-1}; ——, no external signal present; · · · · analytic result of Eq. (6); —·—·—·, external signal ($E_e=10^{-4}$) present. (a) Noise strength $P=4\times10^{-3}$ s^{-1} (dye laser). (b) Noise strength $P=4\times10^{-8}$ s^{-1} (solid-state laser).

shows the time development of the laser intensity. The intensity is seen to grow from a very small value to steady state and then drops rapidly when the laser cavity Q is reduced abruptly by the Q switch. The area under the time trace of the intensity in the absence of the external signal is denoted A_0. When an external signal E_e that is to be detected is coupled into the laser cavity as shown in Fig. 1, the time for growth of the laser intensity is reduced, as shown by the dashed curve in Fig. 2(a). The area A_e under this time trace is greater than A_0; it depends on the magnitude of the external signal. A quantitative measure of the external signal is obtained from the ratio of the area under the two curves A_e/A_0. The ratio of the areas (which we will call the receiver output) depends also on the time at which we choose to quench the laser intensity. Thus the quench frequency is an important parameter that determines the behavior of the super-regenerative receiver. The ability of a super-regenerative receiver to detect very weak signals arises from the repeated amplification of the weak external signal by the active medium in the laser cavity. Our recent experimental and theoretical studies of switching transients in lasers[2] provide the necessary theoretical background for the analysis presented here.

We demonstrate that it should be possible to detect optical signals that are only slightly above the ambient noise level in a laser. This implies that certain types of lasers are better suited to this purpose than others; we will study the dependence of the detection sensitivity on the level of spontaneous emission noise in the active medium. The technique can be highly wavelength selective, eliminating noise in other regions of the spectrum. The response of the super-regenerative receiver can be linear or logarithmic with respect to the external signal, depending on the manner in which the quenching of the oscillator is performed. The conditions for both kinds of behavior will be explored. Section II of this paper describes the basic laser model used. In Sec. III the results of computer simulations of the stochastic laser equations are presented for various parameter regimes. Section IV is a discussion of the results; we consider the limits of detectability of the super-regenerative detector, and the errors involved. The effect of pump noise on the threshold of detection will be discussed.

II. THE LASER MODEL

To analyze the process of super-regenerative detection, we utilize a model of laser operation that has successfully accounted for both transient and steady-state behavior of the laser output. It is assumed that the laser is single mode, and the effect of spontaneous emission is introduced though a Langevin noise source, as is customary.[2,3] The noise term is taken to be δ correlated, a reasonable assumption when the fluctuations due to spontaneous emission occur on a time scale much faster than for macroscopic changes in the intensity. We do include the effect of saturation on the active medium, since we are interested in laser operation at high intensities, far above threshold. We will assume that for the laser under consideration, it is possible to adiabatically eliminate the population inversion and describe the laser dynamics with a single complex equation for the field. Thus, relaxation oscillations are not considered in this treatment. Further, we assume that the external signal and the laser oscillation frequency are resonant. This simplifying assumption can easily be realized experimentally. It is worthwhile pointing out that nonlinear dynamical phenomena of great interest occur when the adiabatic elimination of the inversion and the dipole moment are not possible and when the resonance condition is not satisfied. Many studies of injection locking and chaotic dynamics in lasers have investigated the situation when the injected signal is detuned by different amounts from the laser frequency and relaxation oscillations are not inhibited.[4,5] These phenomena are not our focus in this paper, and so we have chosen to study the simplest possible case. The complex, scaled, dimensionless laser field within the laser cavity in the absence of the external signal is then obtained from the stochastic differential equation[2]

$$dE/dt = -\kappa E + [FE/(1+AI/F)] + q(t) , \quad (1)$$

where

$$\langle q^*(t) q(t') \rangle = 2Q\delta(t-t') . \quad (2)$$

κ is the decay rate of laser radiation in the cavity. It includes the output coupling of the laser light, as well as losses internal to the laser cavity. F is the gain factor for the active medium, and A is the saturation parameter of the active medium.[3] I ($\equiv |E|^2$) is the laser intensity and $q(t)$ is the complex random-force term that represents the noise in the single longitudinal mode. In this paper we have assumed for concreteness the parameter values determined in previous experiments on the transients in a single-mode ring dye laser. The particular laser system used (dye, semiconductor, etc.) will determine the specific values in an actual receiver. The value of the noise strength Q as determined from previous experiments[2] is $\cong 0.004$. The saturation parameter A was found to possess a value of $\cong 2.6\times 10^6\ \mathrm{s}^{-1}$.

In the presence of an external signal, Eq. (1) needs to be modified. This signal is easily included as follows:[6]

$$dE/dt = -(\kappa E - \kappa_e E_e) + [FE/(1+AI/F)] + q(t) , \quad (3)$$

where E_e is the complex external field, and κ_e is the coupling of the laser cavity to the external signal. κ_e is typically of the same order as κ, the total cavity loss, which includes both output coupling and internal losses. In the calculations, we have taken $\kappa = 1.2\times 10^7\ \mathrm{s}^{-1}$ and $\kappa_e = 6\times 10^6\ \mathrm{s}^{-1}$. The conclusions presented here do not depend critically on the values of the parameters.

Equation (3) allows us to investigate the transient growth of the laser field in the presence of the external signal. Let us now make an estimate of the intensity in the laser cavity. If the external field is absent, the average intensity in the cavity with the laser far below threshold may be obtained from a linearized version of Eq. (1), which has been converted to the corresponding equation for the intensity,[7]

$$dI/dt = 2(F-\kappa)I + Q \ . \tag{4}$$

In the steady state,

$$I_{NB} = Q/(2\kappa - 2F) \ , \tag{5}$$

where I_{NB} refers to the intensity due to the noise background. When the laser is operated below threshold, $\kappa > F$. In our analysis, the pumping of the active medium is assumed to remain constant, and the cavity loss is switched from a high to a low value, to turn the laser on. If in the high loss condition we take $2(\kappa_{HL} - F) \cong 10^8 \ s^{-1}$, then $I_{NB} \cong 4\times 10^{-11}$, where we have used $\kappa_{HL} = 7\times 10^7 \ s^{-1}$, $F = 1.4\times 10^7 \ s^{-1}$, and $Q = 4\times 10^{-3} \ s^{-1}$. For given values of the parameters, Eq. (5) thus provides us with a lower limit of the intensity of the external field that would be detectable with a super-regenerative detector. We will verify this estimate in our simulations, as reported in Sec. III.

The analysis of Ref. 2 also allows us to estimate the time scale of growth of the intensity in the absence of the external signal for different operating points of the laser. The mean time for the laser to grow to I_0 ($I_0 \equiv I_{ref}/I_{SS}$), where I_{ref} is a reference intensity and I_{SS} is the steady-state average intensity) is given by

$$\langle t \rangle = (1/2a_0)\{C + \ln(a_0^2/PA) + \ln[I_0/(1-I_0)]\} \tag{6}$$

while the variance of the times of arrival at the reference intensity is

$$\langle \Delta t^2 \rangle = (\pi^2/24a_0^2) \ . \tag{7}$$

Here, $C = 0.5772\ldots$ is Euler's constant and a_0 is the small-signal net gain equal to $(F-\kappa)$. These results are obtained by expansion of the saturation denominator in Eq. (1) to third order in the field, and are valid not too far above threshold. We will compare the estimates from these equations to the results of the numerical simulations in Sec. III.

Pump noise has not been included in the analysis here since it does not affect the mean growth trajectory of the intensity, and since it would vary drastically from system to system. It should be noted that its presence could affect the accuracy with which an external signal would be detected, and it may be necessary to include pump noise for a comparison with experimental results.

III. NUMERICAL SIMULATIONS

The method for numerical integration of the stochastic differential equation (3) is straightforward.[2] The white noise is accounted for by generating Gaussian random numbers with the required mean (zero) and variance ($2Q$) on the computer using the Box-Mueller algorithm. A Taylor series approach is used with a small enough step size to ensure accurate integration. The intensity is allowed to grow, and then switched off. The point at which the laser is switched off determines the quench frequency and gain of the super-regenerative receiver. The average values are determined from a set of ten thousand trajectories with different initial seeds for the random numbers. The area under the trajectories is determined by a simple integration procedure. The ratio A_e/A_0, where A_0 is the area under the intensity trajectory without the external signal and A_e is the area in its presence, is taken as the measure of the detector output.

In Fig. 2, we have shown the growth of the laser intensity for the case of a laser pumped about 20% above threshold, with a value of $a_0 = 2.16\times 10^6 \ s^{-1}$. For Fig. 2(a) we have taken a spontaneous emission noise strength of $P = 4\times 10^{-3} \ s^{-1}$ which is typical for our ring dye laser, while in Fig. 2(b) we have $P = 4\times 10^{-8} \ s^{-1}$, which corresponds to typical values for a titanium-doped sapphire laser.[8] The curves plotted are actually obtained from averaging 10 000 trajectories, each determined by different random numbers. It is seen that in the absence of an external signal the time taken for the laser intensity to reach the steady state is shorter for the dye laser than for the solid-state laser. This is to be expected. The dotted curves are plotted from Eq. (6), the analytic expression for the mean first passage time. Very good agreement is found in this case, for the onset time of the laser oscillation, though as the laser approaches steady state, a slight deviation is noticeable. This is due to the fact that the analytic result assumes a cubic saturation behavior, while the computations account for saturation much more completely.

In the presence of the external signal, the growth occurs at earlier times. In these figures we have taken the strength of the external field to be 10^{-4}. The external signal intensity is a factor of 10^{-8} smaller than the steady-state laser intensity ($I_{SS} \cong 1$) and a hundred times larger than the noise background intensity of the dye laser, estimated in Sec. II. The noise background intensity for the solid-state laser would be five orders of magnitude smaller than for the dye laser.

In Fig. 3 the receiver output defined as $S = A_e/A_0$ is plotted as a function of the strength of the external signal field. These curves were obtained from the averaged trajectories of Fig. 2. The laser intensity was allowed to grow to steady state before being switched off. It is clear that the solid-state laser is capable of detecting a far smaller external field than is the dye laser. Both curves

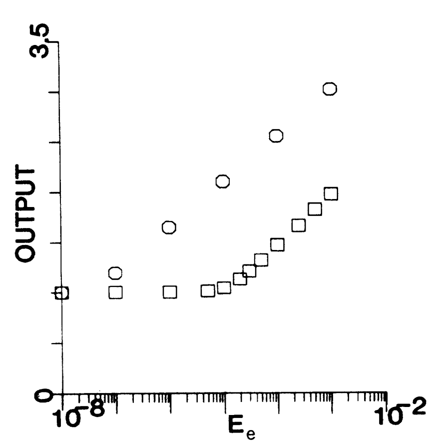

FIG. 3. Receiver output A_e/A_0 as a function of external signal strength. □, dye laser; ○, solid-state laser.

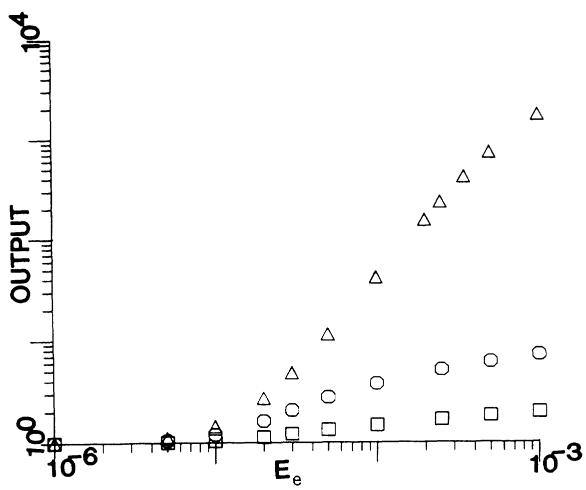

FIG. 4. Output vs external signal for three different quench times. △, 3 μs; ○, 5 μs; □, 7 μs.

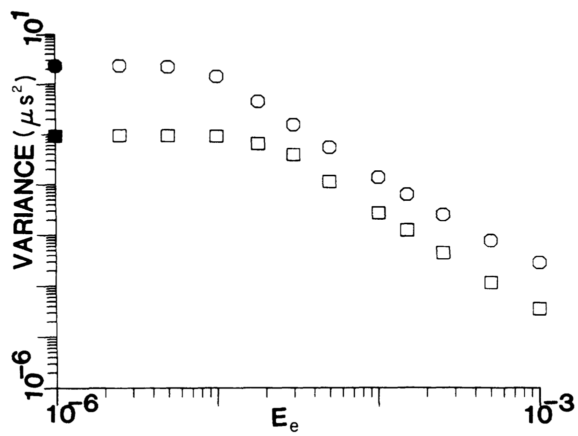

FIG. 6. Dependence of variance of passage time distributions for the laser to reach the half steady-state intensity point. □, $a_0=2.16\times10^6$ s^{-1}; ○, $a_0=4.32\times10^5$ s^{-1}. The closed circle and square are the variances obtained from the analytic result, Eq. (7), in the absence of the external signal.

show the same trend of behavior. For an extremely weak external field, the output value is seen to approach unity, i.e., the receiver is incapable of effective detection in this regime. For larger external signals, still extremely weak compared to the steady-state laser intensity, the output grows to values appreciably greater than unity. After an initially flat portion, the curves become approximately linear. This implies that the output of the super-regenerative receiver is logarithmic in response to the external signal in this mode of operation when the intensity is allowed to reach steady state. The limiting values of the external signals that can be detected are found to agree with the estimates of the noise background intensities for both the laser systems.

What quench frequency should one select for the operation of the super-regenerative receiver? This question is of practical importance, and also determines the variation of the receiver output with respect to the external signal. To find an answer, we varied the quench time for the dye laser. The results are shown in Fig. 4. The

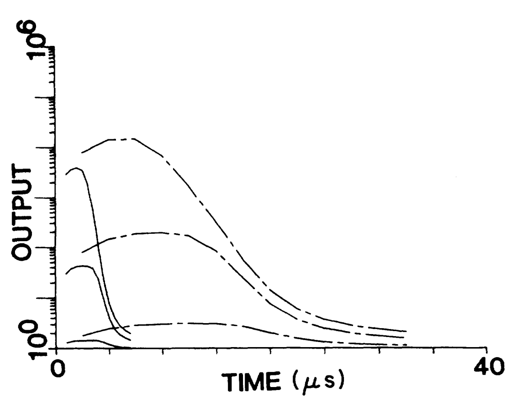

FIG. 5. Output as a function of quench time, for two levels of excitation; ——, $a_0=2.16\times10^6$ s^{-1}; – – – -, $a_0=4.32\times10^5$ s^{-1}. The existence of an optimum quench frequency is evident.

squares correspond to a quench time of 7 μs (the same as for Fig. 3). The circles and triangles are for 5 and 3 μs, respectively. A large gain in the output can be obtained by choosing the quench frequency optimally. The restriction on short quench times is due to the fact that the areas underneath the trajectories becomes very small and difficult to measure accurately if the quench time is too short. There is thus an optimum quench frequency which has to be determined for a given operating point of the laser.

A much more detailed study of the effect of quench time is shown in Fig. 5, for two different operating points of the laser, and three different external signal levels. It is clearly seen that there is an optimal quench frequency, approximately 4 and 10 μs for excitation levels of $a_0=2.16\times10^6$ and 4.32×10^5 s^{-1}. It appears from these plots that the output is increased as the laser is operated closer to threshold. However, the uncertainty in the determination of the areas under the intensity trajectories also increases as we operate closer to threshold.

A measure of the uncertainty in determination of the area is given by the variance of the passage time distribution for the laser to reach half the steady-state average intensity. Figure 6 shows the change in the variance for the two operating points as a function of the external signal level. When the external signal is absent, the variance calculated from Eq. (7) agrees well with the results from the simulations. The variance is seen to decrease sharply with increasing signal level. It is also uniformly higher when the laser is operated near threshold.

IV. DISCUSSION

The analysis of the previous sections demonstrate that it is possible to detect very low-level external signals by operating a laser as a super-regenerative receiver. The characteristics of such a receiver have been described. A logarithmic response is obtained when the laser oscillation is allowed to build up to steady state, while a roughly linear response is obtained if the oscillation is quenched at an earlier stage. The sensitivity of the receiver is very high; signals very close to the noise level for the laser can be detected. This noise level depends, of course, on the spontaneous emission rate for the active medium. Solid-state lasers are proposed as good candidates for super-regenerative receivers, because of their long spontaneous emission lifetimes.

The quench frequency of the receiver is found to be an important parameter that affects the output of the receiver strongly. We have studied the output as a function of the quench frequency and found that there is an optimum frequency for a given level of laser excitation.

It should be possible to test the predictions of this analysis experimentally and to determine if there are other issues that need to be considered. For example, we have not included the effect of laser pump noise in our analysis here, since it does not affect the averaged trajectory of the intensity.[2] However, this source of noise may be of concern in the practical operation of such a receiver where it may influence the accuracy with which the area under the intensity trajectory can be determined. The values of the laser parameters chosen here are mainly for illustration. The coupling of the laser cavity to the external signal would be dependent on mode matching conditions. For lasers in which the adiabatic elimination of the population inversion is not possible, the coupled equations for the field and inversion need to be studied. Finally, it may be of interest to examine the effect of detuning between the external signal and the laser oscillation frequency.

ACKNOWLEDGMENTS

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