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# One-dimensional kinetic Ising model with competing dynamics: Steady-state correlations and relaxation times

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A one-dimensional kinetic Ising model with dynamics characterized by a combination of spins flips at temperature $T$ and spin exchanges at $T=\infty$ is studied. The two-spin correlations in the steady state are calculated exactly and the decay times describing the relaxation of both the magnetization and the two-spin correlations are also given. We find that neither the steady-state nor the dynamic quantities show any sign of a phase transition that could exist in this one-dimensional, nonequilibrium system. Two remarkable features of the solution are that (i) the correlation length in the steady state with random spin exchanges is larger than the correlation length in the corresponding equilibrium state without spin exchanges, and (ii) a fluctuation-dissipation theorem is satisfied in the nonequilibrium steady state.

## I. INTRODUCTION

The Ising model played an important role in the theory equilibrium phase transitions and its kinetic generalizations, such as the one-spin-flip Glauber model[1] or the spin-exchange Kawasaki model,[2,3] were equally instrumental in sorting out the questions about the dynamics of fluctuations near critical points. It was thus natural that the kinetic Ising models were further generalized[4,5] with the aim of modeling phase transitions in far-from-equilibrium steady states. The importance of developing these new models lies in the fact that a general description of far-from-equilibrium steady states is lacking and, as a consequence, nonequilibrium phase transitions are usually described by phenomenological rate equations.

Generalizations of the kinetic Ising models followed two lines in the past few years. One of them[4] was to consider the Kawasaki model as a model of lattice gas at temperature $T$ and to switch on a uniform field $E$ which biases the spin exchanges in the direction of the field. As an effect of the field, a current carrying steady state was produced and, for attractive nearest-neighbor interactions between the particles, an anisotropic segregation type ordering occurred at low enough temperature. The transition between the disordered and ordered phases has been found[4,6,7] to be a mean-field type though there is some evidence[8] for the behavior being nonclassical.

Another way of constructing a model which has a nonequilibrium steady state is to assume that spin-flip and spin-exchange processes take place independently and that the spin flips alone bring the system to equilibrium with a heath bath at temperature $T$ while the spin exchanges try to equilibrate the system with a heath bath at another temperature usually taken to be $T=\infty$ (random exchanges of nearest neighbors).[5] The distinguishing feature of this model is that the current in the steady state is local, it is the current of energy between the two heat baths which are connected to every sites of the lattice. Computer simulations and mean-field-like theories of this system indicate[5,9,10] that the phase transition which is of second order without the spin exchange is altered from second to first order as the rate of spin exchange is increased beyond a critical value.

Since the analytical results for the above models come from mean-field-type theories or from considerations of limits where mean-field-type approximations are valid, the effects of fluctuations have remained largely unexplored. One of the main effects of the fluctuations in equilibrium systems is the inhibition of ordering in low dimensions. Landau-Pierls–type arguments[11] can be used for example to exclude the existence of long-range order in one-dimensional systems with short-range forces. Since we do not have similar arguments for orderings in nonequilibrium steady states and since there are examples[12] of nonequilibrium phase transitions in one dimension, at present one can decide the question of existence of phase transition in a steady state only by solving the problem exactly.

An example where ordering in a nonequilbrium system has been excluded by explicit calculation is the one-dimensional kinetic Ising model[13] in which spin flips at $T=0$ compete with random ($T=\infty$) "bond flips" (flips of all the spins to one side of a randomly chosen site). Identifying domain walls with particles, this model describes diffusion-controlled annihilation in the presence of particle sources and, in the long-time limit, it has a steady state in which particle production is balanced by the annihilation of pairs of particles. For this system one can calculate[13,14] the relaxation of the one- and two-spin correlations as well as their steady-state value and the absence of long-range order can be seen directly.

Below we shall carry out a similar program for the one-dimensional kinetic Ising model in which the competition is between the spin flips and spin exchanges.[5] After introducing the model in Sec. II, we shall show that both one- and two-spin correlation functions satisfy a closed set of linear differential equations (Secs. III and IV) which can be solved and thus the steady-state and relaxational properties can be calculated exactly. For the absence of any sign of critical slowing down in the dynamics and of any sign of nonanalyticity in the steady-state correlations we then conclude that there is no phase transition in the system. For the first sight, this result is not unexpected and a simple reason for it is that ordering would only occur at $T=0$ in the spin-flip model and the random exchanges should just lead to further disordering. This is, however, a misleading argument. As we shall see, the long-range correlations in the steady state are *increased* by the introduction of the random spin-exchange process. Finally, in Sec. V we discuss the fluctuation-dissipation theorem which is shown to be satisfied in the far-from-equilibrium steady state of this model.

## II. THE MODEL

We consider a one-dimensional kinetic Ising model whose state $\{\sigma\}\equiv\{\ldots,\sigma_i,\sigma_{i+1},\ldots\}$ at time $t$ is specified by stochastic Ising variables $\sigma_i(t)=\pm 1$ assigned to lattice sites $i=1,2,\ldots,N$. Periodic-boundary conditions are used thus we have $\sigma_{N+1}=\sigma_1$. The dynamics of the system which consists of spin flips and nearest-neighbor spin exchanges is assumed to be governed by the following master equation for the probability distribution $P(\{\sigma\},t)$:

$$\frac{\partial P(\{\sigma\},t)}{\partial t}=\sum_{i=-\infty}^{\infty}\sum_{\alpha=1}^{2}[w_i^{(\alpha)}(\{\sigma\}_i^{\alpha})P(\{\sigma\}_i^{\alpha},t)-w_i^{(\alpha)}(\{\sigma\})P(\{\sigma\},t)] . \quad (1)$$

Here the state $\{\sigma\}_i^1$ differs from $\{\sigma\}$ by a flipping of the $i$th spin and the flip rate is given by

$$w_i^{(1)}(\sigma)=\frac{1}{2\tau_1}\left[1-\frac{\gamma}{2}\sigma_i(\sigma_{i+1}+\sigma_{i-1})\right] . \quad (2)$$

If no other processes were present ($w_i^{(2)}\equiv 0$) then Eqs. (1) and (2) would define the exactly solvable Glauber model[1] which relaxes to the equilibrium state of the Ising model at temperature $T$ provided $\gamma$ is chosen to be $\gamma=\tanh(2J/kT)$ where $J$ is the strength of the nearest-neighbor interaction. A nonequilbrium situation is now created by introducing random exchanges of spins at nearest-neighbor sites. The process is described by the $\alpha=2$ terms in the master equation where the state denoted by $\{\sigma\}_i^2$ is different from $\{\sigma\}$ by the exchange of spins at sites $i$ and $i+1$. The exchanges are assumed to take place independently of the spin flips and the rate of the process is given by

$$w_i^{(2)}(\sigma)=\frac{1}{2\tau_2}(1-\sigma_i\sigma_{i+1}) . \quad (3)$$

Without the spin-flip process ($w_i^{(1)}\equiv 0$), Eqs. (1) and (3) define the Kawasaki model[2] at temperature $T=\infty$. It is again an exactly solvable model which may be used to describe diffusion in lattice gases as well as surface evolution in deposition processes.[15] When both spin-flip and exchange processes are present we have a nonequilbrium model since the system is in contact with two heath baths which are at temperatures $T$ and $T=\infty$. As we shall see below, this model is also solvable in the sense that the time evolution of the one- and two-spin correlations can be calculated exactly.

In principle, one should proceed now by solving the master equation for an arbitrary initial distribution $P(\{\sigma\},0)$ and then calculate the averages of various physical quantities $A(\{\sigma\})$ through

$$\langle A\rangle=\sum_{\{\sigma\}}A(\{\sigma\})P(\{\sigma\},t) . \quad (4)$$

This can be done rarely[16] and the usual course of action is the derivation and solution of the equations for the averages of physical interest. The quantities we shall be interested in are the one- and two-spin functions, $\langle\sigma_i\rangle$ and $\langle\sigma_i\sigma_j\rangle$, which can indicate if any homogeneous or modulated magnetic order occurs in the system. By restricting ourselves to the above quantities we may, in principle, miss some ordering which can be seen only in higher order correlations. The simplicity of the system under consideration makes such ordering, however, rather unlikely.

The derivation of the equations for $\langle\sigma_i\rangle$ and $\langle\sigma_i\sigma_j\rangle$ is of the same order of complexity as the corresponding calculation for the Glauber model.[1] It does not contain any new technical element thus we shall just present and discuss the results.

## III. TIME EVOLUTION OF THE MAGNETIZATION

Multiplying both sides of Eq. (1) by $\sigma_i$ and summing over all configurations $\{\sigma\}$ we find that the average values of the magnetization at different sites $\langle\sigma_i\rangle$ are related by the following set of differential equations:

$$\frac{\partial\langle\sigma_i\rangle}{\partial t}=-\frac{1}{\tau_1}\left[\langle\sigma_i\rangle-\frac{\gamma}{2}(\langle\sigma_{i+1}\rangle+\langle\sigma_{i-1}\rangle)\right]+\frac{1}{\tau_2}(\langle\sigma_{i+1}\rangle-2\langle\sigma_i\rangle+\langle\sigma_{i-1}\rangle) , \quad (5)$$

where the contributions from the spin flips ($1/\tau_1$ term on the right-hand side) and the spin exchanges ($1/\tau_2$ term) are clearly separated. This equation can be solved by Fourier transformation with the results that (a) for $\gamma\neq 1$, we have $\langle\sigma_i\rangle=0$ in the steady state, (b) all perturbations of the magnetization decay exponentially, and (c) the relaxation times $\tau_q$ of perturbations of wave number $q$ ($q=2\pi n/N$; $n=0,1,\ldots,N-1$) are given as

$$\frac{1}{\tau_q}=\frac{1}{\tau_1}[1-\gamma\cos q+\kappa(1-\cos q)] , \quad (6)$$

where we introduced $\kappa=2\tau_1/\tau_2$. Clearly, the relaxation times show no sign of a finite-temperature phase transition. They are smooth functions of the temperature and critical slowing down occurs only for the homogeneous mode ($q=0$) and only as we approach $T=0$ ($\gamma=1$). Note that the relaxation time of the slowest ($q=0$) mode is equal to that of the Glauber model $\tau_0=\tau_1/(1-\gamma)$. This is a consequence of the facts that the spin exchanges do not affect the total magnetization and that the equation for the time evolution of the total magnetization does not contain higher-order correlations in the case of the Glauber model.

A remarkable feature of Eq. (5) is that by collecting the corresponding terms and introducing a renormalized time constant

$$\frac{1}{\tau'}=\frac{1}{\tau_1}+\frac{2}{\tau_2}=\frac{1}{\tau_1}(1+\kappa)\ , \tag{7}$$

and a renormalized flip-rate parameter

$$\gamma'=\frac{\gamma+2\tau_1/\tau_2}{1+2\tau_1/\tau_2}=\frac{\gamma+\kappa}{1+\kappa}\ , \tag{8}$$

we arrive to the corresponding equation of the single-flip Glauber model:

$$\frac{\partial\langle\sigma_i\rangle}{\partial t}=-\frac{1}{\tau'}\left[\langle\sigma_i\rangle-\frac{\gamma'}{2}(\langle\sigma_{i+1}\rangle+\langle\sigma_{i-1}\rangle)\right]. \tag{9}$$

Thus, apart from the change of timescale (6), the perturbations of magnetization behave as those in the Glauber model in equilibrium with a heath bath at temperature $T'$ determined from $\gamma'=\tanh(2J/kT')$. The surprise here is that since $\gamma'>\gamma$ we have $T'<T$ contrary to the intuition that if the spin flips and the spin exchanges were in equilibrium with heath baths at temperatures $T$ and $T=\infty$, respectively, then the combination of these processes would only lead to an equivalent spin-flip process in equilbrium with a heath bath at temperature $T<T'<\infty$. Before discussing this result, let us examine whether the above picture of an equivalent spin-flip system at a temperature $T'$ would be supported by the calculation of the two-spin correlations.

## IV. TWO-SPIN CORRELATIONS

The equations for $\langle\sigma_i\sigma_j\rangle$ are derived by multiplying both sides of (1) by $\sigma_i\sigma_j$ and summing over all configurations $\{\sigma\}$. After some algebra one finds that $\langle\sigma_i\sigma_j\rangle$ satisfy a closed set of differential equations. For $i\neq j\pm1$ these equations are of the form

$$\frac{\partial\langle\sigma_i\sigma_j\rangle}{\partial t}=-2\left[\frac{1}{\tau_1}+\frac{2}{\tau_2}\right]\langle\sigma_i\sigma_j\rangle+\left[\frac{\gamma}{2\tau_1}+\frac{1}{\tau_2}\right](\langle\sigma_{i+1}\sigma_j\rangle+\langle\sigma_i\sigma_{j+1}\rangle+\langle\sigma_{i-1}\sigma_j\rangle+\langle\sigma_i\sigma_{j-1}\rangle) \tag{10}$$

which can be reduced to the corresponding equations of the Glauber model ($\kappa\to0$) by introducing $\tau'$ and $\gamma'$ defined in Sec. II. The equation for $j=i\pm1$, however, is slightly different

$$\frac{\partial\langle\sigma_i\sigma_{i+1}\rangle}{\partial t}=-2\left[\frac{1}{\tau_1}+\frac{1}{\tau_2}\right]\langle\sigma_i\sigma_{i+1}\rangle+\left[\frac{\gamma}{2\tau_1}+\frac{1}{\tau_2}\right](\langle\sigma_i\sigma_{i+2}\rangle+\langle\sigma_{i-1}\sigma_{i+1}\rangle)+\frac{\gamma}{\tau_1}\ , \tag{11}$$

thus the description of the system in terms of a one-spin-flip model at an effective temperature $T'$ discussed in Sec. II breaks down at the level of two-spin correlations. The corrections introduced by Eq. (11), however, are rather trivial as we shall see now by calculating the steady-state correlations.

Since there is no process in the system which would reinforce any inhomogeneous fluctuation, the steady-state and the longest-living perturbations are expected to be translationally invariant. Thus we shall simplify our task of solving Eqs. (10) and (11) by assuming that the initial distribution $P(\{\sigma\},0)$ is translationally invariant. Then $\langle\sigma_i\sigma_j\rangle$ depends only on $i-j$ at all times. Introducing $r_n=\langle\sigma_i\sigma_{i+n}\rangle$ we can rewrite Eqs. (10) and (11) as follows:

$$\tau'\dot r_n=-2r_n+\gamma'(r_{n-1}+r_{n+1})\ (n\geq2)\ , \tag{12}$$

$$\tau'\dot r_1=-\frac{2+\kappa}{1+\kappa}r_1+\gamma' r_2+\gamma'-\frac{\kappa}{1+\kappa}\ . \tag{13}$$

The stationary solution $\bar r_n$ is now found by setting the time derivatives to zero and making the following ansatz:

$$\bar r_n=\bar r_1\eta^{n-1}\ . \tag{14}$$

All the $n\geq2$ equations are satisfied by choosing $\eta$ to be the nearest-neighbor correlation of the Ising model at temperature $T'$

$$\eta=\frac{1}{\gamma'}[1-(1-\gamma'^2)^{1/2}] \tag{15}$$

and the remaining equation (13) is used to determine the value of $\bar r_1$:

$$\bar r_1=\frac{(1+\kappa)\gamma'-\kappa}{1+(1+\kappa)(1-\gamma'^2)^{1/2}}=\frac{\gamma}{1+[1-\gamma^2+2\kappa(1-\gamma)]^{1/2}}\ . \tag{16}$$

It follows from Eqs. (14) and (15) that the steady-state correlations decay with distance exponentially

$$\bar r_n=\frac{\bar r_1}{\eta}\exp(-n/\xi)\ , \tag{17}$$

just as it happens in the equilibrium Ising model $\bar r_n^{(\mathrm{eq})}=\exp(-n/\xi)$ and the correlation length

$$\xi=-1/\ln\eta \tag{18}$$

is equal to that of the Ising model at temperature $T'$. Since $T'<T$ and consequently $\xi(T')>\xi(T)$ we arrive at the result that the distant spins are correlated more in the steady state of the system with random spin exchanges than in the equilibrium system without the exchanges.

At small distances, of course, the factor in front of the exponential in (17) takes over and the effect of the random exchanges is to make the correlations weaker than in the corresponding equilibrium system (note, for example, that $\bar{r}_1 < \bar{r}_1^{(\mathrm{eq})}$).

The weakening of the correlations at short distances is easily understood. The spin flips alone align the spins within domains of size $\xi(T)$ and the direction of the magnetization of the domains is randomly up or down. As a result of spin exchanges, the spins of neighboring domains are mixed. If the magnetizations of the neighboring domains are in the same direction then there is no effect but if the magnetizations are in the opposite direction then the process clearly decreases the order.

It is harder to explain the increase of order at larger distances since it is a consequence of the fact that the spin exchanges conserve the magnetization. In order to see the effect of the conservation law let us assume that a magnetization fluctuation occurs on a scale which is larger than the domain size $\xi(T)$ in equilibrium. If only the spin flips would be present, this fluctuation would decay locally. The spin exchanges, however, spread this fluctuation by diffusion and thus create correlations at distances larger than $\xi(T)$. The effect is clearly enhanced if the rate of exchanges is increased and indeed we can see from Eqs. (8), (15), and (18) that the correlation length increases with the increase of the ratio $\kappa = 2\tau_1/\tau_2$. The above argument may seem strange if we think in terms kinetic Ising models which relax to equilibrium since the equilibrium correlation length in those models is independent of the dynamics. We should have in mind, however, that we are considering a nonequilbrium steady state and this state may, and indeed does, depend on the details of the dynamics such as the conservation laws satisfied by some of the underlying processes.

In closing the discussion of the steady-state properties, we note that despite the increase in long-distance correlations, the correlation length diverges only at $T=0$ as can be seen from Eqs. (8), (15), and (18). Thus the two-spin correlations in the steady state do not indicate the presence of a finite-temperature ordering in the system.

We turn now to the study of the dynamics of the two-spin correlations. The decay times of homogeneous perturbations are obtained by seeking the solution of Eqs. (12) and (13) in the form

$$r_n = \bar{r}_n + Q_n \exp(-\lambda t/\tau') \ . \tag{19}$$

The solvability condition of the set of linear equations for $Q_n$ resulting from substituting (19) into (12) and (13) yields the possible values of $\lambda$ as

$$\lambda = 2 - \gamma' \lambda' \tag{20}$$

where $\lambda'$ is given by the eigenvalues of the following $(N-1)\times(N-1)$ tridiagonal matrix

$$\hat{Q} = \begin{vmatrix} a & 1 & & & & \\ 1 & 0 & 1 & & & \\ & 1 & 0 & & & \\ & & & \ddots & & \\ & & & & 0 & 1 \\ & & & & 1 & a \end{vmatrix} \ , \quad . \tag{21}$$

with $a = \kappa/(\gamma+\kappa)$. Following standard methods of matrix diagonalization[17] one finds now that the eigenvalues of this matrix can be written as

$$\lambda' = -2\cos\theta \tag{22}$$

and the possible values of $\theta$ are determined from the following two transcendental equations:

$$a = -\frac{\sin(\frac{1}{2}N\theta)}{\sin[\frac{1}{2}(N-2)\theta]} \ , \tag{23}$$

$$a = -\frac{\cos(\frac{1}{2}N\theta)}{\cos[\frac{1}{2}(N-2)\theta]} \ . \tag{24}$$

It can be easily shown that for $a<1$ [which is the case since $a = \kappa/(\gamma+\kappa)$] Eqs. (23) and (24) have $N-1$ distinct solutions in the interval $0<\theta<\pi$. Thus by solving these equations we find all the eigenvalues of $\hat{Q}$. It follows then from (22) that $-2<\lambda'<2$ and consequently Eq. (20) may be used to give a lower limit $\lambda \ge 2(1-\gamma')$ for $\lambda$. This inequality in turn yields an upper limit for the relaxation times, of two-spin correlations

$$\tau = \frac{\tau'}{\lambda} \le \frac{\tau'}{2(1-\gamma')} \ . \tag{25}$$

Since $\gamma'$ approaches 1 only as $T\to 0$ we arrive to one of our results, namely that no critical slowing down occurs in the dynamics of two-spin correlations at finite temperature.

## V. FLUCTUATION-DISSIPATION THEOREM

The fluctuation-dissipation theorem relates the correlation function of fluctuations to the corresponding susceptibilities (Green's functions).[11] This theorem can be used effectively for simplifying the studies of fluctuations by diagrammatic expansions[18] thus it is an important question whether the theorem holds or not for a far-from-equilibrium steady state. There are examples where the theorem can be shown to be valid[18,19] as well as examples where it seems to break down.[20] Below we shall add the model considered in this paper to the list of nonequilibrium systems where the theorem is valid by showing that the steady-state fluctuations of the magnetization are related to the zero-field susceptibility of the system in the manner required by the fluctuation-dissipation theorem. More precisely, introducing the average magnetization as

$$m = \frac{1}{N}\langle M\rangle = \frac{1}{N}\sum_i \langle \sigma_i \rangle \tag{26}$$

and defining the susceptibility $\chi$ in the limit of vanishing external field $h = H/kT \to 0$ through

$$m = \chi h \ , \tag{27}$$

we show that the following relation is valid in the steady state:

$$\chi = \frac{1}{N}(\langle M^2\rangle - \langle M\rangle^2) \ . \tag{28}$$

The calculation of the right-hand side of (28) is rather simple since $\langle M\rangle = 0$ in the steady state and $\langle M^2\rangle$ can be expressed through the two-spin correlations

$$\frac{1}{N}\langle M^2\rangle = 1+2\sum_{n=1}^{\infty}\bar{r}_n = 1+\frac{2\bar{r}_1}{1-\eta} . \tag{29}$$

In order to find the susceptibility we need the steady-state properties in the presence of an infinitesimal magnetic field $H$. Thus we have to return to the master equation and examine the changes made by such a field. The rate of spin exchanges can be assumed to be unaffected since the change of energy caused by an exchange is independent of the magnetic field. The rate of spin flips, however, should be modified since it becomes energetically advantageous for the spins to align with the field. One has some freedom in choosing the appropriate flip rate which obeys the detailed balance condition at temperature $T$ and in the presence of a field $H$. We shall use a simple form suggested by Glauber[1]

$$w_i^{(1)}(\sigma) = \frac{1}{2\tau_1}\left[1-\frac{\gamma}{2}\sigma_i(\sigma_{i+1}+\sigma_{i-1})\right](1-h\sigma_i) . \tag{30}$$

Note that this form is valid only in the limit $h \to 0$. For finite field $h$ must be replaced by $\tanh h$.

Once the rates of the processes are specified in Eq. (1), it is straightforward to derive an equation for the average magnetization

$$\tau_1 \dot{m} = -(1-\gamma)m + (1-\gamma r_1)h . \tag{31}$$

The steady-state value of $m$ in the limit of $h \to 0$ is then found by setting $\dot{m}=0$ and replacing $r_1$ with $\bar{r}_1$ calculated for the system without the field. As a result, the susceptibility is obtained in the form

$$\chi = \frac{m}{h} = \frac{1-\gamma\bar{r}_1}{1-\gamma} . \tag{32}$$

Using Eqs. (8), (15), and (16), this expression can now be shown to be equal to $\langle M^2\rangle/N$ as given by (29), thus we have completed the proof of the fluctuation-dissipation theorem (28). One point, however, remains to be discussed. The simple form of the theorem (28) is valid since we defined the susceptibility through Eq. (27) with the reduced field $h = H/kT$ which is appropriate for an equilibrium system at the "spin-flip" temperature $T$. As we saw in Secs. III and IV, the steady state of our system is more like that of an Ising model at an effective temperature $T'$. Thus one might say that a more appropriate reduced field would be $h = H/kT'$ and then the simple form of the fluctuation-dissipation theorem would be complicated by the introduction of an extra factor $T/T'$. We do not see how to resolve this ambiguity. It seems that this type of lack of uniqueness will always exist in nonequilibrium systems where the temperature is not well defined.

In summary, we have seen a one-dimensional, analytically tractable model which exhibits a nonequilibrium steady state. Although the system is too simple to show phase transition in one dimension, it nevertheless displays some remarkable properties. One of them is the increase of correlations in the steady state due to random exchanges and another one is the validity of a fluctuation dissipation theorem in the steady state. We believe that this type of model deserves further attention, and hope that its study will contribute to our understanding of the physics of far-from-equilibrium systems.

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