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Phase sensitivity in atom-field interaction via coherent superposition

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Considering a system consisting of a two-level atom, initially prepared in a coherent superposition of upper and lower levels, interacting with a coherent state of the field, we show that the dynamics of the atom as well as the spectrum of the field are sensitive to the relative phase between the atomic dipole and the cavity field. It is shown that, for a certain choice of this phase, "coherent trapping" occurs in two-level atoms. In the case of spectra, for the same choice of the phase, instead of a three-peaked symmetric spectrum, we have an asymmetric two-peaked spectrum.

I. INTRODUCTION

Linear superposition is one of the fundamental principles of quantum mechanics. Some very interesting phenomena have either been predicted or observed to arise due to this principle. Examples include quantum beats and interference of quantum-mechanical amplitudes. The idea of preparing the atomic system in a coherent superposition of states has become quite popular, particularly because of its applications to noise quenching by correlated spontaneous emission,[1] quantum beats,[2] and noise-free amplification.[3]

Recent studies have also been directed towards the possibility of observing the quantum-mechanical features in macroscopic systems. The observation of macroscopic superposition of quantum states is difficult because of the "collapse" of the wave function. This is evidently due to the influence of the environment in the form of dissipation on the system. Suggestions have, however, been made to modify the environment by use of a "squeezed bath" instead of a thermal bath to model the dissipation, i.e., every single mode of the field with which the system might interact is assumed to be squeezed. It has been shown in Ref. 4 that a macroscopic superposition of coherent states is preserved when the bath is such a "squeezed vacuum."

In this paper we consider a two-level atom, initially prepared in a coherent superposition of the upper and lower levels, interacting with a single mode coherent state of the field in an ideal cavity. We show that the population inversion and the spectrum of the field exhibit a phase sensitivity and undergo dramatic changes with the change in the relative phase between the atomic dipole and the coherent field. In particular, we show that for a certain choice of the relative phase, the population inversion essentially remains unaffected. The results are quite surprising in view of the general belief that coherent trapping does not occur in two-level atoms.[5] In the case of the spectrum, for the same choice of the phase of the dipole, we have an asymmetric two-peaked spectrum instead of the symmetrically placed three peaks. All these effects arise due to the phase sensitivity of the system which manifests itself in the macroscopic observables and will be discussed in subsequent sections. Such a phase sensitive system has also been proposed in Ref. 6 in which a stream of atoms in a coherent superposition of states is injected into a maser cavity. It is shown in that paper that the upper level probability of the atoms leaving the cavity depends on the relative phase angle between the atoms and the field. Measurement of the excitation probability, therefore, is a tool to probe the coherence produced in the field by the atoms. With the recent advances in extremely cold, high Q cavities, it is now possible to perform such single atom experiments.[7]

It should be mentioned here that a similar system, i.e., the two-level Jaynes-Cummings model (JCM), with the atom in a coherent superposition of the two states, interacting with "squeezed vacuum," was considered in Ref. 8. Contrary to the expectations, this model does not have a phase sensitivity which is because of a lack of coherent coupling between the one-photon transition and the "two-photon" squeezed state. It was shown in that paper that under such conditions, the results are similar to those for a thermal field. Such a similarity was also pointed out in Ref. 9.

It is also worthwhile to mention that the spontaneous decay of an atom,[10] and the fluorescence spectrum of an atom in the squeezed vacuum environment,[11] also exhibit a dependence on the phase of the squeezed field. Once again, some very interesting and novel effects occur due to a change in the phase.

The present paper is organized as follows. In Sec. II we define the model and calculate the population inversion. Numerical results are presented for various conditions of the relative phase. In Sec. III we calculate the spectrum and finally, Sec. IV contains a discussion of our results and conclusions.

II. POPULATION INVERSION

We consider the system of a single two-level atom interacting with a single mode quantized radiation field inside an ideal cavity. We consider the atom to be initially in a coherent superposition of the excited and the ground states, i.e.,

$$|\psi\rangle_{\text{atom}}=\cos(\theta/2)|a\rangle+\sin(\theta/2)e^{-i\phi}|b\rangle\ , \tag{1}$$

where $|a\rangle$ and $|b\rangle$ are the upper and lower levels, respec-

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tively. The Hamiltonian for the system, in the rotating wave approximation, is

$$H=\tfrac{1}{2}\hbar\omega_0\sigma_3+\hbar\omega_0(a^\dagger a+\tfrac{1}{2})+\hbar g(a^\dagger\sigma_-+\sigma_+a)\ . \qquad (2)$$

Here ω_0 is the frequency of the cavity eigenmode which we have taken to be resonant with the atomic transition frequency, $a^\dagger$ and a are the usual photon creation and annihilation operators, and σ_+ and σ_- are the atomic flipping operators given by

$$\sigma_+=|a\rangle\langle b|,\quad \sigma_-=|b\rangle\langle a|\ , \qquad (3)$$

and g is the atom-field coupling constant.

The Heisenberg operator solutions for the Jaynes-Cummings model were given in Ref. 12. We shall use them to calculate the population inversion. The operator $\sigma_-(t)$ is given by

$$\sigma_-(t)=e^{i\omega_0 t}e^{iCt}\left[\left[\cos\gamma t+iC\frac{\sin\gamma t}{\gamma}\right]\times\sigma_-(0)-ig\frac{\sin\gamma t}{\gamma}a(0)\right]\ , \qquad (4)$$

where C and γ are operators defined as

$$C=g(a^\dagger\sigma_-+\sigma_+a)\ , \qquad (5)$$

$$\gamma=(C^2+g^2)^{1/2}\ . \qquad (6)$$

For a general initial state of the system,

$$|\psi\rangle_{A-F}=\sum_{n=0}^{\infty}\langle n|\alpha\rangle[\cos(\theta/2)|n,a\rangle+\sin(\theta/2)e^{-i\phi}|n,b\rangle]\ , \qquad (7)$$

the upper level probability can be calculated in a straightforward manner, which is

$$\langle\sigma_+\sigma_-\rangle_t=\sum_{n=0}^{\infty}|\langle\alpha|n\rangle\cos(\theta/2)\cos(g\sqrt{n+1}t)+i\langle\alpha|n+1\rangle\sin(\theta/2)e^{i\phi}\times\sin(g\sqrt{n+1}t)|^2\ . \qquad (8)$$

For the coherent state $|\alpha\rangle$ with $\alpha=|\alpha|e^{i\psi}$ and $|\alpha|^2=\bar{n}$, where $\bar{n}$ is the mean number of photons in the coherent state, the population inversion is given by

$$W(t)=2\sum_{n=0}^{\infty}\frac{\bar{n}^{\,n}e^{-\bar{n}}}{n!}\left[\cos^2(\theta/2)\cos^2(\mu_n t)+\frac{\bar{n}}{(n+1)}\sin^2(\theta/2)\sin^2(\mu_n t)+\left[\frac{\bar{n}}{n+1}\right]^{1/2}\sin(\theta/2)\cos(\theta/2)\times\sin(\psi-\phi)\sin(2\mu_n t)\right]-1\ , \qquad (9)$$

with $\mu_n=g\sqrt{n+1}$. In the case of incoherent excitation, i.e., for $\theta=0$, Eq. (9) reduces to the well-known results obtained in Ref. 12 (Fig. 1). The third term in the above equation is the "interference" term and depends on the relative phase between the dipole and the coherent state. In Figs. 2 and 3 we have plotted the population inversion for two different values of the relative phase. Note that for $\psi-\phi=0$, the amplitude of the oscillations becomes extremely small. This is also apparent from Eq. (9) wherein, for large value of $\bar{n}$, the Poissonian function peaks sharply around $\bar{n}$. The first and the second terms add up to 1 and the population inversion essentially remains unaffected. Indeed in a semiclassical limit, the atom does not "see" any field. For $\psi-\phi=\pi/2$, the dynamics of the atom is the same as that for the initially excited case.

A possible explanation for such behavior can be as follows. Whereas, in the case of three-level atoms, the coherent trapping can be explained semiclassically, as the destructive interference between the two transitions, i.e., a dipole-dipole destructive interference, in the case of two-level atoms, the atomic dipole interferes destructively with the cavity eigenmode. This inhibits the transition between the two levels. The trapping phenomenon can

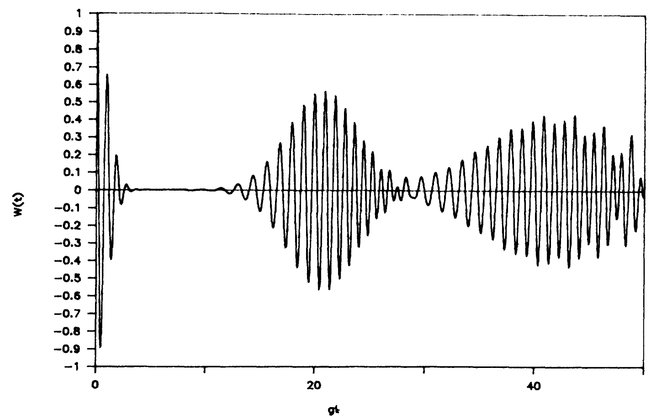

FIG. 1. Plot of population inversion against a dimensionless time gt when the atom is initially in an excited state. The initial state of the field is Poissonian with $\bar{n}=10$.

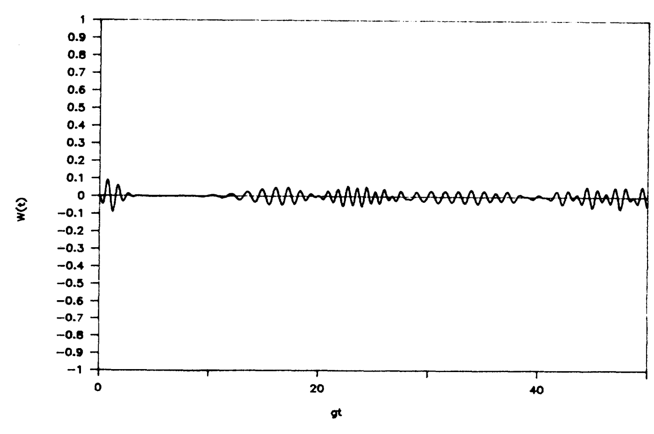

FIG. 2. Plot of population inversion when the atom is initially prepared in a coherent superposition of the two states. The amplitudes of the two states are equal, i.e., $\theta=\pi/2$ and the relative phase $\psi-\phi=0$.

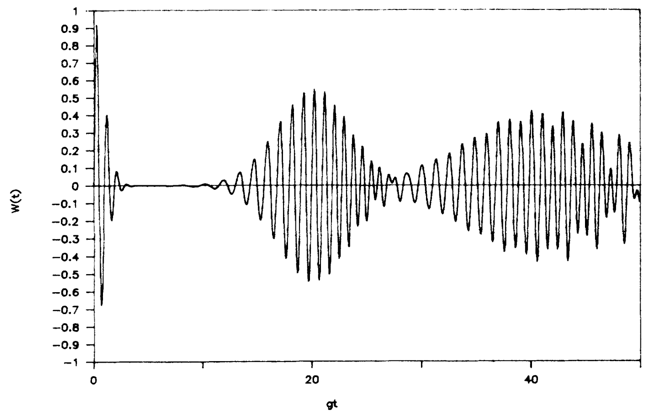

FIG. 3. Same as Fig. 2, except for $\psi-\phi=\pi/2$.

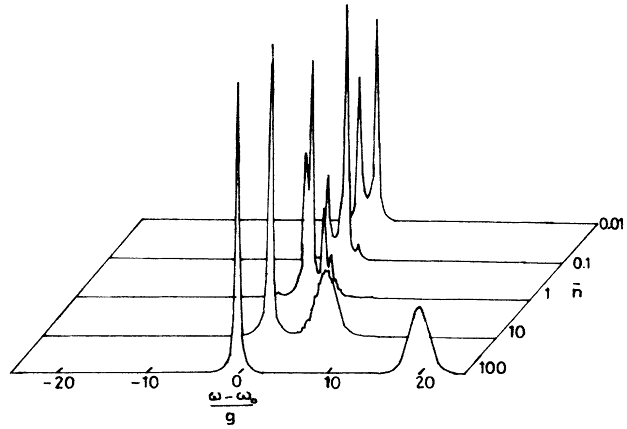

FIG. 4. Spectrum of the field for $\theta=\pi/2$ and $\psi-\phi=0$. Interaction time $T=20g$ and the detectors response time $\Gamma=0.2g$.

also be visualized in the Bloch vector representation.[13] The Bloch equation, describing the interaction of a two-level system with nearly resonant electromagnetic field, in the absence of relaxation is

$$\frac{d\mathbf{R}}{dt}=\mathbf{\Omega}_R\times\mathbf{R}\ , \tag{10}$$

where $\mathbf{R}=(R_1,R_2,R_3)$ is the Bloch vector and $\Omega_R=(-p\cdot\epsilon/\hbar,0,\Delta)$ is the field vector. The components of the Bloch vector are related to the density matrix elements by

$$R_1=\rho_{12}e^{i\omega t}+\rho_{21}e^{-i\omega t}\ , \tag{11a}$$

$$R_2=i(\rho_{12}e^{i\omega t}-\rho_{21}e^{-i\omega t})\ , \tag{11b}$$

$$R_3=\rho_{11}-\rho_{22}\ , \tag{11c}$$

and p and Δ are atomic dipole matrix element and atom-field detuning, respectively. Generally, the vector $\mathbf{R}$ precesses about $\mathbf{\Omega}_R$ in a cone. The motion of $\mathbf{R}$ is largest when $\mathbf{R}$ and $\mathbf{\Omega}_R$ are orthogonal. However, if the atomic system is initially prepared in such a way that $\mathbf{R}$ and $\mathbf{\Omega}_R$ are parallel (or antiparallel), $\mathbf{R}$ remains stationary. Under these conditions, the atom and field are obviously decoupled. It has been pointed out that such configurations correspond to the dressed states of the atom-field system.[14]

III. PHASE DEPENDENCE OF THE SPECTRUM

It is obvious that the emission characteristics of the coherently excited atoms would be different from those for the incoherently excited atoms. As a first consequence, one would expect the spectrum to be dependent on the phase of the dipole. We now proceed to investigate the effect of the relative phase on the emission spectrum. The main difficulty with the ideal cavity treatment is obvious; in the absence of losses, the emitted radiation is not stationary. To this end, the "physical spectrum" of Eberly and Wódkiewicz[15] can be used. Moreover, recent studies show that for small values of the leakage parameter in nonideal cavities, the spectrum of the transmitted light and the cavity electrodynamics are very close to those for the ideal cavity.[16]

The emission spectrum is given by the Fourier transform of the dipole-dipole correlation function, weighted by the detector response function[17]

$$\langle\psi|\sigma_+(t_1)\sigma_-(t_2)|\psi\rangle\ , \tag{12}$$

where $|\psi\rangle$ is the initial state of the atom-field system given by Eq. (7). The transient spectrum is given by the expression

$$\begin{aligned}S(\omega)=2\Gamma\int_0^T dt_1\int_0^T dt_2 \exp[-(\Gamma-i\omega)(T-t_1)\\ -(\Gamma+i\omega)(T-t_2)]\\ \times\langle\psi|\sigma_+(t_1)\sigma_-(t_2)|\psi\rangle\ .\end{aligned} \tag{13}$$

Here T is the interaction time and $1/\Gamma$ is the detector's response time. The correlation function (12) has been calculated in Ref. 8. We will use those results in Eq. (13). Using the notation of that paper, with minor modifications,

$$\begin{aligned}\langle\psi|\sigma_+(t_1)\sigma_-(t_2)|\psi\rangle=\sum_{n=0}^{\infty}\exp[i\omega_0(t_1-t_2)]\cos[\mu_n'(t_1-t_2)]\\ \times[\cos(\theta/2)\cos(\mu_n t_1)\langle\alpha|n\rangle+\sin(\theta/2)\sin(\mu_n t_1)e^{i\phi}\langle\alpha|n+1\rangle]\\ \times[\cos(\theta/2)\cos(\mu_n t_2)\langle n|\alpha\rangle+\sin(\theta/2)\sin(\mu_n t_2)e^{-i\phi}\langle n+1|\alpha\rangle]\ .\end{aligned} \tag{14}$$

with $\mu_n=g\sqrt{n+1}$ and $\mu_n'=g\sqrt{n}$. On inserting Eq. (14) in Eq. (13) and performing the integrations, we obtain

$$S(\omega)=\frac{\Gamma}{4}\sum_{n=0}^{\infty}|[F(\mu_n,\mu_n')+F(-\mu_n,\mu_n')][\cos(\theta/2)\langle n|\alpha\rangle+\sin(\theta/2)e^{-i\phi}\langle n+1|\alpha\rangle]|^2+(\mu_n'\rightarrow-\mu_n')\ , \tag{15}$$

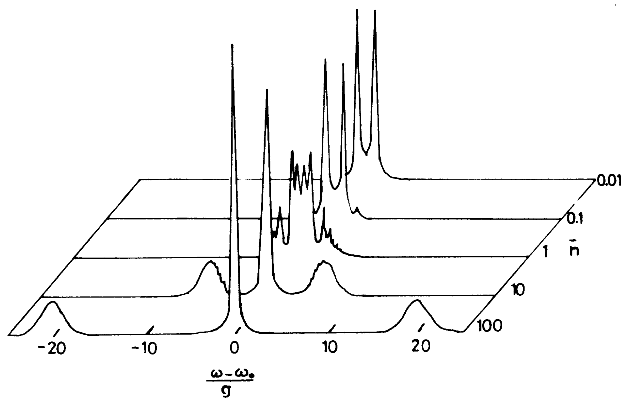

FIG. 5. All parameters same as Fig. 4 except $\psi-\phi=\pi/2$.

with the function $F(\mu_n,\mu_n')$ defined as

$$F(\mu_n,\mu_n')=\frac{\exp[i(\mu_n+\mu_n'-\omega+\omega_0)T]-\exp(-\Gamma T)}{\Gamma+i(\mu_n+\mu_n'-\omega+\omega_0)} . \tag{16}$$

Equation (15) readily reduces to the special case of Ref. 17 for the atom in the excited state initially ($\theta=0$). For $\theta=\pi/2$, the interference terms are also present in Eq. (15), whose phase is the relative phase between the dipole and the cavity field.

Figures 4 and 5 show the spectrum for two different phases. Of particular interest is Fig. 4 ($\psi-\phi=0$). Here the two-peaked vacuum Rabi splitting becomes asymmetric. This asymmetry persists as the mean number of photons in the initial Poissonian state is increased. Comparison with Figs. 5 and 6 reveals that the most striking effect of superposition and phase change appears for larger values of mean number of photons. The asymmetric fluorescence peak develops into a single sideband at a distance $2g(\bar{n})^{1/2}$ away from the central peak at ω_0 and the emission in the other sideband is inhibited. Note that the intensity of this sideband is also doubled as compared to the one for an initially excited atom or for a relative phase of $\pi/2$. For the choice of the relative phase $\psi-\phi=\pi$, the spectrum is just the reverse of Fig. 4 and the sideband appears on the other side of the central peak.

The emission characteristics in Figs. 5 and 6 are similar, which is to be expected from the dynamics of the atom under the same conditions. The only difference between the two spectra is lower intensity for a lesser value of $\bar{n}$ in Fig. 5. This is because of the lesser excitation energy available as compared to the case of the initially excited state of the atom.

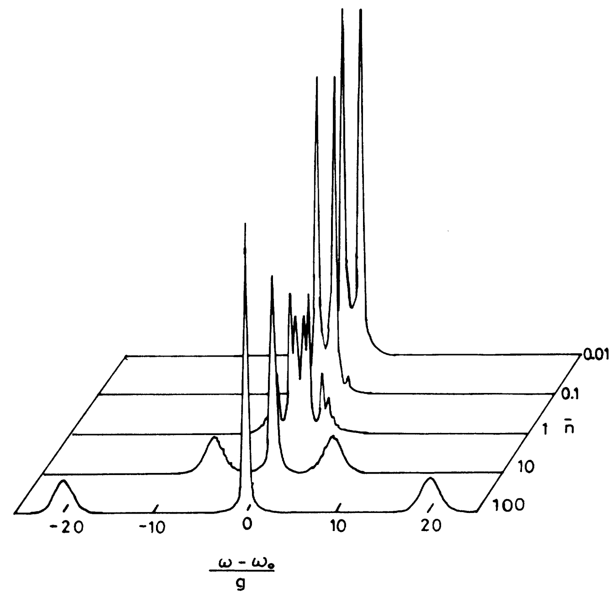

FIG. 6. Emission spectrum of the cavity field for atom initially in excited state ($\theta=0$). All other parameters same as Fig. 4.

IV. CONCLUSION

In conclusion, the dynamics of the atom, i.e., the population inversion, as well as the emission characteristics become phase sensitive when the atom is initially prepared in a coherent superposition of the upper and lower levels in the two-level JCM interacting with a coherent field. For a particular choice of the phase and in a semiclassical limit, "coherent trapping" occurs in two-level atoms. This can be interpreted as the result of a destructive interference between the dipole wave and the cavity eigenmode.

The microscopic superposition also manifests itself in the emission spectrum, which again exhibits a phase sensitivity. In particular, unlike the ordinary case, the two-peaked vacuum Rabi splitting does not evolve into the three-peaked spectrum for higher values of mean number of photons. Instead, one of the side modes is completely quenched and the other is doubled in intensity.

ACKNOWLEDGMENT

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