Phase-sensitive population decay: The two-atom Dicke model in a broadband squeezed vacuum

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We show that atoms interacting with a squeezed radiation field reservoir can exhibit phase-dependent population decay and relax into a highly correlated pure state, in striking difference to decay in a heat bath. We use a Heisenberg operator treatment of radiative transitions in a broadband squeezed light field described as a quantum white-noise field with correlated fluctuations to describe the modified decay of a single atom, and of two atoms cooperatively decaying in a squeezed vacuum. We compare this decay with that generated by a broadband thermal field which lacks the essential field-mode correlations characteristic of squeezing.

I. INTRODUCTION

Squeezed states are states of the radiation field with phase-dependent noise, i.e., with an amount of noise in one quadrature which is below the quantum vacuum limit[1] (this, of course, is at the expense of an increased noise in the conjugate quadrature of the field). In the two-mode squeezed state this is realized by a strong correlation between the modes, so that squeezing properties manifest themselves only in the expectation value of field operators acting on both modes.[2] On the other hand the expectation value of field operators acting only on one of the two modes show thermal-like features. For this reason the two-mode squeezed state is a thermofield representation of thermal statistics.[3] A broadband squeezed vacuum can be used to describe a reservoir with a phase-sensitive white noise.[4] A similar reservoir contains strong internal correlations, a feature completely absent in a heat bath at finite temperature. The interaction between a single atom and a broadband squeezed vacuum has been studied previously, concentrating on the Lamb shift and the decay of population and polarization of a two-level atom in a squeezed reservoir and on modified probe absorption spectroscopy in a broadband squeezed vacuum.[5]

In this paper we study the dynamics of two atoms interacting with broadband squeezed light. We obtain a phase-dependent decay of the atomic *population* near resonance, and find the quadrature phase dipole operators decay at different rates because their decay is stimulated by the different amount of noise contained in the two quadratures of the field. The population decay is increased by the stimulated emission due to the nonzero number of photons present in the reservoir, but for a single atom does not show phase sensitivity. As far as the population is concerned, the single two-level atom decay is analogous to a decay in a heat bath at finite temperature, due to the single photon nature of the process. With a single two-level atom it is impossible to change the population by two-photon absorption or emission. This is not the case in the interaction between a two-atom Dicke system and the field.[6] We show that this system exhibits phase-sensitive population decay when interacting with a squeezed reservoir, a feature completely absent in the single two-level atom decay. This is due to the fact that two-photon process are now possible, due to the cross correlation between population and polarization between the two atoms. Another very interesting feature of this system is that the final equilibrium atomic state is a highly correlated pure state in which the two atoms are both excited or both in their ground state. These states are known as two-atom squeezed states or fermionic thermofields.[7] They have properties similar to the two-mode squeezed state, in that the expectation value of single-atom operators shows single-atom thermal statistics, but the expectation value of product of operators acting on both atoms shows the strong correlation existing between the two atoms. The final equilibrium atomic state is far from being a state of thermal equilibrium because correlations and phase information are transferred from the reservoir to the atomic system. This is very different from the behavior of a two-atom Dicke system interacting with a heat bath. In this case the bath contains no phase information or internal correlation between the modes and the final atomic state is simply a state of thermal equilibrium.

The plan of the paper is as follows: in Sec. II we discuss the decay of a single atom in a squeezed vacuum; in Sec. III we examine the decay of two atoms in an ordinary heat bath at finite temperature. In Sec. IV we report our results on the decay of two closely positioned atoms interacting with a broadband squeezed vacuum and demonstrate that this system relaxes to a highly correlated fermionic thermofield or two-atom squeezed state. Two Appendixes summarize the properties of the radiation-field broadband squeezed state and the two-atom squeezed state.

II. SINGLE-ATOM DECAY IN A SQUEEZED VACUUM

In order to illustrate the technique we will use to obtain the equations of motion for the two-atom decay in a squeezed vacuum, we first treat the single-atom decay in a broadband squeezed vacuum[5] with our Heisenberg equation technique. We assume that the system is described by the following Hamiltonian:

$$H=\tfrac{1}{2}\omega_0\sigma_3-i\sum_\lambda g_\lambda(\sigma_++\sigma_-)(a_\lambda-a_\lambda^\dagger)+\sum_\lambda \omega_\lambda a_\lambda^\dagger a_\lambda , \tag{1}$$

where we use units such that $\hbar=1$. The atomic system is modeled by a two-level system. The pseudospin operators σ obey Pauli algebra

$$\begin{aligned}&[\sigma_+,\sigma_-]=\sigma_3 ,\\ &[\sigma_3,\sigma_+]=2\sigma_+ ,\\ &\sigma_+=\sigma_-^\dagger ,\end{aligned} \tag{2}$$

and a_λ and $a_\lambda^\dagger$ are boson annihilation and creation operators for the radiation field. The index λ includes both polarization and wave vector values, and

$$\begin{aligned}&[a_\lambda,a_{\lambda'}^\dagger]=\delta_{\lambda\lambda'} ,\\ &[a_\lambda,a_{\lambda'}]=0 .\end{aligned} \tag{3}$$

The Heisenberg equations of motion are

$$\begin{aligned}&\dot\sigma_3=-2\sum_\lambda(\sigma_+-\sigma_-)(a_\lambda-a_\lambda^\dagger)g_\lambda ,\\ &\dot\sigma_-=-i\omega_0\sigma_-+\sum_\lambda g_\lambda\sigma_3(a_\lambda-a_\lambda^\dagger) ,\\ &\dot a_\lambda=-i\omega_\lambda a_\lambda+g_\lambda(\sigma_++\sigma_-) .\end{aligned} \tag{4}$$

This is a set of coupled operator differential equations. Assuming that the coupling is weak we adopt an iterative solution in powers of g_λ.[8] Formally integrating Eq. (4) we obtain

$$\begin{aligned}&\sigma_3=-2\sum_\lambda g_\lambda\int_0^t dt'[\sigma_+(t')-\sigma_-(t')][a_\lambda(t')-a_\lambda^\dagger(t')] ,\\ &\sigma_-(t)=\sigma_-(0)e^{-i\omega_0 t}\\ &\qquad+\sum_\lambda g_\lambda\int_0^t dt'\sigma_3(t')[a_\lambda(t')-a_\lambda^\dagger(t')]e^{i\omega_0(t'-t)} ,\\ &a_\lambda(t)=a_\lambda(0)e^{-i\omega_\lambda t}\\ &\qquad+g_\lambda\int_0^t dt'[\sigma_+(t')+\sigma_-(t')]e^{i\omega_\lambda(t'-t)} .\end{aligned} \tag{5}$$

If we look for an iterative solution we can suppose in first approximation

$$\begin{aligned}&\sigma_+(t')\simeq\sigma_+(t)e^{+i\omega_0(t'-t)} ,\\ &a_\lambda(t')\simeq a_\lambda(t)e^{-i\omega_\lambda(t'-t)} .\end{aligned} \tag{6}$$

Substituting Eq. (6) in Eq. (5) we obtain the first-order solution,

$$\begin{aligned}\sigma_3(t)\simeq-2\sum_\lambda g_\lambda\Bigg[&\sigma_+(t)a_\lambda(t)\left[\frac{iP}{\omega_\lambda-\omega_0}+\pi\delta(\omega_\lambda-\omega_0)\right]+a_\lambda^\dagger(t)\sigma_-(t)\left[\frac{-iP}{\omega_\lambda-\omega_0}+\pi\delta(\omega_\lambda-\omega_0)\right]\\ &-\sigma_-(t)a_\lambda(t)\left[\frac{iP}{\omega_\lambda+\omega_0}+\pi\delta(\omega_\lambda+\omega_0)\right]-a_\lambda^\dagger(t)\sigma_+(t)\left[\frac{-iP}{\omega_\lambda+\omega_0}+\pi\delta(\omega_\lambda+\omega_0)\right]\Bigg] ,\\ a_\lambda(t)\simeq a_\lambda(0)e^{-i\omega_\lambda t}+g_\lambda&\left[\sigma_+(t)\left[\frac{-iP}{\omega_\lambda+\omega_0}+\pi\delta(\omega_\lambda+\omega_0)\right]+\sigma_-(t)\left[\frac{-iP}{\omega_\lambda-\omega_0}+\pi\delta(\omega_\lambda-\omega_0)\right]\right] ,\\ \sigma_-(t)\simeq\sigma_-(0)e^{-i\omega_0 t}+\sum_\lambda g_\lambda&\left[\sigma_3(t)a_\lambda(t)\left[\frac{iP}{\omega_\lambda-\omega_0}+\pi\delta(\omega_\lambda-\omega_0)\right]-a_\lambda^\dagger(t)\sigma_3(t)\left[\frac{-iP}{\omega_\lambda+\omega_0}+\pi\delta(\omega_\lambda+\omega_0)\right]\right] ,\end{aligned} \tag{7}$$

where we have used the identity[9]

$$\int_0^t dt'\exp[i(\omega_\lambda-\omega_0)(t'-t)]\simeq\pi\delta(\omega_\lambda-\omega_0)-i\frac{P}{\omega_\lambda-\omega_0} , \tag{8}$$

valid for times $t\gg\omega_\lambda^{-1},\omega_0^{-1}$, and P denotes the Cauchy principal part.

We use normal order so that we have kept all creation (annihilation) operators on the extreme left (right) in any operator product. We could have chosen any other ordering scheme, but once a scheme is chosen it must be maintained during the whole calculation.[8] Substituting the first-order solution, Eq. (7), back in (4) we obtain a set of equations correct to second order in the coupling constant g_λ. These can be solved with the following method: firstly we use the Heisenberg equations to derive Langevin equations for the expectation value of the atomic operators. We neglect all nonsecular terms, i.e., all terms oscillating too fast [at $\pm(\omega_0+\omega_\lambda)$] in a rotating frame.[10] We define the slowly varying operators $\bar\sigma_-(t)$ and $\bar a_\lambda(t)$ by the relations

$$\sigma_-(t)=\bar{\sigma}_-(t)e^{-i\omega_0 t} ,$$
$$a_\lambda(t)=\bar{a}_\lambda(t)e^{-i\omega_\lambda t} .$$

If we assume that the carrier frequency of the squeezed reservoir is resonant with the frequency of the atomic transition we obtain the following Langevin equations for the slowly varying operators:

$$\frac{d}{dt}\begin{bmatrix}\langle\bar{\sigma}_-(t)\rangle\\ \langle\bar{\sigma}_+(t)\rangle\end{bmatrix}=\begin{bmatrix}-\gamma(2N+1) & -2\gamma M\\ -2\gamma M & -\gamma(2N+1)\end{bmatrix}\times\begin{bmatrix}\langle\bar{\sigma}_-(t)\rangle\\ \langle\bar{\sigma}_+(t)\rangle\end{bmatrix}, \tag{9a}$$

$$\frac{d}{dt}\langle\sigma_3(t)\rangle=-2\gamma(2N+1)\langle\sigma_3(t)\rangle-2\gamma , \tag{9b}$$

where

$$\langle a^\dagger(\omega)a(\omega')=N(\omega)\delta(\omega-\omega') ,$$
$$\langle a(\omega)a(\omega')\rangle=M^*(\omega)\delta(\omega'-2\Omega+\omega) , \tag{10}$$
$$M=M(\omega)|_{\omega=\omega_0},\quad N=N(\omega)|_{\omega=\omega_0} .$$

In (9a) we have neglected the imaginary part associated with the principal value part of expression (8) which can be shown to be negligibly small on resonance.[5] We note the presence of off-diagonal elements in Eq. (9a) which would be absent if the reservoir were not squeezed. Their origin is due to the presence of secular terms of the form $\langle\sigma_-a_\lambda^\dagger a_{\lambda'}^\dagger\rangle$ which are absent in a heat bath with phase insensitive random Gaussian noise (in Appendix A we summarize the relationship between intermode correlations and squeezing). The equation of motion of the inversion $\langle\sigma_3\rangle$ contains no phase-sensitive terms.

The solutions of the final "squeezed Bloch equations" (9) are given by

$$\langle\sigma_3(t)\rangle=\left[\langle\sigma_3(0)\rangle+\frac{1}{1+2N}\right]e^{-2\gamma(1+2N)t}-\frac{1}{1+2N} , \tag{11a}$$

$$\langle\sigma_x(t)\rangle=\langle\sigma_x(0)\rangle e^{-\gamma(2N+2M+1)t} ,$$
$$\langle\sigma_y(t)\rangle=\langle\sigma_y(0)\rangle e^{-\gamma(2N-2M+1)t} , \tag{11b}$$

where

$$\sigma_+=\sigma_x+i\sigma_y .$$

The physical interpretation of Eq. (11b) is straightforward: the quadrature phase atomic dipole operators σ_x and σ_y are stimulated by the different amount of noise in the two quadrature of the field and causes them to decay at different rates. In other words, they show phase sensitive decay. On the other hand $\langle\sigma_3\rangle$ shows no phase sensitivity but behaves as if the atoms were interacting with a heat bath at finite temperature. Indeed, we can define a fictitious temperature for the reservoir by defining

$$\exp(\omega_0/kT_{\text{fict}})=(N+1)/N . \tag{12}$$

The absence of phase sensitivity in the population decay is due to the impossibility of having phase sensitive secular terms in the equation of motion of $\langle\sigma_3\rangle$ because $\langle\sigma_+^2a_\lambda a_{\lambda'}\rangle=\langle\sigma_-^2a_\lambda^\dagger a_{\lambda'}^\dagger\rangle=0$. This means that the interaction between a single two-level atom and the radiation field is fundamentally a combination of one-photon processes so that it is not possible to change the population by simultaneous two-photon absorption or emission.

III. TWO-ATOM DECAY IN A HEAT BATH AT FINITE TEMPERATURE

The impossibility of having phase-sensitive population decay with a squeezed reservoir leads us to consider the decay of a two-atom Dicke model.[6,11] In this system two-photon processes changing the total population are possible through interactions which couple to both atoms. In this section we will study the decay of the total atomic population of a two-atom system interacting with a heat bath at finite temperature, and in Sec. IV we will study the same system interacting with a broadband squeezed vacuum, stressing the striking differences between the two decay processes.

We assume that the system is described by the following Hamiltonian:

$$H=\tfrac{1}{2}\omega_0(\sigma_3^{(a)}+\sigma_3^{(b)})+\sum_\lambda\omega_\lambda a_\lambda^\dagger a_\lambda -i\sum_\lambda g_\lambda(\sigma_+^{(a)}+\sigma_+^{(b)}+\sigma_-^{(a)}+\sigma_-^{(b)})(a_\lambda-a_\lambda^\dagger) , \tag{13}$$

where the σ_a refers to atom a and σ_b refers to atom b. The Hamiltonian is again of the electric dipole form in which the atomic dipole **d** and the electric field operator **E** are coupled in the form **d**·**E** form in dipole approximation, and we have assumed that the distance between the two atoms is much smaller than c/ω_0. If this is so, we can assume that the two atoms are close enough to neglect the spatial variation of the resonant mode.[6] However, we assume also that any exchange interaction between the two atoms due to overlap of the atomic wave function is completely negligible. This model is known as the Dicke model.[6] The only states taking part in the dynamics are the triplet states ($|e\rangle,|g\rangle$ describe the excited and ground states of the two identical atoms)

$$|1\rangle=|e_a,e_b\rangle ,$$
$$|2\rangle=\frac{1}{\sqrt{2}}(|e_a,g_b\rangle+|e_b\,g_a\rangle) , \tag{14a}$$
$$|3\rangle=|g_a,g_b\rangle ,$$

and the singlet state

$$|4\rangle=\frac{1}{\sqrt{2}}(|g_a,e_b\rangle-|e_a,g_b\rangle) \tag{14b}$$

is completely decoupled from the triplet state: the Hamiltonian (13) is invariant under the exchange $a\leftrightarrow b$ which implies that the symmetry of the atomic state is a constant of motion. The singlet state is antisymmetric while the triplet states are symmetric, which means that they cannot be coupled by the interaction with the field.

To study the atomic dynamics we will again work in the Heisenberg picture. The equation of motion of the operators are

$$\begin{aligned}
\dot\sigma_3^{(a)} &= -2\sum_\lambda g_\lambda(\sigma_+^{(a)}-\sigma_-^{(a)})(a_\lambda - a_\lambda^\dagger)\ ,\\
\dot\sigma_3^{(b)} &= -2\sum_\lambda g_\lambda(\sigma_+^{(b)}-\sigma_-^{(b)})(a_\lambda - a_\lambda^\dagger)\ ,\\
\dot\sigma_-^{(a)} &= -i\omega_0\sigma_-^{(a)} + \sum_\lambda g_\lambda \sigma_3^{(a)}(a_\lambda - a_\lambda^\dagger)\ ,\\
\dot\sigma_-^{(b)} &= -i\omega_0\sigma_-^{(b)} + \sum_\lambda g_\lambda \sigma_3^{(b)}(a_\lambda - a_\lambda^\dagger)\ ,\\
\dot a_\lambda &= -i\omega_\lambda a_\lambda + g_\lambda(\sigma_+^{(a)}+\sigma_+^{(b)}+\sigma_-^{(a)}+\sigma_-^{(b)})\ .
\end{aligned} \tag{15}$$

These equations are used to derive Langevin equations correct to second order in g_λ. The radiation field at finite temperature is characterized by the following expectation values for the annihilation and creation operators:

$$\begin{aligned}
\langle a(\omega)a(\omega')\rangle &= 0\ ,\\
\langle a^\dagger(\omega)a(\omega')\rangle &= N(\omega)\delta(\omega-\omega')\ ,\\
\langle a(\omega)a^\dagger(\omega')\rangle &= [N(\omega)+1]\delta(\omega-\omega')\ ,
\end{aligned} \tag{16}$$

where

$$N(\omega) = [1+\exp(\omega_0/kT)]^{-1}\ . \tag{17}$$

We obtain the following Langevin equations:

$$\frac{d}{dt}\begin{bmatrix} \langle\sigma_3^{(a)}\rangle \\ \langle\sigma_3^{(b)}\rangle \\ \langle\sigma_+^{(a)}\sigma_-^{(b)}\rangle \\ \langle\sigma_+^{(b)}\sigma_-^{(a)}\rangle \\ \langle\sigma_3^{(a)}\sigma_3^{(b)}\rangle \end{bmatrix} = \begin{bmatrix} -2\gamma_1 & 0 & -2(\gamma+i\delta) & -2(\gamma-i\delta) & 0 \\ 0 & -2\gamma_1 & -2(\gamma-i\delta) & -2(\gamma+i\delta) & 0 \\ \frac{1}{2}(\gamma-i\delta) & \frac{1}{2}(\gamma+i\delta) & -2\gamma_1 & 0 & \gamma_1 \\ \frac{1}{2}(\gamma+i\delta) & \frac{1}{2}(\gamma-i\delta) & 0 & -2\gamma_1 & \gamma_1 \\ -2\gamma & -2\gamma & 4\gamma_1 & 4\gamma_1 & -4\gamma_1 \end{bmatrix} \begin{bmatrix} \langle\sigma_3^{(a)}\rangle \\ \langle\sigma_3^{(b)}\rangle \\ \langle\sigma_+^{(a)}\sigma_-^{(b)}\rangle \\ \langle\sigma_+^{(b)}\sigma_-^{(a)}\rangle \\ \langle\sigma_3^{(a)}\sigma_3^{(b)}\rangle \end{bmatrix} + \begin{bmatrix} -2\gamma \\ -2\gamma \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{18a}$$

where

$$\begin{aligned}
\gamma - i\delta &= \int \omega^2 d\omega \left[\frac{iP}{\omega+\omega_0} + \frac{iP}{\omega-\omega_0} + \pi\delta(\omega-\omega_0)\right] g^2(\omega)\ ,\\
\gamma_1 &= (2N+1)\gamma\ .
\end{aligned} \tag{18b}$$

As we are interested only in the decay of the total population we can reduce the system of equations to the following simpler form:

$$\frac{d}{dt}\begin{bmatrix} E(t) \\ C(t) \\ F(t) \end{bmatrix} = \begin{bmatrix} -2\gamma_1 & -4\gamma & 0 \\ \gamma & -2\gamma_1 & 2\gamma_1 \\ -2\gamma & 4\gamma_1 & -4\gamma_1 \end{bmatrix} \begin{bmatrix} E(t) \\ C(t) \\ F(t) \end{bmatrix} + \begin{bmatrix} -4\gamma \\ 0 \\ 0 \end{bmatrix}, \tag{19}$$

where

$$\begin{aligned}
E(t) &= \langle\sigma_3^{(a)}(t)+\sigma_3^{(b)}(t)\rangle\ ,\\
C(t) &= \langle\sigma_+^{(a)}(t)\sigma_-^{(b)}(t)+\sigma_+^{(b)}(t)\sigma_-^{(a)}(t)\rangle\ ,\\
F(t) &= \langle\sigma_3^{(a)}(t)\sigma_3^{(b)}(t)\rangle\ .
\end{aligned} \tag{20}$$

These equations can be solved easily by using Laplace transform techniques. We denote by $\tilde E(s)$, $\tilde C(s)$, and $\tilde F(s)$ the transforms of $E(t)$, $C(t)$, and $F(t)$ and we obtain the following linear system:

$$\begin{bmatrix} (s+2\gamma_1) & 4\gamma & 0 \\ -\gamma & (s+2\gamma_1) & -2\gamma_1 \\ 2\gamma & -4\gamma_1 & (s+4\gamma_1) \end{bmatrix} \begin{bmatrix} \tilde E(s) \\ \tilde C(s) \\ \tilde F(s) \end{bmatrix} = \begin{bmatrix} E(0)-4\gamma/s \\ C(0) \\ F(0) \end{bmatrix}. \tag{21}$$

Using Cramer's rule we obtain the following results for the various states of interest. (a) For the initially inverted state $|1\rangle$, we have the initial conditions $E(0)=2$, $C(0)=0$, $F(0)=1$,

$$\tilde E(s) = \left[\frac{-32\gamma\gamma_1}{(s-\lambda_1)(s-\lambda_2)s} + \frac{4(3\gamma_1-\gamma)+s}{(s-\lambda_1)(s-\lambda_2)}\right], \tag{22}$$

with

$$\begin{aligned}
\lambda_1 &= 4\gamma_1 + 2(\gamma_1^2-\gamma^2)^{1/2}\ ,\\
\lambda_2 &= 4\gamma_1 - 2(\gamma_1^2-\gamma^2)^{1/2}\ .
\end{aligned} \tag{23}$$

The Laplace inversion leads to the following expression for the time-dependent two-atom inversion:

$$E(t)=-\frac{32\gamma_1\gamma}{\lambda_1\lambda_2}+\frac{1}{\lambda_1-\lambda_2}\left[e^{-\lambda_1 t}\left[6\gamma_1-\gamma-\lambda_1+\frac{32\gamma\gamma_1}{\lambda_1}\right]-e^{-\lambda_2 t}\left[6\gamma_1-\gamma-\lambda_2+\frac{32\gamma\gamma_1}{\lambda_1}\right]\right] . \quad (24)$$

(b) For the state $|2\rangle$, the Dicke symmetric state initially excited, we have the initial conditions $E(0)=0$, $C(0)=1$, and $F(0)=1$. We find for the two-atom inversion in Laplace space

$$\tilde{E}(s)=-\left[\frac{32\gamma\gamma_1}{(s+\lambda_1)(s+\lambda_2)s}+\frac{8\gamma}{(s+\lambda_1)(s+\lambda_2)}\right] , \quad (25a)$$

and inverting we obtain for the time-dependent inversion

$$E(t)=-\frac{32\gamma\gamma_1}{\lambda_1\lambda_2}-\frac{8\gamma}{(\lambda_2-\lambda_1)}\left[e^{-\lambda_1 t}\left[-\frac{4\gamma_1}{\lambda_1}+1\right]+e^{-\lambda_2 t}\left[+\frac{4\gamma_1}{\lambda_2}-1\right]\right] . \quad (25b)$$

The physical interpretation of the process is simple: the atomic system relaxes into a state of thermal equilibrium. The decay constants show the typical superradiant behavior characteristic of the Dicke model.[6] Indeed, if the heat bath is at zero temperature we see that $\lambda_1=\lambda_2=4\gamma$, which is twice the natural decay rate of a single two-level atom. The presence of the thermal noise merely increases the decay constants by stimulated emission. The final equilibrium atomic state is, of course, a state of thermal equilibrium. Indeed it is simple to verify that

$$E_{\text{therm. equil.}}=2\frac{(e^{-\omega_0/kT}-e^{\omega_0/kT})}{(e^{-\omega_0/kT}+1+e^{\omega_0/kT})}=\frac{-2(2N+1)}{3N^2+3N+1} , \quad (26)$$

where we have used the fact that in thermal equilibrium the average number photons is given by Eq. (17). We note that $E_{\text{therm. equil.}}$ is identical to $E(\infty)$ given by Eq. (24) and (25), and that at zero temperature our results coincide with Lehmberg's[12] for which $\lambda_1=\lambda_2=4\gamma$ and for the initially fully excited state

$$E(t)=4e^{-4\gamma t}(1+2\gamma t)-2 \quad (27)$$

and for the initially excited symmetric state

$$E(t)=2(e^{-4\gamma t}-1) . \quad (28)$$

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We now study the decay process of the same two-atom system interacting with a squeezed-vacuum reservoir. The Hamiltonian describing our model is again (13). Using the same techniques we used to study the decay of the two-level atom we obtain the following Langevin equations:

$$\frac{d}{dt}\mathbf{v}=\underline{M}\mathbf{v}+\mathbf{u} , \quad (29a)$$

where the column vectors $\mathbf{v}$ and $\mathbf{u}$ are given by

$$\mathbf{v}=\begin{bmatrix}\langle\sigma_3^{(a)}\rangle\\ \langle\sigma_3^{(b)}\rangle\\ \langle\sigma_+^{(a)}\sigma_-^{(b)}\rangle\\ \langle\sigma_+^{(b)}\sigma_-^{(a)}\rangle\\ \langle\sigma_+^{(a)}\sigma_+^{(b)}\rangle\\ \langle\sigma_-^{(a)}\sigma_-^{(b)}\rangle\\ \langle\sigma_3^{(a)}\sigma_3^{(b)}\rangle\end{bmatrix} , \quad \mathbf{u}=\begin{bmatrix}-2\gamma_1\\ -2\gamma_1\\ 0\\ 0\\ 0\\ 0\\ 0\end{bmatrix} \quad (29b)$$

and

$$\underline{M}=\begin{bmatrix}-2\gamma_1 & 0 & -2(\gamma+i\delta) & -2(\gamma-i\delta) & 0 & 0 & 0\\ 0 & -2\gamma_1 & -2(\gamma-i\delta) & -2(\gamma+i\delta) & 0 & 0 & 0\\ \frac{1}{2}(\gamma-i\delta) & \frac{1}{2}(\gamma+i\delta) & -2\gamma_1 & 0 & -(\gamma_2+i\delta_2) & -(\gamma_2-i\delta_2) & \gamma_1\\ \frac{1}{2}(\gamma+i\delta) & \frac{1}{2}(\gamma-i\delta) & 0 & -2\gamma_1 & -(\gamma_2+i\delta_2) & -(\gamma_2-i\delta_2) & \gamma_1\\ 0 & 0 & -(\gamma_2-i\delta_2) & -(\gamma_2-i\delta_2) & -2(\gamma_1+i\delta_1) & 0 & (\gamma_2-i\delta_2)\\ 0 & 0 & -(\gamma_2+i\delta_2) & -(\gamma_2+i\delta_2) & 0 & -2(\gamma_1-i\delta_1) & (\gamma_2+i\delta_2)\\ -2\gamma & -2\gamma & 4\gamma_1 & 4\gamma_1 & 4(\gamma_2+i\delta_2) & 4(\gamma_2-i\delta_2) & -4\gamma_1\end{bmatrix} , \quad (29c)$$

where $\gamma - i\delta$ is given by Eq. (18b) and the new decay parameters are

$$\frac{1}{2}(\gamma_2+i\delta_2)=\int \omega_\lambda^2 d\omega_\lambda M(\omega_\lambda) \times\left[\frac{iP}{\omega_\lambda-\omega_0}+\pi\delta(\omega_\lambda-\omega_0)\right] \times g(\omega_\lambda)g(2\Omega-\omega_\lambda)\ ,$$
$$(\gamma_1+i\delta_1)=\int \omega_\lambda^2 d\omega_\lambda \times\left[\frac{-iP}{\omega_\lambda+\omega_0}-\frac{iP}{\omega_0-\omega_\lambda}+\pi\delta(\omega_\lambda-\omega_0)\right] \times g^2(\omega_\lambda)[2N(\omega_\lambda)+1]\ . \tag{30}$$

We note again the presence of phase sensitive terms due to the nonzero expectation value of $\langle a_\lambda a_{\lambda'}\rangle$ and $\langle a_\lambda^\dagger a_{\lambda'}^\dagger\rangle$. As we are interested only in the decay of the total atomic population we can concentrate our attention on the following simpler system of equations:

$$\frac{d}{dt}\begin{bmatrix}E(t)\\C(t)\\S(t)\\F(t)\end{bmatrix}=\begin{bmatrix}-2\gamma_1 & -4\gamma & 0 & 0\\ \gamma & -2\gamma_1 & -2\gamma_2 & 2\gamma_1\\ 0 & -2\gamma_1 & -2\gamma_1 & 2\gamma_2\\ -2\gamma & 4\gamma_1 & 4\gamma_2 & -4\gamma_1\end{bmatrix}\times\begin{bmatrix}E(t)\\C(t)\\S(t)\\F(t)\end{bmatrix}+\begin{bmatrix}-4\gamma\\0\\0\\0\end{bmatrix}\ , \tag{31}$$

where $E(t)$, $C(t)$, and $F(t)$, are the same as in (20) and $S(t)=\langle\sigma_+^{(a)}\sigma_+^{(b)}+\sigma_-^{(a)}\sigma_-^{(b)}\rangle$. In obtaining (31) from (29) we have neglected the shifts δ_1 and δ_2 which can be shown to be negligibly small when the atoms are resonant with the carrier frequency of the squeezed reservoir.[5] From the preceding results it is already possible to appreciate the new features shown by the equations of motion describing the total population decay. The two-atom inversion $E(t)$ is coupled to $S(t)$ through $C(t)$. This means that now, because of the cross coupling between population and polarization of the two molecules, $E(t)$ shows a phase-sensitive decay, in strong contrast with the case of a single two-level atom. This is due to the possibility of changing the atomic population by two-photon absorption or emission, described by terms such as $\langle\sigma_+^{(a)}\sigma_+^{(b)}a_\lambda a_{\lambda'}\rangle$ in (29).

The two-atom Bloch equations (31) can be solved by Laplace transform techniques. By analogy with Eq. (21) we can now write

$$\begin{bmatrix}(s+2\gamma_1) & 4\gamma & 0 & 0\\ -\gamma & (s+2\gamma_1) & 2\gamma_2 & -2\gamma_1\\ 0 & 2\gamma_2 & (s+2\gamma_1) & -2\gamma_2\\ 2\gamma & -4\gamma_1 & -4\gamma_2 & (s+4\gamma_1)\end{bmatrix}\times\begin{bmatrix}\tilde{E}(s)\\ \tilde{C}(s)\\ \tilde{S}(s)\\ \tilde{F}(s)\end{bmatrix}=\begin{bmatrix}E(0)-4\gamma/s\\ C(0)\\ S(0)\\ F(0)\end{bmatrix}\ . \tag{32}$$

(a) For the initially inverted state $E(0)=2$, $F(0)=1$, and $C(0)=S(0)=0$, and we find

$$\tilde{E}(s)=\left[\frac{2s-4\gamma}{s(s+2\gamma_1)}-\frac{8\gamma(\gamma+\gamma_1)}{(s+2\gamma_1)(s+\lambda_1)(s+\lambda_2)}\right]\ , \tag{33}$$

with

$$\lambda_1=-4(\gamma_1+\gamma_2)\ ,\qquad \lambda_2=-4(\gamma_1-\gamma_2)\ . \tag{34}$$

The inverse Laplace transform gives the time-dependent inversion as

$$E(t)=-2\frac{\gamma}{\gamma_1}+2e^{-2\gamma_1 t}\left[\frac{\gamma+\gamma_1}{\gamma_1}+\frac{\gamma(\gamma+\gamma_1)}{4\gamma_2^2-\gamma_1^2}\right] -\frac{\gamma}{2}\frac{\gamma+\gamma_1}{\gamma_1}\left[\frac{e^{-4(\gamma_1+\gamma_2)t}}{2\gamma_2+\gamma_1}+\frac{e^{-4(\gamma_1-\gamma_2)t}}{2\gamma_2-\gamma_1}\right]\ . \tag{35}$$

(b) For the initially-excited Dicke symmetric state $E(0)=S(0)=0$, $C(0)=1$, and $F(0)=-1$, and we find

$$\tilde{E}(s)=-4\gamma\left[\frac{1}{(s+2\gamma_1)s}+\frac{1}{(s+\lambda_1)(s+\lambda_2)}+\frac{2\gamma_1}{(s+2\gamma_1)(s+\lambda_1)(s+\lambda_2)}\right]\ , \tag{36a}$$

$$E(t)=-\frac{2\gamma}{\gamma_1}+\frac{8\gamma}{\gamma_1}\frac{\gamma_2^2}{(4\gamma_2^2-\gamma_1^2)}e^{-2\gamma_1 t} +\frac{\gamma}{2\gamma_2}\left[\left[\frac{\gamma_1+\gamma_2}{\gamma_1+2\gamma_2}\right]e^{-4(\gamma_1+\gamma_2)t} +\left[\frac{3\gamma_2-\gamma_1}{2\gamma_2-\gamma_1}\right]e^{-4(\gamma_1-\gamma_2)t}\right]\ . \tag{36b}$$

The final equilibrium population is

$$E(\infty)=\frac{-2}{2N+1}\ . \tag{37}$$

It is also straightforward to verify that irrespective of the initial conditions, the interatomic dipole correlation $S(t)$ relaxes to

$$S(\infty)=\frac{\gamma_2}{\gamma_1}=\frac{2M}{2N+1}\ . \tag{38}$$

It is now possible to see in detail the strong differences between the decay in a squeezed reservoir and the decay in a heat bath. Comparing (34) with (23) it is evident that the decay constants are now phase sensitive and that the final atomic state is not a state of thermal equilibrium. Indeed, $E(\infty)$ is twice the value of a single two level atom inversion for atoms interacting with a heat bath and this does not describe a state of thermal equilibrium [compare [Eq. (26) with Eq. (37)]. We note that the final state contains internal correlations as evidenced by the nonvanishing $S(\infty)$. The internal correlations characterizing a squeezed reservoir are transferred to the atomic system. The final equilibrium atomic state is a *pure* state of the form

$$|\psi_{eq}\rangle = \cos\theta|g_a\, g_b\rangle + \sin\theta|e_a e_b\rangle \ , \tag{39}$$

where

$$\cos\theta = \left[\frac{N+1}{2N+1}\right]^{1/2}, \quad \sin\theta = \left[\frac{N}{2N+1}\right]^{1/2} .$$

These states are known as two-atom squeezed states.[7] They describe a strongly correlated pure quantum system exhibiting thermal statistics in the expectation values of single-atom operators, but showing strong correlations when the expectation value of operators acting on both atoms is taken. These properties are characteristic also of the two-mode squeezed state.[3]

V. DISCUSSION

We have studied the decay of the total atomic population of (a) a two-atom Dicke system interacting with a broadband squeezed vacuum and we have compared it with the decay of (b) a single two-level atom and also with (c) a two-atom system interacting with a heat bath at finite temperature. In case (b) the atomic polarization shows phase sensitive decay but the population does not show any phase sensitivity. Its decay is analogous to decay in a heat bath at finite temperature. The final population is also characteristic of thermal equilibrium. In case (c) the heat bath does not contain any phase-dependent noise so the atomic population decay is not phase sensitive and the final equilibrium atomic state is merely a state of thermal equilibrium. Case (a) exhibits features completely different from the one described above in that the atomic population decay is now phase sensitive due to the possibility of two-photon processes in the cross coupling between population and polarization of the two atoms. Equivalently, we may say that the individual dipole fields *are* phase sensitive and these couple to the inversion of their partner atoms to create a phase sensitivity to the cooperative inversion decay. Moreover, the final state is not a state of thermal equilibrium but it is a highly correlated pure state known as a two-atom squeezed state, in which both atoms are both excited or both in their ground state. This means the internal correlations contained in the bath are now transferred to the atomic system. We want to stress that because the triplet state of the two atom Dicke model behaves as a three-level ladder system, in which the ground state and the most excited state are decoupled it would be easier to study experimentally the latter system which contains some of the characteristics of the model we have studied, including for instance the possibility of containing phase information through internal correlation among the atomic states.

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APPENDIX A: THE BROADBAND SQUEEZED STATE

The two-mode squeezed-vacuum state is defined as (e.g., Ref. 1)

$$|0(\eta)\rangle = S(\eta)|0\rangle \ , \tag{A1}$$

where the two-mode Bogoliubov transformation generating squeezing is given by

$$S(\eta) = \exp[\eta^* a^\dagger(\omega) a^\dagger(2\Omega-\omega) - \eta a(\omega) a(2\Omega-\omega)] \ , \quad \eta = re^{i\phi} \ . \tag{A2}$$

It is easy to show that

$$S^\dagger(\eta) a(\omega) S(\eta) = \cosh r\, a(\omega) + \sinh r\, e^{i\phi} a^\dagger(2\Omega-\omega) \ . \tag{A3}$$

From (A3) we obtain

$$\langle a^\dagger(\omega) a^\dagger(\omega')\rangle = M(\omega)\delta(\omega'-2\Omega+\omega) \ , \tag{A4}$$

$$\langle a(\omega) a(\omega')\rangle = M^*(\omega)\delta(\omega'-2\Omega+\omega) \ , \tag{A5}$$

$$\langle a^\dagger(\omega) a(\omega')\rangle = N(\omega)\delta(\omega-\omega') \ , \tag{A6}$$

$$\langle a(\omega) a^\dagger(\omega')\rangle = [N(\omega)+1]\delta(\omega-\omega') \ , \tag{A7}$$

where

$$M(\omega) = \cosh r \sinh r\, e^{i\phi} \ , \tag{A8}$$

$$N(\omega) = \sinh^2 r \ . \tag{A9}$$

There is no loss of generality in assuming the phase angle of the squeezing $\phi=0$. Two-mode squeezed states are used as thermofield representations of boson fields in thermal equilibrium. If we calculate the expectation value of single-mode field operators we obtain thermal Bose-Einstein statistics if we relate the squeeze parameter r to the effective temperature T through[3]

$$\cosh r = [1-\exp(\omega/kT)]^{-1/2} \ , \tag{A10}$$

$$\sinh r = [\exp(\omega/kT)-1]^{-1/2} \ . \tag{A11}$$

On the other hand if we calculate the expectation value of operators acting on both modes we obtain results showing clearly the correlations between the modes. Equation (A7) is a clear example of these characteristics of a two-mode squeezed state. The two-mode squeezed vacuum is characterized by time-stationary quadrature phase noise, so that if we write the electric field as

$$E(t) = E_1(t)\cos(\Omega t) + E_2(t)\sin(\Omega t) \tag{A12}$$

the quadrature phase field operators $E_1(t)$ and $E_2(t)$ are characterized by time-independent Gaussian noise. A squeezed state generated by Eq. (A1) is a minimum uncertainty state in which the amount of noise contained in the two quadratures is different. If the functions $N(\omega)$ and $M(\omega)$ are broad functions of ω, i.e., if their spectrum is much broade₁ than the linewidth of the atomic transition we are interested in, the squeezed vacuum can be considered a reservoir with white noise.[4]

APPENDIX B: THE TWO-ATOM SQUEEZED STATE

By analogy with the definition of a two-mode squeezed state it is possible to define a two-atom squeezed state. They have been introduced by Takahashi and Umezawa[7] with the name of fermion thermofields and have been recently utilized by Barnett and Dupertuis in connection with quantum optical problems.[7]

The two-atom squeezed state is defined as

$$|\eta\rangle = S(\eta)|g_a\, g_b\rangle \ , \tag{B1}$$

where the atomic squeezing transformation $S(\eta)$ is given by

$$S(\eta)=\exp(\eta^* \sigma_-^{(a)}\sigma_-^{(b)} - \eta\sigma_+^{(a)}\sigma_+^{(b)}), \quad \eta=\theta e^{i\phi} \ . \tag{B2}$$

It is possible to show that[7]

$$|\eta\rangle = \cos\theta|g_a\, g_b\rangle - \sin\theta\, e^{i\phi}|e_a e_b\rangle \ . \tag{B3}$$

If A is an operator acting only on one atom we obtain

$$\langle\eta|A|\eta\rangle = \cos^2\theta\langle g|A|g\rangle + \sin^2\theta\langle e|A|e\rangle \ , \tag{B4}$$

and if

$$\cos^2\theta = [1+\exp(-\omega_0/kT)]^{-1} \ ,$$

$$\sin^2\theta = \exp(-\omega_0/kT)/[1+\exp(-\omega_0/kT)] \ ,$$

then $\langle\eta|A|\eta\rangle$ reproduces the single-atom thermal average. This is not the case if the operator acts on both atoms. As an example, we take

$$|\eta\rangle = \cos\theta|g_a\, g_b\rangle + \sin\theta|e_a e_b\rangle \ , \tag{B5}$$

then

$$\langle\eta|\sigma_3^{(a)}|\eta\rangle = \frac{\exp(-\omega_0/kT)-1}{1+\exp(-\omega_0/kT)} \ , \tag{B6a}$$

$$\langle\eta|\sigma_+^{(a)}\sigma_+^{(b)}|\eta\rangle = \frac{\exp(-\tfrac{1}{2}\omega_0/kT)}{1+\exp(-\omega_0/kT)} \ . \tag{B6b}$$

If we write

$$\exp(\omega_0/kT) = (N+1)/N \ , \tag{B7}$$

where N is the thermal average number of photons at temperature T [Eq. (17)], we obtain

$$\langle\sigma_3^{(a)}\rangle = \langle\sigma_3^{(b)}\rangle = -1/(2N+1) \ , \tag{B8}$$

$$\langle\sigma_+^{(a)}\sigma_+^{(b)}\rangle = \langle\sigma_-^{(a)}\sigma_-^{(b)}\rangle = \frac{[N(N+1)]^{1/2}}{2N+1} \ , \tag{B9}$$

which are identical with Eqs. (37) and (38).

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