

Nonresonant interaction of a three-level atom with cavity fields.

III. Photon-number probabilities and fluctuations

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We study the photon statistics in the interaction of a three-level atom with cavity fields of arbitrary detunings. A number of new features of photon-number distributions and fluctuations are found and discussed. The possibility of sub-Poissonian and antibunching effects is also discussed.

I. INTRODUCTION

It is well known that fluctuations exist in every type of electromagnetic field. There are always random uncertainties in the amplitude and phase of the light. In practical cases, these fluctuations may be regarded as a result of the environmental influence on the light source. However, the field still fluctuates even if all the environmental effects are removed. This is because the quantum-mechanical uncertainties are intrinsic, and the noise that exists in light propagation can never be removed by any means.

In general, the fluctuation of any variable quantities F can be expressed by its variance $\langle (\Delta F)^2 \rangle = \langle F^2 \rangle - \langle F \rangle^2$. The variance of the photon number is usually expressed in terms of Mandel's Q parameter¹ defined by

$$Q = \frac{\langle (\Delta n)^2 \rangle}{\langle n \rangle} - 1. \quad (1)$$

When the light field is in the coherent state, $Q = 0$ and the photon-number probability distribution is Poissonian. When $Q > 0$, the photon-number fluctuation is large. The probability distribution spreads wide and is called super-Poissonian. When $Q < 0$, the photon-number fluctuation is small and we have sub-Poissonian probability distribution which has a narrow peak. There is a lower bound $Q = -1$ corresponding to a pure number state.

In a system of an atom interacting with the radiation field, the photon statistical properties have been of great interest. The sub-Poisson photon distribution in resonance fluorescence is now well established.¹⁻⁶ Recent works of Filipowicz *et al.*⁷ and Davidovich *et al.*⁸ have shown theoretically that the photon-number statistics of a one-photon or a two-photon micromaser exhibit sub-Poisson distribution. Kim and Knight⁹ have studied fluctuations in fluorescence intensity produced by a three-level atom in a two-state random telegraph signal. Many

authors¹⁰⁻¹⁶ have investigated the photon statistical distribution in the Jaynes-Cummings (JC) model. Meystre *et al.*¹¹ studied photon statistics for a two-level atom interacting with an initially coherent cavity mode and concluded that, for long times, the atom acts as a nonlinear filter on the coherent properties of the cavity mode. In a recent review, Loudon and Knight¹⁴ considered a two-level atom interacting with an initially squeezed cavity mode. Their conclusion is that the photon distribution properties depend strongly upon the squeezing parameter. Different choices of the squeezing parameter can result in a sub-Poisson or super-Poisson distribution of the photon-number probability. In the micromaser experiments of Rempe and Walther,¹⁶ the quantum collapse and revival phenomena predicted by the JC model were demonstrated for the first time. It is also expected in these experiments that the cavity field will develop significant sub-Poisson statistics.

In the first paper of this series¹⁷ (hereafter referred to as I), the problem of nonresonant interaction of a three-level atom with one- or two-mode cavity fields of arbitrary detuning was formulated and the atomic level occupation probabilities discussed in detail. The coherent properties of the stimulated field were studied in the second paper¹⁸ (this paper will be referred to as II from now on). It is found in II that the idea of a nonlinear filter discussed in Ref. 11 is no longer true in general. We investigate in the present paper the photon-number distribution and fluctuation of the field in the off-resonance interaction of cavity modes with the atom.

We consider two cases of interaction similar to what we have done in the previous papers of this series: a Ξ -type atom with one-mode field and a Λ -type atom with two-mode field. The time dependence of the photon-number probability distribution function $p(n, t)$ or $p(n_1, n_2, t)$ as well as the Q parameter for various detuning parameters are calculated. We find that for one-mode

Ξ type sub-Poisson and super-Poisson distributions appear alternately as time develops, and for two-mode Λ type the distribution is always super-Poissonian.

II. THEORY

The general formalism of a three-level atom interacting with cavity fields is given in I. Here we merely outline what is essential for our present discussion of photon statistics. We consider the two typical cases: one-mode Ξ type and two-mode Λ type. The relevant atomic level configurations are shown in Fig. 1.

The Hamiltonian is, in the interaction picture,

$$H = \hbar(H_0 + H_1), \quad (2)$$

where, for the one-mode Ξ type,

$$H_0 = \sum_{\eta=a,b,c} \omega_\eta A_\eta^\dagger A_\eta + \Omega a^\dagger a, \quad (3)$$

$$H_1 = \lambda_1 e^{i\Delta_1 t} a A_b^\dagger A_a + \lambda_2 e^{i\Delta_2 t} a A_a^\dagger A_c + \text{H.c.}, \quad (4)$$

$$\Delta_1 = -(\Omega - \omega_b + \omega_a), \quad \Delta_2 = \Omega - \omega_a + \omega_c;$$

and for two-mode Λ type,

$$H_0 = \sum_{\eta=a,b,c} \omega_\eta A_\eta^\dagger A_\eta + \sum_{i=1,2} \Omega_i a_i^\dagger a_i, \quad (5)$$

$$H_1 = \lambda_1 e^{-i\Delta_1 t} a_1 A_a^\dagger A_b + \lambda_2 e^{-i\Delta_2 t} a_2 A_a^\dagger A_c + \text{H.c.}, \quad (6)$$

$$\Delta_1 = \Omega_1 - \omega_a + \omega_b, \quad \Delta_2 = \Omega_2 - \omega_a + \omega_c.$$

The operators in the Hamiltonian are defined as follows. A_η^\dagger creates an atom in the state $|\eta\rangle$, a^\dagger creates a photon, λ_i are the usual coupling constants, and Δ_i are the detuning parameters.

As has been shown in I, the wave equation can be solved by the state vector

$$|\Psi(t)\rangle = \sum_n Q(n) [A(n_a, t) |a, n_a\rangle + B(n_b, t) |b, n_b\rangle + C(n_c, t) |c, n_c\rangle] \quad (7a)$$

for one mode or

$$|\Psi(t)\rangle = \sum_{n_1, n_2} Q_1(n_1) Q_2(n_2) \times [A(n_{1a}, n_{2a}, t) |a, n_{1a}, n_{2a}\rangle + B(n_{1b}, n_{2b}, t) |b, n_{1b}, n_{2b}\rangle + C(n_{1c}, n_{2c}, t) |c, n_{1c}, n_{2c}\rangle] \quad (7b)$$

for two mode, with the corresponding initial conditions

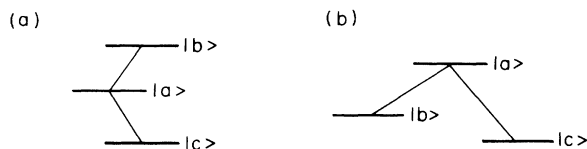


FIG. 1. Atomic level configuration for (a) Ξ type and (b) Λ type.

$$|\Psi(0)\rangle = |\eta, \xi\rangle = |\eta\rangle \sum_n Q(n) |n\rangle \quad (8a)$$

for one mode and

$$|\Psi(0)\rangle = |\eta, \xi_1, \xi_2\rangle = |\eta\rangle \sum_{n_1, n_2} Q_1(n_1) Q_2(n_2) |n_1, n_2\rangle \quad (8b)$$

for two mode where n_η is the photon number when the atom is in the level η , and $n_{i\eta}$ is the photon number referring to the mode i . The probability amplitudes in (7) are

$$A = -e^{i\Delta_2 t} \sum_{i=1}^3 U_i \mu_i e^{i\mu_i t}, \quad (9a)$$

$$B = \frac{1}{V_1} e^{i(\Delta_1 - \Delta_2)t} \sum_{i=1}^3 U_i (\mu_i^2 - \Delta_2 \mu_i - V_2^2) e^{i\mu_i t}, \quad (9b)$$

$$C = V_2 \sum_{i=1}^3 U_i e^{i\mu_i t}, \quad (9c)$$

where

$$\mu_1 = -\frac{1}{3}x_1 + \frac{2}{3}(x_1^2 - 3x_2)^{1/2} \cos\theta, \quad (10a)$$

$$\mu_2 = -\frac{1}{3}x_1 + \frac{2}{3}(x_1^2 - 3x_2)^{1/2} \cos(\theta + \frac{2}{3}\pi), \quad (10b)$$

$$\mu_3 = -\frac{1}{3}x_1 + \frac{2}{3}(x_1^2 - 3x_2)^{1/2} \cos(\theta + \frac{4}{3}\pi), \quad (10c)$$

$$\theta = \frac{1}{3} \cos^{-1} \left[\frac{9x_1 x_2 - 2x_1^3 - 27x_3}{2(x_1^2 - 3x_2)^{3/2}} \right], \quad (10d)$$

and

$$x_1 = \Delta_1 - 2\Delta_2, \quad (11a)$$

$$x_2 = -[V_1^2 + V_2^2 + \Delta_2(\Delta_1 - \Delta_2)], \quad (11b)$$

$$x_3 = (\Delta_2 - \Delta_1)V_2^2. \quad (11c)$$

The probability amplitudes for one and two mode take the same expressions (9) and depend on the photon number in different modes through the coupling strength parameters V_1 and V_2 . The explicit forms of these parameters are listed in Table I of I for different cases.

III. PHOTON-NUMBER DISTRIBUTION AND Q PARAMETER

The photon-number probability distributions are most easily calculated from the density matrix which is defined by

$$\rho(t) = |\Psi(t)\rangle \langle \Psi(t)| \quad (12)$$

with the state vector given by (7). We now proceed to investigate the time evolution of photon-number probabilities and Mandel's Q parameter of two typical cases.

A. One-mode Ξ type

We consider a three-level atom with Ξ -type energy-level configuration. The initial condition is that the atom starts in the upper state $|b\rangle$ and the one-mode cavity field is in the coherent state. Thus

$$p_c(n) = e^{-\bar{n}} \bar{n}^n / n! , \quad (13)$$

where \bar{n} is the initial mean photon number. From Table I of I we find

$$\begin{aligned} U_1 &= V_1 / \mu_{12} \mu_{13} , \\ U_2 &= V_1 / \mu_{23} \mu_{21} , \\ U_3 &= V_1 / \mu_{31} \mu_{32} , \end{aligned} \quad (14)$$

where $V_1^2 = \lambda_1^2(n+1)$, $V_2^2 = \lambda_2^2(n+2)$, $\mu_{ij} = \mu_i - \mu_j$ ($i, j = 1, 2, 3$). Substituting the V 's in (11) and (14) and then making use of the resulting (14) together with (9), (10),

and (11), we find the probability that n photons exist in the cavity at time t ,

$$\begin{aligned} p(n, t) &= \sum_{\alpha=a,b,c} \langle \alpha, n | \rho(t) | \alpha, n \rangle \\ &= p_0(n-1) |A(n, t)|^2 + p_0(n) |B(n, t)|^2 \\ &\quad + p_0(n-2) |C(n, t)|^2 , \end{aligned} \quad (15)$$

where $p_0(n) = |Q(n)|^2$ is the initial photon distribution. In the present case, the cavity field is assumed to be in the coherent state and hence $p_0(n) = p_c(n)$. More explicitly, we have from (12), (13), and (15)

$$p(n, t) \begin{cases} e^{-\bar{n}} |B(0, t)|^2 & \text{for } n=0 \\ e^{-\bar{n}} [|A(1, t)|^2 + \bar{n} |B(1, t)|^2] & \text{for } n=1 \\ p_c(n) \left[\frac{n}{\bar{n}} |A(n, t)|^2 + |B(n, t)|^2 + \frac{n(n-1)}{\bar{n}^2} |C(n, t)|^2 \right] & \text{for } n \geq 2 . \end{cases} \quad (16a)$$

$$(16b)$$

$$(16c)$$

With the probabilities $p(n, t)$ given by (16), the Q parameter can be calculated directly from (1) if we recall that the mean values are given by

$$\langle n \rangle = \sum_n n p(n, t) , \quad (17)$$

$$\langle n^2 \rangle = \sum_n n^2 p(n, t) . \quad (18)$$

We first study the time evolution of the photon distribution for different detunings. Throughout this paper, we have assumed $\lambda_1 = \lambda_2 = \lambda$ in our numerical computation. The detuning parameters Δ_i ($i=1, 2$) are measured in the unit of λ , and the time t is measured in $1/\lambda$. In Fig. 2 we plot the time evolution of the photon probability distribution for a fixed Δ_1 and different Δ_2 . The figures depict how the probability variation with time is influenced by changing detunings. It is observed that as long as $t > 0$, the photon distribution curve changes shape all the time. This implies that the nonlinear coupling with the atom has changed statistical properties of the field. Generally speaking, the $p(n, t)$ curve becomes alternatively flatter and sharper than the original coherent state distribution. The most probable photon number also changes oscillatory with time. When $p(n, t)$ is flatter than the initial Poisson distribution, it is super-Poissonian with a large fluctuation, and a sharper curve means sub-Poissonian distribution with small fluctuation.

In the short-time regime, a larger most probable photon number than its initial value corresponds to the radiation process, while a smaller most probable photon number corresponds to the absorption process. As t becomes greater than π , multipeak structure appears in the $p(n, t)$ curve. This reflects the coherence in summing over the coefficients of the atomic states. As can be seen from Eq. (16c), the photon distribution is a sum of three terms corresponding to the three atomic level occupation probabilities. Each of these terms involves three sets of

beat frequencies as has been discussed in great detail in I.

As t increases further, the distribution curve recovers a Poissonian shape. This approximately periodic appearance of nearly coherent state is consistent with the results of II. The situation is illustrated in Fig. 3, in which we plot the long-time behavior of $p(n, t)$.

To study the detuning dependence of the photon distribution, we calculate for fixed n and Δ_1 , $p(n, t)$ as a function of Δ_2 as shown in Figs. 4 and 5. Both the figures demonstrate the strong interference effect when t is large. It is observed that all the curves in Fig. 4 for $\Delta_1 = 0$ are symmetric with respect to $\Delta_2 = 0$, and those in Fig. 5 for $\Delta_1 = 5$ are all asymmetric with respect to $\Delta_2 = 0$. In other words, a dispersion phenomenon^{17,19} appears when mode 1 is off resonance. We note that the atomic level occupation probabilities have similar features, as has been discussed in I.

A straightforward way to determine whether the photon distribution is sub-Poissonian or super-Poissonian is to look at the Q parameter which can be obtained by plugging (17) and (18) in (1). In what follows we calculate Q numerically as a function of time for various cases. The results are presented in Figs. 6 and 7. It is clear that the photon distribution oscillates between super-Poissonian ($Q > 0$) and sub-Poissonian ($Q < 0$). Q is a nearly periodic function of the time. When the curve intersects the time axis, it is nearly Poisson distribution in agreement with the results of II. The period of oscillation becomes shorter as $|\Delta_1 - \Delta_2|$ increases, namely, as the system deviates farther from the two-photon resonance. At two-photon resonance, the amplitude of oscillation is modulated as in Figs. 6(d) and 7(c). It is also seen from these figures that minimum photon-number fluctuation can be achieved at certain times at the two-photon resonance. Thus it appears clear that the two-photon processes result in sub-Poissonian distribution. When the field-atom coupling is far from one-photon resonance but satisfies two-photon resonance, the coupling

is mediated only via two-photon processes and the resulting photon distribution is sub-Poissonian.

The long-time behavior of Q is depicted in Fig. 8 for two different cases, i.e., off and on (two-photon) resonance. It is observed that the oscillatory variation of the photon-number fluctuation with time shows collapse and revival phenomenon. During the time between collapse and revival, the field stays in sub-Poissonian distribution for off-resonance and in super-Poissonian distribution for on-resonance cases. A comparison of Figs. 6 and 3 reveals that fluctuations remain more or less unchanged for

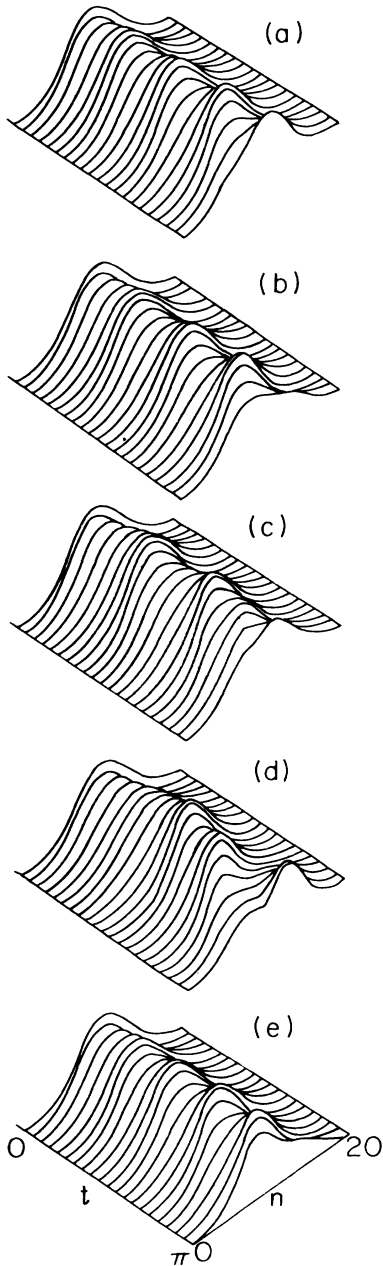


FIG. 2. Time evolution of the photon-number distribution for $\Delta_1=5$ and $\bar{n}=10$. (a) $\Delta_2=-10$, (b) $\Delta_2=-5$, (c) $\Delta_2=0$, (d) $\Delta_2=5$, (e) $\Delta_2=10$.

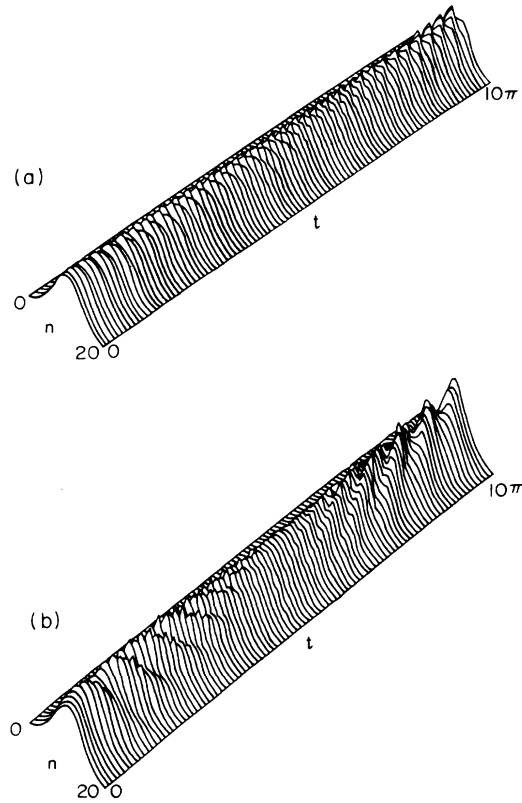


FIG. 3. Long-time behavior of the photon-number distribution for $\Delta_1=5$, $\bar{n}=10$. (a) $\Delta_2=-5$ and (b) $\Delta_2=5$.

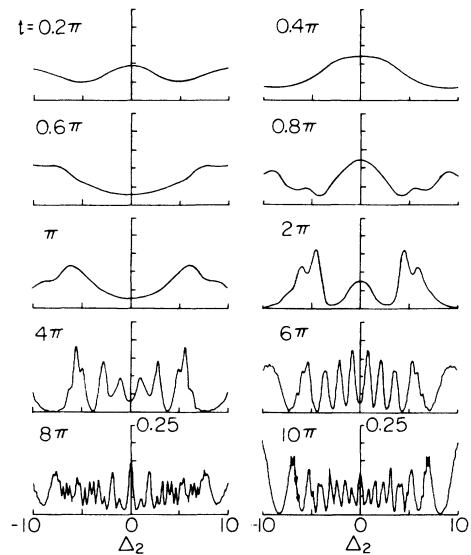


FIG. 4. $p(n,t)$ vs Δ_2 for $\Delta_1=0$ (at resonance), $n=10$, and $\bar{n}=10$ at different times.

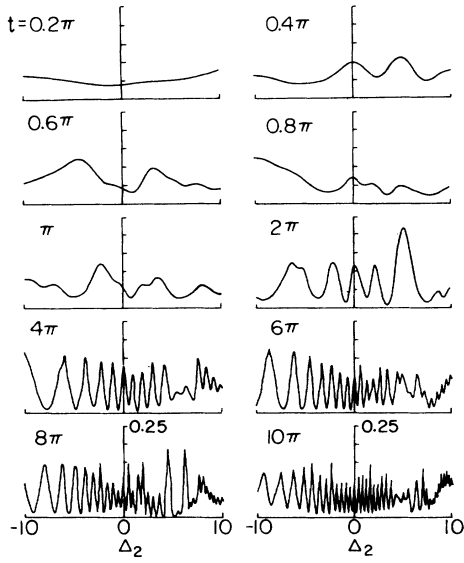


FIG. 5. $p(n, t)$ vs Δ_2 for $\Delta_1 = 5$ (off resonance), $n = 10$, and $\bar{n} = 10$ at different times.

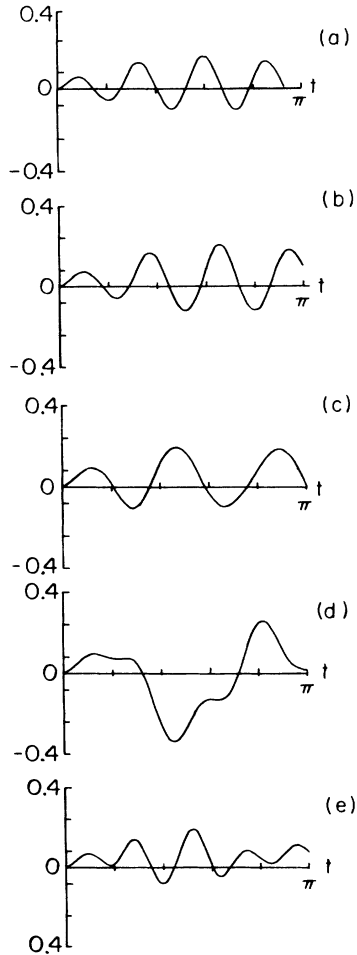


FIG. 6. Q parameter vs t for $\Delta_1 = 5$ and $\bar{n} = 10$. (a) $\Delta_2 = -10$, (b) $\Delta_2 = -5$, (c) $\Delta_2 = 0$, (d) $\Delta_2 = 5$, (e) $\Delta_2 = 10$.

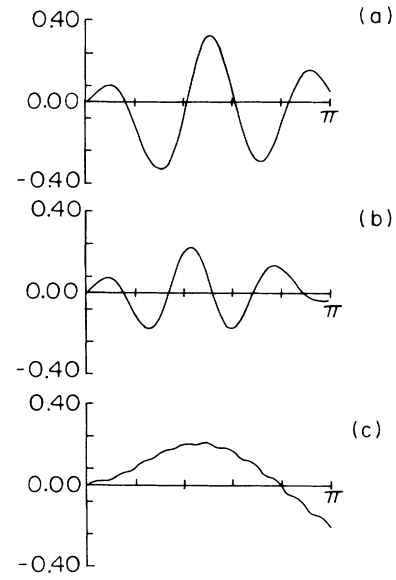


FIG. 7. Q parameter vs t for $\bar{n} = 10$. (a) $\Delta_1 = \Delta_2 = 0$, (b) $\Delta_1 = 0$, $\Delta_2 = \pm 5$, (c) $\Delta_1 = \Delta_2 = 20$.

a long time after the collapse of oscillatory varying Q , but the probability distribution keeps changing all the time.

B. Two-mode Λ type

We now consider the atom with Λ -type level structure interacting with a two-mode cavity field. The initial conditions are that the atom is in the state $|a\rangle$ and the field is again in the coherent state with initial photon distribution

$$p_c(n_1, n_2) = e^{-(\bar{n}_1 + \bar{n}_2)} \bar{n}_1^{n_1} \bar{n}_2^{n_2} / n_1! n_2! . \quad (19)$$

We first look up from Table I of I

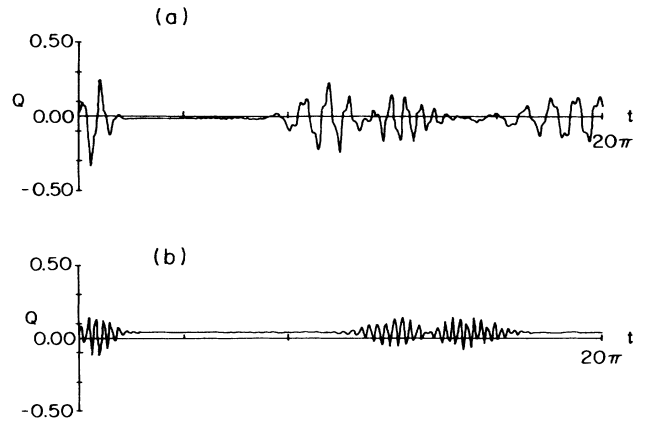


FIG. 8. Long-time behavior of Q for $\Delta_1 = 5$ and $\bar{n} = 10$. (a) $\Delta_2 = -5$, (b) $\Delta_2 = 5$.

$$\begin{aligned}
 U_1 &= -(\mu_1 + \Delta_{12})/\mu_{12}\mu_{13}, \\
 U_2 &= -(\mu_2 + \Delta_{12})/\mu_{21}\mu_{23}, \\
 U_3 &= -(\mu_3 + \Delta_{12})/\mu_{31}\mu_{32},
 \end{aligned}
 \tag{20}$$

where $\Delta_{12} = \Delta_1 - \Delta_2$, $V_i^2 = \lambda_i^2(n_i + 1)$, $i = 1, 2$. The photon-number distribution for a two-mode field is

$$\begin{aligned}
 p(n_1, n_2, t) &= \sum_{\alpha=a,b,c} \langle \alpha, n_1, n_2 | p(t) | \alpha, n_1, n_2 \rangle \\
 &= p_c(n_1, n_2) \left\{ |A(n_1, n_2, t)|^2 \right. \\
 &\quad + \frac{n_1}{\bar{n}_1} |B(n_1, n_2, t)|^2 \\
 &\quad \left. + \frac{n_2}{\bar{n}_2} |C(n_1, n_2, t)|^2 \right\}.
 \end{aligned}
 \tag{21}$$

Like what has been done in II, we consider only mode 1 for simplicity. Thus we shall calculate numerically in this paper the photon probability distribution

$$p(n_1, t) = \sum_{n_2} p(n_1, n_2, t) \tag{22}$$

and the Q parameter

$$Q = \frac{\langle n_1^2 \rangle - \langle n_1 \rangle^2}{\langle n_1 \rangle} - 1, \tag{23}$$

where

$$\langle n_1^2 \rangle = \sum_{n_1, n_2} n_1^2 p(n_1, n_2, t), \tag{24}$$

$$\langle n_1 \rangle = \sum_{n_1, n_2} n_1 p(n_1, n_2, t). \tag{25}$$

Once more, the units employed in our calculation are λ for energy and $1/\lambda$ for time, and we still assume $\lambda_1 = \lambda_2 = \lambda$. In Fig. 9 we plot the photon distribution for different cases. The curve changes shape more appreciably and the multipeak structure appears earlier than the corresponding one-mode case. It is also noted that the peak never narrows, implying super-Poissonian distribution all the time. The peak height, however, oscillates as a function of time. The period of oscillation is shorter when Δ_1 and Δ_2 have the same sign than it is when the detunings have opposite signs. This is probably due to the ac Stark effect.²⁰

We now turn our attention to fluctuations and plot Q computed from Eq. (23) as a function of time in Fig. 10. Clearly, $Q > 0$ all the time in every case. Hence the fluctuation is always larger than its initial condition, and photon probability distribution remains super-Poissonian once the interaction starts. In general, Q is an oscillatory function of time and the frequency increases as the coupling deviates from the two-photon resonance, or as $|\Delta_1 - \Delta_2|$ increases. We also note that Q has a very different time dependence at two-photon resonance as in Fig. 10(d).

Finally, we investigate the dependence of the photon probability distribution upon the photon number n and detuning parameters. Figure 11 shows for fixed Δ_1 and n_2 the distribution $p(n_1, n_2)$ as a function of n_1 for various Δ_2 and t . The curve starts with a Poissonian form, changes continuously because of the interaction, and gradually develops into multiple peaks. At the beginning, the influence of Δ_2 on coherence effects between

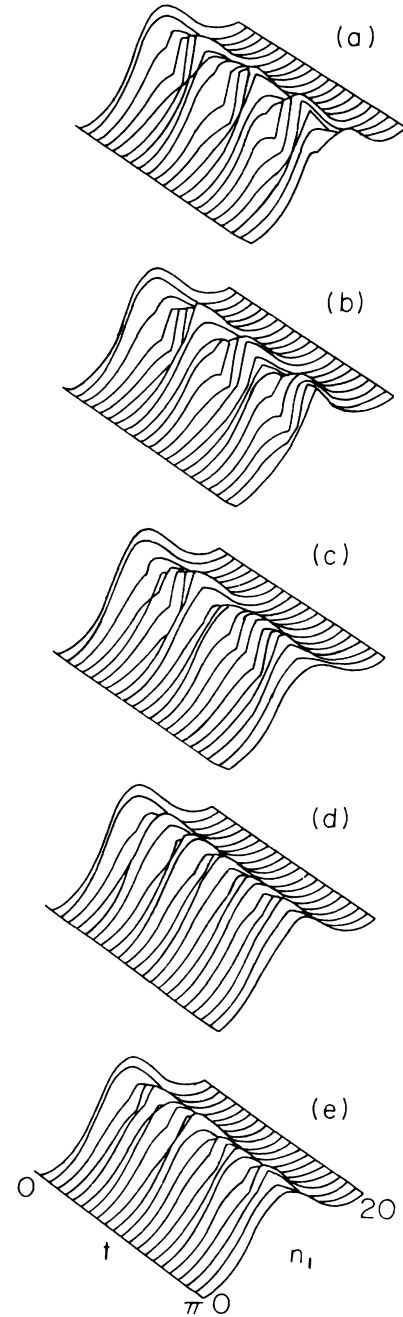


FIG. 9. Time evolution of photon-number distribution of mode 1 for $\Delta_1 = 5$, $\bar{n}_1 = 10$, $\bar{n}_2 = 20$. (a) $\Delta_2 = -10$, (b) $\Delta_2 = -5$, (c) $\Delta_2 = 0$, (d) $\Delta_2 = 5$, (e) $\Delta_2 = 10$.

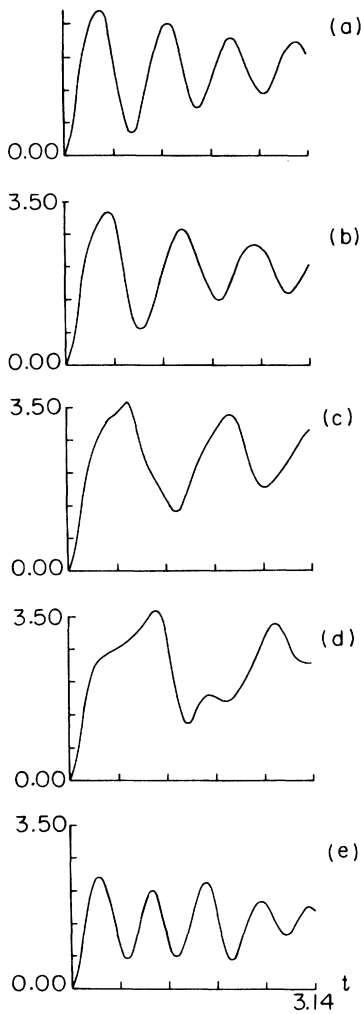


FIG. 10. Q of mode 1 vs t for $\bar{n}_1=10$, $\bar{n}_2=20$, and $\Delta_1=5$. (a) $\Delta_2=-10$, (b) $\Delta_2=-5$, (c) $\Delta_2=0$, (d) $\Delta_2=5$, (e) $\Delta_2=10$.

atomic levels is not very significant. Gradually, such an interference effect becomes remarkable, and the distributions corresponding to different Δ_2 values are totally different even though they are calculated for the same time with the same Δ_1 . In Fig. 12 we also show the probability distribution as a function of detuning parameters at a given instant of time. It is seen that at any time, the photon distribution is symmetric with respect to $\Delta_1=0$ for $\Delta_2=0$, asymmetric for $\Delta_2 \neq 0$, and antisymmetric with respect to $\Delta_2=0$. We recall from I that the atomic level occupation probabilities possess the same symmetry properties. We are not able to offer any simple explanations at this stage. We would, however, like to point out that a similar situation exists for the one-mode coupling even though we have not shown the curves explicitly. For instance, the mirror image of Fig. 5 can be obtained by setting $\Delta_1=-5$ instead of 5 in the computation.

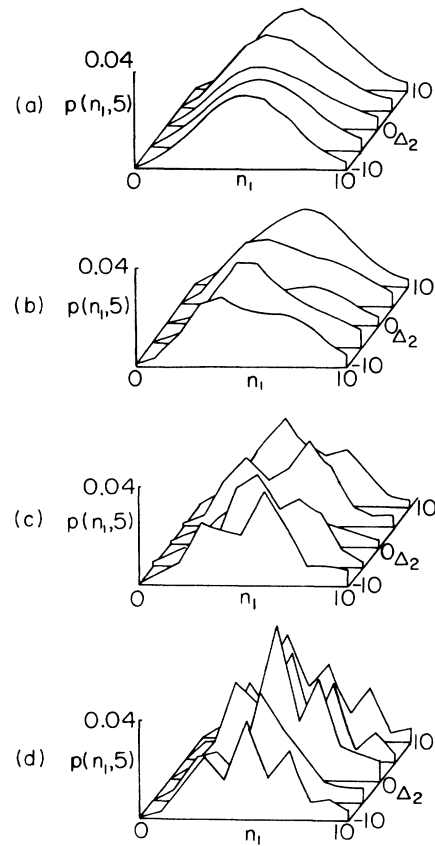


FIG. 11. $p(n_1, n_2)$ vs n_1 for $\bar{n}_1=\bar{n}_2=5$, $\Delta_1=5$, $n_2=5$, and various choices of time. (a) $t=\pi/2$, (b) $t=\pi$, (c) $t=2\pi$, (d) $t=4\pi$.

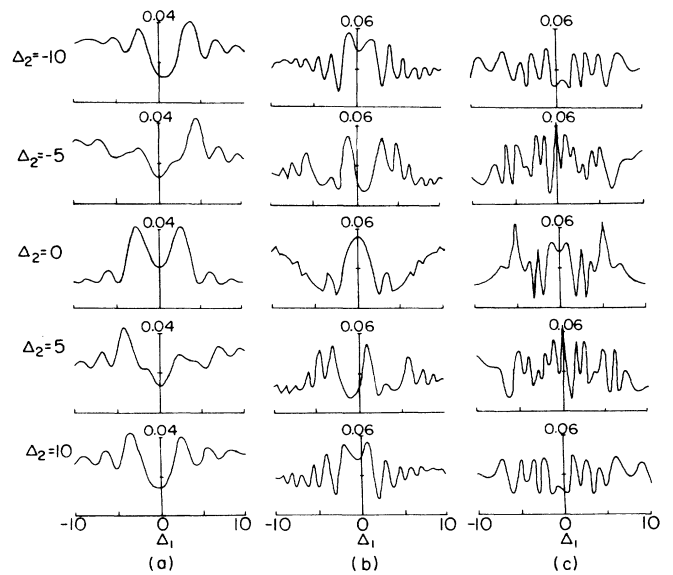


FIG. 12. $p(5, 5)$ vs Δ_1 for various choices of t and Δ_2 where $\bar{n}_1=\bar{n}_2=5$. (a) $t=\pi$, (b) $t=2\pi$, (c) $t=4\pi$.

IV. CONCLUSIONS

We have investigated the photon probability distribution and fluctuation in the interaction of a three-level atom with one- or two-mode cavity fields of arbitrary detunings. A large number of new results are obtained. In addition to what has been discussed above, we can also determine the bunching or antibunching effects in the interacting system. Since the normalized Glauber correla-

tion function $g_{(2)}(0)$ is related to the Q parameter by¹⁴

$$Q = \langle n \rangle [g^{(2)}(0) - 1], \quad (26)$$

we can draw the conclusion from our study that the degenerate two-photon processes result in sub-Poissonian distribution of the field and at the same time exhibit antibunching effects. Therefore photons involved in the one-mode Ξ -type interaction exhibit antibunching and in the two-mode Λ -type interaction exhibit bunching.

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