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Preparation of a pure number state and measurement of the photon statistics in a high-Q micromaser

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Two schemes to prepare pure number states in a high-Q micromaser with Rydberg atoms are proposed. In the first experiment the atoms are probed after the interaction with the cavity field and the number state is obtained via state reduction. In the second experiment the interaction time of ionic Rydberg atoms with the maser field can be controlled via an electric accelerating field. The velocity of the ions is adjusted in such a way that every ion emits a photon and the total number of photons is exactly known via the total number of passing ions. It is also shown how the photon statistics in the micromaser in general can be probed via the outgoing atoms.

I. INTRODUCTION

The photon statistical distribution in lasers and masers is of fundamental interest. We show how the recently developed micromaser[1] can be used to give new insight into the interdependence of photon statistics and measurement. As a special case we consider the preparation of a pure number state, which is interesting both from the point of view of measurement theory and precision measurements.

In a micromaser, two-level Rydberg atoms are prepared in the upper maser level and injected into a maser cavity with a high quality factor Q. It has been shown experimentally[1] that the maser oscillation can be sustained even when the atomic flux is so low that only one atom is in the cavity at a time. The theoretical description of the micromaser[2,3] shows that the photon statistics is essentially determined by the duration of the atom-field interaction, which is the atoms' time of flight through the cavity. In the experiment, this time can be controlled by a velocity selector for the atoms. For certain values of this parameter, the photon statistics is found to be sub-Poissonian. It is the aim of this paper to demonstrate how the statistics of the photon field in the cavity (which cannot be directly measured) is projected onto the statistics of the outgoing Rydberg atoms.

The present paper deals with two selected but separate topics. First we consider the inference of the maser photon statistics by "looking" at the Rydberg atoms as they exit the cavity. Next we consider how the atomic information may, in some conditions, be used to prepare a number state. In both cases we make use of the quantity $P_n^{|k\rangle}(m)$ which is the probability to measure n atoms in the lower state out of m atoms injected into the cavity in their upper states when the field is initially in a number state with k photons. The basic idea of our experimental scheme is to infer from the number of atoms in the lower state the number of photons emitted in the cavity.[4] To experimentally achieve a state of the field where the number of photons is exactly known, two conditions have to be fulfilled.

The first condition concerns the temperature. Thermal photons have to be suppressed because they do induce decay and influence the statistics so that a superposition of number states is obtained. We can eliminate thermal photons by cooling the cavity to a low enough temperature. The mean number of thermal photons for a frequency ν of about 20 GHz is 3×10^{-5} at $T=0.1$ K.

As a second condition we must not lose photons stored in the cavity for the duration of the experiment, i.e., we need a cavity in which losses can be neglected for this time. The photon lifetime is determined by a decay rate $\lambda\simeq\nu/Q$. The quality factor Q of the cavity can reach values of up to 10^{11}, and with a microwave frequency of about 20 GHz this results in photon lifetimes of several seconds. The effects of statistical losses on the photon statistics of the field are investigated in the following section. In Sec. III we look at the interaction between m

atoms and a field number state, and the result is used in Sec. IV to measure the photon statistics of the micromaser in Sec. V to prepare a number state via state reduction. In Sec. VI the preparation of a number state is reconsidered using a different scheme.

II. STATISTICAL DECAY OF NUMBER STATES

To understand the extent to which cavity losses would be detrimental to n-state preparation, we consider the decay of the photons in the cavity at a rate λ. The density-matrix equation of motion describing such a situation is given by

$$\frac{d}{dt}\rho_{m,m} = -\lambda m \rho_{m,m}(t) + \lambda(m+1)\rho_{m+1,m+1}(t) \ . \tag{1}$$

When this equation is solved, we then find the probability of being in the state $|m\rangle$, given we started with $|n\rangle$, to be[5]

$$\rho^{(n)}_{m,m} = \begin{bmatrix} n \\ m \end{bmatrix} e^{-\lambda m t}(1-e^{-\lambda t})^{n-m} \ . \tag{2}$$

This can be verified by substitution into Eq. (1).

Thus we see that the state $|n\rangle$ becomes an impure mixed state $\rho_{m,m}$ when cavity absorption takes place. However, if $\lambda t \ll 1$, i.e., if $\lambda \simeq \nu/Q$ is small, then we have

$$\rho^{(n)}_{m,m} \simeq \delta_{nm} \ .$$

This result is in agreement with the quantum theory of the beam splitter.[6] In such a device we consider two modes with equal frequency which are characterized by wave vector and polarization $(\mathbf{k},\lambda)$ and $(\mathbf{K},\Lambda)$, respectively. If $(\mathbf{k},\lambda)$ describes the incoming beam, the transmitted beam is described by $(\mathbf{k},\lambda)$ as well, while (K,Λ) describes the reflected beam.

The action of the beam splitter on the incoming beam can be described by an unitary operator

$$u_{\mathbf{k}\lambda} = \exp(a^{\dagger}_{\mathbf{k}\lambda}a_{\mathbf{K}\Lambda} - a^{\dagger}_{\mathbf{K}\Lambda}a_{\mathbf{k}\lambda}) \ . \tag{3}$$

If the initial state is a two-mode coherent state, the resulting state is again a coherent state, and the amplitudes behave like the classical amplitudes. If the initial state is given by a number state $|n;m\rangle$, where n refers to the mode $(\mathbf{k},\lambda)$ and m to $(\mathbf{K},\Lambda)$, then we have[6]

$$|\eta\rangle' = u_{\mathbf{k}\lambda}|n;m\rangle$$

with

$$|\eta\rangle' = \frac{1}{\sqrt{n!m!}}(a^{\dagger}_{\mathbf{k}\lambda}\cos\psi + a^{\dagger}_{K\Lambda}\sin\psi)^n \times(-a^{\dagger}_{\mathbf{k}\lambda}\sin\psi + a^{\dagger}_{\mathbf{K}\lambda}\cos\psi)^m|0;0\rangle \ . \tag{4}$$

Here the transmission coefficients are $r(\mathbf{k},\lambda) = r(\mathbf{K},\Lambda) = \cos\psi$ and the reflection coefficients $\rho(\mathbf{k},\lambda) = -\rho(\mathbf{K},\Lambda) = \sin\psi$. For $\psi = \pi/4$, we have a 50-50 beam splitter, and the number state $|2;0\rangle$, for example, will be transferred into the superposition state

$$|\eta\rangle' = \tfrac{1}{2}(|2;0\rangle + |0;2\rangle + \sqrt{2}|1;1\rangle) \ . \tag{5}$$

Hence we see that both absorption [Eq. (2)] and transmission losses are detrimental to the preparation of a number state $|n\rangle$.

III. INTERACTION OF THE ATOMS WITH A NUMBER STATE

The basic interest of our experiment is to obtain information about the photon statistics of the field in the cavity from the outgoing atoms and to use this information for manipulation of the experimental parameters. Therefore we have to look at the basic problem of the interaction of a single atom with a radiation field in a pure number state first.

We assume that our experiment takes place within a time interval $t \ll \lambda^{-1}$, where $\lambda \simeq \nu/Q$ is the cavity decay rate, and consider Rydberg two-level atoms initially prepared in their upper levels. These Rydberg atoms are then injected into the micromaser cavity at a very low rate such that only one atom is in the cavity at a given time. We assume that it interacts with only one cavity mode via a coupling constant g.

The Rydberg atoms are probed by a static electric field after they have left the cavity. The field is adjusted in such a way that all the atoms in the upper level are ion-

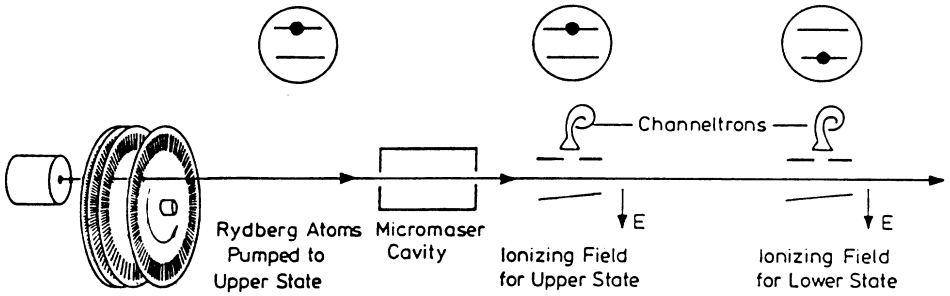

FIG. 1. Setup for the state reduction experiments: Two-level Rydberg atoms prepared in their upper levels are injected into a high-Q micromaser cavity. To obtain a fixed interaction time of the atoms they pass a Fizeau velocity selector before they enter the cavity. After they leave the cavity the atoms in the upper state and the atoms in the lower state are detected separately in different ionizing fields.

ized. The ground-state atoms which are not ionized have emitted a photon, and by counting those, the total number of photons emitted in the cavity can be inferred (cf. Fig. 1). It should be emphasized, however, that the number of atoms in the lower state is equal to the number of emitted photons only for a lossless cavity. It is also important to note that the detection efficiency, which is always less than one, should be as close to unity as possible to allow for a clear interpretation of the measurements.

When we assume that the field is in a number state $|\nu\rangle$ with ν photons (which can be the vacuum state $|0\rangle$), the time development operator for the interaction of a single two-level atom with the field is[7]

$$U(\tau)=\begin{bmatrix} \cos(g\tau\sqrt{\nu+1}) & -i\sin(g\tau\sqrt{\nu+1}) \\ -i\sin(g\tau\sqrt{\nu+1}) & \cos(g\tau\sqrt{\nu+1}) \end{bmatrix}, \tag{6}$$

where τ is the interaction time. Since the atom is in the upper level $|a\rangle$ at $t=0$, the combined density operator for atom and field is given by

$$\rho(0)=|a,\nu\rangle\langle a,\nu| , \tag{7}$$

while at $t=\tau$ the combined density operator reads

$$\begin{aligned}\rho(\tau)&=U(\tau)|a,\nu\rangle\langle a,\nu|U^{\dagger}(\tau)\\ &=\cos^2(g\tau\sqrt{\nu+1})|a,\nu\rangle\langle a,\nu|\\ &\quad+\sin^2(g\tau\sqrt{\nu+1})|b,\nu+1\rangle\langle b,\nu+1|\\ &\quad+i\sin(g\tau\sqrt{\nu+1})\cos(g\tau\sqrt{\nu+1})\\ &\quad\times(|b,\nu+1\rangle\langle a,\nu|-|a,\nu\rangle\langle b,\nu+1|) ,\end{aligned} \tag{8}$$

where $|b\rangle$ denotes the lower atomic level and $U(\tau)$ is the time development operator in the Jaynes-Cummings model.

By the interaction of the atom with the field the state of the field is changed to a superposition of the two number states $|\nu\rangle$ and $|\nu+1\rangle$. This result is obtained if we only know that an atom passed and we trace over all atomic states (as is usually done in maser theory). In this experiment, however, we get to know the state of the atom after it has left the cavity. As it will be shown later, it is not necessary in principle to perform this measurement before the next atom enters the cavity but we assume this here for simplicity.

If the state of the outgoing atom is now determined, the density matrix of the radiation field is reduced to

$$\rho(\tau)=|\nu\rangle\langle\nu| \tag{9a}$$

if the atom is in the upper state $|a\rangle$ or to

$$\rho(\tau)=|\nu+1|\rangle\langle\nu+1| \tag{9b}$$

if it is in the lower state $|b\rangle$. The first possibility (9a), i.e., the field remains in the state $|\nu\rangle$, is obtained with a probability $c(\nu)\equiv\cos^2(g\tau\sqrt{\nu+1})$, and the second (9b), i.e., the field makes a transition to the state $|\nu+1\rangle$, with a probability $s(\nu)\equiv\sin^2(g\tau\sqrt{\nu+1})$. The superposition of states $|\nu\rangle$ and $|\nu+1\rangle$ which is the state of the field after the interaction with the atom is then reduced to one of the states $|\nu\rangle$ or $|\nu+1\rangle$, depending on the result of the measurement.[4]

From this we obtain the probability $P_n^{|k\rangle}(m)$ to obtain n atoms in the lower state $|b\rangle$ out of m atoms that have passed, given that the field was initially in the number state $|k\rangle$. That is, we have initially $P_0^{|k\rangle}(0)=1$ and $P_n^{|k\rangle}(0)=0$ for $n\neq 0$. After one atom has passed, the only probabilities different from 0 are $P_0^{|k\rangle}(1)=c(k)$ and $P_1^{|k\rangle}(1)=s(k)$. For two atoms one has

$$\begin{aligned}P_0^{|k\rangle}(2)&=c(k)P_0^{|k\rangle}(1)=[c(k)]^2 ,\\ P_1^{|k\rangle}(2)&=c(k+1)P_1^{|k\rangle}(1)+s(k)P_0^{|k\rangle}(1)\\ &=s(k)[c(k)+c(k+1)] ,\\ P_2^{|k\rangle}(2)&=s(k+1)P_1^{|k\rangle}(1)=s(k)s(k+1) ,\end{aligned}$$

and for three atoms this becomes

$$\begin{aligned}P_0^{|k\rangle}(3)&=c(k)P_0^{|k\rangle}(2)=[c(k)]^3 ,\\ P_1^{|k\rangle}(3)&=c(k+1)P_1^{|k\rangle}(2)+s(k)P_0^{|k\rangle}(2)\\ &=s(k)\{[c(k)]^2+c(k)c(k+1)+[c(k+1)]^2\} ,\\ P_2^{|k\rangle}(3)&=c(k+2)P_2^{|k\rangle}(2)+s(k+1)P_1^{|k\rangle}(2)\\ &=s(k)s(k+1)[c(k)+c(k+1)+c(k+2)] ,\\ P_3^{|k\rangle}(3)&=s(k+2)P_2^{|k\rangle}(2)=s(k)s(k+1)s(k+2) .\end{aligned}$$

When $m-1$ atoms have passed, the field is in the state $|k+n\rangle$ with a probability $P_n^{|k\rangle}(m-1)$ and in the state $|k+n-1\rangle$ with a probability $P_{n-1}^{|k\rangle}(m-1)$. Then the probability that the field is in the state $|k+n\rangle$ after m atoms have traversed the cavity is simply

$$\begin{aligned}P_n^{|k\rangle}(m)&=c(k+n)P_n^{|k\rangle}(m-1)\\ &\quad+s(k+n-1)P_{n-1}^{|k\rangle}(m-1) .\end{aligned} \tag{10}$$

With this recursion relation we can find a general expression

$$P_n^{|k\rangle}(m)=\prod_{i=0}^{n-1}s(k+i)\sum_{\substack{i_j=0\\(i_{m-1}\le\cdots\le i_n)}}\prod_{j=n}^{m-1}c(k+i_j) , \tag{11a}$$

where the definition is

$$\sum_{\substack{i_j=0\\(i_{m-1}\le\cdots\le i_n)}}^{n}\prod_{j=n}^{m-1}c(k+i_j)\equiv\begin{cases}\displaystyle\sum_{i_n=0}^{n}\sum_{i_{n+1}=0}^{i_n}\cdots\sum_{i_{m-1}=0}^{i_{m-2}}\prod_{j=n}^{m-1}c(k+i_j) & \text{for } m>n ,\\ 1 \text{ for } m=n , \\ 0 \text{ for } m<n .\end{cases} \tag{11b}$$

The proof of this equation, which is quite elementary, is given in the Appendix.

From this it follows as well that we do not know prior to the experiment which number of atoms in the lower state we are going to prepare. We are only able to calculate an *a priori* probability to obtain a certain number of atoms in the lower state. Therefore this experiment has to be performed repeatedly with a constant number of atoms so that the probability distribution can be measured and the prediction checked. However, this *a priori* probability must not be confused with the photon statistics of the radiation field. After the experiment, the field is always in a number state whose photon statistics are described by a δ function.

The distribution of the *a priori* probabilities can be evaluated numerically, and two examples are shown in Fig. 2. The distribution becomes very narrow for larger atom numbers because the probability $s(n)$ to add one photon to the field gets very small for $g\tau\sqrt{n+1}\simeq j\pi(j=1,2,\ldots)$. If it has an exact zero at some natural number n_0, a number state is obtained there in the steady state. This has been proposed as a scheme

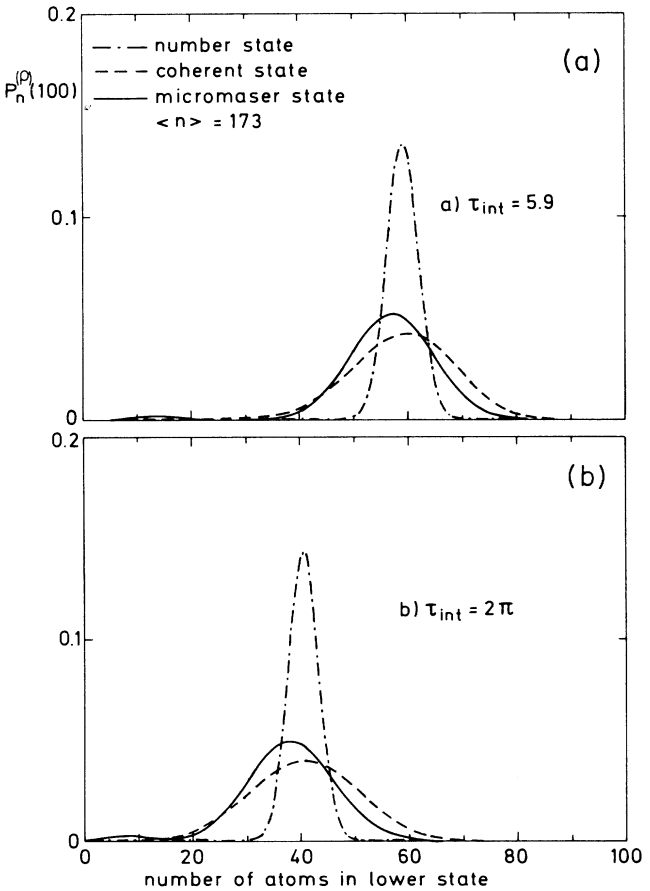

FIG. 2. Probability of finding n atoms in the lower level after $m=100$ atoms have passed the cavity, calculated for a number state, a coherent state, and a micromaser state (Refs. 2 and 3): (a) scaled interaction time $g\tau\sqrt{r/\gamma}\equiv\tau_{\rm int}=5.9\simeq 1.9\pi$; (b), $\tau_{\rm int}=2\pi$.

to prepare pure number states by Filipowicz *et al.*[8] For such an experiment, the precision of the atomic velocity is crucial, as opposed to our state reduction scheme. If $s(n)$ has an approximate zero at n_0, another peak in the probability distribution will build up at a larger photon number.

IV. MEASUREMENT OF THE PHOTON STATISTICS IN A MICROMASER

In this section we discuss an idealized sequence of measurements which allow us to infer the photon distribution by looking at two-level atoms which have interacted with the maser field as they exit the cavity.

Recall that the passage of active atoms through the cavity results in the generation of a steady-state photon distribution. That is to say, the atoms passed through the cavity, acting in concert with the cavity dissipation, yield a steady-state photon distribution as discussed by Filipowicz *et al.*[2] and by Lugiato *et al.*.[3] Given then that we have such a steady-state photon distribution we ask what is the probability that of m atoms injected in this upper state n are in the ground state upon leaving the microwave cavity. In particular we consider the experimental setup shown in Fig. 1. The atoms are probed for excitation only during a time window. If this time is short enough losses in the cavity can be neglected. After m atoms have passed the cavity, the rest of the beam is injected for a long time to restore the steady-state photon distribution.

In this way the experiment can be performed repeatedly to measure the statistics of the state distribution of the outgoing probe atoms. In this regard we are in conformity with the approach used to determine a statistical distribution of an ensemble of measurements which were made at various times and thus invoking the ergodic theorem to infer the distribution in a statistical sense.

We turn now to the special situation in which we have a number state for the radiation field. As we have shown in the preceding section the probability that an initially excited atom remains in the excited state is given by

$$P_a^{|k\rangle}=\cos^2(g\tau\sqrt{k+1})\ ,$$

and the probability that the atom makes a transition to the lower state is

$$P_b^{|k\rangle}=\sin^2(g\tau\sqrt{k+1})\ .$$

Here g is again the one photon Rabi frequency, τ the duration of the interaction, and k the total number of photons in the cavity. In our idealized experiment we neglect cavity losses, i.e., the total number of photons k is increased by the photons emitted by the atoms which make a transition to the lower state. The number of atoms in the lower state can be found by the recursion relation (10):

$$P_n^{|k\rangle}(m)=P_n^{|k\rangle}(m-1)\cos^2(g\tau\sqrt{n+k+1})+P_{n-1}^{|k\rangle}(m-1)\sin^2(g\tau\sqrt{n+k}\,) \qquad (12)$$

with the initial condition $P_n^{|k\rangle}(0)=\delta_{0n}$.

In general the radiation field is not in a pure number

state but it is described by a field density operator $\rho_F=\sum_{k,k'}\rho_{k,k'}|k\rangle\langle k'|$, and we are interested in the probability of finding n atoms in the lower state out of m atoms that have been injected in the upper state. We first look at the simple case $m=2$ and then generalize the result.

When two atoms pass the cavity one after another, we need two time development operators $U_1(\tau)$ and $U_2(\tau)$ which describe the interaction of the first and of the second atom with the field, respectively. Both operators contain a time step function to ensure that the atoms interact with the field one after another. We then start with the density matrix

$$\rho(0)=\sum_{k,k'}\rho_{k,k'}|k,a,a\rangle\langle k',a,a| , \tag{13}$$

i.e., both atoms are initially in the upper state. We calculate $\rho(t_f)=U_2(\tau)U_1(\tau)\rho(0)U_1^{\dagger}(\tau)U_2^{\dagger}(\tau)$ and trace over the field states to obtain the combined density operator for the two atoms:

$$\begin{aligned}\rho_A(t_f)=\sum_k\rho_{k,k}[&\cos^4(g\tau\sqrt{k+1})|a,a\rangle\langle a,a|+\sin^2(g\tau\sqrt{k+1})\cos^2(g\tau\sqrt{k+1})|a,b\rangle\langle a,b|\\&+\sin^2(g\tau\sqrt{k+1})\cos^2(g\tau\sqrt{k+2})|b,a\rangle\langle b,a|+\sin^2(g\tau\sqrt{k+1})\sin^2(g\tau\sqrt{k+2})|b,b|\rangle\langle b,b|\\&+\sin^2(g\tau\sqrt{k+1})\cos(g\tau\sqrt{k+1})\cos(g\tau\sqrt{k+2})(|a,b\rangle\langle b,a|+|b,a\rangle\langle a,b|)] .\end{aligned} \tag{14}$$

From this we immediately obtain the probabilities (i) to find both atoms in the upper state, $P_0^{(\rho)}(2)=\sum_k\rho_{k,k}\cos^4(g\tau\sqrt{k+1})$, (ii) to find one atom in the upper and one atom in the lower state,

$$P_1^{(\rho)}(2)=\sum_k\rho_{k,k}\sin^2(g\tau\sqrt{k+1})[\cos^2(g\tau\sqrt{k+1})+\cos^2(g\tau\sqrt{k+2})] ,$$

and (iii) to find both atoms in the lower state,

$$P_2^{(\rho)}(2)=\sum_k\rho_{k,k}\sin^2(g\tau\sqrt{k+1})\sin^2(g\tau\sqrt{k+2}) .$$

Obviously this is the result of the recursion relation (12) for an initial number state $|k\rangle$, multiplied by the probability $\rho_{k,k}$ of this number state, and summed over all k. The same result can be obtained if one assumes that the field density operator is changed by the passage of an atom according to

$$\rho_F^{(a)}=N_a\sum_{k,k'}\cos(g\tau\sqrt{k+1})\cos(g\tau\sqrt{k'+1})\rho_{k,k'}|k\rangle\langle k'| \tag{15a}$$

for an atom that leaves the cavity in the upper state and

$$\rho_F^{(b)}=N_b\sum_{k,k'}\sin(g\tau\sqrt{k+1})\sin(g\tau\sqrt{k'+1})\rho_{k,k'}|k\rangle\langle k'| \tag{15b}$$

for an atom which leaves the cavity in the lower state.

$$N_a=\left[\sum_k\cos^2(g\tau\sqrt{k+1})\rho_{k,k}\right]^{-1}$$

and

$$N_b=\left[\sum_k\sin^2(g\tau\sqrt{k+1})\rho_{k,k}\right]^{-1}$$

are normalization factors.[9]

Although the first method is the correct one, e.g., allowing for the measurement of the first atom after the second without changing the result, the second method helps in generalizing the result. One finds that the probability $P_n^{(\rho)}(m)$ for a field characterized by a density matrix ρ is given by the corresponding probabilities for the number states:

$$P_n^{(\rho)}(m)=\sum_k\rho_{k,k}P_n^{|k\rangle}(m) , \tag{16}$$

where $P_n^{|k\rangle}(m)$ can be found from Eq. (11).

In Fig. 2 we have indicated a typical result for a number state, and we show the calculation for a coherent state whose diagonal matrix elements are given by

$$\rho_{k,k}^{|\alpha\rangle}=\frac{1}{k!}|\alpha|^{2k}\exp(-|\alpha|^2) \tag{17}$$

with

$$|\alpha|^2=\bar{k} .$$

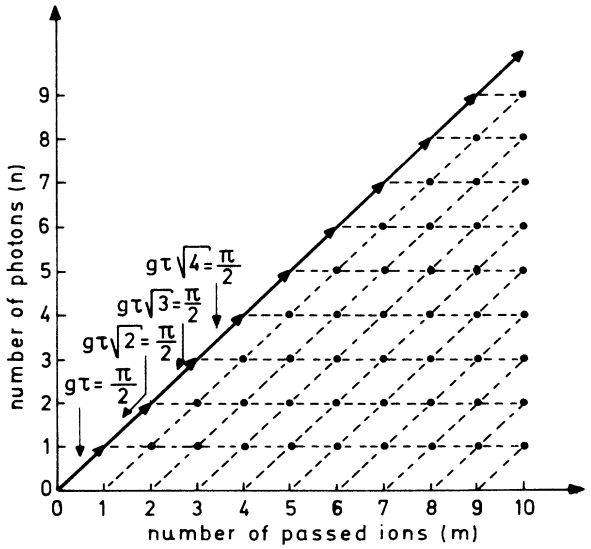

FIG. 3. Creation of pure number states in the cavity by adjusting the emission probabilities via manipulation of atomic velocities.

The field of the micromaser is characterized by the field distribution[7,8]

$$\rho_{k,k}^{\mu\ \text{maser}} = N \prod_{l=1}^{k} \left[\left[n_b \gamma + r \frac{\sin^2(g\tau\sqrt{l})}{l} \right] \frac{1}{\gamma(n_b+1)} \right] , \tag{18}$$

where r is the rate of incoming atoms, γ the cavity decay constants, and N a normalization constant. The result of the calculation for the micromaser is depicted in Fig. 2 as well.

V. n-STATE PREPARATION BY STATE REDUCTION

Two conceptually simple ways to create an n photon state are illustrated in this and the next section, which demonstrate the feasibility of n-state preparation in a micromaser.

In a first way, as discussed in this section, we are able to obtain a pure number state via the state reduction scheme[4] described in Sec. III, which was also applied to the measurement of the photon statistics in the cavity in the preceding section. Contrary to this experiment we now start the experiment with an empty cavity, i.e., the field is initially in the number state $|0\rangle$.

We inject Rydberg atoms in their upper levels and probe them by a static electric field after they have left the cavity. The atoms which are not ionized have emitted a photon, and by counting them, the total number of photons in the cavity can be inferred. Again we would like to emphasize the importance of avoiding cavity losses during the time of the experiment.

From this it follows as in the case of Sec. III that we do not know prior to the experiment which number state we are going to prepare. We are only able to calculate an *a priori* probability to obtain a certain number state. Therefore also this experiment has to be performed repeatedly with a constant number of atoms, and the measured probability distribution has to be compared to the prediction. However, as discussed before, this *a priori* probability must not be confused with the photon statistics. After the experiment, the field is always in a number state whose photon statistics are described by a δ function.

It is now easy to calculate the *a priori* probabilities from Eqs. (10) and (11) by setting $k=0$ there. The distribution of the *a priori* probabilities can be evaluated numerically, and an example is shown in Ref. 4 for different numbers of atoms.

VI. GENERATION OF A NUMBER STATE BY MANIPULATION OF ATOMIC VELOCITIES

When we are able to control the velocity of the atom beam, there is another possibility to prepare number states in a more straightforward way. We first look at the probability that the atom emits a cavity photon, given that the field is in a number state with n photons:

$$s(n) = \sin^2(g\tau\sqrt{n+1}) . \tag{19}$$

To obtain a probability of unity, for a given g and n, we have only to control the interaction time τ, which is the time of flight of the atom through the cavity. The change in atomic velocity necessary for our purposes cannot easily be imposed on neutral atoms. Therefore we envision using a beam of ions (e.g., alkaline earth atoms) which have lost one electron and the outermost electron is excited to the upper level of a Rydberg state pair. We can now accelerate these two-level Rydberg ions with an ejectable dc voltage. For the first atom, which enters the cavity when the field is in the vacuum state $|0\rangle$, we adjust the time of flight to be $\tau=\pi/2g$, so that the probability of emitting a photon is one. When the next atom enters the cavity, the field is in the state $|1\rangle$, and we change the velocity (i.e., change the dc voltage) so that $\tau=\pi/2g\sqrt{2}$ and that another photon is added to the field. Continuing, we have $\tau=\pi/2g\sqrt{n}$ for the nth ion, which will then add the nth photon to the field. This is summarized in Fig. 3.

We thus create a state with n photons by injecting n ions with selected velocities into the cavity. One way to prepare a suitable ion beam would be, e.g., to use a miniature storage ring. This allows one to prepare a bunch of ions with a predetermined ion number. In addition, the storage ring allows one to produce an ion beam with equally spaced ions if they are, e.g., cooled by photon recoil with nearly resonant laser radiation. It has been shown in molecular-dynamics calculations[10] that in such a case a regular array of ions can be obtained identical to a one-dimensional Coulomb lattice. In this way it can be guaranteed that the time between two successive ions is constant and always larger than the interaction time in the cavity, i.e., there are not two or more ions in the cavity at the same time.

In order to prepare the proper dc field for each ion we have only to add a constant increment to the field after the passage of each ion. In this way we produce an accelerating voltage proportional to the number of single ions that have passed through the cavity. The kinetic energy of the accelerated ions is then proportional to n, and therefore we have the desired result

$$v = \frac{2gl\sqrt{n}}{\pi}$$

and thus

$$\tau = \frac{\pi}{2g\sqrt{n}} , \tag{20}$$

where l is the length of the cavity.

In summary, we have shown how an n photon state can be produced via a conceptually simple experimental arrangement. Compared with our previous proposition[4] in the second scheme larger photon numbers, i.e., higher intensities, can be achieved within the time given by the cavity decay rate by the present means.

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APPENDIX

We prove Eq. (11a) with the help of the definition (11b). Together with the definition of the product symbol $\prod_{i=0}^{-1} s(i)=1$ the initial condition $P_n^{|k\rangle}(0)=\delta_{n0}$ is obtained for $m=0$. For $m=1$ we have

$$P_0^{|k\rangle}(1)=\prod_{i=0}^{-1} s(k+i)\sum_{i_0=0}^{0}\prod_{j=0}^{0} c(k+i_j)=c(k)\,,$$

$$P_1^{|k\rangle}(1)=\prod_{i=0}^{0} s(k+i)\times 1=s(k)\,,$$

and $P_n^{|k\rangle}(1)=0$ for $n\neq 0,1$.

Assuming that Eq. (11a) is correct for $m-1$, we get with the recursion relation (10):

$$\begin{aligned}
P_n^{|k\rangle}(m)&=c(k+n)\prod_{i=0}^{n-1} s(k+i)\sum_{\substack{i_j=0\\(i_{m-2}\le\cdots\le i_n)}}^{n-1}\prod_{j=n}^{m-2} c(k+i_j)+s(k+n-1)\prod_{i=0}^{n-2} s(k+i)\sum_{\substack{i_j=0\\(i_{m-2}\le\cdots\le i_{n-1})}}^{n-1}\prod_{j=n-1}^{m-1} c(k+i_j)\\
&=\prod_{i=0}^{n-1} s(k+i)c(k+n)\sum_{\substack{i_j=0\\(i_{m-1}\le\cdots\le i_{n+1})}}^{n}\prod_{j=n+1}^{m-1} c(k+i_j)+\prod_{i=0}^{n-1} s(k+i)\sum_{\substack{i_j=0\\(i_{m-1}\le\cdots\le i_n)}}^{n-1}\prod_{j=n-1}^{m-1} c(k+i_j)\\
&=\prod_{i=0}^{n-1} s(k+i)\Bigg[c(k+n)\sum_{i_{n+1}=0}^{n}\sum_{i_{n+2}=0}^{i_{n+1}}\cdots\sum_{i_{m-1}=0}^{i_{m-2}}\prod_{j=n+1}^{m-1} c(k+i_j)\\
&\qquad+\sum_{i_n=0}^{n-1} c(k+i_n)\sum_{i_{n+1}=0}^{i_n}\cdots\sum_{i_{m-1}=0}^{i_{m-2}}\prod_{j=n+1}^{m-1} c(k+i_j)\Bigg]\\
&=\prod_{i=0}^{n-1} s(k+i)\Bigg[\sum_{i_n=0}^{n} c(k+i_n)\sum_{i_{n+1}=0}^{i_n}\cdots\sum_{i_{m-1}=0}^{i_{m-2}}\prod_{j=n+1}^{m-1} c(k+i_j)\Bigg]\\
&=\prod_{i=0}^{n-1} s(k+i)\sum_{\substack{i_j=0\\(i_{m-1}\le\cdots\le i_n)}}^{n}\prod_{j=n}^{m-1} c(k+i_j)\,,
\end{aligned}$$

which proves Eq. (11a).

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