Statistical theory of Raman amplification and spontaneous generation in dispersive media pumped with a broadband laser

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We present a statistical theory of forward and backward Raman amplification and spontaneous generation in dispersive media pumped by a phase-diffusion field of arbitrary bandwidth. It is shown that, unless the Stokes input is correlated with the pump, the Stokes output and the pump are statistically orthogonal, even if the Stokes field builds up spontaneously. The growth of the average Stokes field amplitude is not affected by dispersion. A general expression is given for the average Stokes intensity which is valid both above and below the critical pump intensity for overcoming the decorrelating effects of group-velocity dispersion. If the length of the medium is much greater than the coherence length associated with the dephasing of the two waves, then the average intensity of stimulated Raman scattering (SRS) depends on the pump bandwidth. For relatively short medium lengths the average intensity of spontaneously initiated SRS is essentially independent of the bandwidth of a phase-diffusion pump field.

I. INTRODUCTION

The effect of group-velocity dispersion in stimulated Raman and Brillouin scattering (SRS and SBS) of broadband laser radiation has been the subject of numerous papers.[1–10] Yet all the theoretical treatments of this problem are approximate. The problem arises in the use of high-power broadband laser systems in important applications of these two nonlinear-optical processes such as generation of frequency-shifted laser radiation, pulse compression, and generation of phase-conjugate waves.[10] Morever, it is known that the undesirable presence and effects of SRS and SBS in other laser applications, such as laser-induced fusion[11] and fiber-optic communications,[12] can be suppressed by using broadband lasers and dispersive media. These are the reasons for the continuing theoretical interest in this problem and the motivation for improved theoretical treatments. Note that, in the case of forward SRS, if the variation of the index of refraction with frequency is small, the effects of dispersion can be neglected.[13–19] In the case of backward scattering, however, even if the medium is assumed to be nondispersive, the counter propagation itself introduces a group-velocity mismatch that cannot be neglected.

In the absence of group-velocity dispersion the Maxwell-Bloch equations for SRS in the one dimensional approximation, neglecting depletion of the pump and of the ground-state population, can be solved analytically for an arbitrary fluctuating pump field, and with a random Langevin force acting on the medium to simulate spontaneous Raman scattering.[14–19] From the analytic solutions one can easily then calculate, by averaging, observables such as the average amplitude (coherent component), average intensity, spectrum, and two-time intensity correlation of the Stokes field, as well as the pump-Stokes field cross correlation. Extensive theoretical work in this area has been carried out by Raymer and coworkers[15–18] for a phase diffusion and a chaotic pump field of arbitrary bandwidth. In the presence of dispersion, the Maxwell-Bloch equations for SRS and the mathematically similar SBS cannot be solved analytically for a fluctuating pump field. The prescribed theoretical approach[9] in this case is to, first, derive the equations of motion for the quantum operators corresponding to the various observables mentioned above; second, average these equations over all fluctuations, quantum and classical; and third, solve the resulting deterministic equations. The first significant theoretical work on SRS of broadband laser radiation in dispersive media was published by D'yakov.[1,2] He considers a Raman amplifier pumped by a phase-diffusion field, and uses a Dyson-type equation iterative method to calculate the average amplitude and intensity of the Stokes output field. The calculation is carried out in the first (Bourret) approximation and predicts, in agreement with experimental observation,[2,3] the existence of a critical pump intensity needed to overcome the decorrelating effects of dispersion. In subsequent papers,[4–7] a nonstatistical coupled mode theory is developed where the pump and Stokes fields are written as sums over discrete equidistant modes. The mode spacing and, hence, the pump bandwidth are assumed to be much greater than the Raman linewidth of the medium, and the interaction between cross modes of the pump and the Stokes fields is neglected. This model also predicts the existence of a critical pump intensity. Another treatment[8] that uses a Karhunen-Loeve expansion to study SRS of a chaotic pump with continuous spectrum is valid only for pump bandwidths much smaller than the Raman linewidth. Under this condition there is no critical pump intensity. In a previous paper on Raman amplification in dispersive media[9] we presented a mathematical technique for averaging exactly the integral equation of motion for the Stokes field amplitude over the fluctuations of a Markovian chaotic pump of arbitrary bandwidth. The average intensity, spectrum, and two-time intensity correlation, however, were calculated approximately using a Markovian expansion for the Stokes field amplitude, and not by averaging the corresponding equations of motion for these observables. In this approximation, it is predicted correctly that in the case of a narrow-band chaotic pump the Stokes gain is higher than in the case of a coherent pump.[9,17] Moreover, the amplification of a Stokes input that is perfectly correlated to the pump is described correctly for both narrow-band and broadband chaotic pumps. But, in the case of the pump bandwidth being much larger than the Raman linewidth, the Markovian modeling of the Stokes field cannot account for the growth, out of a coherent Stokes seed, of a non-Markovian field component that is quasicorrelated to the pump and grows with nearly the same gain as in the case of a coherent pump. As a consequence of this, the Stokes amplification calculated in Ref. 9 for the last case accounts only for the intensity of the Stokes field component that is uncorrelated to the pump, and whose gain is less than that of the quasicorrelated component. Also, the corresponding spectrum and two-time intensity correlation of the Stokes field should tend to reproduce those of the pump, just as in the case of the correlated Stokes input.

In this paper we study the Raman amplification of a Stokes signal and the concurrent spontaneous generation of Stokes radiation in a dispersive medium pumped by a phase-diffusion field of arbitrary bandwidth. Note that previous theoretical papers[1–10] on SRS in dispersive media do not deal with spontaneously initiated SRS. In Sec. II we describe the mathematical model giving the relevant Maxwell-Bloch equations, the initial condition of the medium, and the statistical properties of the pump and the intrinsic noise of the medium. In Sec. III we calculate the average amplitudes of the Stokes field and the Raman transition. We also calculate the cross correlation of the Stokes and the pump field. Finally, in Sec. IV we derive the equations of motion for the Stokes intensity operator and calculate the average intensity.

II. BASIC EQUATIONS

Consider a broadband laser field propagating through a dispersive Raman active medium. In the plane-wave approximation and assuming negligible depletion, the electric field of this wave, referred to as the pump, can be written as

$$\mathbf{E}_p(z,t)=E_p(t\pm z/v_p)e^{i(\omega_p t\pm k_p z)}+\text{c.c.}\ , \tag{1}$$

where $\omega_p=k_p v_p$ is the center frequency of the spectrum, v_p the group velocity, and $E_p(t\pm z/v_p)$ a fluctuating complex amplitude which is treated as a classical stochastic process. Note that for either forward or backward Raman scattering, the Stokes field is taken to propagate in the positive z direction, while the direction of the pump field is positive z for forward scattering and negative z for backward scattering. In this paper we model $E_p(t\pm z/v_p)$ with the well-known phase-diffusion field which has a constant real amplitude, while its phase is a Wiener-Levy Markovian process.[15,20] The mean value and the autocorrelation of the complex field amplitude are

$$\langle E_p(t\pm z/v_p)\rangle=0\ , \tag{2a}$$

$$\langle E_p(t\pm z/v_p)E_p^*(t'\pm z'/v_p)\rangle =E_{p0}^2\exp[-\tfrac{1}{2}\gamma_p|t-t'\pm(z-z')/v_p|]\ , \tag{2b}$$

where γ_p is the full width at half maximum (FWHM) of the Lorentzian spectrum, and E_{p0} the constant real amplitude.

The electric field of the Stokes radiation is treated as a quantum-mechanical operator and is written as

$$\hat{\mathbf{E}}_s(z,t)=\hat{E}_s^\dagger(z,t)e^{i(\omega_s t-k_s z)}+\hat{E}_s(z,t)e^{-i(\omega_s t-k_s z)}\ , \tag{3}$$

where $\omega_s=k_s v_s$ is the center frequency of the spectrum and v_s the group velocity. The quantum field amplitudes $\hat{E}_s^\dagger(z,t)$ and $\hat{E}_s(z,t)$ are effective multimode photon creation and annihilation operators, respectively, for the Stokes field.

The Maxwell-Bloch equations (or Heisenberg equations of motion) for the Stokes field operator $\hat{E}_s^\dagger$ and the slowly varying amplitude $\hat{Q}^\dagger$ of the collective atomic raising operator corresponding to the two-photon Raman transition in the medium can be written in the form[2,9,16]

$$\left[\frac{\partial}{\partial z}+\frac{\alpha}{2}\right]\hat{E}_s^\dagger(z,\tau)=-i\kappa_2 E_p(\tau+\mu z)\hat{Q}^\dagger(z,\tau)\ , \tag{4}$$

$$\left[\frac{\partial}{\partial \tau}-i\Delta+\frac{\Gamma}{2}\right]\hat{Q}^\dagger(z,\tau) =i\kappa_1 E_p^*(\tau+\mu z)\hat{E}_s^\dagger(z,\tau)+\hat{F}^\dagger(z,\tau)\ , \tag{5}$$

where $\tau=t-z/v_s$ is a retarded time and $\mu=(1/v_s \pm 1/v_p)$ measures the group velocity mismatch of the two waves for forward $(-)$ and backward $(+)$ Raman scattering. The constant α is the linear absorption coefficient of the medium at the Stokes frequency; Γ is the FWHM of the Raman line, and $\Delta=\omega_p-\omega_s-\omega_R$ the detuning from resonance, with ω_R being the center frequency of the Raman line. The coupling parameters κ_1 and κ_2 are defined in Refs. 13 and 16 for vibrational and electronic Raman scattering, and the relation between them in the meter-kilogram-second-ampere (MKSA) system of units is $\kappa_2=N\hbar\omega_s\kappa_1^*/(2\epsilon_s v_s)$, where N is the number density of the medium and ϵ_s its permittivity at the Stokes frequency. $\hat{F}^\dagger(z,\tau)$ is a collective quantum Langevin force operator describing the intrinsic noise of the medium.[21] The statistical properties of the Langevin force are described by the relations[2,16]

$$\langle \hat{F}^\dagger(z,\tau)\rangle=0\ , \tag{6a}$$

$$\langle \hat{F}^\dagger(z,\tau)\hat{F}(z',\tau')\rangle=\frac{\Gamma}{\rho_L}\delta(z-z')\delta(\tau-\tau')\ , \tag{6b}$$

$$\langle \hat{F}^\dagger(z,\tau)\hat{F}^\dagger(z',\tau')\rangle=\langle \hat{F}(z,\tau)\hat{F}(z',\tau')\rangle=0\ , \tag{6c}$$

where $\rho_L=AN$ is the linear number density of the medium, with A being the cross-sectional area of the interaction volume. The medium at position z is assumed to be in its ground state until the leading edge of the pump pulse arrives there at $\tau_0=-\mu z$. This initial state of the medium is described by the relations[16]

$$\langle \hat{Q}^{\dagger}(z,\tau_0)\rangle = \langle \hat{Q}(z,\tau_0)\rangle = 0 , \quad (7a)$$

$$\langle \hat{Q}^{\dagger}(z,\tau_0)\hat{Q}(z',\tau_0)\rangle = \frac{1}{\rho_L}\delta(z-z') , \quad (7b)$$

$$\langle \hat{Q}(z,\tau_0)\hat{Q}^{\dagger}(z',\tau_0)\rangle = 0 . \quad (7c)$$

It is assumed that during the interaction the depletion of the ground-state population is negligible.

III. AVERAGE AMPLITUDES AND CROSS CORRELATIONS WITH THE PUMP

The Heisenberg equations of motion (4) and (5) can be integrated to yield the following two integral equations:

$$\begin{aligned}\hat{E}_s^{\dagger}(z,\tau) = {} & \hat{E}_s^{\dagger}(0,\tau)e^{\frac{1}{2}\alpha z} - i\kappa_2\int_0^z e^{\frac{1}{2}\alpha(z_1-z)}E_p(\tau+\mu z_1)\hat{Q}^{\dagger}(z_1,\tau_0)e^{(-i\Delta+\Gamma/2)(\tau_0-\tau)}dz_1 \\ & - i\kappa_2\int_0^z e^{\frac{1}{2}\alpha(z_1-z)}dz_1\int_{\tau_0}^{\tau} e^{(-i\Delta+\Gamma/2)(\tau_1-\tau)}E_p(\tau+\mu z_1)\hat{F}^{\dagger}(z_1,\tau_1)d\tau_1 \\ & + \kappa_1\kappa_2\int_0^z e^{\frac{1}{2}\alpha(z_1-z)}dz_1\int_{\tau_0}^{\tau} e^{(-i\Delta+\Gamma/2)(\tau_1-\tau)}E_p(\tau+\mu z_1)E_p^*(\tau_1+\mu z_1)\hat{E}_s^{\dagger}(z_1,\tau_1)d\tau_1 , \end{aligned} \quad (8)$$

$$\begin{aligned}\hat{Q}^{\dagger}(z,\tau) = {} & \hat{Q}^{\dagger}(z,\tau_0)e^{(-i\Delta+\Gamma/2)(\tau_0-\tau)} + \int_{\tau_0}^{\tau} e^{(-i\Delta+\Gamma/2)(\tau_1-\tau)}\hat{F}^{\dagger}(z,\tau_1)d\tau_1 \\ & + i\kappa_1\int_{\tau_0}^{\tau} e^{(-i\Delta+\Gamma/2)(\tau_1-\tau)}E_p^*(\tau_1+\mu z)\hat{E}_s^{\dagger}(0,\tau_1)e^{-\frac{1}{2}\alpha z}d\tau_1 \\ & + \kappa_1\kappa_2\int_0^{\tau} e^{(-i\Delta+\Gamma/2)(\tau_1-\tau)}d\tau_1\int_0^z e^{\frac{1}{2}\alpha(z_1-z)}E_p^*(\tau_1+\mu z)E_p(\tau_1+\mu z_1)\hat{Q}^{\dagger}(z_1,\tau_1)dz_1 . \end{aligned} \quad (9)$$

From these equations we can calculate exactly the average amplitudes, as well as the cross correlations of the Stokes field and the atomic raising operator with the pump field.

A. Average Stokes field amplitude

On taking quantum expectation values and averaging over the classical fluctuations of the pump,[20] Eq. (8) gives

$$\langle E_s(z,\tau)\rangle = \langle E_s(0,\tau)\rangle e^{-\frac{1}{2}\alpha z} + \kappa_1\kappa_2E_{p0}^2\int_0^z e^{\frac{1}{2}\alpha(z_1-z)}dz_1\int_{\tau_0}^{\tau} e^{[-i\Delta+(\Gamma+\gamma_p)/2](\tau_1-\tau)}\langle E_s(z_1,\tau_1)\rangle d\tau_1 , \quad (10)$$

where the quantum expectation value is denoted by dropping the circumflex, and the classical average over the pump fluctuations by angular brackets. In carrying out the average over the phase-diffusion field we have used the relation

$$\langle E_p^*(\tau+\mu z_1)E_p(\tau_1+\mu z_1)E_s(z_1,\tau_1)\rangle = \langle E_p^*(\tau+\mu z_1)E_p(\tau_1+\mu z_1)\rangle\langle E_s(z_1,\tau_1)\rangle ,$$

which is exact owing to the statistical independence[20] of the phase increments $\varphi_p(\tau+\mu z_1)-\varphi_p(\tau_0)$ and $\varphi_p(\tau_1+\mu z_1)-\varphi_p(\tau_0)$, and the fact that $E_s(z_1,\tau_1)$ depends only on the latter of the two. Note that the decorrelation above is not valid for other types of stochastic pump fields, such as the chaotic field.[9] Equation (10) can be solved by Laplace transform techniques. The steady-state solution for $\Delta=0$ is

$$\langle E_s(z)\rangle = \langle E_s(z=0)\rangle \exp\left[\frac{1}{2}\left\{gI_p\frac{\Gamma}{\Gamma+\gamma_p}-\alpha\right\}z\right] , \quad (11)$$

where $I_p=2v_p\epsilon_pE_{p0}^2$ is the pump intensity and $gI_p=4\kappa_1\kappa_2E_{p0}^2/\Gamma$ is the well-known steady-state gain coefficient per unit length in the case of coherent pump and Stokes fields. As can be seen, for a phase-diffusion pump the growth of the coherent component of the Stokes field is not affected by dispersion. This is not so in the case of a chaotic pump, where the fluctuations in the intensity of the pump give higher gain than in Eq. (11), but the dispersion reduces it.[9] From Eq. (11) it can also be seen that if the Stokes input is completely incoherent, i.e., $\langle E_s(z=0)\rangle=0$, then the Stokes output is also completely incoherent.

B. Pump-Stokes field cross correlation

Multiplying both sides of Eq. (8) by $E_p^*(\tau+\mu z)$ and averaging,[20] we obtain the integral equation

$$\begin{aligned}\langle E_p^*(\tau+\mu z)E_s(z,\tau)\rangle = {} & \langle E_p^*(\tau)E_s(0,\tau)\rangle e^{-\frac{1}{2}(\alpha+|\mu|\gamma_p)z} \\ & + \kappa_1\kappa_2E_{p0}^2\int_0^z e^{\frac{1}{2}(\alpha+|\mu|\gamma_p)(z_1-z)}dz_1\int_{\tau_0}^{\tau} e^{(-i\Delta+\Gamma/2)(\tau_1-\tau)}\langle E_p^*(\tau_1+\mu z_1)E_s(z_1,\tau_1)\rangle d\tau_1 , \end{aligned} \quad (12)$$

where again the implicit field decorrelation is exact for the same reasons as in the case of Eq. (10). For $\Delta=0$, the steady-state solution for the pump-Stokes field cross correlation is

$$\langle E_p^*(z)E_s(z)\rangle=\langle E_p^*(z=0)E_s(z=0)\rangle \times \exp[\tfrac{1}{2}(gI_p-|\mu|\gamma_p-\alpha)z] \ . \tag{13}$$

In a lossless nondispersive medium, the gain for the cross correlation and, consequently, for the Stokes field component that is correlated to the pump is bandwidth independent, and equal to that in the case of coherent Stokes and pump fields. The effect of dispersion, through the time lag that it causes between the two waves, is to reduce this gain. Note that if $gI_p<|\mu|\gamma_p$, the cross correlation decreases with z, but this does not imply that the Stokes field decreases in intensity. Actually, it experiences a small gain which in the presence of dispersion cannot be extracted from Eq. (13), but must be calculated from the average total Stokes output intensity. If there is no correlation between the Stokes field and the pump at the input, i.e., $\langle E_p^*(z=0)E_s(z=0)\rangle=0$, then according to Eq. (13), no correlation develops through either the amplification or the spontaneous Raman generation process. This can also be seen, in the case of nondispersive media, by multiplying the analytic expression for the Stokes field in Eq. (19) of Ref. 16 by $E_p^*(\tau)$ and averaging. In the case of a nondispersive Raman generator pumped by a broadband ($\gamma_p \gg \Gamma$) pump,[14,16] the Stokes field that grows from noise tends to reproduce the pump spectrum, but there is no correlation between the two fields in the strict statistical sense.[22] In fact, as we have seen, the two waves are statistically orthogonal, i.e., $\langle E_p^*(z)E_s(z)\rangle=0$. The small difference in the two spectra is associated with a slow stochastic phase difference in the time domain which makes the stationary cross correlation equal to zero. Indeed, the fact that (i) the Stokes intensity in a nondispersive Raman generator pumped by either a coherent or a phase-diffusion field and (ii) the amplification of a coherent Stokes input in a nondispersive Raman amplifier pumped by a phase-diffusion field are both proportional[14–16] to $\exp(gI_pz)/(\pi gI_pz)^{1/2}$ is a manifestation of the lack of cross correlation between the fields. The Stokes field amplitude does not follow exactly the fluctuations of the pump, and this reduces the gain by the factor $(\pi gI_pz)^{1/2}$ compared to the case where the Stokes field amplitude is an exact replica of the fluctuating pump amplitude. Therefore, in the two cases above we can only speak of the Stokes field as being quasicorrelated with the pump.

C. Statistics of the Raman transition amplitude

Averaging Eq. (9) in a procedure similar to that used in the case of Eq. (8), we obtain

$$\begin{aligned}\langle Q^*(z,\tau)\rangle=&\, i\kappa_1\int_{\tau_0}^{\tau} e^{(-i\Delta+\Gamma/2)(\tau_1-\tau)}\langle E_p^*(\tau_1+\mu z)E_s(0,\tau_1)\rangle e^{-\frac{1}{2}(\alpha+|\mu|\gamma_p)z}d\tau_1\\ &+\kappa_1\kappa_2E_{p0}^2\int_{\tau_0}^{\tau} e^{(-i\Delta+\Gamma/2)(\tau_1-\tau)}d\tau_1\int_0^z e^{\frac{1}{2}(\alpha+|\mu|\gamma_p)(z_1-z)}\langle Q^*(z_1,\tau_1)\rangle dz_1 \ ,\end{aligned} \tag{14}$$

where the quantum expectation value of the operator $\hat{Q}^\dagger(z,\tau)$ is denoted by the conjugate amplitude $Q^*(z,\tau)$. For $\Delta=0$, the steady-state solution is

$$\begin{aligned}\langle Q^*(z)\rangle&=i\frac{2\kappa_1}{\Gamma}\langle E_p^*(z=0)E_s(z=0)\rangle\exp[\tfrac{1}{2}(gI_p-|\mu|\gamma_p-\alpha)z]\\ &=i\frac{2\kappa_1}{\Gamma}\langle E_p^*(z)E_s(z)\rangle \ .\end{aligned} \tag{15}$$

Thus, the average Raman transition amplitude is proportional to the pump-Stokes field cross correlation. Likewise, we can show that the steady-state pump-Raman transition amplitude cross correlation is given by

$$\langle E_p(z)Q^*(z)\rangle=i\kappa_2^{-1}\tfrac{1}{2}gI_p\frac{\Gamma}{\Gamma+\gamma_p}\langle E_s(z)\rangle \ . \tag{16}$$

Relations (15) and (16) reflect the parametric nature of stimulated Raman scattering. If the Stokes field has a component that is correlated with the broadband pump, Eq. (15) says that the Raman excitation (phonon or electronic) of the medium has a corresponding coherent monochromatic component. In the case of spontaneously initiated SRS, as the Stokes field tends to reproduce the pump spectrum ($\gamma_p \gg \Gamma$), the phonon wave tends to become monochromatic. On the other hand, when a coherent Stokes seed is amplified from a broadband pump below the critical pump intensity (see discussion in Sec. IV), and the Stokes output is essentially monochromatic, Eq. (16) says that the phonon wave is correlated with the pump. Of course, this does not imply that the phonon spectrum reproduces the pump spectrum. Unlike the Stokes field, the spectral width of the medium excitation cannot exceed the Raman linewidth Γ.

IV. AVERAGE STOKES INTENSITY

From Eqs. (4) and (5) we can derive the following coupled equations for the Stokes intensity Hermitian operator

$\hat{I}_s(z,\tau)=2v_s\epsilon_s\hat{E}_s^\dagger(z,\tau)\hat{E}_s(z,\tau)$

and the field-matter product operator $\hat{C}(z,\tau)=\hat{E}_s^{\dagger}(z,\tau)\hat{Q}(z,\tau)$:

$$\left[\frac{\partial}{\partial z}+\alpha\right]\hat{I}_s(z,\tau)=i\kappa_2'[E_p^*(\tau+\mu z)\hat{C}(z,\tau)-E_p(\tau+\mu z)\hat{C}^{\dagger}(z,\tau)] , \quad (17)$$

$$\left[\frac{\partial}{\partial \tau}+i\Delta+\frac{\Gamma}{2}\right]\hat{C}(z,\tau)=-i\kappa_1'E_p(\tau+\mu z)\hat{I}_s(z,\tau)+\hat{E}_s^{\dagger}(z,\tau)\hat{F}(z,\tau)+\left[\frac{\partial}{\partial \tau}\hat{E}_s^{\dagger}(z,\tau)\right]\hat{Q}(z,\tau) , \quad (18)$$

where $\kappa_1'=\kappa_1/(2\epsilon_s v_s)$ and $\kappa_2'=2\epsilon_s v_s\kappa_2$. The last term on the right-hand side (rhs) of Eq. (18) involves the time derivative of $\hat{E}_s^{\dagger}(z,\tau)$, which can be evaluated from Eq. (8) using Leibnitz's rule for differentiation of integrals. Integrating then the two equations above and performing a formal average leads to the following integral equations:

$$\langle I_s(z,\tau)\rangle=I_{s0}e^{-\alpha z}+i\kappa_2\int_0^z e^{\alpha(z_1-z)}[\langle E_p^*(\tau+\mu z_1)C(z_1,\tau)\rangle-\text{c.c.}]dz_1 , \quad (19)$$

$$\begin{aligned}
&\langle E_p^*(\tau+\mu z)C(z,\tau)\rangle\\
&\quad=\int_{\tau_0}^{\tau}e^{(i\Delta+\Gamma/2)(\tau_1-\tau)}\\
&\qquad\times\Bigg[\Bigg[\langle E_p^*(\tau+\mu z)\langle\hat{E}_s^{\dagger}(z,\tau_1)\hat{F}(z,\tau_1)\rangle_q\rangle\\
&\qquad\quad-i\kappa_2\int_0^z e^{\frac{1}{2}\alpha(z_1-z)}\Bigg\{\langle E_p^*(\tau+\mu z)E_p(\tau_1+\mu z_1)\langle\hat{F}^{\dagger}(z_1,\tau_1)\hat{Q}(z,\tau_1)\rangle_q\rangle\\
&\qquad\qquad+\int_{\tau_0}^{\tau}e^{(-i\Delta+\Gamma/2)(\tau_2-\tau_1)}\\
&\qquad\qquad\quad\times\Big\langle E_p^*(\tau+\mu z)\left[\left[i\Delta-\frac{\Gamma}{2}+\frac{\partial}{\partial\tau_1}\right]E_p(\tau_1+\mu z_1)\right]\\
&\qquad\qquad\quad\times\langle\hat{F}^{\dagger}(z_1,\tau_2)\hat{Q}(z,\tau_1)\rangle_q\Big\rangle d\tau_2\Bigg\}dz_1\Bigg]\\
&\qquad\quad-i\kappa_1'I_{s0}e^{-\alpha z}\langle E_p^*(\tau+\mu z)E_p(\tau_1+\mu z)\rangle\\
&\qquad\quad+\kappa_1\kappa_2\int_0^z e^{\alpha(z_1-z)}[\langle E_p^*(\tau+\mu z)E_p(\tau_1+\mu z)E_p^*(\tau_1+\mu z_1)C(z_1,\tau_1)\rangle\\
&\qquad\qquad-\langle E_p^*(\tau+\mu z)E_p(\tau_1+\mu z)E_p(\tau_1+\mu z_1)C^*(z_1,\tau_1)\rangle]dz_1\\
&\qquad\quad+\kappa_1\kappa_2\int_0^z e^{\frac{1}{2}\alpha(z_1-z)}\Bigg\{\langle E_p^*(\tau+\mu z)E_p(\tau_1+\mu z_1)E_p^*(\tau_1+\mu z_1)\langle\hat{E}_s^{\dagger}(z_1,\tau_1)\hat{Q}(z,\tau_1)\rangle_q\rangle\\
&\qquad\qquad+\int_{\tau_0}^{\tau_1}e^{(-i\Delta+\Gamma/2)(\tau_2-\tau_1)}\\
&\qquad\qquad\quad\times\Big\langle E_p^*(\tau+\mu z)\left[\left[i\Delta-\frac{\Gamma}{2}+\frac{\partial}{\partial\tau_1}\right]E_p(\tau_1+\mu z_1)\right]E_p^*(\tau_2+\mu z_1)\\
&\qquad\qquad\quad\times\langle\hat{E}_s^{\dagger}(z_1,\tau_2)\hat{Q}(z,\tau_1)\rangle_q\Big\rangle d\tau_2\Bigg\}dz_1\Bigg]d\tau_1 ,
\end{aligned} \quad (20)$$

where $\langle\ \rangle_q$ denotes quantum average, and for simplicity in this section we assume that the Stokes input field is coherent and its intensity is I_{s0}. Note that in obtaining Eq. (20) we have neglecting a transient term corresponding to the time derivative of the second term in Eq. (8) because we are interested in the calculation of the average Stokes intensity only in the steady-state case.

The three correlation functions on the rhs of Eq. (20) which involve the Langevin force can be evaluated exactly. Multiplying Eq. (8) by $\hat{F}(z,\tau)$ and averaging gives

$$\begin{aligned}
\langle\hat{E}_s^{\dagger}(z,\tau)\hat{F}(z,\tau)\rangle_q&=-i\kappa_2\int_0^z e^{\frac{1}{2}\alpha(z_1-z)}dz_1\int_{\tau_0}^{\tau}e^{(-i\Delta+\Gamma/2)(\tau_1-\tau)}E_p(\tau+\mu z_1)\langle\hat{F}^{\dagger}(z_1,\tau_1)\hat{F}(z,\tau)\rangle d\tau_1\\
&=-i\kappa_2\frac{\Gamma}{4\rho_L}E_p(\tau+\mu z) .
\end{aligned} \quad (21)$$

Note that there is no correlation between $\hat{F}(z,\tau)$ and $\hat{E}_s^\dagger(z_1,\tau_1)$ for $z_1<z$ and $\tau_1<\tau$ in the last term of Eq. (8), because the generated Stokes field at a given point in spacetime does not depend on the Langevin force at later times. Multiplying the Hermitian conjugate of Eq. (9) by $\hat{F}^\dagger(z_1,\tau_1)$ and averaging gives

$$\langle \hat{F}^\dagger(z_1,\tau_1)\hat{Q}(z,\tau_1)\rangle_q=\int_{\tau_0}^{\tau_1} e^{(i\Delta+\Gamma/2)(\tau_2-\tau_1)}\langle \hat{F}^\dagger(z_1,\tau_1)\hat{F}(z,\tau_2)\rangle d\tau_2 = \frac{\Gamma}{2\rho_L}\delta(z-z_1)\ . \tag{22}$$

For the third term on the rhs of Eq. (20) which involves the Langevin force the averaging leads to

$$\left\langle E_p^*(\tau+\mu z)\left[\left[i\Delta-\frac{\Gamma}{2}+\frac{\partial}{\partial\tau_1}\right]E_p(\tau_1+\mu z_1)\right]\langle \hat{F}^\dagger(z_1,\tau_2)\hat{Q}(z,\tau_1)\rangle_q\right\rangle$$
$$=\frac{\Gamma}{\rho_L}e^{(i\Delta+\Gamma/2)(\tau_2-\tau_1)}\delta(z-z_1)\left[i\Delta-\frac{\Gamma}{2}+\frac{\partial}{\partial\tau_1}\right]\langle E_p^*(\tau+\mu z)E_p(\tau_1+\mu z)\rangle$$
$$=\frac{\Gamma}{\rho_L}e^{(i\Delta+\Gamma/2)(\tau_2-\tau_1)}\delta(z-z_1)[i\Delta-\tfrac{1}{2}(\Gamma-\gamma_p)]E_{p0}^2e^{-\frac{1}{2}\gamma_p(\tau-\tau_1)}\ , \tag{23}$$

where we have used the fact that the order of differentiation $(\partial/\partial\tau_1)$ and averaging can be interchanged.[22] Using Eqs. (21)–(23) in conjunction with Eq. (2b), the expression for the first three correlations on the rhs of Eq. (20) reduces to

$$-i\frac{\kappa_2}{2\rho_L}E_{p0}^2e^{-\frac{1}{2}\gamma_p(\tau-\tau_1)}\{[i\Delta+\tfrac{1}{2}(\Gamma+\gamma_p)] -[i\Delta-\tfrac{1}{2}(\Gamma-\gamma_p)]e^{-\Gamma(\tau_1-\tau_0)}\}\ . \tag{24}$$

The two correlations on the rhs of Eq. (20) which involve $C(z_1,\tau_1)$ and $C^*(z_1,\tau_1)$ can be also evaluated exactly and they reduce to

$$E_{p0}^2e^{-\frac{1}{2}\gamma_p(\tau-\tau_1)}[\langle E_p^*(\tau_1+\mu z_1)C(z_1,\tau_1)\rangle-\text{c.c.}]\ . \tag{25}$$

For the first correlation function inside the last set of curly brackets on the rhs of Eq. (20) we have

$$\langle E_p^*(\tau+\mu z)E_p(\tau_1+\mu z_1)E_p^*(\tau_1+\mu z_1)\times\langle \hat{E}_s^\dagger(z_1,\tau_1)\hat{Q}(z,\tau_1)\rangle_q\rangle =E_{p0}^2\langle E_p^*(\tau+\mu z)\langle \hat{E}_s^\dagger(z_1,\tau_1)\hat{Q}(z,\tau_1)\rangle_q\rangle\ . \tag{26}$$

The average $\langle \hat{E}_s^\dagger(z_1,\tau_1)\hat{Q}(z,\tau_1)\rangle_q$ is a two-point, one-time cross correlation and cannot be evaluated exactly. We can, however, relate it, to a good approximation, with the one-point, one-time cross correlation $\langle \hat{C}(z_1,\tau_1)\rangle_q$ as follows:

$$\langle \hat{E}_s^\dagger(z_1,\tau_1)\hat{Q}(z,\tau_1)\rangle_q \simeq\langle \hat{C}(z_1,\tau_1)\rangle_q\exp[\tfrac{1}{2}(gI_p-|\mu|\gamma_p-\alpha)(z-z_1)]\ , \tag{27}$$

where we assume that $\hat{Q}(z,\tau_1)$ grows from $\hat{Q}(z_1,\tau_1)$ exponentially, in the same way that $\langle Q(z)\rangle$ grows from $\langle Q(z_1)\rangle$ according to Eq. (15). The justification for this approximation is based on the following analysis. Consider the classical triple correlation $\langle E_p^*E_sQ\rangle$. If we write $E_s=\langle E_s\rangle+\tilde{E}_s$ and $Q=\langle Q\rangle+\tilde{Q}$, where $\tilde{E}_s$ and $\tilde{Q}$ are fluctuating parts with zero mean value, then we have

$$\langle E_p^*E_sQ\rangle=\langle E_p^*\rangle\langle E_s\rangle\langle Q\rangle+\langle E_p^*\tilde{E}_s\rangle\langle Q\rangle +\langle E_p^*\tilde{Q}\rangle\langle E_s\rangle+\langle E_p^*\tilde{E}_s\tilde{Q}\rangle\ .$$

The first term on the rhs is zero since $\langle E_p^*\rangle=0$, and the last term can be neglected compared to the other two. From the results of Sec. III it follows that the dominant term is the second term,

$$\langle E_p^*\tilde{E}_s\rangle\langle Q\rangle=\langle E_p^*E_s\rangle\langle Q\rangle\ ,$$

which has the largest gain coefficient. Hence, it is justified to approximate $\hat{Q}(z,\tau_1)$ in the triple correlation on the rhs of Eq. (26) with its average value and obtain thus relation (27). The effect of this approximation on the calculation of the average Stokes intensity will be discussed later on in this section. Using relation (27), Eq. (26) reduces to

$$E_{p0}^2\langle E_p^*(\tau+\mu z)\langle \hat{E}_s^\dagger(z_1,\tau_1)\hat{Q}(z,\tau_1)\rangle_q\rangle \simeq E_{p0}^2e^{-\frac{1}{2}\gamma_p(\tau-\tau_1)}\times\exp[\tfrac{1}{2}(gI_p-2|\mu|\gamma_p-\alpha)(z-z_1)] \times\langle E_p^*(\tau_1+\mu z_1)C(z_1,\tau_1)\rangle\ . \tag{28}$$

Similarly, we can show that the second correlation function inside the last set of curly brackets on the rhs of Eq. (20) reduces to

$$\left\langle E_p^*(\tau+\mu z)\left[\left[i\Delta-\frac{\Gamma}{2}+\frac{\partial}{\partial\tau_1}\right]E_p(\tau_1+\mu z_1)\right]E_p^*(\tau_2+\mu z_1)\langle\hat{E}_s^\dagger(z_1,\tau_2)\hat{Q}(z,\tau_1)\rangle_q\right\rangle$$

$$=\left[\left[i\Delta-\frac{\Gamma}{2}+\frac{\partial}{\partial\tau_1}\right]e^{-\frac{1}{2}\gamma_p(\tau-\tau_1)}\right]\langle E_p^*(\tau_1+\mu z)E_p(\tau_1+\mu z_1)E_p^*(\tau_2+\mu z_1)\langle\hat{E}_s^\dagger(z_1,\tau_2)\hat{Q}(z,\tau_1)\rangle_q\rangle$$

$$=[i\Delta+\tfrac{1}{2}(\gamma_p-\Gamma)]E_{p0}^2e^{-\frac{1}{2}\gamma_p[(\tau-\tau_1)+|\mu|(z-z_1)]}\langle E_p^*(\tau_2+\mu z_1)\langle\hat{E}_s^\dagger(z_1,\tau_2)\hat{Q}(z,\tau_1)\rangle_q\rangle$$

$$\simeq[i\Delta+\tfrac{1}{2}(\gamma_p-\Gamma)]E_{p0}^2e^{-\frac{1}{2}\gamma_p(\tau-\tau_1)}\exp[\tfrac{1}{2}(gI_p-2|\mu|\gamma_p-\alpha)(z-z_1)]\langle E_p^*(\tau_2+\mu z_1)C(z_1,\tau_2)\rangle\ , \quad (29)$$

where in the first step we have interchanged the order of time differentiation and statistical averaging. Note that for a phase-diffusion field the average of the product $E_p^*(\tau_1+\mu z)E_p(\tau_1+\mu z_1)$ is not a function of τ_1. In the last step we have employed again relation (27) and, in addition, have assumed that $Q(z_1,\tau_1)\simeq Q(z_1,\tau_2)$. The latter assumption of stationarity is justified since we are interested in the calculation of steady-state average values only.

Substituting Eqs. (24)–(29) into Eq. (20) and rewriting the first term on the rhs we obtain

$$\begin{aligned}\langle E_p^*(\tau+\mu z)C(z,\tau)\rangle=&-i\frac{\kappa_2}{2\rho_L}E_{p0}^2\Gamma\int_{\tau_0}^{\tau}e^{\Gamma(\tau_1-\tau)}d\tau_1\\&-i\kappa_1'E_{p0}^2I_{s0}e^{-\alpha z}\int_{\tau_0}^{\tau}e^{[i\Delta+\frac{1}{2}(\Gamma+\gamma_p)](\tau_1-\tau)}d\tau_1\\&+\kappa_1\kappa_2E_{p0}^2\int_{\tau_0}^{\tau}e^{[i\Delta+\frac{1}{2}(\Gamma+\gamma_p)](\tau_1-\tau)}d\tau_1\int_0^z e^{\alpha(z_1-z)}[\langle E_p^*(\tau_1+\mu z_1)C(z_1,\tau_1)\rangle-\text{c.c.}]dz_1\\&+\kappa_1\kappa_2E_{p0}^2\int_{\tau_0}^{\tau}e^{[i\Delta+\frac{1}{2}(\Gamma+\gamma_p)](\tau_1-\tau)}d\tau_1\\&\times\int_0^z e^{(\alpha+|\mu|\gamma_p-\frac{1}{2}gI_p)(z_1-z)}\\&\times\Big[\langle E_p^*(\tau_1+\mu z_1)C(z_1,\tau_1)\rangle\\&+[i\Delta+\tfrac{1}{2}(\gamma_p-\Gamma)]\int_{\tau_0}^{\tau_1}e^{(-i\Delta+\Gamma/2)(\tau_2-\tau_1)}\\&\times\langle E_p^*(\tau_2+\mu z_1)C(z_1,\tau_2)\rangle d\tau_2\Big]dz_1\ . \quad (30)\end{aligned}$$

The first term on the rhs of the equation above is a source term associated with spontaneous Raman scattering and is independent of the detuning and the laser bandwidth, as expected. It arises from the contributions of the three different correlations on the rhs of Eq. (20) which involve the Langevin force. The second term is a source term associated with the input Stokes field. The third term depends on the pump bandwidth and is associated with amplification of the Stokes field component that is uncorrelated with the pump. The last term, which involves the approximation in Eq. (27), is associated with the Stokes field component that grows either spontaneously or from the coherent seed and tends to become correlated with the pump field, in accordance with our discussion following Eq. (13).

Equation (30) can be solved by two-dimensional Laplace transform techniques. In the steady-state case and for zero detuning ($\Delta=0$), the Laplace transform $F(k)$, where k is the transform variable corresponding to z, of the difference

$$f(z,\tau)=\langle E_p^*(\tau+\mu z)C(z,\tau)\rangle-\text{c.c.}$$

is given by the quotient

$$F(k)=X(k)/Y(k)\ , \quad (31a)$$

where

$$X(k)=(k+\alpha+|\mu|\gamma_p-\tfrac{1}{2}gI_p)\times\left[-i\frac{\kappa_2}{\rho_L}\frac{(k+\alpha)}{k}E_{p0}^2-i\kappa_1'E_{p0}^2I_{s0}\frac{4}{\Gamma+\gamma_p}\right]\ , \quad (31b)$$

and

$$Y(k)=(k+\alpha+|\mu|\gamma_p-\tfrac{1}{2}gI_p)\left[k+\alpha-gI_p\frac{\gamma_p}{\Gamma+\gamma_p}\right]-\tfrac{1}{2}gI_p\frac{\gamma_p}{\Gamma+\gamma_p}(k+\alpha)\ . \quad (31c)$$

Taking the inverse Laplace transform of $F(k)$ and substituting it into Eq. (19), the steady-state average value of the Stokes intensity is found to be

$$\langle I_s(z)\rangle = I_{s0} + \left[\frac{\Gamma\hbar\omega_s}{4A} + I_{s0}\frac{\Gamma}{\Gamma+\gamma_p}\right] \times \left[\frac{(\eta_1-1/2+1/r)}{\eta_1(\eta_1-\eta_2)}(e^{G_1 z}-1) + \frac{(\eta_2-1/2+1/r)}{\eta_2(\eta_2-1)}(e^{G_2 z}-1)\right], \quad (32)$$

where $r = gI_p/|\mu|\gamma_p$ and $G_{1,2} = gI_p\eta_{1,2}$, with the relative gain coefficients $\eta_{1,2}$ being the roots of the quadratic polynomial $Y(gI_p\eta)$ given above. For $\alpha=0$, this polynomial can be rewritten in terms of dimensionless variables as

$$Y(\eta) = \eta^2 - \left[1 - \frac{1}{r} + \frac{b}{2}\right]\eta + \frac{b}{2}(1-2/r), \quad (33)$$

where $b = \Gamma/(\Gamma+\gamma_p)$. Note that the quantity $\Gamma\hbar\omega_s/4A$ in Eq. (32) plays the role of an effective input intensity for spontaneously initiated Raman scattering.

In the case of forward Raman scattering in a nondispersive medium ($r\to\infty$), Eq. (32) becomes

$$\langle I_s(z)\rangle = I_{s0} + \left[\frac{\Gamma\hbar\omega_s}{4A} + I_{s0}\frac{\Gamma}{\Gamma+\gamma_p}\right]\frac{\Gamma+\gamma_p}{\Gamma+2\gamma_p} \times \left\{(e^{gI_p z}-1) + 2\frac{\gamma_p}{\Gamma}\left[\exp\left[\tfrac{1}{2}gI_p\frac{\Gamma}{\Gamma+\gamma_p}z\right]-1\right]\right\}. \quad (34)$$

The two gain coefficients are the same as those in the exact theory of Raymer *et al.*[15] for nondispersive media. The first term inside the curly brackets gives the growth of the intensity of the Stokes field component that is quasicorrelated to the pump. The second term is due to interference between, on the one hand, the amplitude of the Stokes field component that is uncorrelated with the pump and, on the other hand, the Stokes input or part of the spontaneous Stokes field. This term vanishes when the pump becomes monochromatic ($\gamma_p=0$). For very low gain ($gI_pz \ll 1$), Eq. (34) reduces to

$$\langle I_s(z)\rangle = I_{s0}\left[1 + gI_p\frac{\Gamma}{\Gamma+\gamma_p}z\right] + \frac{\Gamma\hbar\omega_s}{4A}gI_pz, \quad (35)$$

which agrees exactly with the results of theories for nondispersive media.[15,16] At this point, we should mention that a quadratic polynomial determining the gain coefficients $G_{1,2}$ has been obtained also by D'yakov using a Dyson-type equation method in the Bourret approximation, neglecting spontaneous Raman generation.[1,2] In our notation, his polynomial takes the form

$$Y_{\mathrm{D'yakov}}(\eta) = \eta^2 - \left[1 - \frac{1}{r} - \frac{\Gamma}{\gamma_p}\frac{1}{r}\right]\eta - \frac{\Gamma}{\gamma_p}\frac{1}{r}. \quad (36)$$

Note that its coefficients have a different dependence on the laser bandwidth and the parameter r from our polynomial in Eq. (33). For forward SRS in a nondispersive medium ($r\to\infty$) it gives $G_1 = gI_p$ and $G_2=0$. According to this, the Stokes field that grows from the coherent seed is perfectly correlated with the pump, and there is no interference term in the expression for the average Stokes intensity, in disagreement with exact nondispersive theories.[15] Moreover, in the case of a monochromatic pump ($\gamma_p=0$) it gives $G_1=gI_p$ and $G_2=-|\mu|\Gamma$. The fact that the effect of dispersion does not disappear in this case, as is expected, is unphysical. In these respects, the present theory is an improvement. Both theories, however, as well as the nonstatistical coupled mode theory[4–7] miss the factor $(\pi gI_pz)^{-1/2}$ which in the nondispersive case multiplies e^{G_1z} and accounts for the lack of perfect correlation between the Stokes field and the pump. In any case, for high gain this factor becomes unimportant (see Fig. 4 of Ref. 16).

In the general case of $\mu\neq 0$ and for high pump intensities ($r \gg 1$), the two gain coefficients are given approximately by

$$G_1 = gI_p - \tfrac{1}{2}|\mu|\gamma_p[1+(1-3b/2)/(1-b/2)], \quad (37a)$$

and

$$G_2 = \tfrac{1}{2}gI_p\frac{\Gamma}{\Gamma+\gamma_p} - \tfrac{1}{2}|\mu|\gamma_p[1-(1-3b/2)/(1-b/2)]. \quad (37b)$$

If $\gamma_p \gg \Gamma$, then the expressions above simplify to $G_1 = gI_p - |\mu|\gamma_p$ and $G_2 = \frac{1}{2}gI_p\Gamma/(\Gamma+\gamma_p)$, with G_1 being much greater than G_2 and determining essentially the Stokes gain. The last expression for G_1 agrees with previous results.[1–6,10] In this high pump intensity regime ($I_p \gg |\mu|\gamma_p/g$) the effect of group-velocity dispersion is negligible, and the Stokes field is quasicorrelated with the pump. For low pump intensities ($I_p \ll |\mu|\gamma_p/g$), the gain coefficients are given approximately by

$$G_1 = gI_p\frac{\Gamma}{\Gamma+\gamma_p}\left[1 + \frac{1}{2}\frac{gI_p}{|\mu|\gamma_p}\right], \quad (38a)$$

and

$$G_2 = -|\mu|\gamma_p + gI_p - \tfrac{1}{2}gI_p\frac{\Gamma}{\Gamma+\gamma_p} < 0. \quad (38b)$$

The Stokes gain is again determined by G_1, and can be high ($\gamma_p < \Gamma$) or low ($\gamma_p \gg \Gamma$). Note that G_1 is approximately equal to twice the gain coefficient for the average Stokes field amplitude given in Eq. (11). In the low pump intensity regime, the Stokes field that grows from either a coherent seed or spontaneous Raman scattering is uncorrelated with the pump.

Figure 1 shows the dependence of $G_{1,2}$ on the ratio $gI_p/|\mu|\gamma_p$, for four different pump bandwidths. It can be seen that G_1 is always positive and greater than G_2. The latter, which is associated with an interference term in the average Stokes intensity, becomes negative if $I_p < 2|\mu|\gamma_p/g$. For very large pump bandwidths ($\gamma_p \gg \Gamma$), the curve for G_1 displays a steep increase at

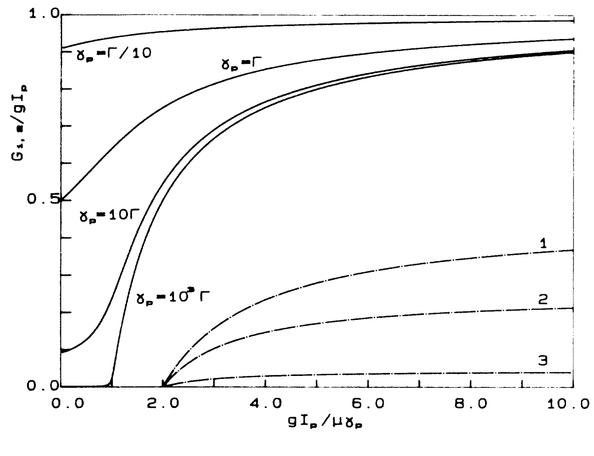

FIG. 1. Stokes gain coefficients G_1 (solid line) and G_2 (dot-dashed line) vs $gI_p/|\mu|\gamma_p$ for four different pump bandwidths, $\gamma_p=\Gamma/10$, Γ, 10Γ, and $10^3\Gamma$, where Γ is the Raman linewidth. The curves for G_2 numbered 1, 2, and 3 correspond to the first three pump bandwidths. The curve for G_2 with $\gamma_p=10^3\Gamma$ falls on the horizontal axis and is not drawn.

the critical pump intensity, $I_{cr}\equiv|\mu|\gamma_p/g$, and there is a rapid growth of the Stokes field component that is quasicorrelated to the pump. For small bandwidths ($\gamma_p<\Gamma$), there is no such critical intensity and the quasicorrelated Stokes field component develops continuously from $I_p=0$. We should mention here that the curves for G_1 in Fig. 2 of Ref. 2 show a trend similar to ours, but there are quantitative differences due to the differences in the polynomials of Eqs. (33) and (36).

The next three figures show the Stokes amplification, $\langle I_s(z)\rangle/I_{s0}$, and spontaneous Stokes generation, $\langle I_s(z)\rangle/(\Gamma\hbar\omega_s/4A)$, versus $gI_p/|\mu|\Gamma$ for three different lengths of the dispersive medium. Figure 2 is for $z=100$ L_{coh}, where $L_{coh}\equiv1/|\mu|\Gamma$ is a mean coherence length for the pump-Stokes field cross correlation in the case of zero gain. As can be seen, for this length we can obtain five orders of magnitude of Stokes amplification with pump intensities $I_p<0.1I_{cr}$, for all three different pump bandwidths. This case corresponds to the low pump intensity regime in Fig. 1, where G_1 is most sensitive to γ_p and the Stokes field is uncorrelated with the pump. If in this case, the intensity I_{s0} of the coherent seed is much greater than $\Gamma\hbar\omega_s/4A$, then the Stokes output is essentially monochromatic and we have conversion of broadband radiation into monochromatic radiation.[2] The incoherence of the pump is filtered by the medium, which is left excited with an energy spread equal to $\hbar\Gamma$. The conversion process burns a hole at the center (for $\Delta=0$) of the pump spectrum with width Γ. In order to have high-energy conversion efficiency, we must have $\Gamma\approx\gamma_p$. The broadband to narrow-band conversion could be observed more easily in backward Raman or Brillouin amplification, and the competing process would be depletion of the pump from forward scattering. In Fig. 3 the length of the medium is $z=L_{coh}$; that is one hundred times shorter than in the previous figure. Therefore, in order to get five orders of magnitude of Stokes amplification as before, the pump

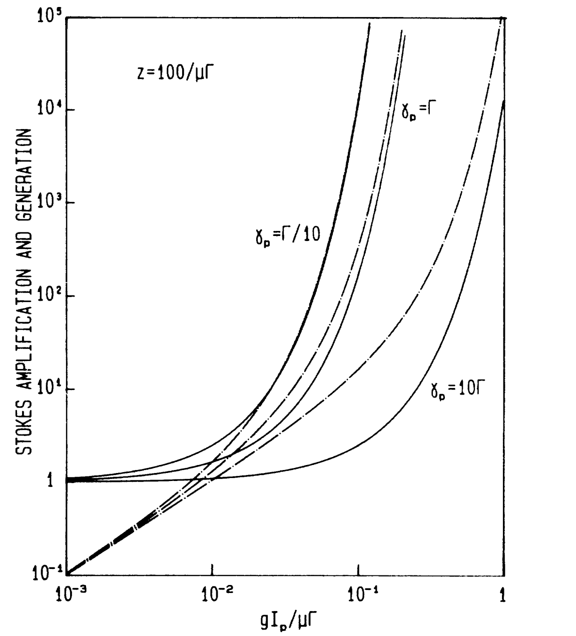

FIG. 2. Stokes amplification (solid line), $\langle I_s(z)\rangle/I_{s0}$, and generation (dot-dashed line), $\langle I_s(z)\rangle/(\Gamma\hbar\omega_s/4A)$, vs $gI_p/|\mu|\Gamma$, for three different pump bandwidths. The length of the medium is $z=100/|\mu|\Gamma$.

intensity must be increased above the critical intensity. In this intensity regime, G_1 is less sensitive to γ_p, and this is reflected in the smaller spread of the threshold intensities in Fig. 3 compared to the previous figure. Lastly, in Fig. 4 the length of the medium is $z=L_{coh}/100$. To get the same amplification and Stokes generation as

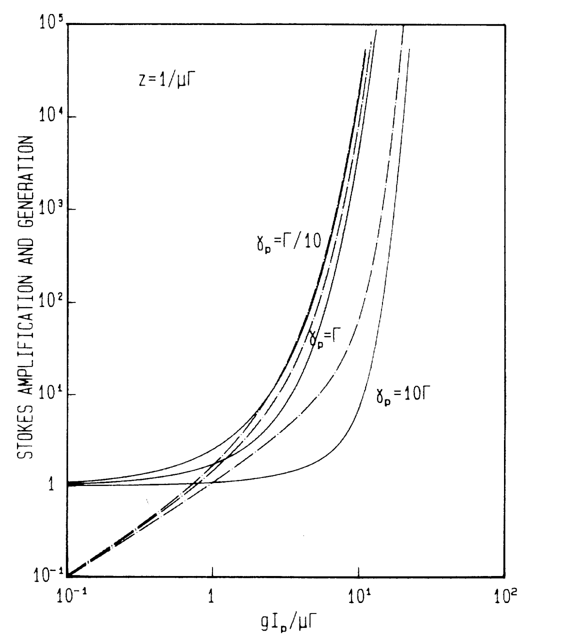

FIG. 3. Stokes amplification (solid line) and generation (dot-dashed line) for $z=1/|\mu|\Gamma$.

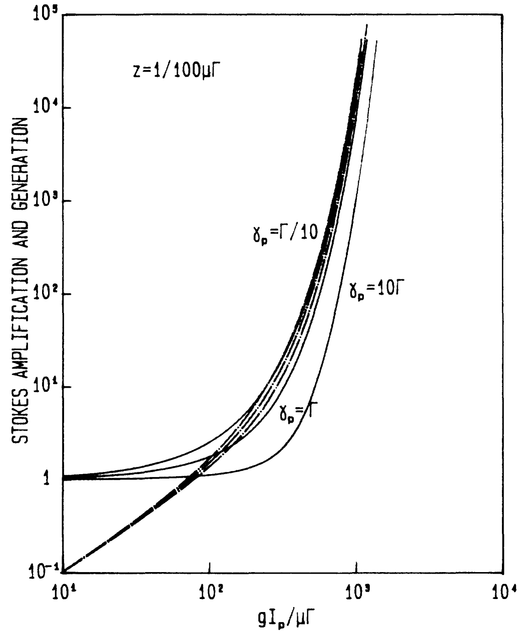

FIG. 4. Stokes amplification (solid line) and generation (dot-dashed line) for $z=1/100|\mu|\Gamma$.

before, the pump intensity must be increased to 10^4, 10^3, and 10^2 times the critical intensity corresponding to the three bandwidths, $\gamma_p=\Gamma/10$, Γ, and 10Γ, respectively. For these pump intensities the effect of group-velocity dispersion is negligible and $G_1 \approx gI_p$. The small spread in the three curves for the intensity of the spontaneously initiated Stokes radiation is due to the slight bandwidth dependence (a factor of 2 variation) of the factor multiplying $e^{G_1 z}$. It is under the conditions of Fig. 4 ($z \ll L_{\rm coh}, \gamma_p \gg \Gamma$, and $I_p \gg I_{\rm cr}$), that one would operate a Brillouin phase conjugate mirror pumped by a broadband pump, in order to obtain high-fidelity wave-front inversion.[10]

Figures 2–4 demonstrate very clearly the well-known (from experimental observations) asymmetry for forward and backward Raman scattering in the case of a broadband pump. Assume that the group velocities for the pump and the Stokes waves differ by two percent, in which case $\mu_+=100|\mu_-|$. If Fig. 3 corresponds to forward SRS, then Fig. 2 corresponds to backward SRS for the same medium length. The range of the absolute pump intensity in the two figures is the same. Likewise, if Fig. 4 corresponds to forward SRS, then Fig. 3 corresponds to backward SRS. As can be seen, the asymmetry depends on the length of the medium, and becomes more pronounced as the length is increased. Another point that is demonstrated by the three figures is the fact that the intensity of spontaneous Raman scattering (linear growth region) does not depend either on the pump bandwidth or the scattering direction.

Comparing Figs. 2–4 with Fig. 1 of Ref. 9 and Fig. 2 of Ref. 17 for SRS in nondispersive media pumped by a chaotic field, we expect that the effect of intensity fluctuations in the pump would be to cause a larger spread in the threshold intensities for the different pump bandwidths than in the case of a phase-diffusion pump. The increase in the spread will be on the narrow-band side ($\gamma_p \leq \Gamma$) of the figures.

V. CONCLUSIONS

We have presented an improved theoretical treatment of Raman amplification in dispersive media pumped by a phase-diffusion field. In addition, we have treated, for the first time, the problem of spontaneously initiated SRS in dispersive media using a Langevin approach. The average Stokes field amplitude and the pump-Stokes field cross correlation have been calculated exactly. One of the new results is the fact that, unless the Stokes input is correlated with the pump, the Stokes output and the pump are statistically orthogonal, even if the Stokes field builds up spontaneously. This is related to the fact that the reproduction of the pump spectrum by the Stokes spectrum is never perfect due to the finite Raman linewidth. Hence, we can only speak of quasicorrelation between the pump and the Stokes field. The average Stokes intensity is determined by the triple correlation

$$\langle E_p^*(\tau+\mu z)\hat{E}_s^\dagger(z,\tau)\hat{Q}(z,\tau)\rangle ,$$

between the pump, the Stokes field, and the density matrix element for the two-photon Raman transition. The calculation of this correlation is quite complicated, and new averaging techniques must be developed in order to calculate the growth of the average Stokes intensity more precisely. Nonetheless, the present treatment is an improvement over previous works.[1–10] Other unsolved problems related to the effects of group-velocity dispersion are (i) the exact evolution of the Stokes spectrum, (ii) the calculation of the two-time intensity correlation of the Stokes field, (iii) the calculation of the cross correlation between the pump and the Stokes intensities, and (iv) the effects of pump intensity fluctuations. The solution of these problems will provide a more complete understanding of the statistical properties of the Stokes radiation and their dependence on dispersion and pump statistics.

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[1]Yu. E. D'yakov, Krat. Soobshch. fiz. (FIAN) **4**, 23 (1973).

[2]S. A. Akhmanov, Yu. E. D'yakov, and L. I. Pavlov, Zh. Eksp. Teor. Fiz. **66**, 520 (1974) [Sov. Phys.—JETP **39**, 249 (1974)].

[3]I. G. Zubarev, A. B. Mironov, and S. I. Mikhailov, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 697 (1976) [JETP Lett. **23**, 642 (1976)].

[4]G. P. Dzhotyan, Yu. E. D'yakov, I. G. Zubarev, A. B. Mironov, and S. I. Mikhailov, Zh. Eksp. Teor. Fiz. **73**, 822 (1977) [Sov. Phys.—JETP **46**, 431 (1977)]; Kvant. Elektron. (Moscow) **4**, 1377 (1977) [Sov. J. Quantum Electron. **7**, 783 (1977)].

[5]I. G. Zubarev and S. I. Mikhailov, Kvant. Elektron. (Moscow) **5**, 2383 (1978) [Sov. J. Quant. Electron. **8**, 1338 (1978)].

[6]I. M. Bel'dyugin, I. G. Zubarev, and S. I. Mikhailov, Kvant. Elektron. (Moscow) **7**, 1471 (1980) [Sov. J. Quant. Electron. **10**, 847 (1980)].

[7]J. Eggleston and R. L. Byer, IEEE J. Quantum Electron. **16**, 850 (1980).

[8]G. S. Agarwal, Opt. Commun. **35**, 267 (1980).

[9]A. T. Georges, Opt. Commun. **41**, 61 (1982).

[10]For a review of SBS with broadband lasers see G. C. Valley, IEEE J. Quant. Electron. **22**, 704 (1986).

[11]S. E. Bodner, Phys. Today **41** (1), s-65 (1988).

[12]R. H. Stolen and A. M. Johnson, IEEE J. Quantum Electron. **22**, 2154 (1986).

[13]R. L. Carman, F. Shimizu, C. S. Wang, and N. Bloembergen, Phys. Rev. A **2**, 60 (1970).

[14]S. A. Akhmanov, K. N. Drabovich, A. P. Sukhorukov, and A. S. Chirkin, Zh. Eksp. Teor. Fiz. **59**, 485 (1970) [Sov. Phys.—JETP **32**, 266 (1971)].

[15]M. G. Raymer, J. Mostowski, and J. L. Carlsten, Phys. Rev. A **19**, 2304 (1979).

[16]M. G. Raymer and J. Mostowski, Phys. Rev. A **24**, 1980 (1981).

[17]M. Trippenbach, K. Rzazewski, and M. G. Raymer, J. Opt. Soc. Am. B **1**, 671 (1984).

[18]L. A. Westling and M. G. Raymer, Phys. Rev. A **36**, 4835 (1987).

[19]M. Trippenbach and C. Long Van, J. Opt. Soc. Am. B **3**, 879

[20]A. T. Georges, Phys. Rev. A **21**, 2034 (1980).

[21]W. H. Louisell, *Quantum Statistical Properties of Radiation* (Wiley, New York, 1973).

[22]See, for example, A. Papoulis, *Probability, Random Variables, and Stochastic Processes* (McGraw-Hill, New York, 1965).