Influence of the Fock-state field on the many-atom radiation processes in a damped cavity

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Radiation effects from two-level atoms caused by the presence of a Fock-state field in a damped single-mode cavity are examined. An exact recurrence differential equation for density-matrix elements valid for any number of atoms or photons is derived. Numerical solutions for different cavity dampings (low- and high-Q cavities) and different photon numbers of the Fock-state field being present initially in the cavity are presented. Collective effects caused by the presence of many atoms are investigated. It is shown that new quasistationary states as well as collective radiation inhibition appear in the time behavior of the atomic energy expectation value. These effects arise from averaging over an ensemble of which each member consists of N identical two-level atoms placed into a resonant single-mode cavity. A comparison of the obtained results with those for an initial coherent or thermal field is also given.

I. INTRODUCTION

The radiation effects from two-level atoms caused by the presence of a Fock-state field inside a cavity have not been examined sufficiently in the literature. The reason for this is that until very recently no experimental realization of photon-number (Fock) states was in sight.

However, recently Hong and Mandel[1] have described the realization of a one-photon state. Later, Filipowicz *et al.*[2] suggested a method for creation of high photon-number (Fock) states. The method consists in the injection of two-level atoms into a lossless single-mode cavity. The inverted atoms are injected at such a low rate that at most one atom at a time is present in the cavity. If the injected atoms spend a quite definite time interval in the cavity, namely, $T=(2\pi/g)/(1/\sqrt{p+1})$ (g is the atom-field coupling constant), then after a large number of atoms has passed through the cavity a Fock state with p photons will be created.

This and the fact that within the scope of a stimulating action of the European Communities the experimental groups of the Laboratoire de Spectroscopie Hertzienne de l'E.N.S. in Paris and the Institut für Quantenoptik in München are concentrated on production of photon-number states, indicates to us that the experimental realization will be possible in the near future.

Since in experiments with Rydberg atoms[3] the interaction of two-level atoms with a single-mode radiation field is realized, it seems to us important to examine this problem theoretically in the case of a Fock-state field inside a realistic cavity with damping.

The single two-level atom in a damped cavity has been already treated extensively in the literature.[4,5] In our previous work[6] we have treated the many-atom spontaneous emission in a damped cavity. Therefore in our present paper we examine the influence of the Fock-state field being present initially in the cavity on the many-atom radiation processes. By comparing these results for the average atomic energy with those obtained for the initial vacuum field (spontaneous emission) the new effect of the quasistationary states as well as some new aspects (collapse and revival phenomena) of the already known effect of the cooperatively inhibited average radiation (see the results in lossless cavities given by Senitsky[7] in analytic form and Abate and Haken[8] in numerical form) are pointed out.

In Sec. II we derive an exact recurrence equation for density-matrix elements which is valid for an arbitrary number of atoms and photons in the Fock state; furthermore it is also valid for any cavity damping. In Sec. III numerical results for different cavity dampings (low- and high-Q cavities) and different initial number of photons are presented.

In Sec. IV we discuss the statistical aspects of the obtained quantum-mechanical results. Such discussions are not made often enough in the literature. Senitzky[7,9] was the first to point out that a quantum-mechanical expectation value (EV) does not necessarily give even a qualitative description of a single experiment. Namely, EV's describe an average over an ensemble of identical systems, where the differences among the members (quantum fluctuations) may add up in such a way as to make the average behavior appear greatly different from the behavior of an individual member of the ensemble.

In Sec. V a discussion of the obtained results is given. Moreover, the appearance of quasistationary states in the case of an initial coherent field and their nonappearance in the case of an initial thermal field is also demonstrated.

II. EXACT RECURRENCE EQUATION OF MOTION FOR DENSITY-MATRIX ELEMENTS

Since Dicke's original paper[10] great interest has been awakened in treating a sample of N identical two-level atoms occupying a region of dimensions small compared with the wavelength λ of the atomic transition. However, conditions which are very close to this model have been achieved only recently in experiments with Rydberg atoms inside a resonant single-mode cavity.[3] The Rydberg-level transition wavelength falls in the millimeter range and therefore it is not difficult to prepare many atoms lying in a volume small compared with λ^3.

The Liouvillian for a pointlike Dicke model inside a resonant single-mode cavity in the rotating-wave approximation reads as

$$L=L_0+L_{AR}+i\Lambda_R \ , \tag{1}$$

$$L_0=[H_0,\ldots],\ \ L_{AR}=[H_{AR},\ldots] \ , \tag{2}$$

with corresponding Hamiltonians ($\hbar=1$)

$$H_0=H_A+H_R=\omega(R^z+a^\dagger a) \ , \tag{3}$$

$$H_{AR}=g(a\otimes R^+ + a^\dagger\otimes R^-) \ , \tag{4}$$

$$R^z=\sum_{l=1}^{N} R_l^z,\ \ R^\pm=\sum_{l=1}^{N} R_l^\pm \ , \tag{5}$$

and the field-damping Liouvillian

$$\Lambda_R(\cdots)=K\{[a(\cdots),a^\dagger]+[a,(\cdots)a^\dagger]\} \ , \tag{6}$$

where $R_l^z, R_l^\pm$ are the population inversion and dipole-moment operators of the lth atom, respectively, ω is the frequency of the atomic transition and the resonant field mode, $a^\dagger, a$ are the photon creation and annihilation operators for the resonant field mode, g is the atom-field coupling constant, and K is the cavity damping factor.

The statistical density operator $\rho(t)$ obeys the Liouville equation

$$\frac{d\rho(t)}{dt}=-iL\rho(t) \ . \tag{7}$$

In this paper we shall treat the special initial case where the atomic system (A) and the radiation field (R) are statistically independent

$$\rho(0)=\rho_A(0)\otimes\rho_R(0) \ , \tag{8}$$

the atoms are initially totally inverted

$$\rho_A(0)=\left|r=\frac{N}{2},m=\frac{N}{2}\right\rangle_{A\ A}\left\langle r=\frac{N}{2},m=\frac{N}{2}\right| \ , \tag{9}$$

and the radiation field is in a Fock state

$$\rho_R(0)=|p\rangle_{R\,R}\langle p| \ , \tag{10}$$

where p is the number of photons.

As a consequence of the special form of the Liouvillian [cf. Eqs. (1)–(6)] and the special initial condition (8), the time evolution of the statistical density operator

$$\rho(t)=\exp(-itL_0)\exp[-it(L_{AR}+i\Lambda_R)]\rho(0) \tag{11}$$

is restricted to the subspace spanned by the state vectors:

$$|n(k)\rangle=|r,r-n\rangle_A\otimes|k\rangle_R=|n\rangle_A\otimes|k\rangle_R \ , \quad r=\frac{N}{2},\ n=0,1,\ldots,2r,\ k=0,1,\ldots,n+p \tag{12}$$

$$\langle n(k)|n'(k')\rangle=\delta_{nn'}\delta_{kk'} \ , \quad \sum_{n=0}^{N}\sum_{k=0}^{n+p}|n(k)\rangle\langle n(k)|=I \ , \tag{13}$$

where the total number of state vectors is $(N+1)(N/2+p+1)$, $\delta_{nn'},\delta_{kk'}$ are the Kronecker deltas, and I is the unit operator in the subspace.

By using the Liouville equation (7) in the interaction picture and the relations (12) and (13) a closed set of exact linear differential equations for the density-matrix elements in the interaction picture can be obtained:

$$\begin{aligned}\frac{d\rho^I_{n(k),l(m)}}{dt}=&-ig\{[(N-n)(n+1)(k+1)]^{1/2}\rho^I_{n+1(k+1),l(m)}\\&-[(N-l)(l+1)(m+1)]^{1/2}\rho^I_{n(k),l+1(m+1)}+[(N-n+1)nk]^{1/2}\rho^I_{n-1(k-1),l(m)}\\&-[(N-l+1)lm]^{1/2}\rho^I_{n(k),l-1(m-1)}\}\\&+2K[(k+1)(m+1)]^{1/2}(1-\delta_{n+p,k})(1-\delta_{l+p,m})\rho^I_{n(k+1),l(m+1)}-K(k+m)\rho^I_{n(k),l(m)} \ ,\\ &n=0,1,\ldots,N,\ k=0,1,\ldots,n+p,\ l=\begin{cases}n-k,n-k+1,\ldots,N & \text{for } n-k\geq 0\\ 0,1,\ldots,N & \text{for } n-k<0,\end{cases}\\ &m=l-n+k \ .\end{aligned} \tag{14}$$

The total number of equations (which is equal to the number of unknowns)

$$N_{\text{tot}}=2\sum_{j=2}^{N}\frac{j!}{2(j-2)!}+\frac{(p+1)(N+1)!}{2(N-1)!}+(N+1)\left[\frac{N}{2}+p+1\right] \tag{15}$$

can be reduced by introducing new unknowns:

$$X_{n(k),l(m)}=\begin{cases}[\rho^I_{n(k),l(m)}+\rho^I_{l(m),n(k)}] & \text{for } n-l \text{ even}\\ -i[\rho^I_{n(k),l(m)}-\rho^I_{l(m),n(k)}] & \text{for } n-l \text{ odd,}\end{cases} \tag{16}$$

$$X_{n(k),l(m)}=(-1)^{n-l}X_{l(m),n(k)}\ , \tag{17}$$

$$X^*_{n(k),l(m)}=X_{n(k),l(m)}\ . \tag{18}$$

Namely, using Eq. (16), Eq. (14) can be transformed to real equations with a reduced number of unknowns:

$$\begin{aligned}\frac{dX_{n(k),l(m)}}{dt}=&(-1)^{n-l}g\{[(N-n)(n+1)(k+1)]^{1/2}X_{n+1(k+1),l(m)}\\ &-[(N-l)(l+1)(m+1)]^{1/2}X_{n(k),l+1(m+1)}+[(N-n+1)nk]^{1/2}X_{n-1(k-1),l(m)}\\ &-[(N-l+1)lm]^{1/2}X_{n(k),l-1(m-1)}\}\\ &+2K[(k+1)(m+1)]^{1/2}(1-\delta_{n+p,k})(1-\delta_{l+p,m})X_{n(k+1),l(m+1)}-K(k+m)X_{n(k),l(m)}\ ,\\ &n=0,1,\ldots,N,\ k=0,1,\ldots,n+p,\ l=\begin{cases}n-k,n-k+1,\ldots,n & \text{for } n-k\geq 0\\ 0,1,\ldots,n & \text{for } n-k<0\end{cases}\\ &m=l-n+k\ ,\end{aligned} \tag{19}$$

with the initial condition (8)–(10)

$$X_{n(k),l(m)}(0)=2\delta_{n0}\delta_{l0}\delta_{kp}\delta_{mp} \tag{20}$$

and

$$N_{\text{tot}}=\sum_{j=2}^{N+1}\frac{j!}{2(j-2)!}+(p+1)(N+1)N+(N+1)\left[\frac{N}{2}+p+1\right]\ . \tag{21}$$

III. TIME EVOLUTION OF THE ATOMIC POPULATION INVERSION AND THE MEAN PHOTON NUMBER

Since Eq. (19) cannot be treated analitically, we solve this equation numerically. In Figs. 1–7 we plot the time-dependent results for the EV's of the atomic population inversion operator

$$Z(\tau)=\frac{1}{N}\langle R^z\rangle_t=\sum_{n=0}^{N}\sum_{k=0}^{n+p}\frac{1}{4N}(N-2n)X_{n(k),n(k)},\quad \tau=gt \tag{22}$$

and photon number operator

$$n(\tau)=\langle a^\dagger a\rangle_t=\tfrac{1}{2}\sum_{n=0}^{N}\sum_{k=0}^{n+p}kX_{n(k),n(k)}\ ,$$

$$\bar{n}(\tau)=\frac{\langle a^\dagger a\rangle_t}{p}\ . \tag{23}$$

In Fig. 1 for large and small cavity dampings ($\kappa=K/g=5$, 0.5), we plot the time evolution of the atomic population inversion for different numbers of

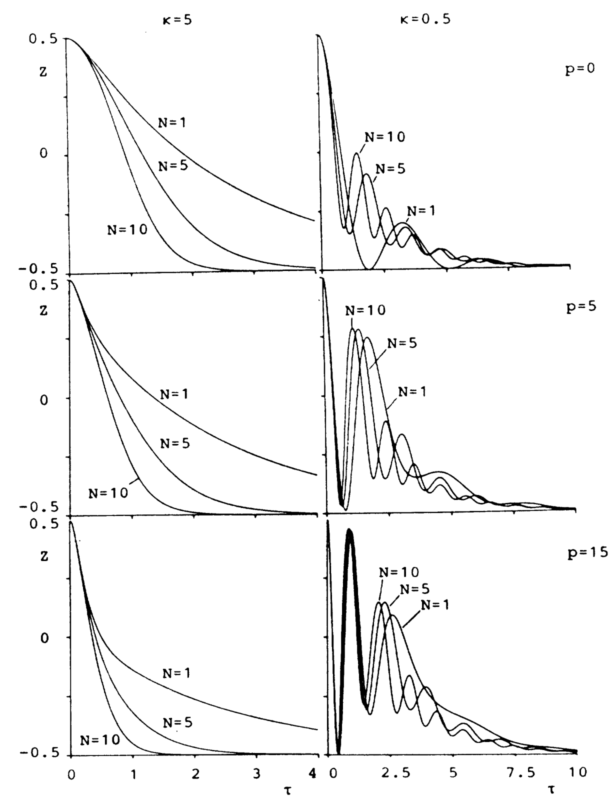

FIG. 1. Atomic population inversion $Z(\tau)=\langle R^z\rangle_t/N$ as a function of the scaled time $\tau=gt$ for different numbers of atoms, $N=1$, 5, 10 and cavity dampings, $\kappa=5$, 0.5. The radiation field in the cavity is initially in a Fock state with different photon numbers, $p=0$, 5, 15.

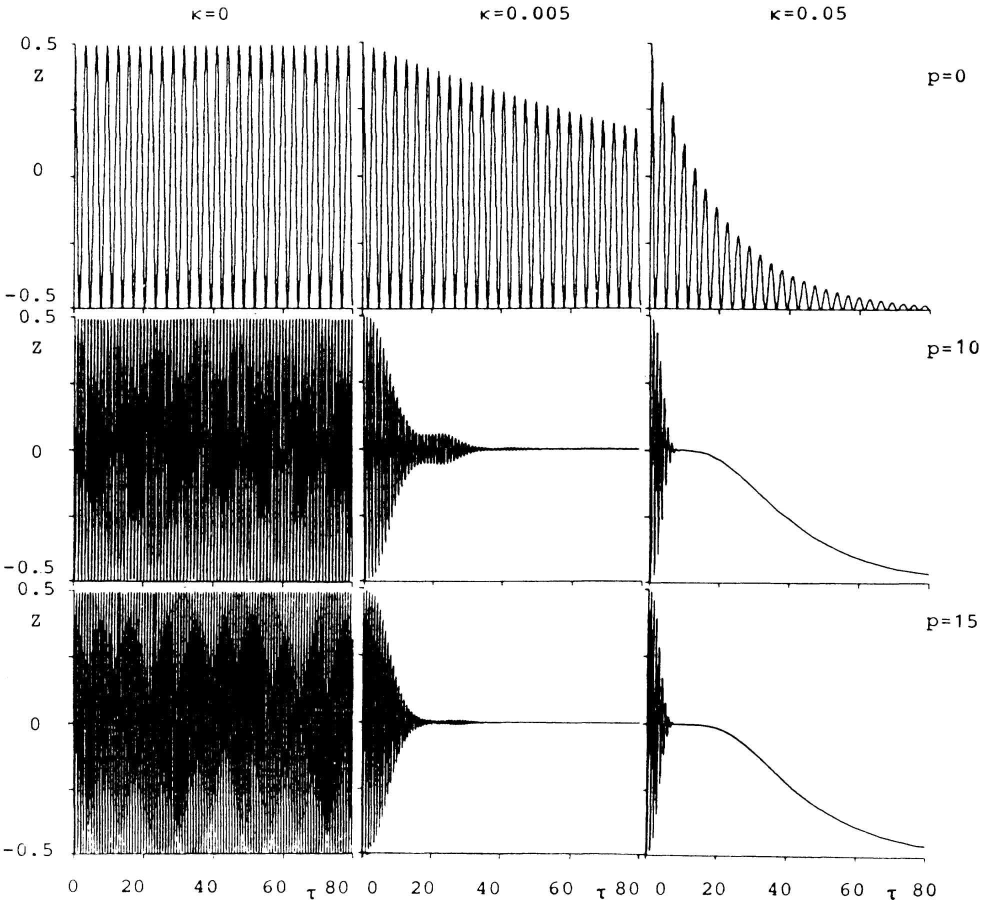

FIG 2. Time evolution of the atomic population inversion $Z(\tau)$ in the case of a single atom ($N=1$) interacting with an initial Fock-state field (with the photon number, $p=0$, 10, 15) inside a cavity with damping, $\kappa=0$, 0.005, 0.05.

atoms, $N=1,5,10$ and initial photon numbers, $p=0$, 5, 15. In the case of large cavity damping $\kappa=K/g=5$ (low-Q cavity), the *collective radiation effects* cause a more rapid decay of the EV of the atomic population inversion. The rapidity of the decay increases with the number N of atoms. This can be seen particularly in the case of spontaneous emission ($p=0$), where superfluorescent effects arise. In addition to these effects, induced emission occurs in the case of an initial Fock-state field ($p=5,15$). At the beginning induced emission (which is proportional to the initial number of photons p) predominates over spontaneous emission to such an extent that all collective radiation effects are suppressed. This means that the atomic system begins to radiate by normal induced emission (no correlations between different atoms exist initially). Much later, after almost the half of the atoms has decayed on the average ($p=15$), dipole-dipole correlations will be created which give rise to cooperative radiation effects (cf. Fig. 1). At the same time the spontaneous emission begins to contribute appreciably and predominates at later times, where in consequence of the strong cavity damping the mean number of photons has decreased almost to zero.

In the case of smaller cavity damping, $\kappa=0.5$ (cf. Fig. 1), the photons are stored long enough in the cavity to be reabsorbed by the atoms and thus the EV of the atomic population inversion undergoes damped Rabi oscillations. In the case of spontaneous emission ($p=0$) the collective effects cause a higher Rabi oscillation frequency. Furthermore, *collective inhibition* effects of the *average energy radiated* appear. These effects prevent a total deexcitation of all members of the ensemble to the ground state ($Z=-\frac{1}{2}$) during the Rabi oscillations (see Refs. 7 and 8 for $\kappa=0$). However, an individual member consisting of N atoms placed into the cavity may be totally deexcited (more about this in Sec. IV). In the case of an initial Fock-state field ($p=15$), as can be clearly seen from Fig. 1, induced emission predominates and at least during the first oscillation period no collective effects occur, i.e., the frequency will neither be increased as compared to the single-atom case, nor will energy inhibition effects be caused.

We now analyze the numerical results for different numbers of atoms, $N=1$, 5, 10 and initial photon numbers, $p=0$, 10, 15 in the case of an ideal cavity ($\kappa=0$) and high-Q cavities ($\kappa=0.005$, 0.05). The corresponding results for the EV of the atomic population inversion $Z=\langle R^z\rangle_t/N$ and mean photon number $\bar{n}=\langle a^\dagger a\rangle_t/p$ we plot in Figs. 2–7.

In the ideal cavity case ($\kappa=0$) the single-atom Rabi os-

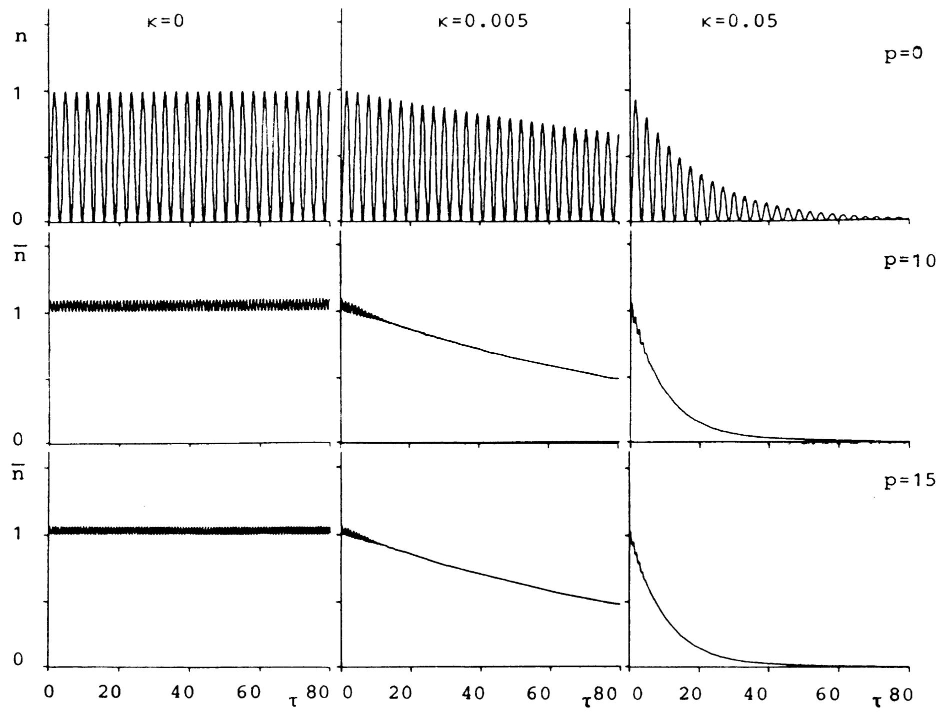

FIG. 3. Time evolution of the mean photon number, in $n(\tau)=\langle a^{\dagger}a\rangle_t$ for initial vacuum field ($p=0$) and $\bar{n}(\tau)=\langle a^{\dagger}a\rangle_t/p$ for initial Fock-state field ($p=10$, 15) in the case of a single atom ($N=1$) inside a cavity with damping, $\kappa=0$, 0.005, 0.05.

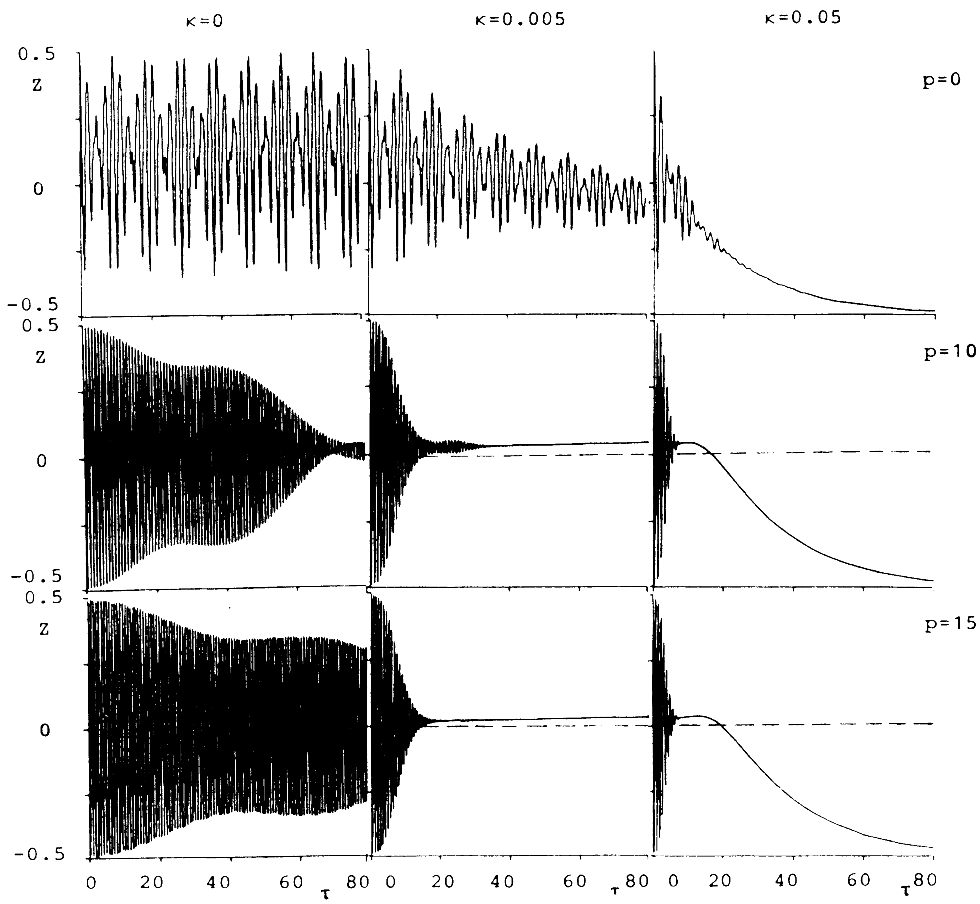

FIG. 4. Same as Fig. 2 for $N=5$.

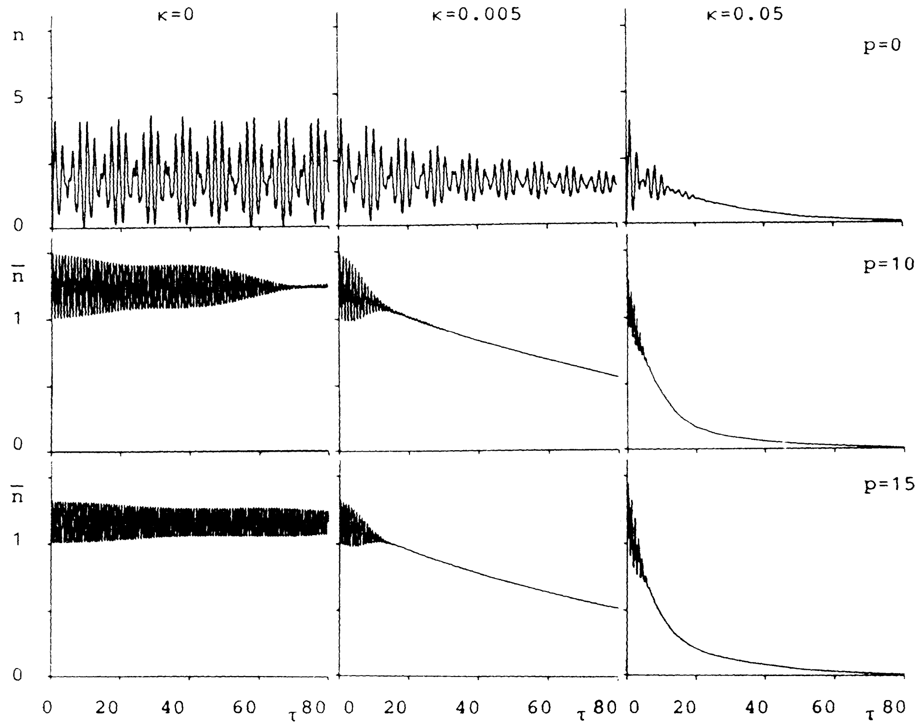

FIG. 5. Same as Fig. 3 for $N=5$.

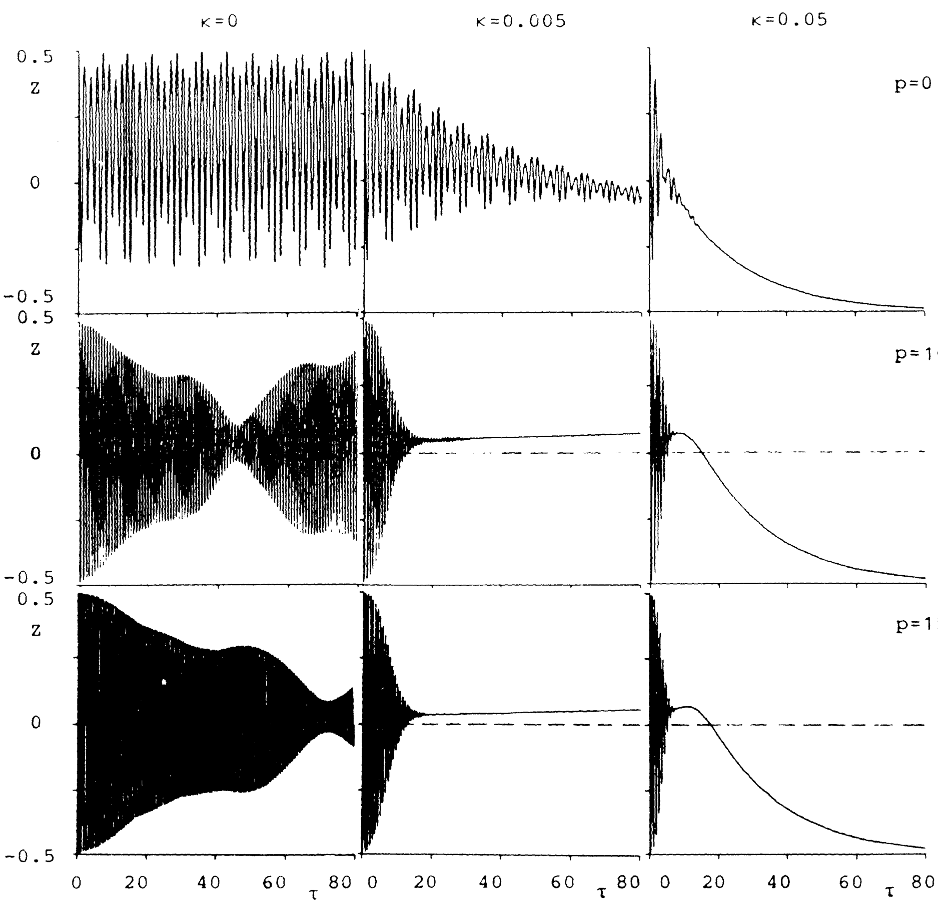

FIG. 6. Same as Fig. 2 for $N=10$.

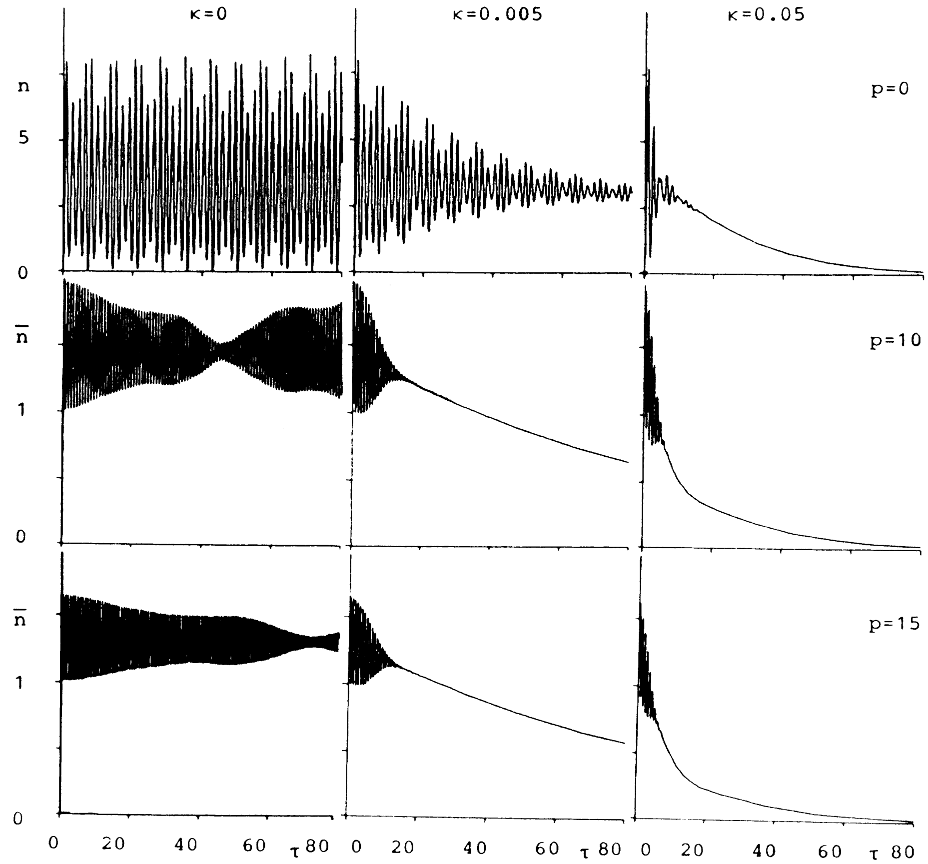

FIG. 7. Same as Fig. 3 for $N=10$.

cillations are sinusoidal and their frequencies are increased by a factor of $(p+1)^{1/2}$ (cf. Fig. 2 for $p=0$, 10, 15). In the many-atom case ($N=5$, 10) the Rabi frequencies increase with the number N of atoms also (cf. Figs. 4 and 6). Since the average number of photons in the cavity is of order of $N+p$, the average field amplitude is proportional to $(N+p)^{1/2}$ and therefore the Rabi frequency is approximately proportional to $(N+p)^{1/2}$ (see Ref. 11). At the same time *collectively inhibited average-radiation effects* which destroy the symmetry of the single-atom case regarding the average energy oscillation between excited (inverted atomic system, $Z=\frac{1}{2}$) and ground state (total deexcitation , $Z=-\frac{1}{2}$), occur (the average radiation emission is inhibited and the ground-state value $-\frac{1}{2}$ cannot be reached). The larger the number of atoms, the stronger the average energy inhibition becomes. However, as already mentioned above, these effects are totally suppressed by the induced emission ($p=10,15$) during the first Rabi oscillation period. Afterwards, the collective inhibition effects become increasingly pronounced and cause a decay (collapse) of the envelopes of the Rabi oscillations in spite of the fact that the model is exactly lossless ($\kappa=0$). However, a subsequent revival of the envelopes occurs quasiperiodically. The increasing number of atoms makes the revival periods of the envelopes shorter, whereas the increasing number of photons p makes it longer (cf. Figs. 4 and 6 for $\kappa=0$, $p=0$, 10, 15, $N=5$, 10 and also see Fig. 9 in Sec. V).

In the ideal cavity ($\kappa=0$), because of the energy balance equation [which follows from the Liouville equation (7)]:

$$\frac{d\langle R^z\rangle_t}{dt}+\frac{d\langle a^\dagger a\rangle_t}{dt}=-2K\langle a^\dagger a\rangle_t \ , \qquad (24)$$

it holds that

$$\langle R^z\rangle_t+\langle a^\dagger a\rangle_t=\frac{N}{2}+p \quad \text{for } K=0 \ . \qquad (25)$$

Therefore the time evolution of the mean photon number $\bar{n}=\langle a^\dagger a\rangle_t/p$ (cf. Figs. 3, 5, and 7) is quite analogous to that of the EV of the atomic population inversion (cf. Figs. 2, 4, and 6).

In the case of small cavity damping ($\kappa=0.005$, 0.05) the field damping operator Λ_R begins to play a role, and therefore novel effects can be observed. First, under the influence of the interaction Liouvillian L_{AR} on the states $|n(k)\rangle$ of the density operator $\rho^I(t)$, Rabi oscillations arise. However, these oscillations are damped by the field damping operator

$$\begin{aligned} e^{\Lambda_R t}&|n(k)\rangle\langle n(k)| \\ &=\sum_{l=0}^{k}\binom{k}{l}e^{-2K(k-l)t} \\ &\qquad\times(1-e^{-2Kt})^l|n(k-l)\rangle\langle n(k-l)| \\ &\approx e^{-2Kkt}|n(k)\rangle\langle n(k)| \quad \text{for } Kt\ll 1 \ . \end{aligned} \qquad (26)$$

The damping increases with the number of photons k being present in the state $|n(k)\rangle$. Therefore, despite small cavity damping, if the number of photons is large enough, the Rabi oscillations of the EV of the atomic population inversion will be damped to a *quasistationary state* after a short time interval Δt, where $K\Delta t \ll 1$ and Eq. (26) still holds. The numerical results for average atomic population inversion and mean photon number are plotted in Figs. 2–7 for $\kappa=0.005$ and 0.05.

The lifetime of the quasistationary state of the EV of the atomic population inversion can be explained by examining the energy balance equation (24). Under the influence of damping κ and large photon number [cf. Eq. (26)] the Rabi oscillations of the EV of the atomic population inversion will be damped to the extent that its time-change rate will be negligibly small compared to the time-change rate of the mean photon number $d\langle a^\dagger a\rangle_t/dt$ caused by the cavity field damping:

$$2K\langle a^\dagger a\rangle_t \gg \left|\frac{d\langle R^z\rangle_t}{dt}\right| . \tag{27}$$

This leads to the conclusion that the mean number of photons decays exponentially in the quasistationary state. The numerical calculations in Figs. 3, 5, and 7 confirm this conclusion. By looking at Eqs. (24) and (27) the lifetime of the quasistationary state can be also explained. That is to say, the quasistationary state begins to decay when the number of photons has decreased to a value where the above relation (27) is not valid any more. Therefore, the increasing initial number of photons leads to an earlier appearance and longer lifetimes of quasistationary states (compare $p=10$ and $p=15$ in Figs. 2, 4, and 6 for $\kappa=0.005$, 0.05).

In the case of a *single two-level atom* there is a symmetry in the Rabi oscillation ($\kappa=0$). Namely, the EV of the atomic population inversion oscillates between values $\frac{1}{2}$ (upper level) and $-\frac{1}{2}$ (lower level) in an ideal cavity. Therefore, the leakage of photons from the cavity causes a damping to the quasistationary state, which lies in the middle between these two values and corresponds to the zero value of the atomic population inversion, $\langle R^z\rangle_t=0$ (cf. Fig. 2). As easily can be seen (and is confirmed by numerical calculations) the density operator in the quasistationary state has the following form:

$$\rho^I(t)=\sum_{n=0}^{1}\sum_{k=0}^{p+1}\rho^I_{n(k),n(k)}(t)|n(k)\rangle\langle n(k)| ,$$
$$\rho^I_{n(k),n(k)}(t)=\rho^I_{n-1(k-1),n-1(k-1)}(t) \tag{28}$$

where $\rho^I_{n(k),n(k)}(t)$ is the probability to find an individual member of an ensemble of identical systems, consisting of a single atom placed in a resonant single-mode damped cavity, in state $|n(k)\rangle$. Therefore, during the lifetime of the quasistationary state, which arises from averaging over an ensemble, the probabilities $\rho^I_{n(k),n(k)}(t)$ remain constant and it holds that

$$L_{AR}\rho^I(t)=0 . \tag{29}$$

In the *many-atom case* the situation is more complex because the number of states $|n(k)\rangle$, $n=0,1,\ldots,N$ with a fixed photon number k increases appreciably as compared to the single-atom case ($N=1$). This gives rise to *collective inhibition effects* destroying the symmetry of the single-atom case regarding the average energy oscillations between upper and lower states (compare Figs. 2, 4, and 6 for $\kappa=0$). As already mentioned, the collective inhibition effects prevent a total deexcitation of all individual members of the ensemble ($Z=-\frac{1}{2}$). Therefore, the quasistationary value of the EV of the atomic population inversion, which corresponds to the time-averaged value over many Rabi oscillations, lies in the middle between the maximum value ($Z_{\max}=\frac{1}{2}$) and certain minimum value ($Z_{\min}>-\frac{1}{2}$). The larger the number of atoms, the stronger the average-energy inhibition becomes and therefore the shift of the quasistationary state above the zero value, $Z_{qs}>0$, becomes also larger. The increasing number of initial photons p may weaken this effect a little (compare Figs. 4 and 6 for $p=10,15$). Since inhibition effects of the average radiation damp the Rabi oscillations additionally to the already existing damping through cavity losses and the presence of photons in the field, the quasistationary state will be reached much earlier than in the single-atom case.

IV. STATISTICAL ASPECTS OF OBTAINED QUANTUM-MECHANICAL RESULTS

As Senitzky[7,9] has pointed out, a quantum-mechanical EV does not necessarily give even qualitatively a good description of a single experiment. Only probability predictions can be made about the result of each single experiment. If one carries out a measurement of a dynamical variable on an individual system being member of the quantum-statistical ensemble, usually large quantum fluctuations of the result of each measurement around the EV arise.

Our EV of the atomic population inversion (energy) $Z=\langle R^z\rangle_t/N$ describes the average over an ensemble of which each member consists of N identical two-level atoms placed in a resonant single-mode damped cavity.

The invariance of the Dicke Hamiltonian [cf. Eqs. (1)–(5)] and our initial conditions [cf. Eq. (9)] with respect to the permutation of indices $i=1,\ldots,N$ of the operators $R_i^z, R_i^\pm$ leads to the relations:

$$\langle R_i^z\rangle_t=\frac{\langle R^z\rangle_t}{N} ,$$
$$\langle R_i^zR_j^z\rangle_t=\frac{\sum_{l,m=1,\ l\neq m}^{N}\langle R_l^zR_m^z\rangle_t}{N(N-1)} ,\quad i\neq j . \tag{30}$$

The total atomic energy (population inversion) fluctuations are given by

$$Z_f(\tau)=\frac{1}{N^2}[\langle (R^z)^2\rangle_t-(\langle R^z\rangle_t)^2]$$
$$=\frac{Z_f^{(1)}(\tau)}{N}+\frac{N-1}{N}Z_c^{(2)}(\tau) , \tag{31}$$

where

$$Z_f^{(1)}(\tau)=\langle (R_i^z)^2\rangle_t-(\langle R_i^z\rangle_t)^2 \quad (32)$$

are the single-atom fluctuations, and

$$Z_c^{(2)}(\tau)=\langle R_i^z R_j^z\rangle_t-\langle R_i^z\rangle_t\langle R_j^z\rangle_t,\quad i\neq j \quad (33)$$

are the atom-atom (two-atom) correlations. As can be seen easily the contribution from two-atom correlations dominates for large number N of atoms:

$$Z_f(\tau)\approx Z_c^{(2)}(\tau)\quad \text{for } N\gg 1\ . \quad (34)$$

To demonstrate the statistical aspects of the results obtained in Sec. III, we treat the five-atom case in more detail. For $N=5$ we have six atomic levels: $|n\rangle_A$, $n=0,1,\ldots,5$ [cf. Eq. (12)]. The corresponding occupation probabilities we denote by $W_1,\ldots,W_6$, where

$$W_{n+1}=\tfrac{1}{2}\sum_{k=0}^{n+p}X_{n(k),n(k)}\ . \quad (35)$$

In the case of a lossless cavity ($\kappa=0$) the sum reduces to only one term: $k=n+p$.

In Fig. 8 we plot the numerical results for $N=5$ and different initial photon numbers $p=0$ and $p=10$ in the case of lossless ($\kappa=0$) and damped ($\kappa=0.005$) cavities. From these results it can be seen that for $p=0$ and $\kappa=0$ the EV of the population inversion does not reach the lowest eigenvalue $-\frac{1}{2}$ (the lowest value lies at about -0.36). This is also exhibited in the time behavior of probabilities for finding the atomic system in any one of the six levels. In contrast with the two-level case, the probability of finding the atomic system in the lowest state never reaches the value 1 [$W_6(\tau)<1$ for all times, cf. Fig. 8]. As compared with the single two-level atom, where only probabilities for upper or lower state exist, here we can speak statistically about *collective inhibition* of the *average energy radiated.* This is exhibited in the fact that in higher than two-level systems, the atomic system does not go completely into the lowest state, as was earlier pointed out by Senitzky[7] analytically and by Abate and Haken[8] numerically. In other words, for spontaneous emission in lossless cavities ($\kappa=0,p=0$) the maximal emitted average radiation $Z(\tau)$ in a three-level system is ($N=2$) about 94.45% (cf. Ref. 7), in a six-level system ($N=5$) about 95.05% (cf. Figs. 4 and 8) and in an eleven-level system ($N=10$) about 91.65% (cf. Fig. 6) of the initial energy $Z(0)=\frac{1}{2}$, respectively.

Statistically speaking, there is an energy inhibition if we average over an ensemble of identical multilevel systems interacting with a single-mode resonant radiation field inside a cavity. But there is *no radiation inhibition in a single multilevel system,* since a finite nonzero probability for reaching the lowest state exist in every single experiment ($0<W_6<1$ in Fig. 8 for $\kappa=0,p=0$).

The collective inhibition of the average energy is caused by strong energy fluctuations $Z_f(\tau)$, which, as Eq. (35) shows, arise from strong atom-atom correlations $Z_f^{(2)}$ [see $Z_f(\tau)$ for $\kappa=0,p=0$ in Fig. 8]. These atom-atom correlations prevent a total deexcitation of all individual systems contained in the ensemble and cause quasiperiodical collapses and revivals of the envelopes of Rabi oscillations of the atomic energy EV [cf. $Z(\tau)$ in Fig. 8 for $\kappa=0,p=0$].

We now discuss the influence of the initial Fock-state field with p photons on the collective radiation effects. From Figs. 8 ($p=10$), 4, and 6 ($p=10,15$), it can be seen that the induced emission effects compete with the collective inhibition effects in lossless cavities. Initially all collective effects are suppressed. The greater the number of initial photons p is, the more the collective effects become suppressed. However, as time goes on strong atom-atom correlations causing large energy fluctuations will be created. This leads to the collapse of the envelopes of the Rabi oscillations. Since these energy fluctuations oscillate quasiperiodically between the zero value and some maximum value, the collapse will occur at the maximum value and the revival at the zero value [compare $Z(\tau)$ and $Z_f(\tau)$ in Fig. 8 for $p=0,10$ and $\kappa=0$]. By comparing the numerical results for the spontaneous ($p=0$) and induced emission ($p=10$), we see that the induced emission ($p=10$) may cause the creation of the atom-atom correlations to be approximately 15 times slower than in the spontaneous emission case. (see also Fig. 9 in Sec. V).

As can be seen from Fig. 8, the cavity damping κ damps the Rabi oscillations to a quasistationary state. This means that the energy fluctuations as well as the probabilities for finding the atomic system in any one of the six levels remain constant in time during the lifetime of the quasistationary state. However, since there are constant nonzero energy fluctuations, it is obvious that energy values measured in a single experiment may differ from the quasistationary EV appreciably. In other words the quasistationary effect is an effect which arise from averaging over an ensemble of identical systems. Such effects do not appear in an individual system. Collective effects do not play any significant role in the creation of quasistationary energy EV's. These effects cause only a shift of the corresponding energy EV above the zero value. This shift means only that the probability to find the atomic system in upper levels is a little greater than in lower levels: $W_1+W_2+W_3>W_4+W_5+W_6$ (cf. Fig. 8 for $\kappa=0.005$).

It should be mentioned that Senitzky[7,9] has shown that in lossless cavities the statistical aspects introduced by the quantum mechanics are preserved in the classical solution. He has pointed out that in a single experiment, the atomic population inversion oscillates approximately like the classical energy. Namely, at $t=0$ both the atomic system as well as the radiation field are in an energy eigenstate (atomic system in the highest energy eigenstate and radiation field in the vacuum state), therefore quantum mechanically there exists an uncertainty in R^+,R^- and $a^\dagger,a$, respectively. The uncertainty in R^+ and R^- will produce an uncertainty in the oscillation amplitudes of the radiated field, whereas the uncertainty in $a^\dagger$ and a produce an uncertainty in the oscillation amplitude of the initial field. The effective field acting on the atomic system is the superposition of the initial field and the radiated field. There exists, thus, an uncertainty in the effective field, which will produce an uncertainty (or spread among the members of the ensemble) in the frequency of R^z.

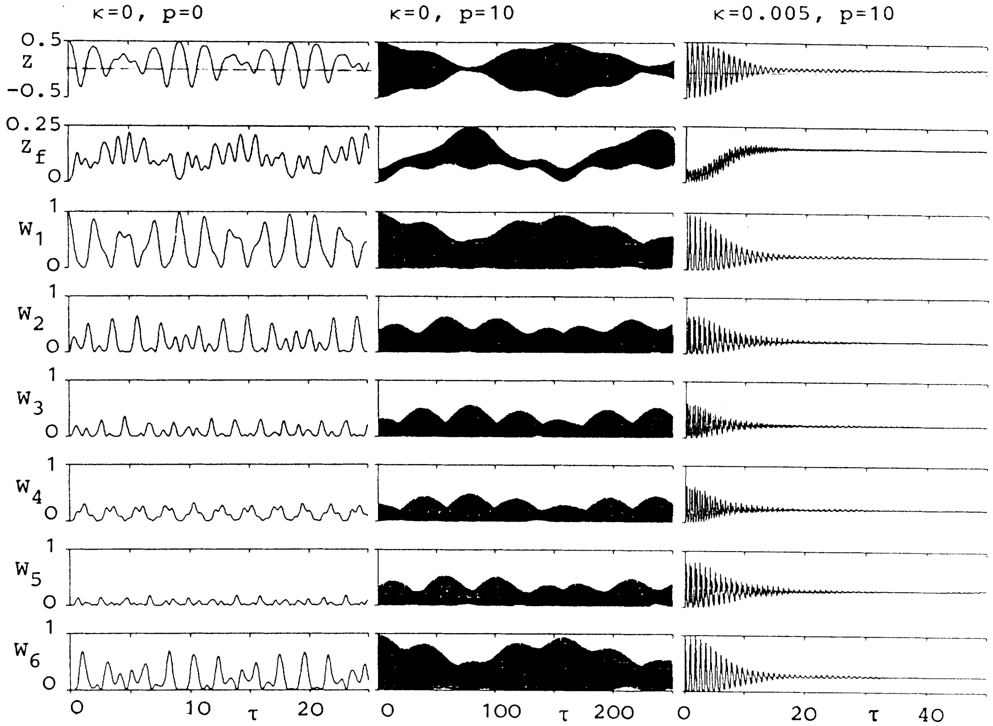

FIG. 8. Time evolutions of energy EV $Z(\tau)$, energy fluctuations $Z_f(\tau)$ and occupation probabilities $W_{n+1}(\tau)$ of atomic levels $|n\rangle_A$, $n=0,1,\ldots,5$ for $N=5$, $p=0,10$, and $\kappa=0,0.005$.

The average over the members of the ensemble gives the EV $\langle R^z\rangle_t$.

V. SUMMARY AND DISCUSSION

In the preceding sections we have examined the influence of the Fock-state field on the many-atom radiation processes in a cavity. First, we have demonstrated some new aspects of *cooperatively inhibited average radiation emission,* namely, the appearance of *collapse* and *revival* phenomena in the Rabi oscillations of the energy EV in a lossless cavity. In Fig. 9, to show that the revivals of the envelopes occur quasiperiodically, we plot the energy EV $Z(\tau)$ and the energy fluctuations $Z_f(\tau)$ for $N=10$ and $p=10,70$ for longer time intervals. This figure shows that the larger number of initial photons increases the revival periods of the envelopes. The collapse and revival of the envelopes is similar to that in the coherent Jaynes-Cummings model[12] with the difference that there is no total collapse to the time-averaged value taken over many Rabi oscillation periods (this value is shifted above zero in the many-atom case), but a complete revival occurs. However, as can be seen from Fig. 9, the increasing number of initial photons p causes a more complete collapse.

Moreover, as was already pointed out in the Sec. IV, from Fig. 9 it can be seen that the strong energy fluctuations Z_f cause quasiperiodical collapses of the envelopes of the Rabi oscillations of Z. The collapse and revival occurs at times where the average value of the energy fluctuations Z_f has its maximum or its minimum, respectively. In other words, collapse and revival phenomena arise from averaging over an ensemble of identical many-atom systems placed into a resonant single-mode cavity. A single many-atom system does not exhibit such a behavior.

Second, to our knowledge we have demonstrated for the first time that small cavity losses damp these Rabi oscillations of the energy EV to a *quasistationary value.* This effect arises from averaging over an ensemble of identical systems and does not appear in the time behavior of a single member of the ensemble. In the single-atom case this quasistationary value equals zero, whereas in the many-atom case, as a consequence of *collective inhibition effects,* this value lies above zero. The mean photon number decays exponentially:

$$\langle a^\dagger a\rangle_t \approx e^{-2Kt}\langle a^\dagger a\rangle_{t_0},\quad 0\le t-t_0\le \Delta t_{\mathrm{qs}} \tag{36}$$

during the lifetime Δt_{qs} of the quasistationary state.

The question about the appearance of quasistationary states in the case of initial field distributions differing from the Fock-state field arises certainly. This question will be discussed here in the case of an initial coherent field and an initial thermal field.

The initial density operator $\rho_R(0)$ for a coherent state field reads as

$$\rho_R(0)=|\alpha\rangle\langle\alpha| = \sum_{p=0}^{\infty}\rho_R^{p,p}(0)+\sum_{\substack{p,p'=0\\ p\neq p'}}^{\infty}\rho_R^{p,p'}(0)\ , \tag{37}$$

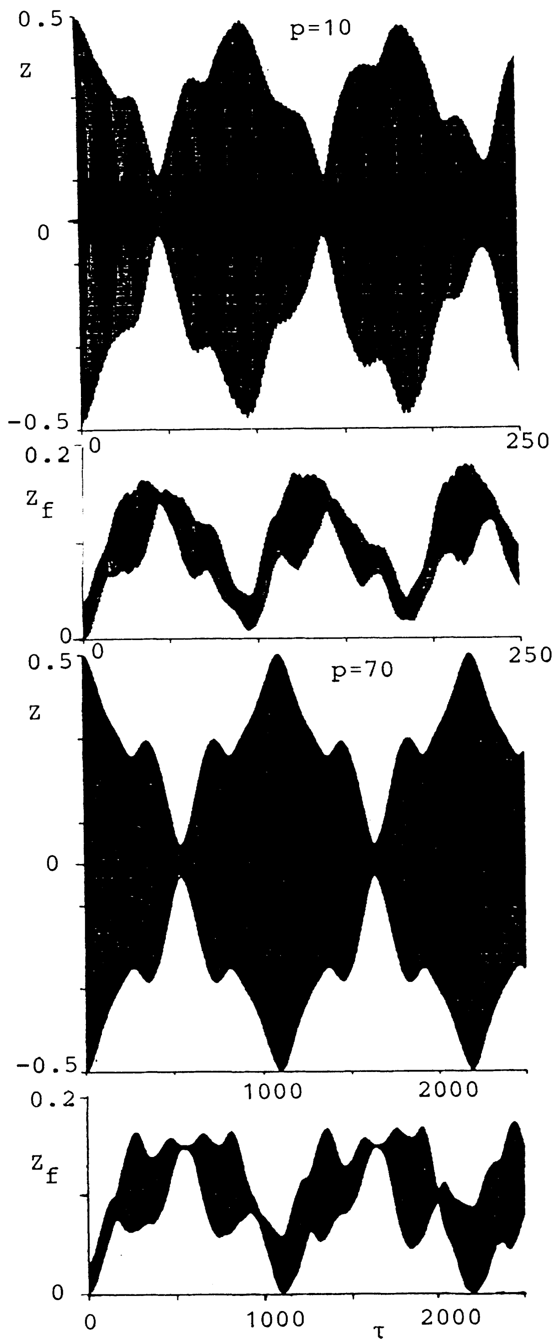

FIG. 9. Time evolution of the energy EV $Z(\tau)$ and energy fluctuations $Z_f(\tau)$ for longer time intervals for $N=10$ and $p=10,70$ in the case of an ideal cavity ($\kappa=0$).

$$\rho_R^{p,p}(0)=w_p|p\rangle\langle p|, w_p=e^{-|\alpha|^2}\frac{(|\alpha|^2)^p}{p!}\ ,$$
$$\sum_{p=0}^{\infty} w_p=1\ , \tag{38}$$

$$\rho_R^{p,p'}(0)=e^{-|\alpha|^2}\frac{\alpha^p(\alpha^*)^{p'}}{\sqrt{p!}\sqrt{p'!}}|p\rangle\langle p'|\ , \tag{39}$$

where w_p is the Poisson distribution of photons over Fock (number) states $|p\rangle$ and $|\alpha|^2=\langle a^\dagger a\rangle_0=n_0$ is the initial mean photon number. The expectation value of the atomic population inversion operator and photon-number operator then reads

$$\langle R^z\rangle_t=\mathrm{Tr}_{AR}[R^z e^{-it(L_{AR}+i\Lambda_R)}\rho_A(0)\otimes\rho_R(0)]$$
$$=\sum_{p=0}^{\infty} w_p\langle R^z\rangle_t^{p,p}\ , \tag{40}$$

$$\langle a^\dagger a\rangle_t=\sum_{p=0}^{\infty} w_p\langle a^\dagger a\rangle_t^{p,p}\ , \tag{41}$$

where

$$\langle R^z\rangle_t^{p,p'}=\mathrm{Tr}_{AR}[R^z e^{-it(L_{AR}+i\Lambda_R)}\rho_A(0)\otimes\rho_R^{p,p'}(0)] \tag{42}$$

and

$$\langle a^\dagger a\rangle_t^{p,p'}=\mathrm{Tr}_{AR}[a^\dagger a e^{-it(L_{AR}+i\Lambda_R)}\rho_A(0)\otimes\rho_R^{p,p'}(0)] \tag{43}$$

are the atomic population inversion and mean photon number, respectively, in the case of an initial field density matrix $\rho_R^{p,p'}(0)$. In obtaining Eqs. (40) and (41) we took into account that

$$\langle R^z\rangle_t^{p,p'}=0,\quad p\neq p' \tag{44}$$

$$\langle a^\dagger a\rangle_t^{p,p'}=0,\quad p\neq p' \tag{45}$$

because

$$\mathrm{Tr}_A[R^z L_{AR}^{2l+1}\rho_A(0)]=0,\quad l=0,1,\ldots \tag{46}$$

$$\mathrm{Tr}_R[L_{AR}^{2l}\rho_R^{p,p'}(0)]=0,\quad l=0,1,\ldots \tag{47}$$

$$\mathrm{Tr}_R[a^\dagger a L_{AR}^{2l}\rho_R^{p,p'}(0)]=0,\quad l=0,1,\ldots \tag{48}$$

$$\mathrm{Tr}_A[L_{AR}^{2l+1}\rho_A(0)]=0,\quad l=0,1,\ldots \tag{49}$$

where $\rho_A(0)$ is given by Eq. (9).

Eqs. (40) and (41) mean that the atomic population inversion $\langle R^z\rangle_t$ as well as the mean photon number $\langle a^\dagger a\rangle_t$ are simple linear combinations of population inversions $\langle R^z\rangle_t^{p,p}$ and mean photon numbers $\langle a^\dagger a\rangle_t^{p,p}$ corresponding to initial Fock-states fields with p photons, respectively. Therefore, we can use our Eq. (19) for numerical calculations in the case of an initial coherent field.

In Figs. 10 and 11 we plot the numerical results for an initial coherent field with the mean photon number $\langle a^\dagger a\rangle_0=10$ in the single- and five-atom case for small cavity damping $\kappa=0.005$, 0.05. The obtained results show that quasistationary states appear. There is much analogy to the quasistationary states appearing in the case of an initial Fock-state field. This can be well understood by considering the Poissonian distribution w_p, which is the probability for finding p photons in the Fock state $|p\rangle$. Namely, in calculation of the population inversion the contribution of Fock states with small p's (where no quasistationary states appear) is extremely small (e.g., $w_0=4.5\times10^{-5}$, $w_1=4.5\times10^{-4}$, $w_5=3.7\times10^{-2}$ for $\langle a^\dagger a\rangle_0=|\alpha|^2=10$). Therefore, only the

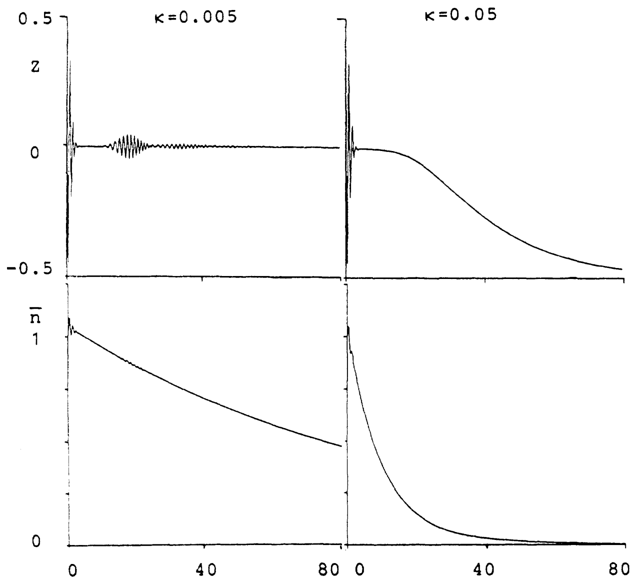

FIG. 10. Time evolution of the atomic population inversion $Z(\tau)$ and mean photon number $\bar{n}(\tau)=n(\tau)/n_0$ in the case of a single atom ($N=1$) interacting with an initial *coherent field* (with the mean photon number $n_0=|\alpha|^2=\langle a^\dagger a\rangle_0=10$) inside a cavity with damping, $\kappa=0.005$, 0.05.

contribution of Fock states with higher photon numbers (where quasistationary states already appear) play a role.

In Figs. 10 and 11 two kinds of quasistationary states can be distinguished. After a short time interval the so-called coherent Jaynes-Cummings quasistationary state,[12] which is due to the special Poissonian photon distribution of the field, appears. That is to say, in the EV of population inversion $\langle R^z\rangle_t$ there is a sum over all EV's $\langle R^z\rangle_t^{p,p}$ corresponding to different initial Fock states, But since each $\langle R^z\rangle_t^{p,p}$ oscillates with a different Rabi frequency after some time interval destructive interference causing a collapse of the envelope occurs. This is the *first kind of quasistationary states.* Since these states are not a consequence of the cavity damping, a subsequent revival of the envelope can be observed.

The *second kind of quasistationary states,* which has been already discussed above, appears later as a consequence of cavity losses. Despite small cavity losses, the Rabi oscillations corresponding to higher photon number states [which give the only relevant contribution at small times, cf., Eq. (26)] will be damped to a quasistationary value after some short time interval. Therefore no revival can occur any more. The *exponential decay* of the mean photon number can be observed in both kinds of quasistationary states (cf. Figs. 10 and 11). Moreover, the comparison of Figs. 10 and 11 shows that *collective average-energy inhibition effects* shift the quasistationary value of the population inversion above zero.

In the case of an initial thermal field the photon distribution reads as

$$w_p^{\mathrm{th}}=\frac{n_0^p}{(1+n_0)^{p+1}}, \quad n_0=\langle a^\dagger a\rangle_0 \tag{50}$$

and the corresponding field density operator reads as

$$\rho_R(0)=\sum_{p=0}^{\infty} w_p^{\mathrm{th}}|p\rangle\langle p|, \quad \sum_{p=0}^{\infty} w_p^{\mathrm{th}}=1 \ . \tag{51}$$

From the thermal photon distribution over Fock states it follows that the contribution of photon number states with no or lower number of photons predominates (e.g., $w_0=\frac{1}{11}$ for $\langle a^\dagger a\rangle_0=n_0=10$). Since according to Eq. (26) the Rabi oscillations corresponding to lower number Fock states are weaker damped, no quasistationary states appear for such initial states. Therefore, it can be con-

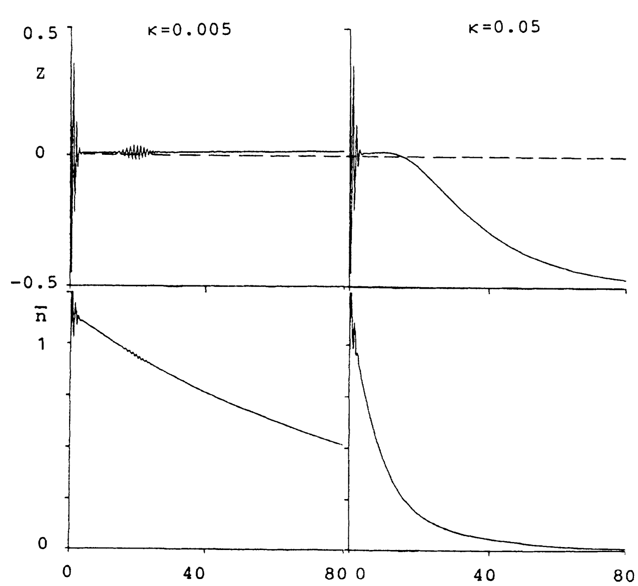

FIG. 11. Same as Fig. 9 for $N=5$.

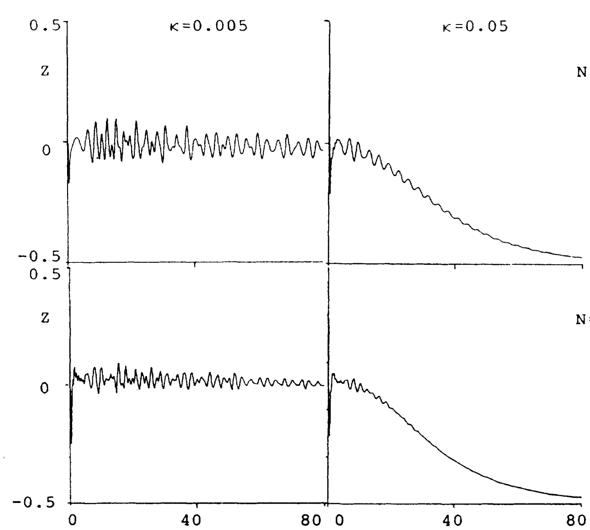

FIG. 12. Time evolution for the atomic population inversion $Z(\tau)$ for $N=1,5$ inside a cavity with an initial *thermal field* ($n_0=\langle a^\dagger a\rangle_0=10$) and damping, $\kappa=0.005$, 0.05.

cluded that since the contribution of terms $\langle R^z \rangle_t^{p,p}$ for $p \gg 1$ is small, no quasistationary states can appear in the case of an initial thermal field. This is also confirmed by numerical calculations which are plotted in Fig. 12 in the single- and five-atom case for $n_0 = 10$ and $\kappa = 0.005$, 0.05.

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[1]C. K. Hong and L. Mandel, Phys. Rev. Lett. **56**, 58 (1986).

[2]P. Filipowicz, J. Javanainen, and P. Meystre, J. Opt. Soc. Am. B **3**, 906 (1986).

[3]J. M. Raimond, P. Goy, M. Gross, C. Fabre, and S. Haroche, Phys. Rev. Lett. **49**, 117 (1982); **49**, 1924 (1982); P. Goy, J. M. Raimond, M. Gross, and S. Haroche, *ibid.* **50**, 1903 (1983); Y. Kaluzny, P. Goy, M. Gross, J. M. Raimond, and S. Haroche, *ibid.* **51**, 1175 (1983); D. Meschede, H. Walther, and G. Müller, *ibid.* **54**, 551 (1985); G. Rempe, H. Walther, and N. Klein, *ibid.* **58**, 353 (1987).

[4]J. Seke, Lett. Nuovo Cimento **43**, 103 (1985); Opt. Lett. **11**, 189 (1986); Phys. Rev. A **33**, 739 (1986); **33**, 4409 (1986); G. Adam and J. Seke, Opt. Commun. **62**, 413 (1987).

[5]S. Sachdev, Phys. Rev. A **29**, 2627 (1984); G. S. Agarwal and R. R. Puri, *ibid.* **33**, 1757 (1986); R. R. Puri and G. S. Agarwal, *ibid.* **33**, 3610 (1986); **35**, 3433 (1987).

[6]J. Seke and F. Rattay, J. Opt. Soc. Am. B **4**, 380 (1987).

[7]I. R. Senitzky, Phys. Rev. A **3**, 421 (1971).

[8]E. Abate and H. Haken, Z. Naturforsch. **19a**, 857 (1964).

[9]I. R. Senitzky, Phys. Rev. A **6**, 1175 (1972); Phys. Rev. Lett. **47**, 1503 (1981).

[10]R. H. Dicke, Phys. Rev. **93**, 99 (1954).

[11]G. Scharf, Helv. Phys. Acta **43**, 806 (1970).

[12]J. H. Eberly, N. B. Narozhny, and J. J. Sanchez-Mondragon, Phys. Rev. Lett. **44**, 1323 (1980); H. I. Yoo, J. J. Sanchez-Mondragon, and J. H. Eberly, J. Phys. A **14**, 1383 (1981); N. B. Narozhny, J. J. Sanchez-Mondragon, and J. H. Eberly, Phys. Rev. A **23**, 236 (1981).