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Experimental measurement of nonlinear plasma wake fields

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We report direct high-resolution observation of nonlinear steepened plasma waves excited in the wake of an intense, self-pinched electron beam. Oscillations in both accelerating and deflecting fields are measured, and analyzed in the context of linear and nonlinear plasma-wave theory. The degree of nonlinearity in the wake fields is shown to be consistent with analytical predictions of the beam self-pinching. The impact of these results on plasma acceleration and focusing schemes is discussed.

The plasma wake-field accelerator (PWFA), a promising ultrahigh-gradient-acceleration scheme which uses longitudinal electric fields in plasma waves driven in the wake of a bunched relativistic electron beam, has been the subject of much recent experimental[1] and theoretical[2-8] investigation. Much of the theoretical effort to date has concentrated on the linear regime of the PWFA, in which the amplitude of the sinusoidal electron density perturbations in the electron plasma waves n_1 is much smaller than the ambient unperturbed plasma density n_0. The predominance of the linear theory is due to its usefulness—it allows calculation of the general three-dimensional variation of the transverse and longitudinal wake fields if the driving electron-beam-current distribution is a known quantity, unchanging in time. Thus the standard linear treatment is inadequate to deal with large-amplitude plasma wake fields, and also ignores the self-pinching of the driver beam,[9] an effect that has been proposed as a powerful final focusing lens (plasma lens[3]) for high-energy linear colliders. On the other hand, previous theoretical analyses of the nonlinear regime of the PWFA have treated only the one-dimensional case, due to the mathematical complexity of the three-dimensional problem.

The only previous experimental test of the PWFA was performed at the Argonne advanced accelerator test facility[10] (AATF) with experimental conditions that satisfied the assumptions of both linear plasma response and negligible beam pinching. The experiments we report here, also performed at the AATF, were executed with much higher driving beam current densities present. These more intense beams undergo a self-pinch in the plasma, increasing the beam-current density further, which in turn increases the amplitude of the wake plasma waves into the nonlinear regime. For moderately nonlinear waves (the plasma electrons become only slightly relativistic[8]), steepened wave profiles develop which can be decomposed into Fourier harmonics of the fundamental frequency, the plasma frequency $\omega_p = (4\pi e^2 n_0/m_e)^{1/2}$. Nonlinear beat-wave excited plasma waves have been indirectly observed by laser scattering off these harmonics in the electron density perturbation[11] and predicted in computer simulations of plasma wake fields.[6] We present below high-resolution measurements of both longitudinal and transverse plasma wake fields which show the clear signature of nonlinearity: steepening and harmonic generation.

As there exist no two- or three-dimensional nonlinear treatments of the PWFA, and since we predict quite dramatic changes in the transverse profile of the driving beam, we must employ a hybrid of linear and nonlinear plasma-wave theories, as well as plasma-focusing theories to understand the experimental results we present. Using theoretical arguments originally derived for the study of beam dynamics[12,13] in the plasma lens, we estimate the expected reduction in beam size for the beam and plasma conditions present. This pinched-beam profile is then used in the linear approximation to calculate the amplitude of the fundamental frequency excitation of the plasma wake fields. Using the harmonic decomposition as a perturbation theory, we then estimate the expected amplitude of the higher harmonics in the wake fields. These calculations are shown to be consistent with the experimental data.

The high-intensity 21-MeV driving electron-beam pulse used in the present experiments has the following characteristics: number of electrons per pulse $N = 2.5\times 10^{10}$ ($Q = 4$ nC), rms pulse length $\sigma_z = 2.1$ mm, initial rms radius $\sigma_r = 1.4$ mm, and emittance $\epsilon = 7\times 10^{-6}$ m rad. The target plasma column is provided by a hollow cathode arc source of length $L = 33$ cm and variable density $n_0 = (0.4\text{–}7)\times 10^{13}$ cm^{-3}. The plasma density is quite uniform, declining by less than 6% along the length of the column. The driving pulse is followed in time through the plasma by a low-intensity, 15-MeV witness beam of approximately the same dimensions, which is variable in delay from about -0.2 to greater than 1.0 nsec. After the plasma interaction both beams are analyzed for energy spectra and transverse deflections by a high-resolution, broad-range spectrometer. At each point in witness-beam delay the centroids of the witness-beam distribution in the energy analyzing and deflecting planes are calculated. The wake-field scans in these experiments consist of many such points incremented in delay by fine time steps over a range of many oscillation lengths. The AATF, plasma source, beam diagnostics, and data acquisition and analysis are described further in Refs. 1 and 10.

For the purpose of illustrating most of the relevant physical phenomena within the scope of this Communication, we show two representative wake-field scans. The first scan, shown in Figs. 1–3, is taken with a relatively low plasma density $n_0 = 7.3\times 10^{12}$ cm^{-3}. The witness and

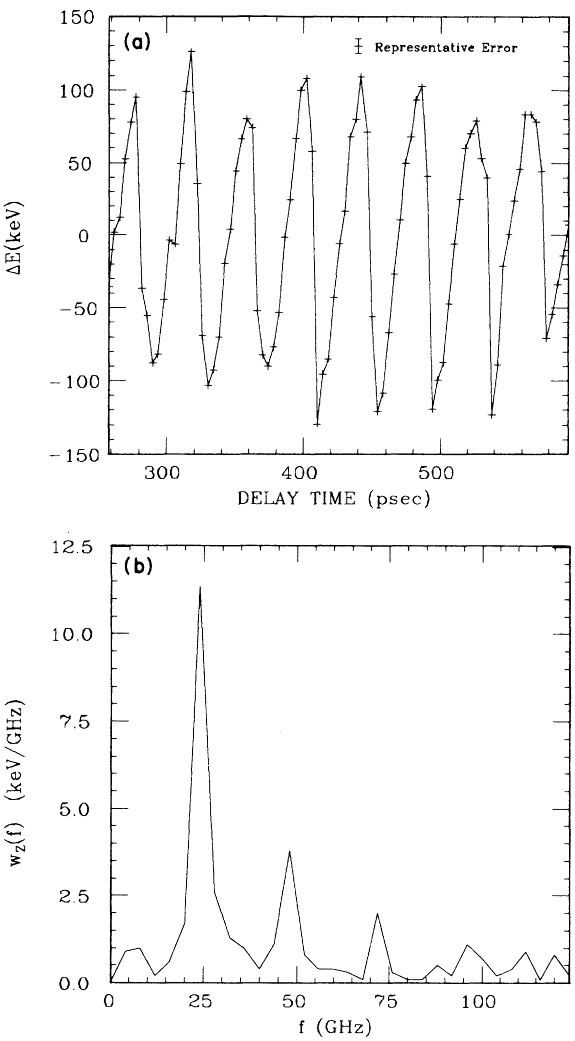

FIG. 1. (a) Longitudinal wake-field-scan–witness-beam energy centroid ΔE motion vs time delay, with plasma density of $n_0 = 7.3 \times 10^{12}$ cm^{-3}. (b) FFT amplitude function $w_z(f)$ for longitudinal wake fields.

driver beams are slightly misaligned in this scan to allow observation of the longitudinal dependence of the transverse wake fields in the nonbend plane of the spectrometer.

Several qualitative remarks can be made upon inspection of Figs. 1(a) and 2. The first is that both the longitudinal and transverse wake fields are stable, oscillatory functions of the distance behind the driving beam $\zeta = ct - z$. Secondly, the longitudinal wake fields W_z have taken on a more saw-tooth appearance, as would be naively expected from the one-dimensional nonlinear theory (cf. Ref. 7). The transverse wake fields (W_r for a cylindrically symmetric driver) show a form consistent with the differential form of the Panofsky-Wenzel theorem,[14] $\partial_r W_z = \partial_z W_r$. It is apparent from inspection that the measured longitudinal wake fields are to a good approximation proportional to the longitudinal derivative of the measured transverse wake fields. In addition to these

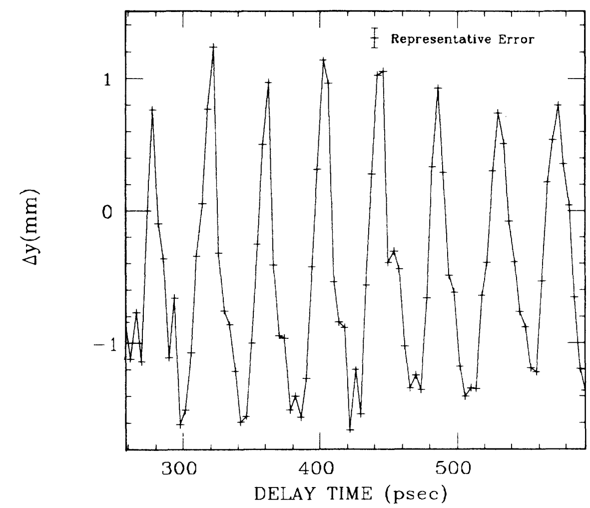

FIG. 2. (a) Transverse wake-field-scan–witness-beam deflection plane Δy centroid vs time delay for the same scan as in Fig. 1.

qualitative remarks on the waveforms, we will find it of use to employ the Fourier spectrum of the longitudinal wakefields in attempting to quantify the physical basis for the nonlinearity of these waves. The fast Fourier transform (FFT) of the longitudinal wakefield shown in Fig. 1(a) is displayed in Fig. 1(b); note that the ratio of first harmonic to the fundamental amplitude in the wakefields is about 0.3.

The one-dimensional nonrelativistic theory of nonlinear plasma waves gives the Fourier decomposition of a wakeplasma electron density wave in terms of the ratio of fundamental frequency amplitude to the ambient density

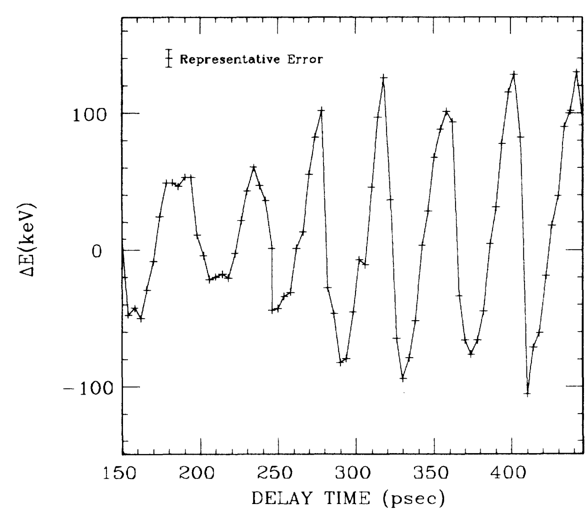

FIG. 3. Longitudinal wake field for the same scan as Fig. 1, with a different delay range. Below 250 psec in delay, the driving beam charge is 2.9 nC. Between 250 and 280 the charge is raised to 4 nC, where it remains for the rest of the scan. The disproportionate rise in the wake-field amplitude is attributable to stronger self-pinching of the driver beam.

n_1/n_0. Note we have used the symbol n_1 here as the fundamental Fourier amplitude in the density perturbation, and we purposefully draw explicit equivalence to our previous usage, where n_1 indicated the calculated perturbation at this frequency due to linear theory. The amplitude of the mth wave harmonic is given by[8] $n_m/n_0=(n_1/n_0)^m m^m/2^{m-1}m!$, and the longitudinal electric field on axis associated with this charge-density wave is, ignoring phase factors,

$$E_z=\sum_{m=1}^{\infty} E_m e^{imk_p\zeta}=4\pi e n_0 \sum_{m=1}^{\infty} \frac{n_m}{n_0}\frac{\eta_{rm}(mk_p\sigma_r)}{mk_p} e^{imk_p\zeta},$$

where $k_p=\omega_p/c$ and $\eta_{rm}(mk_p\sigma_r)$ is a factor less than unity which measures the degree to which the wake fields are longitudinal.[1,4] This factor is a monotonically increasing function of its argument, and thus is an increasing function of harmonic number m. The ratio of first harmonic E_2 to fundamental E_1 amplitude is simply $\frac{1}{2}(n_1/n_0)(\eta_{r2}/\eta_{r1})$. This implies, for example, that the density perturbation at the fundamental frequency in the scan shown in Figs. 1 and 2 is $n_1/n_0\simeq 0.33$.

This wave amplitude is not consistent with the predictions of linear theory if one ignores possible pinching of the driver beam. This is easily seen, as in the linear theory the wake wave amplitude for a bi-Gaussian driver is given by[1,4]

$$n_1/n_0=2Nr_e\exp[-(k_p\sigma_z)^2/2]/k_p\sigma_r^2 .$$

For the present case, with σ_r taken as its initial value, this yields $n_1/n_0=0.08$, which is much smaller than the estimate from harmonic content. Thus we are led to suggest that significant self-pinching of the driver must have occurred.

This hypothesis is supported by several observations. The first is the measurement of large witness beam deflections at zero delay,[1] which indicates the existence of large focusing fields. The second is that the driving beam is aperture limited by the plasma anode with the plasma source off, and 20% of the beam current is not transmitted to the spectrometer. When the plasma is turned on, the full current is transmitted throughout the previously limiting aperture. Thirdly, the driver-beam image in the spectrometer nonbend plane is greatly expanded with plasma present, implying strong overfocusing. Finally, we can examine a different range of the scan, shown in Fig. 3, one in which the beam charge is diminished a factor of 0.72 from 4.0 to 2.9 nC, and we observe a disproportionate reduction in the wake-field amplitude to a factor of approximately 0.35. This indicates that the pinch effect has been taken below a threshold for significant reduction in beam size, with its resultant enhancement of the wake-field amplitude. Note also that the longitudinal wake fields are much more sinusoidal at the lower amplitudes, and that the measured and predicted amplitudes (assuming no driving beam pinching) of n_1/n_0 in this case are 0.09 and 0.06, respectively, in much closer agreement than with the full beam charge present.

To study beam self-pinching in greater detail, we move on to the analysis of a second scan, shown in Fig. 4, in which the plasma is nearly four times as dense as in the first scan. The nonlinearity of these waves is even more pronounced than in the previous scan. The plasma wavelength $\lambda_p=2\pi/k_p$ in this case is twice as short as before (0.65 mm), and is now shorter than the beam length $4\sigma_z=0.84$ mm. This case is in the regime of the plasma lens, where the self-focusing forces on the beam can be well approximated by assuming total space-charge neutralization of the beam and evaluating the magnetic self-forces of the electron beam distribution.[12] Although this produces nonideal focusing forces, an approximate envelope equation for the amplitude or β function $(\beta=\sigma_r^2/\epsilon)$ can be derived to give some insight into the beam dynamics.[13] We are interested in two quantities from this treatment: the distance inside the plasma required to achieve a pinch (one-half of a betatron wavelength) s and equilibrium pinched beam spot size $\sigma_{\rm eq}^2$. In terms of the initial value of the β function $\beta_0=0.3$ m and the invariant $\psi_0=Nr_e/\sqrt{4\pi}\epsilon_n\sigma_z=35$ m^{-1}, where ϵ_n is the normalized emittance, we have $\sigma_{\rm eq}^2=\epsilon/\psi_0=(0.44\text{ mm})^2$ and $s\simeq(\pi/2)\sqrt{\beta_0/\psi_0}=14$ cm. Thus we expect the beam to pinch to approximately one-third its original radius well within the plasma column. This pinching is not uniform along the length of the beam, as in our model it depends on the enclosed current at a given point in ζ, which for our case gives

$$\psi(\zeta)=\psi_0\exp[-(\zeta/\sigma_z)^2/2] .$$

Thus the equilibrium density profile on the axis is approximately proportional to the square of the enclosed current $\sim\exp[-(\zeta/\sigma_z)^2)]$, i.e., the bunch is effectively shortened on the axis by a factor of $\sqrt{2}$.

The FFT of the scan in Fig. 4 gives a relative first harmonic amplitude value of $E_2/E_1\simeq 0.48$. The predicted value, using $\sigma_r=\sigma_{\rm eq}=0.44$ mm and $\sigma_z=2.1/\sqrt{2}$ mm in the linear-response formula, is $E_2/E_1=0.38$, in fairly good agreement. In the first scan, the equilibrium pinched-beam radius necessary for the theoretical estimate to agree with the measured harmonic content $\sigma_{\rm eq}=0.77$ mm is not as small as our calculated value for the second scan. This is due to the fact that the beam

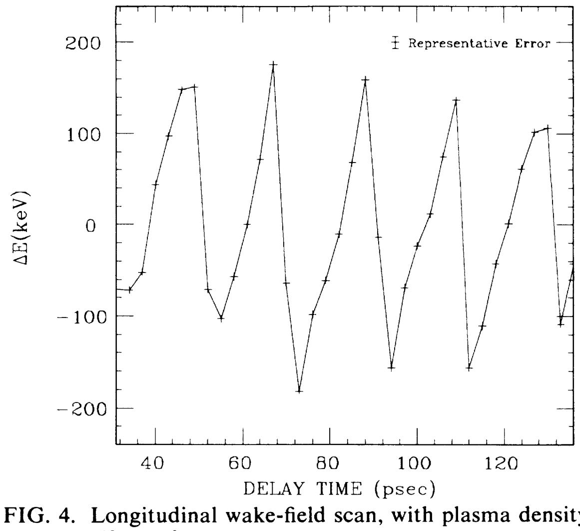

FIG. 4. Longitudinal wake-field scan, with plasma density of $n_0=2.8\times10^{13}$ cm^{-3}.

length is shorter than the plasma wavelength, the the beam does not encounter maximum strength transverse wake fields, as the plasma does not react quickly enough to completely neutralize the beam charge. The driving beam thus takes longer to pinch and does not achieve such small spot sizes.

We have dealt successfully with the data here in a rather phenomenological manner. A more rigorous theoretical and computational understanding of the physics issues would be desirable. The main questions raised by this experiment have to do with the nonlinear plasma response. In these experiments, the beam is much narrower than a plasma wavelength, and thus the plasma electron response and associated wake fields cannot be well explained by a one-dimensional theory. Conversely, one-dimensional theory predicts improved transformer ratios in very nonlinear waves driven by long beams of density $n_0/2$; it is not possible to confirm this prediction by extrapolating our results. On the other hand, the status of the theory and simulation of beam focusing in plasmas is more well understood than the three-dimensional nonlinear plasma motion. These experimental results are consistent with the theoretical picture of the self-pinch dynamics, and can be viewed as a successful test of the concept of the thick plasma lens, as the peak beam density is increased by a factor of 10 inside the plasma.[13]

One surprising aspect of these experiments is that the longitudinal and transverse wake fields are so well behaved under these conditions of moderate nonlinearity. In fact, the wake fields in the first scan were observed out to 18 wavelengths behind the driver with little degradation in form or amplitude. Also, in one-dimensional theory, one would expect plasma electron relativistic effects to cause an increase in the plasma period in waves as steepened as we have presented here. If anything, there appears to be a period *decrease* with larger amplitude (see Fig. 3). One would expect less-severe relativistic effects in our cases, however, as the waves are predominately radial, which causes less energy to be transferred to the plasma electron motion and a concommitant degradation of the longitudinal wake-field amplitude. The measured longitudinal wake-field amplitudes including the effects of the geometrical resolution of the witness beam[1] are, for the scan in Fig. 1, $W_m = 1.48 \pm {}^{0.21}_{0.15}$ MeV/m, and for the scan in Fig. 4, $W_m = 5.30 \pm {}^{0.74}_{0.35}$ MeV/m, which is the largest accelerating gradient obtained thus far in a plasma wave. The issue of period lengthening will be addressed in future experiments.

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