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Theory of a two-photon squeezed laserlike oscillator

R. Ghosh and G. S. Agarwal
School of Physics, University of Hyderabad, Hyderabad-500 134, India
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It is theoretically shown how a two-photon laserlike oscillator with strongly squeezed output can be constructed. The physical mechanism leading to squeezed laserlike oscillator is the strong competition among the nonlinear gain due to resonant four-wave mixing process, nonlinear losses due to the absorption of generated photons, and linear losses from the cavity. In the balanced case, the photon correlations result in a linewidth that is narrower than the Schawlow-Townes limit.

In this paper, we present the theory of a novel kind of two-photon laserlike oscillator whose output is automatically squeezed[1] as a result of strong four-wave mixing in the nonlinear gain medium. In our model, an intense pump laser beam of frequency ω causes the two-photon excitations in an active nonlinear medium which is placed inside a cavity. Two radiation fields of frequencies ω_1 and ω_2 are generated due to four-wave mixing. These photons can get reabsorbed by a two-photon absorption process. We show that a strong competition among four-wave mixing, two-photon absorption, and linear cavity losses leads to lasing action above a certain threshold determined by the nonlinear mixing and the linear damping constants. We also show that the strong correlation between the generated photons leads to a narrower linewidth.

We next present the basic equations[2] for the two-photon squeezed laserlike oscillator. We treat the pump field $E(\omega)$ classically. Let a and b be the annihilation operators for the fields at ω_1 and ω_2, respectively, and $a^\dagger$ and $b^\dagger$ the corresponding creation operators. Under the resonant condition $2\omega=\omega_1+\omega_2$, the dynamical evolution of the fields at ω_1 and ω_2 is described by the master equation for the field density matrix ρ [as in Ref. 2, Eq. (1), but with cavity losses included now]:

$$\frac{\partial\rho}{\partial t}=|G|(a^\dagger b^\dagger\rho-\rho a^\dagger b^\dagger)+|G|(\rho ab-ab\rho)$$
$$-\frac{\kappa}{2}(a^\dagger b^\dagger ab\rho-2ab\rho a^\dagger b^\dagger+\rho a^\dagger b^\dagger ab)-\gamma_a(a^\dagger a\rho-2a\rho a^\dagger+\rho a^\dagger a)-\gamma_b(b^\dagger b\rho-2b\rho b^\dagger+\rho b^\dagger b)\,. \qquad (1)$$

Here the nonlinear gain parameter G is proportional to the lowest-order nonlinear susceptibility $\chi^{(3)}$ for four-wave mixing:[3]

$$G=-2\pi\omega_2\chi^{(3)}(\omega,\omega,-\omega_1)E^2(\omega)$$
$$=-2\pi\omega_1\chi^{(3)}(\omega,\omega,-\omega_2)E^2(\omega)\,. \qquad (2)$$

We have defined a and b in such a way so as to eliminate the phase of G. The nonlinear absorption parameter κ is related to the susceptibility for two-photon absorption:[3]

$$\kappa=8\pi^2\omega_1\omega_2\hbar\,\mathrm{Im}\chi^{(3)}(\omega_2,-\omega_2,\omega_1)/V\,, \qquad (3)$$

where V is the quantization volume for the field mode. The linear-loss parameters γ_a and γ_b arise because of possible leakage from the end mirrors of the cavity and are taken to be slightly different for the two radiation fields at ω_1 and ω_2 in the general case. Because of the single-photon decay terms in this system, the conservation law $\langle(a^\dagger a-b^\dagger b)^p\rangle=0$, $p=1,2,\ldots$ does not hold. This broken symmetry has important consequences, such as the occurrence of a threshold and phase-transition-like behavior of the system. A similar situation is encountered in the treatment of optical bistability where the total cooperation number (corresponding to the angular momentum operator S^2) is not conserved owing to the single-atom relaxation terms. In deriving (1) we have ignored the linear absorption of ω_1 and ω_2 by the atomic medium assuming that ω_1 and ω_2 are far from the frequencies of the intermediate levels. If need be, then this absorption can be accounted for by a redefinition of γ_a and γ_b.

An important consequence of this broken symmetry is also that the destructive interference discussed in Refs. 2 and 3 is no more fully operative. As a result, there is always some population in the excited state.

From Eq. (1) the mean-value equations for the field amplitudes are found to be

$$\langle\dot a\rangle=\mathrm{Tr}(\dot\rho a)=|G|\langle b^\dagger\rangle-\frac{\kappa}{2}\langle ab^\dagger b\rangle-\gamma_a\langle a\rangle\,, \qquad (4)$$

$$\langle\dot b\rangle=\mathrm{Tr}(\dot\rho b)=|G|\langle a^\dagger\rangle-\frac{\kappa}{2}\langle ba^\dagger a\rangle-\gamma_b\langle b\rangle\,. \qquad (5)$$

The steady-state solutions are obtained under the semiclassical approximation, either

$$|\langle a\rangle|^2=0,\quad |\langle b\rangle|^2=0\,, \qquad (6)$$

or

$$|\langle a\rangle|^2=\frac{2}{\kappa}[|G|(\gamma_b/\gamma_a)^{1/2}-\gamma_b]\equiv\alpha^2\,, \qquad (7a)$$

$$|\langle b\rangle|^2=\frac{2}{\kappa}[|G|(\gamma_a/\gamma_b)^{1/2}-\gamma_a]\equiv\beta^2\,. \qquad (7b)$$

Thus, for nonzero values of $\langle a\rangle$ and $\langle b\rangle$, i.e., for nonzero values of the coherent part of the field, we require

$$|G| > \sqrt{\gamma_a \gamma_b}\,. \tag{8}$$

It is clear that (8) defines an above-threshold condition for the laser oscillation in our medium. In the usual laser operation, the threshold is defined by the condition that the linear gain equals the linear absorption. In our case the threshold is defined similarly by the condition that the nonlinear gain (arising from four-wave mixing) equals the geometric mean of the two leakage rates. Equation (6) describes the situation below threshold in the steady-state. Note that in the region above threshold, steady-state is possible only because of two-photon absorption in the medium ($\kappa\neq 0$). If the two photon absorption were to be ignored, then the fields in the medium would grow under the condition (8).

It should be borne in mind that in the present system there is no population inversion which in our view is not absolutely essential for laser action. Laserlike action can result from other mechanisms as long as the system exhibits gain, which in our case arises from parametric processes.

In order to obtain the quantum statistical properties of the generated fields, we linearize the density-matrix Eq. (1) in the vicinity of the steady-state solutions α and β (real and positive) defined in (7). Let us set

$$a=\alpha+A\,,\qquad b=\beta+B\,. \tag{9}$$

Then, using up to quadratic terms in A and B, we get

$$\frac{\partial\rho}{\partial t}=-i[H_{\text{eff}},\rho]-\gamma_a'(A^\dagger A\rho-2A\rho A^\dagger+\rho A^\dagger A)$$
$$-\gamma_b'(B^\dagger B\rho-2B\rho B^\dagger+\rho B^\dagger B)-\kappa'[(A^\dagger B\rho-2B\rho A^\dagger+\rho A^\dagger B)+(B^\dagger A\rho-2A\rho B^\dagger+\rho B^\dagger A)]\,, \tag{10a}$$

where

$$H_{\text{eff}}\equiv i\sqrt{\gamma_a\gamma_b}A^\dagger B^\dagger+\text{H.c.}\,,$$
$$\gamma_a'\equiv|G|(\gamma_a/\gamma_b)^{1/2}\,,$$
$$\gamma_b'\equiv|G|(\gamma_b/\gamma_a)^{1/2}\,,$$
$$\kappa'\equiv|G|-\sqrt{\gamma_a\gamma_b}\,. \tag{10b}$$

Note that Eq. (10) is exactly solvable.[4] From (10), we obtain the linearized equations for the mean values of A, $A^\dagger$, B, $B^\dagger$. The relaxation matrix has eigenvalues (λ) given by

$$\lambda_1=0,\ \lambda_2=-2|G|,\ \lambda_3=-2\gamma,\ \lambda_4=-2(|G|-\gamma)\,, \tag{11}$$

in the balanced case when $\gamma_a=\gamma_b\equiv\gamma$. In the region above threshold, we have $|\lambda_2|>|\lambda_3|$ always. But in a range $2\gamma>|G|>\gamma$, $|\lambda_4|<|\lambda_3|$, thus making $|\lambda_4|$ the smallest eigenvalue; otherwise $|\lambda_3|$ is the smallest of the λ's, giving rise to the narrowest peak in the output spectrum.

One can construct the quantum Langevin equations for the system operators, and from the generalized Einstein relations (Ref. 5, p. 324), one arrives at the following diffusion constants:

$$\langle D_{AA^\dagger}\rangle=|G|(\gamma_a/\gamma_b)^{1/2},\quad \langle D_{BB^\dagger}\rangle=|G|(\gamma_b/\gamma_a)^{1/2}\,,$$
$$\langle D_{AB^\dagger}\rangle=|G|-\sqrt{\gamma_a\gamma_b}=\langle D_{BA^\dagger}\rangle\,, \tag{12}$$

the rest are all zero. In order to look at the linewidth of the laser, we introduce the phase operators φ_a and φ_b for the two radiation fields in the most general way[6]

$$e^{i\varphi_a}=(a^\dagger a+1)^{-1/2}a\,, \tag{13}$$

etc. Thus, the phase-difference is given by

$$\varphi\equiv\varphi_a-\varphi_b\simeq\frac{1}{2i}\left[\frac{A-A^\dagger}{\alpha}-\frac{B-B^\dagger}{\beta}\right]. \tag{14}$$

Using the mean value equations for A and B, we find that the phase ϕ satisfies the equation

$$\dot\phi=-(\gamma_a'-\gamma_b')(\phi_a+\phi_b)+R_\phi(t)\,. \tag{15}$$

The diffusion coefficient $D(\phi)$ associated with the random force R_ϕ can be calculated as follows:

$$\langle 2D(\phi)\rangle=\lim_{\Delta t\to 0}\frac{\langle[\Delta\phi(t)]^2\rangle}{\Delta t}=\frac{1}{2}\left[\frac{\langle D_{AA^\dagger}\rangle}{\alpha^2}+\frac{\langle D_{BB^\dagger}\rangle}{\beta^2}-\frac{\langle D_{AB^\dagger}\rangle}{\alpha\beta}-\frac{\langle D_{BA^\dagger}\rangle}{\alpha\beta}\right]$$
$$=\frac{|G|}{2}\left[\frac{1}{\alpha^2}(\gamma_a/\gamma_b)^{1/2}+\frac{1}{\beta^2}(\gamma_b/\gamma_a)^{1/2}\right]-\frac{(|G|-\sqrt{\gamma_a\gamma_b})}{\alpha\beta}\,. \tag{16}$$

In the balanced case $\gamma_a=\gamma_b=\gamma$, we find the result

$$\dot\phi=R_\phi(t)\,,$$
$$\langle R_\phi(t)R_\phi(t')\rangle=\langle 2D(\phi)\rangle\delta(t-t')\,,$$
$$\langle 2D(\phi)\rangle=\gamma/\alpha^2\,. \tag{17}$$

Thus, in the balanced case the relative phase obeys the diffusion equation. Note that the usual Schawlow-Townes limit for the laser linewidth is given as (Ref. 5, p. 293)

$$\langle 2D(\varphi)\rangle_{\text{ST}}=\frac{|G|+\gamma}{2\alpha^2}\,. \tag{18}$$

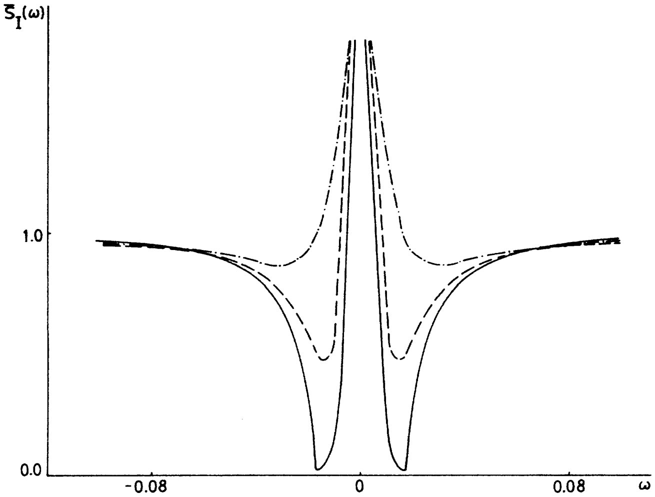

FIG. 1. Plot of the normalized spectrum $\bar{S}_I(\omega)$ [defined by (21)] with $\gamma_a/\gamma_b = 1.125$: for (a) $|G| = 1.1\sqrt{\gamma_a\gamma_b}$ (solid line); (b) $|G| = 1.5\sqrt{\gamma_a\gamma_b}$ (dashed line); (c) $|G| = 2.0\sqrt{\gamma_a\gamma_b}$ (dashed and dotted line).

Since $\gamma < |G|$ for operation above threshold, we conclude that the laser linewidth here is narrower than the Schawlow-Townes limit[7] by a factor of $2\gamma/(|G|+\gamma)$. It is clear from Eq. (16) that the correlations $\langle D_{AB^\dagger}\rangle$ and $\langle D_{BA^\dagger}\rangle$ between the two strongly coupled modes (as they are either generated simultaneously in four-wave mixing or absorbed simultaneously in two-photon absorption process) are responsible for the line narrowing. Note further that G is proportional to pump power and, thus, by increasing the pump power one can move away from threshold.

Finally, we calculate the spectrum of fluctuations in the intensity difference between the two output modes. The two-time correlation functions of the intensities $\hat{I}_j(t)$ outside the cavity and $I_j(t)$ inside the cavity are related as follows:[8]

$$\langle \hat{I}_j(\omega), \hat{I}_k(0)\rangle = 4\gamma_j\gamma_k\langle I_j(\tau), I_k(0)\rangle + 2\delta_{jk}\delta(\tau)\gamma_j\langle I_j(0)\rangle\,, \tag{19a}$$

where

$$I_a(t) = a^\dagger a,\quad I_b(t) = b^\dagger b\,,\qquad \langle x,y\rangle \equiv \langle xy\rangle - \langle x\rangle\langle y\rangle\,. \tag{19b}$$

The spectrum of the fluctuations in the difference $I = I_a - I_b$ of intensities of two output modes is defined as

$$S_I(\omega) = \int_{-\infty}^{\infty} d\tau\, e^{-i\omega\tau}\langle T{:}\hat{I}(\tau),\hat{I}(0){:}\rangle_{\mathcal{SS}}\,, \tag{20}$$

where the subscript $\mathcal{SS}$ stands for steady-state correlations, T stands for time ordering, and : : stands for normal-ordering of the operators. Using Eq. (19) we obtain the following expression for the spectrum (20):

$$S_I(\omega) = S_0 + 4\int d\tau\, e^{-i\omega\tau}\langle T{:}\gamma_a I_a(\tau) - \gamma_b I_b(\tau), \gamma_a I_a(0) - \gamma_b I_b(0){:}\rangle\,, \tag{21}$$

where

$$S_0 = 2(\gamma_a\langle I_a\rangle_{\mathcal{SS}} + \gamma_b\langle I_b\rangle_{\mathcal{SS}}) = 4\gamma_a\alpha^2 = 4\gamma_b\beta^2\,. \tag{22}$$

The normalized spectrum $\bar{S}_I(\omega) \equiv S_I(\omega)/S_0$ is evaluated numerically using the linearized master Eq. (10). Figure 1 shows the plot of $\bar{S}_I(\omega)$ for three operating points of the laser, one very close to threshold (solid curve), and others further above threshold. The vacuum or shot-noise level is given by $\bar{S}_I(\omega) = 1$, and perfect noise suppression corresponds to $\bar{S}_I(\omega) = 0$. In the region near threshold, one has strong squeezing of phase fluctuations and the squeezing gets reduced as the gain $|G|$ is increased. The behavior is the same in the balanced case. In fact, the analytical result in this case is very simple, $\bar{S}_I(\omega) = \omega^2/(4\gamma^2+\omega^2)$ which goes to zero as $\omega \to 0$ in spite of the presence of the two-photon absorption. This is because the two-photon absorption loss is not a passive loss and is balanced in a dynamical fashion by the gain and cavity losses. These features of our two-photon laser are similar to that of nondegenerate parametric oscillators.[9-11]

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the solutions are readily obtained.

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