Effects of colored pump noise on intensity correlations of a single-mode dye laser

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Recent theoretical analyses of the single-mode laser with colored-noise pump fluctuations have allowed the detailed prediction of the short-time behavior of the intensity autocorrelations of such a laser. In particular, a rounding off of the intensity correlation function on the time scale of the correlation time of the pump fluctuations has been predicted. We present results from experiments performed on a single-mode dye-laser system that clearly confirm these predictions. Numerical simulations of the laser Langevin equations including the colored-noise pump fluctuations are used to fit the experimental results.

I. INTRODUCTION

Investigations of dye-laser intensity correlations have proved to be a useful testing ground for theoretical work in nonlinear dynamics. The inclusion of both quantum- (spontaneous emission) and classical- (pump fluctuation) noise terms in the rather simple nonlinear model of a single-mode laser have led to a wealth of predictions based on various approximations. The single-mode dye laser is an easily accessable experimental system as well and with it one can test the accuracy of some of the theoretical predictions. Recently, two groups have predicted a "rounding off" of the intensity correlation function of a single-mode laser with colored-noise pump fluctuations at short times.[1–7] Measurements reported here confirm the presence of this signature of colored noise in the experimental system.

The single-mode laser has already provided a wealth of literature, both experimental and theoretical, dealing with different aspects of the effects of noise on the system. From the earliest days of successful modeling of the laser the importance of the quantum-noise term, representing spontaneous emission in the model, has been discussed.[8] More recently the importance of a multiplicative-noise term in the model, representing pump fluctuations, has been realized, when the laser in question is a dye laser.[9–13] The presence of such an additional noise term is now fairly well established, and discussion centers about the correlation time of the noise and the relative strength of the additive and multiplicative noises[14–20] as well as on the effects of the correlation time of the multiplicative noise on the system.

Investigations of the system have centered on the calculation and measurements of the one- and two-time intensity autocorrelations[1–7,9–14] and, more recently, on the transient, noise-driven, turn-on statistics of the laser.[14(a),17,18,21] Other effects of the multiplicative colored noise have, however, also been seen in the intensity distributions, leading to a novel first-order phase-transition analogy in the system.[14(b),19,20,22,23]

Early investigations of the intensity correlations had some limitations that were not immediately apparent. The importance of good time resolution at short times was not appreciated and therefore the short-time round-off discussed below was not resolvable in the early experiments.[10,12] Early models required rather large (and rather arbitrary) subtractions from the data in order to fit the long-time tails.[11,15,16] Presently theories are capable of fitting the entire correlation function without such subtractions, as we also show here.[13,19,24]

II. THEORY

The behavior of a laser that includes pump fluctuations is qualitatively different from that of a laser with a constant pump. The fluctuation properties in the vicinity of threshold serve to distinguish them readily. The laser with a fluctuating pump has three distinct regimes of fluctuation behavior. At the lowest pump (and hence, output) powers there is only fluorescence (or spontaneous emission). In this regime the laser mode displays the fluctuation behavior characteristic of thermal light. As the pump is turned up, fluctuations will tend to push the laser on and off for brief periods. In this region the laser displays extremely large intensity fluctuations. This behavior is very different from that of a laser with a constant pump. Finally, as the pumping is increased even further, the pump fluctuations, constant in magnitude, become relatively insignificant in comparison to the mean pump parameter. At this point the laser output approaches that of a coherent state, just as it would in the case of a constant pump parameter.

The model used here, in a rather simple, general form is that of a Langevin equation for the complex field amplitude of the laser, E

$$\begin{aligned}
&\dot{E}=[a+\eta(t)-|E|^2]E+q(t)\ ,\\
&E=x_1+ix_2\ ,\\
&q=q_1+iq_2,\ \ \langle q_i(t)\rangle=0,\ \ \langle q_i(t)q_j(t+\tau)\rangle=2D\delta(\tau)\delta_{ij}\ ,\\
&\eta=\eta_1+i\eta_2,\ \ \langle \eta_i(t)\rangle=0,\ \ \langle \eta_i(t)\eta_j(t+\tau)\rangle=\frac{Q\Gamma}{2}e^{-\Gamma|\tau|}\delta_{ij}\ .
\end{aligned} \tag{1}$$

Here a is the pump parameter and $q(t)$ is an additive random noise that represents the spontaneous emission or quantum noise, and which has a strength D. $\eta(t)$ is a multiplicative random-noise term that represents fluctuations in the pump parameter. This noise has a strength Q and a bandwidth Γ ($\Gamma\rightarrow\infty$ represents an approach to δ-correlated, white noise).

Equivalent models including only the pump fluctuations were studied by San Miguel, Hernandez-Machado, and Katz[1,2,5] and by Jung and Risken.[3,6,19] Later work[4,7,14,20] included the effects of the additive noise term as well. The conclusions of these theoretical studies are the following: (a) In the purely multiplicative colored-noise case (no additive noise term included) the initial decay of the intensity correlation function, over a time comparable to the correlation time of the noise, should be slow. Indeed, the intensity autocorrelation

$$\lambda(\tau)=\frac{\langle I(t)I(t+\tau)\rangle}{\langle I^2\rangle}-1 \tag{2}$$

should have an initial slope of zero [$\dot{\lambda}(\tau\rightarrow 0+)=0$]. The functional form of $\lambda(\tau)$ will then contain a change of curvature at short time.[1-7] (b) When additive white noise is included as well, the initial slope is no longer strictly zero. Instead,

$$\dot{\lambda}(0+)=\frac{-4D}{\langle I\rangle}\ , \tag{3}$$

where D is the spontaneous-emission noise strength given in (1). This implies a discontinuity of $\dot{\lambda}(\tau)$ at $\tau=0$, since $\lambda(\tau)$ must be a symmetric function. (c) Recent work[7] has shown that the effective eigenvalue, $\lambda_{\text{eff}}\equiv-\dot{\lambda}(0+)/\lambda(0)$, is a good measure of the presence of colored noise in a system such as this. In this particular case it is found that $\lambda_{\text{eff}}=4D/\lambda(0)\langle I\rangle$. These theoretical predictions were verified in the experiments discussed below.

Similar conclusions can be drawn about the intensity autocorrelations of each of the modes of a two-mode dye laser.[7] Evidence of the short-time rounding in this case can be seen in Ref. 24, although the interpretation given there is incomplete.

Recent work[23] examining the effects of white-noise gain fluctuations (as opposed to colored-noise loss fluctuations) has shown that most of the above-mentioned effects are reproduced in that system. (White "gain" noise will affect both the linear gain and the nonlinear saturation terms in the equation and, apparently, can mimic many of the effects of colored loss noise, which acts only through the linear gain term in these models.) The one feature that distinguishes the two models is the two-time-scale correlation-function decay which is present in the colored-noise case but not in the white-noise case. (Although we believe that the pump noise in our experimental system is due to "gain" fluctuations we have kept only the lowest-order multiplicative noise term, thereby making the model equivalent to one for loss fluctuations. The important feature is that the fluctuations are treated as a colored noise.)

In the present work the measured correlation functions will be fit by performing Monte Carlo computer simulations of Eqs. (1). The simulations, including both the multiplicative and additive noise terms, follow the method developed in Ref. 25 and used previously in Refs. 15, 17, 18, 22, and 24–26.

III. EXPERIMENTS

The measurements were made using photon-counting techniques and reflect their inherent advantages and disadvantages. The experimental setup is shown in Fig. 1. The hardware correlator that was used has severe dead-time limitations which limited the usable count rates. (It is important to note that the dead-time limitations here are relevant to the instantaneous, and not just the average count rates. Inasmuch as the instantaneous count rates can be a factor of 10^3 or more larger than the average count rates in the large-fluctuation regime, the average count rates must be kept correspondingly low; to the point where the pump-laser stability can limit the counting time.) Statistical fluctuations in the low-intensity data are due to an averaging over a rather limited number of fluctuations in the pump parameter that are large enough to initiate lasing action.

The coincidence method allows the extraction of one point (fixed delay) of $\lambda(\tau)$, while the correlator allows the extraction of 128 points at different delays. The two methods produce estimates of the same correlation function. For the coincidence method with no time delay,

$$N_{\text{coinc}}=\frac{N_1N_2T_R}{T}\left[1.0+\frac{1}{T_R}\int_{-T_R/2}^{T_R/2}\lambda(\tau)d\tau\right]\ , \tag{4}$$

where N_{coinc} is the number of coincidences and N_1 and N_2 are the number of counts in each of the two channels accumulated in a counting interval T. T_R is the resolving time of the coincidence detector. If T_R is small enough, one can replace $\lambda(\tau)$ in Eq. (4) with $\lambda(0)$ and Eq. (4) becomes

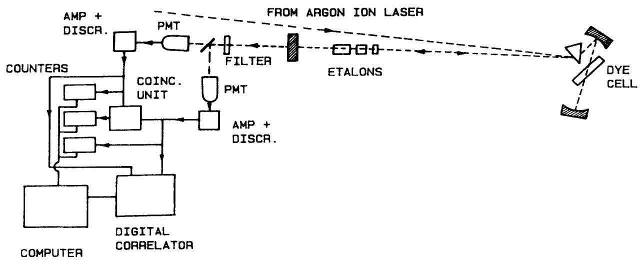

FIG. 1. Diagram of the experimental setup.

$$N_{\text{coinc}} \cong \frac{N_1 N_2 T_R}{T}[1.0+\lambda(0)] \tag{5}$$

or

$$\lambda(0) \cong \frac{N_{\text{coinc}}}{N_{\text{rand}}} - 1.0 ,$$

where $N_{\text{rand}} = N_1 N_2 T_R / T$ is the expected number of "random" coincidences that one would get if the light fields falling on the two detectors were uncorrelated $[\lambda(\tau)=0]$.

In the experimental situation background light that is uncorrelated to the light in the laser mode must also be accounted for. Unfortunately, this is not entirely possible with the information available. This leads to some distortion of the values for $\lambda(0)$ at low intensities because of the excess background.

Measurements of the relative mean-squared intensity fluctuations $\lambda(0)$ versus the mean output intensity of a single-mode standing-wave dye laser have been reported,[26] and similar measurements are shown in Fig. 2. The measurements were taken by the photoelectric coincidence counting method and bear out the general description of the laser-fluctuation behavior given above. $\lambda(0)$ approaches the coherent-state results well above threshold and shows a large-fluctuation peak over a relatively broad range of output intensities. One discrepancy is noticeable, however. That is, at the lowest intensities, $\lambda(0)$ actually approaches zero, and not the single-mode thermal light result of 1.0 that is expected.[27] The explanation for this is found by looking at Eqs. (4) and (5). It is only when the correlation time is much longer than the resolving time of the electronics that $\lambda(\tau)$ can be approximated by $\lambda(0)$ in Eq. (4). This is the theoretical basis of the measurements performed. This condition is, however, no longer satisfied at the lowest intensities. The fluorescence has a correlation time given, approximately, by the spontaneous-emission lifetime of the dye or the inverse of the cavity bandwidth. The correlation function decays from one to zero in that time, therefore it is not surprising that such measurements, performed with coincidence resolving times of about 20 ns, give an answer closer to zero than one.

The dashed curve in Fig. 2 shows the result of the multiplicative (white)-noise-only theory for $\lambda(0)$.[11] The $(1/\langle I \rangle)$-type behavior predicted is remarkably accurate in the higher-intensity ranges shown, but exhibits an unphysical divergence as one goes to low intensities. Inasmuch as the theory is only valid for positive pump parameters the discrepancies are none too surprising. It is clear that the inclusion of additive (spontaneous-emission) noise in the theory is vital if one wants to prevent this unphysical divergence. In fact, the multiplicative- and additive-white-noise theory of Schenzle and Brand[28] shows all of the correct behavior for $\lambda(0)$.

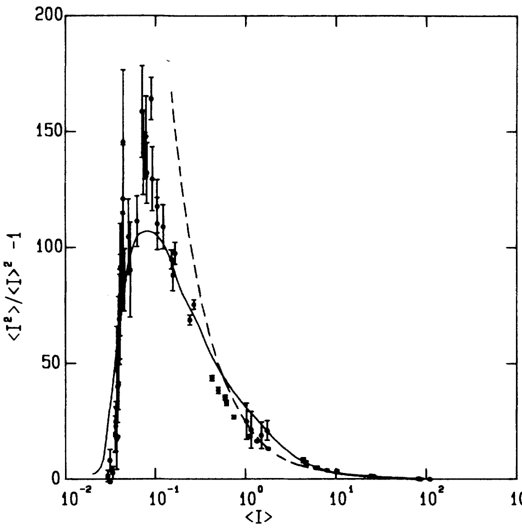

FIG. 2. Relative mean-squared intensity fluctuations, $\lambda(0)$ vs mean output intensity of a single-mode dye laser. Points are experimental measurements with statistical error bars indicated. The dashed curve is the function $25/\langle I \rangle$. The solid curve is from numerical simulations of Eqs. (1) with $Q=500$ and $\Gamma=5$. The x-axis scale is that of the simulations.

Equations (1) were integrated on the computer (setting $Q=500$ and $\Gamma=5$) and an estimate of $\lambda(0)$ was extracted. This is shown as the solid curve in Fig. 2. It is clear that this colored-multiplicative-noise additive-white-noise model satisfactorily reproduces all of the qualitative features mentioned above. Good agreement has also been recently obtained by Fox and Roy,[29] using a slightly different Langevin equation (for the intensity only) that includes additive and multiplicative white noise and an analytical approximation to account for the strongly colored noise.

The time- or delay-dependent correlation functions of a single-mode dye laser were also measured at various operating points of the laser. Plots of the data, taken with the digital correlator, are presented in Figs. 3(a)–3(f) and 4(a) and 4(b). The scatter in the data [see Fig. 3(f)] is somewhat indicative of the error introduced by poor counting statistics, and "wiggles" [Fig. 3(c)] or "bumps" [Fig. 3(e)] in the extracted correlation functions are indicative of poor averaging over the infrequent pulsing of the laser. The scatter is a function of the total number of photons counted, whereas the bumps and wiggles are a

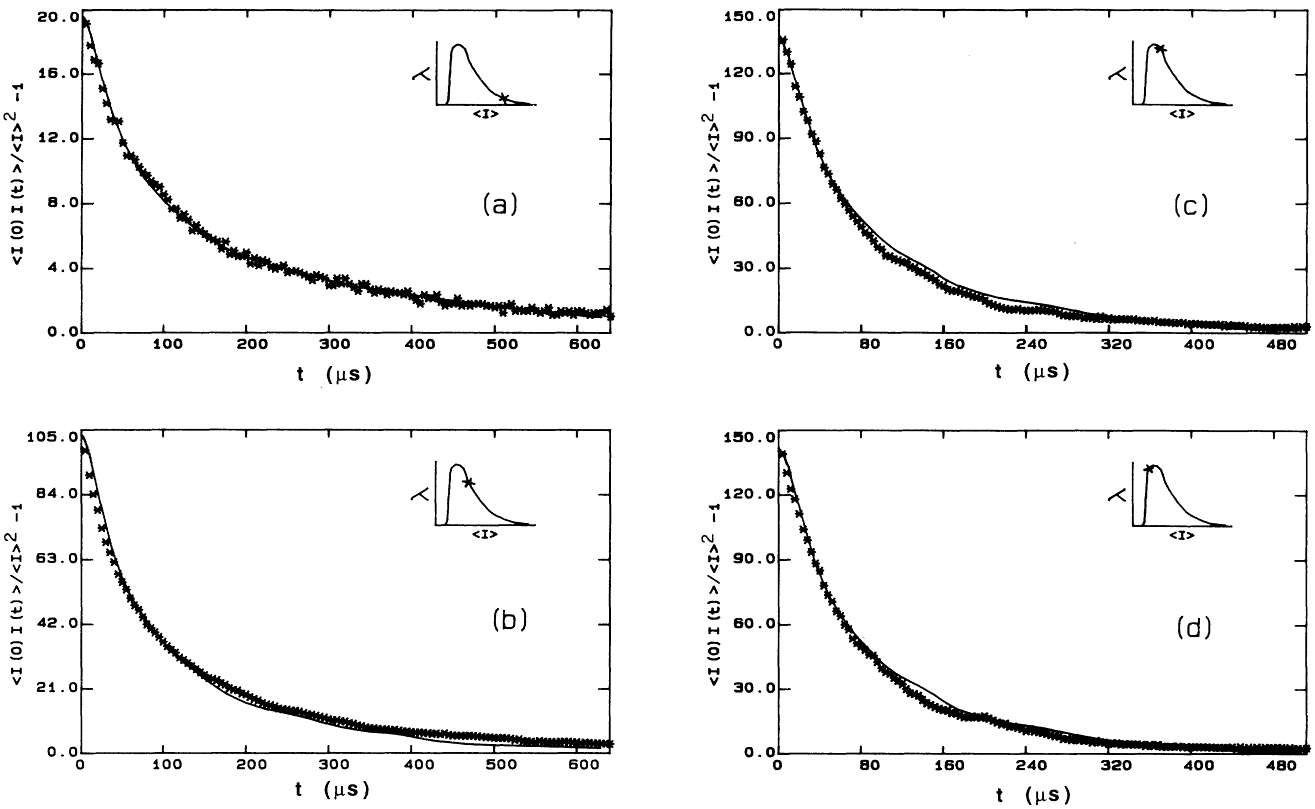

FIG. 3. Time-dependent intensity autocorrelations, $\lambda(t)$ vs time for a single-mode dye laser. The solid curves are from the Monte Carlo simulations of Eqs. (1) with $Q=128$ and $\Gamma=1000$. The time-scale conversion factor is 12.6 ms per unit of dimensionless time used in the simulations. The average pump parameter for each figure is as follows: (a) $a=-40$, (b) $a=-100$, (c) $a=-120$, (d) $a=-130$, (e) $a=-170$, and (f) $a=-270$. The mark on the inset sketch of $\lambda(0)$ vs $\langle I \rangle$ indicates the approximate operating point of the laser for each measurement of $\lambda(\tau)$.

function of the data accumulation time. In the low-intensity regime, or if unusually large spikes appear during the counting interval, these events will cause the anomalous bumps and wiggles in the recovered estimate of the correlation function. Sufficiently long accumulation times would average this out. The working point of the laser is indicated in each figure on the inset sketch of $\lambda(0)$ versus $\langle I \rangle$.

These correlation functions clearly show the initial rounding due to colored-noise pump fluctuations. The initial slope is, however, not zero, indicating that an additive noise is also present in the system; in this case it is spontaneous emission.

The solid curves shown in each of Figs. 3 and 4 are the estimates of the correlation functions extracted from the Monte Carlo simulations of Eqs. (1). Q has been chosen as 128 and Γ as 1000 and these values are fixed in each of the eight figures shown. In fitting these curves a scale factor must also be chosen to relate the dimensionless time of the simulations to the actual times. This constant was chosen to be 12.6 ms per unit of dimensionless time in the simulations and is fixed in value for all eight figures. Finally, a mean pump parameter must also be chosen in each case. These choices are indicated in the figure captions and range from $a=-40$ in Fig. 3(a) down to $a=-270$ in Fig. 3(f). [In general, it is noticed that the value of the average pump parameter at the peak of the $\lambda(0)$ versus $\langle I \rangle$ curve will be $a \cong -Q$. That is, the mean of the fluctuating pump parameter at this value of the average intensity is one standard deviation of the fluctuations in the pump parameter below zero ("threshold"). This property seems to be fairly general and holds even in the white-noise limit.]

Some of the sloppiness of the fits may be accounted for by the statistical uncertainties involved in the simulations as well as in the data. A certain amount of "structure" can be seen in the simulations that is, again, simply a result of not having integrated over a long enough period of time. In addition, any mismatch of the initial point, $\lambda(0)$, will manifest itself at all later times, as is evidenced in Figs. 3(e) and 4(b), in particular.

As was mentioned above, early models of dye-laser correlation functions suffered from two problems. The first was an overshoot near the initial point, and the second was the difficulty encountered in fitting the extended long-time tail. The first problem was dealt with

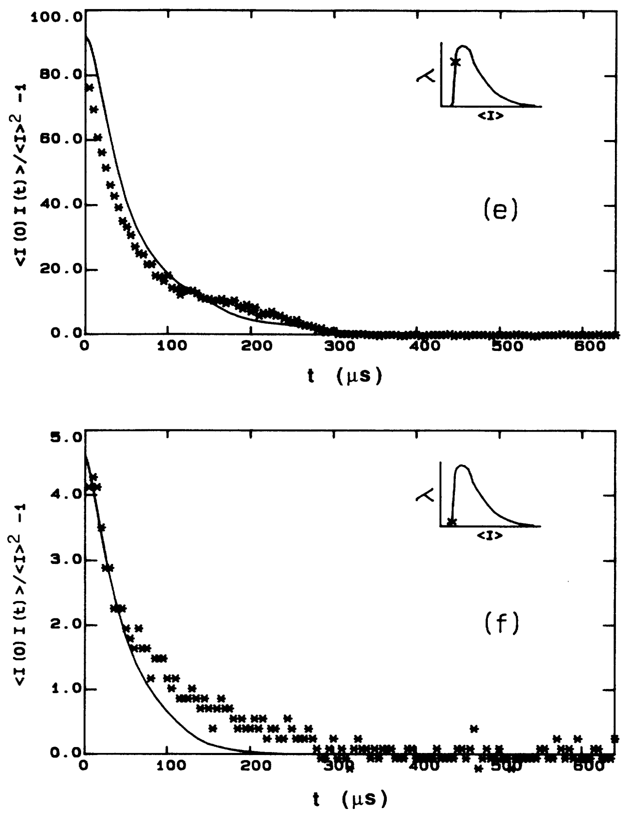

FIG. 3. (*Continued*).

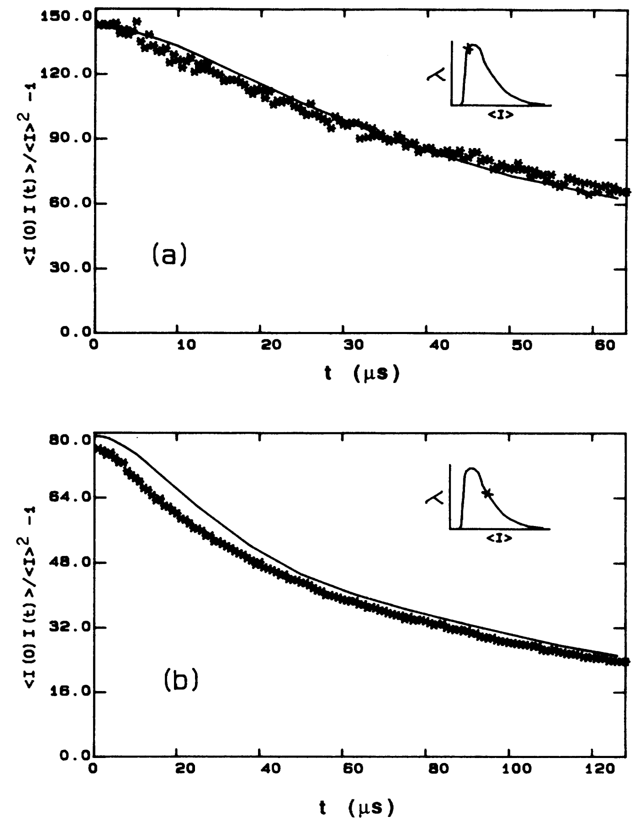

FIG. 4. Same as Fig. 3 except that the measurements were taken with higher-time resolution. Pump parameters in this case were (a) $a = -130$ and (b) $a = -90$. The mark on the inset sketch of $\lambda(0)$ vs $\langle I \rangle$ indicates the approximate operating point of the laser for each measurement of $\lambda(\tau)$.

by introducing a finite correlation time into the pumping fluctuations in the model. The second was dealt with by making a "subtraction" of this long-time tail. These subtractions contributed to the poor parameter choices extracted in at least one case.[15] In the work presented here we have also introduced colored-noise pumping fluctuations into the model but we do not require any subtractions to the data in order to arrive at satisfactory fits.

The correlation measurements reported here were performed with better time resolution than measurements previously reported.[9,10] This has enabled us to resolve the short-time structure in the experimental correlations that was predicted theoretically.[1-7] The short-time rounding off of the correlations is suggested in Fig. 3, although the eye may be influenced by the rounding obviously present in the simulation results. It is clear, however, in Figs. 4(a) and 4(b) that the correlation functions do indeed turn over at short times. Thus, the various numerical and theoretical predictions of this rounding are confirmed. This is also in disagreement with predictions of a model of white-noise gain fluctuations in the system,[23] reinforcing the conclusion that colored noise is responsible.

The pump-noise bandwidth Γ is best determined by adjusting this parameter until the time scale of the "roundoff" is correct. A smaller value of Γ (longer-noise correlation time) yields slower decays (i.e., a longer region of small slope near zero delay in the correlation function). From examining Figs. 4(a) and 4(b) it can be seen that Γ was perhaps chosen somewhat too small here, as the roundoff in the simulations persists a bit too long to match the data perfectly. $\Gamma = 1000$ is, however, close (within a factor of 2).

White-loss-noise theories do not show the initial round-off feature and thus, when one matches the measured $\lambda(0)$ value theoretically, the rest of the correlation function will undershoot the measured values. Although the white-gain-noise model of Ref. 23 reproduces the initially small slope of these correlation functions, however, they have a rather different looking long-time behavior and lack the two distinctive time scales. The long-time tails, which have been subtracted out in the past (with the notable exception of the colored-noise model of Fox, James, and Roy[13]), are also well fit by our model of Eqs. (1), without subtractions.

Figure 5 shows experimental results superimposed on a plot of $\lambda(0)$ versus $\langle I \rangle$ for $Q = 128$ and $\Gamma = 1000$, the same parameters as used in Figs. 3 and 4. It is clear that serious problems are encountered in attempting to fit the intensity dependence of $\lambda(0)$ with the same parameters that fit the time dependent $\lambda(\tau)$'s. If the model is correct

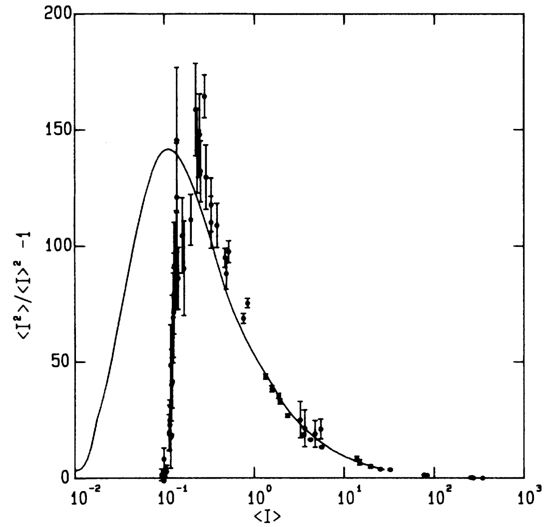

FIG. 5. $\lambda(0)$, the relative mean-squared intensity fluctuations, vs the mean output intensity $\langle I \rangle$. Simulation parameters are, in this case, $Q=128$ and $\Gamma=1000$.

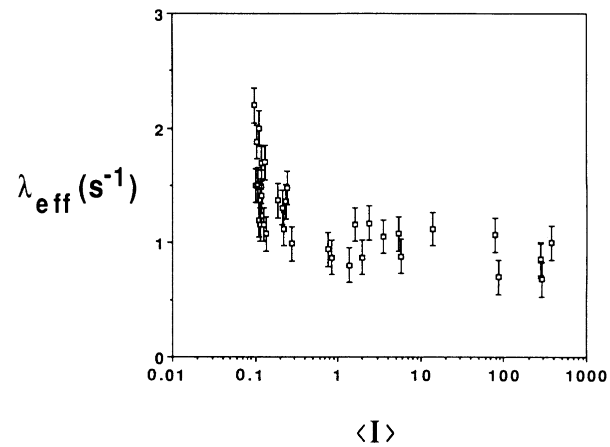

FIG. 7. $\lambda_{\text{eff}} = -\dot{\lambda}(0)/\lambda(0)$ vs $\langle I \rangle$ from the initial slope of many experimental measurements similar to Fig. 4.

in a general sense, then the fitting of both $\lambda(0)$ versus $\langle I \rangle$ and $\lambda(\tau)$ versus τ should be possible, simultaneously. The problem, in this case, may lie in our inability to accurately correct the data for background. If we are convinced by Figs. 3 and 4 that $\Gamma \cong 1000$, then the conclusion is that the peak in the $\lambda(0)$ versus $\langle I \rangle$ data is too narrow. High background levels may distort the low-intensity results. Another possibility is that the actual noise spectrum in the experiments may not be well modeled by that of an Ornstein-Uhlenbeck process and this would show up most dramatically on the short-time behavior of $\lambda(\tau)$. At this time it is not clear whether the background problems or the need for a more precise model of the noise would lead to different estimates of Γ from the time-independent correlations.

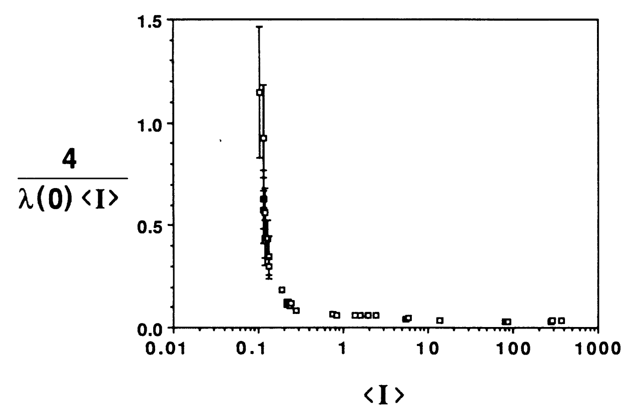

FIG. 6. A plot of $4/\lambda(0)\langle I \rangle$ vs $\langle I \rangle$ as extracted from Fig. 5. $4/\lambda(0)\langle I \rangle$ is proportional to λ_{eff}.

As pointed out by San Miguel *et al.*[7] a comparison of λ_{eff}, as determined by a direct measurement of $\dot{\lambda}(0)/\lambda(0)$ and as determined from the (model-dependent) formula $4D/\lambda(0)\langle I \rangle$, would determine, to some degree, the validity of the model (1). A plot of $\lambda'_{\text{eff}} \propto 1/\lambda(0)\langle I \rangle$ versus $\langle I \rangle$ for the measurements discussed above is presented in Fig. 6. This should be compared with Fig. 1 of Ref. 7. A plot of $\lambda_{\text{eff}} = -\dot{\lambda}(0)/\lambda(0)$ versus $\langle I \rangle$ is shown in Fig. 7. λ_{eff} was extracted from the first 10 μs of the measured $\lambda(\tau)$'s, over a range of $\langle I \rangle$. The error is $\pm 1.5 \times 10^{-5}$ s^{-1}, and was estimated from the linearity of the initial region and the density and scatter of the data. Although the scatter in λ_{eff} is much larger in Fig. 7 the trend is qualitatively similar to Fig. 6 and is much different from the additive-noise-only result (see Fig. 2 of Ref. 7). It is not clear why the range of λ_{eff} is less than predicted in Fig. 6. It is clear that the model of Eqs. (1) cannot be regarded as inappropriate on the grounds of the λ_{eff} versus $\langle I \rangle$ plots.

IV. SUMMARY

Measurements of the two-time correlation functions of the intensity of a single-mode dye laser show an initial period of nearly zero slope. These measurements confirm theoretical predictions of this initial roundoff when pumping fluctuations with a nonzero correlation time are included in the model, as in Eqs. (1). This model was used to produce numerical estimates of $\lambda(\tau)$ and these estimates were found to agree quite well with the experimentally determined correlations. Furthermore, measurements of the effective eigenvalue also support the use of the model (1). The existence of the two time scales in the time-dependent correlation functions would seem to rule out the use of a white-gain-noise model in place of the colored-noise models. Problems encountered fitting $\lambda(0)$ versus $\langle I \rangle$ measurements with the same parameters that fit $\lambda(\tau)$ versus τ well indicate that there are still some remaining difficulties, however.

We point out the close connection between the correlation time of the colored noise and the characteristic roundoff time of the correlation functions. This is perhaps to be expected. The observation that these characteristic times are nearly identical to the decorrelation time (the inverse of the Kolmogorov entropy is about 20 μs), i.e., the time scale for loss of information that was previously reported for this system[30] is, however, somewhat surprising. Although it is not clear that these time scales are actually connected it is speculated that the high-dimensional chaos that was observed due to turbulent dye flow in this system as well as fluctuations in the pump laser[31] cause the pump fluctuations. This evidence for two possible noise sources may also imply a more complex noise spectrum than was modeled here.

ACKNOWLEDGMENTS

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[1] A. Hernandez-Machado, M. San Miguel, and S. Katz, Phys. Rev. A **31**, 2362 (1985).
[2] M. San Miguel and A. Hernandez-Machado, in *Optical Instabilities,* edited by R. Boyd, M. Raymer, and L. Narducci (Oxford University Press, Oxford, England, 1986), p. 373.
[3] P. Jung and H. Risken, in Ref. 2, p. 361.
[4] M. San Miguel, in *Optical Chaos,* Vol. 667 of *SPIE Proceedings,* edited by J. Chrostowski and N. Abraham (SPIE, Bellingham, WA, 1986).
[5] A. Hernandez-Machado, M. San Miguel, and S. Katz, in *Recent Developments in Nonequilibrium Thermodynamics: Fluids and Related Topics,* edited by J. Casas, D. Jou, and J. Rubi (Springer-Verlag, Berlin, 1986).
[6] P. Jung and H. Risken, Z. Phys. B **59**, 469 (1985).
[7] M. San Miguel, L. Pesquera, M. A. Rodriguez, and A. Hernandez-Machado, Phys. Rev. A **35**, 208 (1987).
[8] H. Risken, Z. Phys. **186**, 85 (1965).
[9] J. A. Abate, H. J. Kimble, and L. Mandel, Phys. Rev. A **14**, 788 (1976).
[10] K. Kaminishi, R. Roy, R. Short, and L. Mandel, Phys. Rev. A **24**, 370 (1981).
[11] R. Graham, M. Hohnerbach, and A. Schenzle, Phys. Rev. Lett. **48**, 1396 (1982); A. Schenzle and R. Graham, Phys. Lett. **98A**, 319 1983.
[12] R. Short, L. Mandel, and R. Roy, Phys. Rev. Lett. **49**, 647 (1982).
[13] R. F. Fox, G. E. James, and R. Roy, Phys. Rev. Lett. **52**, 1778 (1984). Phys. Rev. A **30**, 2482 (1984).
[14] (a) P. Jung, Th. Leiber, and H. Risken, Z. Phys. B **66**, 397 (1987); (b) Th. Leiber, P. Jung, and H. Risken, *ibid.* **68**, 123 (1987).
[15] S. N. Dixit and P. S. Sahni, Phys. Rev. Lett. **50**, 1273 (1983).
[16] K. Lindenberg, B. J. West, and E. Cortes, Appl. Phys. Lett. **44**, 175 (1984).
[17] S. Zhu, A. W. Yu, and R. Roy, Phys. Rev. A **34**, 4333 (1986).
[18] R. Roy, A. W. Yu, and S. Zhu, Phys. Rev. Lett. **55**, 2794 (1985).
[19] P. Jung and H. Risken, Phys. Lett. **103A**, 38 (1984).
[20] M. Aguado and M. San Miguel, Phys. Rev. A **37**, 450 (1988).
[21] F. de Pasquale, J. M. Sancho, M. San Miguel, and P. Tartaglia, Phys. Rev. Lett. **56**, 2473 (1986).
[22] P. Lett, E. C. Gage, and T. H. Chyba, Phys. Rev. A **35**, 746 (1987).
[23] M. Aguado, E. Hernandez-Garcia, and M. San Miguel, Phys. Rev. A **38**, 5670 (1988).
[24] P. Lett and L. Mandel, J. Opt. Soc. Am. B **2**, 1615 (1985).
[25] J. M. Sancho, M. Sang Miguel, S. L. Katz, and J. D. Gunton, Phys. Rev. A **26**, 1589 (1982).
[26] P. Lett, R. Short, and L. Mandel, Phys. Rev. Lett. **52**, 341 (1984).
[27] See, for example, R. Loudon, *The Quantum Theory of Light* (Oxford University Press, Oxford, England, 1973), p. 110.
[28] A. Schenzle and H. Brand, Phys. Rev. A **20**, 1628 (1979).
[29] R. F. Fox and R. Roy, Phys. Rev. A **35**, 1838 (1987).
[30] T. H. Chyba, E. C. Gage, R. Ghosh, P. Lett, L. Mandel, and I. A. McMackin, Opt. Lett. **12**, 422 (1987).
[31] A. W. Yu, G. P. Agrawal, and R. Roy, Opt. Lett. **12**, 806 (1987).