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Angular momentum effect and enhanced efficiency in the electron cyclotron maser

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This paper investigates the angular momentum effect of the relativistic electron beam with large orbits in the electron cyclotron maser, where the electrons are injected axially and their equilibrium rotation is supported by a combination of an axial magnetostatic and a radial electrostatic field. Nonlinear simulation shows that the conversion efficiency of the kinetic energy of the electron beam may be enhanced by a proper radial electrostatic field.

I. INTRODUCTION

It is well known that the azimuthal bunching of the electrons plays a dominant role in the electron cyclotron maser and heavily influences the conversion efficiency of the kinetic energy of the beam.[1] This bunching process is mainly determined by both the axial magnetostatic focusing field and the distribution of the rf fields. So methods improved by a tapered axial magnetic field, or by a tapered cavity, or by both, have been used to enhance the conversion efficiency.[2–4] On the other hand, Alexeff and Dyer[5,6] invented the orbitron maser in which the axial magnetic field was replaced by a radial electrostatic field. Lau and Chernin[7–9] investigated in detail the stability of laminar electron layers by means of a two-dimensional (2D) model, where the equilibrium rotation of the electrons was supported either by a radial electric field or an axial magnetic field, or a combination of both in a coaxial waveguide. They found that the most unstable case occurred when the electron beam was focused merely by a radial electric field, and this instability was nonrelativistic. They also calculated the quantity $d\omega_c/d\varepsilon$, called the "negative mass effect," where ω_c and ε were the electron cyclotron frequency and energy, respectively. Obviously, the increased energy of the growing EM wave, in their work, comes from the changes of both the kinetic energy and electrostatic potential energy of the beam. These energy changes, however, were not separated and worked out; why and how the dc electric and magnetic fields affect the maser also were not analyzed specifically.

In this paper we investigate in detail the interaction between the TE_{mn} mode and a relativistic electron beam where the equilibrium beam is focused by both an axial magnetic field and a radial electric field and the electrons rotate around the inner conductor of the coaxial waveguide in circular orbits (the case of noncircular orbits[10] is excluded.) The specific cases where the electrons are focused merely by an axial magnetic field or merely by a radial electric field are also discussed. We found that the angular momentum effect caused by the radial dc electric field is comparable to the relativistic effect caused by the axial dc magnetic field, and the conversion efficiency of the kinetic energy of the beam may be enhanced by the angular momentum effect. Here two points should be pointed out. (1) Our topic is quite different from the relativistic magnetron which was investigated successfully by Bekefi and co-workers.[11,12] Although the equilibrium electrons in both situations are governed by similar dc cross fields, they are injected into the interaction region in quite different ways. In our study the electrons are injected axially from a cathode outside the cavity, while in a relativistic magnetron the electrons are emitted radially from the cathode which is arranged in the cavity. From the mathematical point of view, these two cases have different special solutions because their initial conditions are quite different, although they satisfy the same differential equations. (2) The conclusions of this paper never appeared in the previous papers,[1–12] nor in our early works.[13,14] The present paper may help the reader to understand the physical mechanisms of the prior investigations.

II. QUALITATIVE ANALYSIS

We consider a relativistic cold electron beam which is enclosed in a coaxial waveguide and rotates about the symmetry axis. As shown in Fig. 1, the inner and outer radii are, respectively, R_{in} and R_{out}; the equilibrium electrons with Larmor radius R_0 are focused by an axial magnetic field B_0 and a radial electric field offered by the voltage U_0. We assume $\omega_p \ll \omega$ so that the effect of the space charges can be neglected, where ω_p and ω are, respectively, the plasma frequency and wave frequency.

It is easily found that when forces produced by the focusing fields balance the inertial centrifugal force the electron's cyclotron frequency ω_c is

$$\omega_c=\frac{|e|B_0}{\gamma m_0}+\frac{|e|U_0}{P_\varphi \ln(R_{\text{out}}/R_{\text{in}})} \tag{1}$$

where e and m_0 are the electron charge and rest mass, γ is the relativistic factor, and $P_\varphi = R\gamma m_0 v_\perp$ is the angular momentum ($v_\perp$ being the transverse velocity).

When rf fields are introduced, both γ and P_φ in Eq. (1) will change. For the electrons which are in acceleration field, the transverse velocity $v_\perp$ and energy factor γ increase. Noting that the inertial centrifugal force is greater than the centripetal force with $v_\perp$ increasing, the radial coordinate R has to increase. Namely, both relativistic factor γ and angular momentum P_φ increase,

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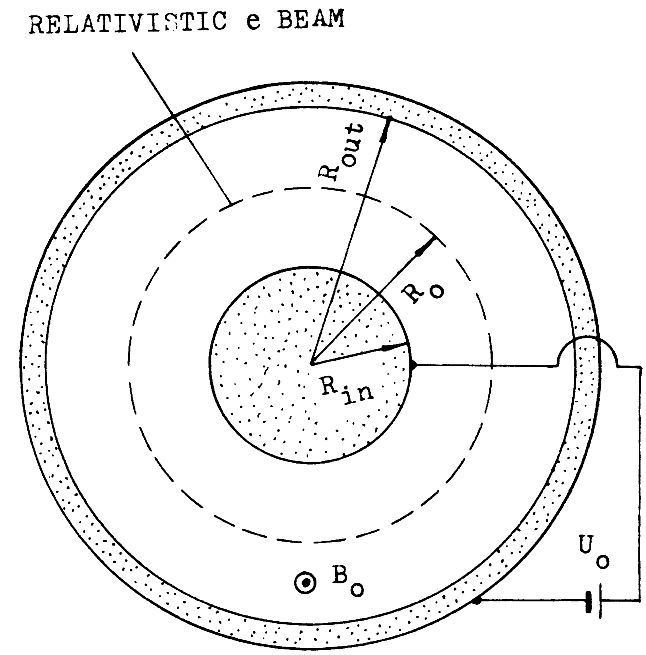

FIG. 1. Model of the equilibrium electron beam with circular orbits about the symmetry of a coaxial waveguide. The electrons are axially injected and supported by an axial magnetic field, or by a radial dc electric field, or by a combination of both.

which leads to the decreasing of cyclotron frequency ω_c according to Eq. (1). Especially, the denominator of the second term on the right-hand side of Eq. (1) changes more rapidly than that of the first one. On the contrary, the cyclotron frequency of those electrons which are in the deceleration field must increase. Summarily, rotation of the electrons in acceleration field becomes slow and rotation of those in deceleration field becomes fast. Thus the azimuthal bunching will appear after several periods. If the bunching center falls into the deceleration field, EM radiation occurs from the electron beam.

Therefore it can be drawn that the azimuthal bunching is influenced by both the axial magnetic field through the relativistic effect and the radial dc electric field through the angular momentum effect. Evidently, in gyrotrons the angular momentum effect disappears since there is no U_0, and its azimuthal bunching is determined merely by the relativistic effect. If there is no B_0 (for example, in an orbitron maser,[5,6] or in helitron[15,16]), the azimuthal bunching is determined merely by the angular momentum effect which is, of course, nonrelativistic in principle.

III. QUANTITATIVE CALCULATION

In this section we briefly derive the angular acceleration in terms of a perturbation approach so as to discuss the azimuthal bunching. Let ξ denote the mixing proportion between the radial dc electric field and the axial dc magnetic field, which is defined as

$$\xi=\frac{|e|B_0}{\gamma_0 m_0\omega_{c0}} \tag{2}$$

where the subscript 0 represents the equilibrium quantities. If $U_0=0$ ($B_0\neq 0$), or $B_0=0$ ($U_0\neq 0$), or $B_0\neq 0\neq U_0$, then $\xi=1$, or $\xi=0$, or $0<\xi<1$, which means that the beam is focused merely by an axial magnetic field, or merely by a radial dc electric field, or by a combination of both.

When rf fields are introduced, the motion of the relativistic electrons is governed by

$$\gamma m_0(\ddot{R}-R\dot{\varphi}^2)+m_0\dot{\gamma}\dot{R}=f_R+eB_0R\dot{\varphi}+\frac{eU_0}{R\ln(R_{\text{out}}/R_{\text{in}})} \tag{3}$$

$$\gamma m_0(2\dot{R}\dot{\varphi}+R\ddot{\varphi})+m_0R\dot{\gamma}\dot{\varphi}=f_\varphi-eB_0\dot{R} \tag{4}$$

$$\gamma m_0\ddot{z}+m_0\dot{\gamma}\dot{z}=f_z \tag{5}$$

$$\gamma=c(c^2-\dot{R}^2-R^2\dot{\varphi}^2-\dot{z}^2)^{-1/2} \tag{6}$$

where f_R, f_φ, f_z denote the radial, azimuthal, and axial forces acting on the electrons by the rf fields (TE_{mn} or TM_{mn} modes), and the overdot represents the derivative with respect to time. Letting $R=R_0+R_1$ ($R_0\gg R_1$), $\varphi=\varphi_0+\varphi_1$ ($\varphi_0\gg\varphi_1$), $z=z_0+z_1$ ($z_0\gg z_1$), $\gamma=\gamma_0+\gamma_1$ ($\gamma_0\gg\gamma_1$), where the subscripts 0 and 1 denote the equilibrium and perturbation quantities, and then substituting these into Eqs. (3)–(6), we obtain

$$-\gamma_0[\Omega^2+2(1-\xi)\omega_{c0}^2]R_1-j\Omega\gamma_0 v_{\perp 0}(\xi-2)\varphi_1 - v_{\perp 0}\gamma_1\omega_{c0}=f_R/m_0\ , \tag{7}$$

$$-j\Omega(2-\xi)\gamma_0\omega_{c0}R_1-\Omega^2\gamma_0R_0\varphi_1-j\Omega v_{\perp 0}\gamma_1=f_\varphi/m_0\ , \tag{8}$$

$$-\gamma_0\Omega^2 z_1-j\Omega v_{\parallel 0}\gamma_1=f_z/m_0\ , \tag{9}$$

$$\gamma_0^3 v_{\perp 0}\omega_{c0}R_1-j\Omega\gamma_0^3 v_{\perp 0}R_0\varphi_1-j\Omega\gamma_0^3 v_{\parallel 0}z_1-c^2\gamma_1=0\ , \tag{10}$$

$$\Omega=\omega-k_\parallel v_{\parallel 0}-m\omega_{c0}\ , \tag{11}$$

where the assumption has been made that all the perturbation quantities have the normal-mode factor $\exp[j(k_\parallel z-\omega t+m\varphi)]$ ($k_\parallel$ being the axial wave number), and only the first-order and zero-order perturbation quantities are kept. Carefully solving Eqs. (7)–(10), we get the displacements of the electrons as follows:

$$R_1=(\gamma_0^2 m_0 K\Omega)^{-1}\{f_\varphi j\gamma_0\omega_{c0}[(2-\beta_{\perp 0}^2)-\xi(1-\beta_{\perp 0}^2)]+f_z j\beta_{\parallel 0}\beta_{\perp 0}\gamma_0\omega_{c0}(\xi-1)+f_R\gamma_0\Omega\}\ , \tag{12}$$

$$\varphi_1=(\gamma_0 m_0 R_0 K\Omega^2)^{-1}\{f_\varphi[(1-\beta_{\perp 0}^2)\Omega^2+(2-\beta_{\perp 0}^2)\omega_{c0}^2-2\xi(1-\beta_{\perp 0}^2)\omega_{c0}^2] + f_z\beta_{\parallel 0}\beta_{\perp 0}(\xi\omega_{c0}^2-\Omega^2)-f_R j[(2-\beta_{\perp 0}^2)-\xi(1-\beta_{\perp 0}^2)]\omega_{c0}\Omega\}\ , \tag{13}$$

$$z_1=(\gamma_0^3 m_0 K\Omega^2)^{-1}\{f_\varphi\beta_{\parallel 0}\beta_{\perp 0}\gamma_0^2(\xi\omega_{c0}^2-\Omega^2)+f_z[(1+\gamma_0^2\beta_{\perp 0}^2)(\Omega^2-\omega_{c0}^2)-(1-\xi)^2\omega_{c0}^2] \\ +f_R j\gamma_0^2\beta_{\parallel 0}\beta_{\perp 0}(1-\xi)\omega_{c0}\Omega\}\ , \tag{14}$$

where

$$K=(A\omega_{c0})^2-\Omega^2\ , \tag{15}$$

$$A=[(2-\beta_{\perp 0}^2)-\xi(2-\xi)(1-\beta_{\perp 0}^2)]^{1/2}\ , \tag{16}$$

$$\beta_{\perp 0}=v_{\perp 0}/c,\quad \beta_{\parallel 0}=v_{\parallel 0}/c\ , \tag{17}$$

where c is the light speed in vacuum, $v_{\parallel 0}$ and $v_{\perp 0}$ are the axial and transverse velocities in equilibrium, respectively.

Making use of the cyclotron resonance condition ($\Omega\approx 0$) and the normal mode assumption, we can get from Eq. (13) the angular acceleration

$$\ddot{\varphi}=\frac{-1}{\gamma_0 m_0 R_0^2 A^2}\times\{2(1-\xi)M_z+\beta_{\perp 0}[M_z\beta_{\perp 0}(2\xi-1)+\xi\beta_{\parallel 0}R_0 f_z]\}\ , \tag{18}$$

where $M_z=Rf_\varphi$ is the axial torque. Here the first-order perturbation quantities M_z and f_z have the normal-mode factor $\exp[j(k_\parallel z-\omega t+m\varphi)]$. Therefore the electrons in a half plane must rotate faster ($\ddot{\varphi}>0$), and those in the other half plane rotate more slowly ($\ddot{\varphi}<0$). After several periods, the azimuthal bunching appears. It is well known that the axial torque M_z represents the change of the angular momentum. So Eq. (18) indicates that the azimuthal bunching comes from the angular momentum effect (the first term in curly brackets) and the relativistic effect (the second term in curly brackets). It can readily be shown that Eq. (18) is identical to Eq. (2) of Ref. 9 in the limit $\beta_{\parallel 0}\to 0$.

If the beam is focused merely by an axial magnetic field (in gyrotrons), i.e., $\xi=1$, Eq. (18) becomes

$$\ddot{\varphi}=\frac{-\beta_{\perp 0}}{\gamma_0 m_0 R_0^2 A^2}(M_z\beta_{\perp 0}+\beta_{\parallel 0}R_0 f_z)\ , \tag{19}$$

which means that the azimuthal bunching results from the relativistic effect of the transverse motion ($\beta_{\perp 0}$ must be nonzero). On the contrary, if the beam is focused merely by a radial dc electric field (in orbitron maser or helitron), i.e., $\xi=0$, Eq. (18) becomes

$$\ddot{\varphi}=\frac{-M_z}{\gamma_0 m_0 R_0^2 A^2}(2-\beta_{\perp 0}^2)\ , \tag{20}$$

which clearly demonstrates that the azimuthal bunching comes from the angular momentum effect and is, in principle, nonrelativistic ($\beta_{\perp 0}$ is permitted to be zero). Obviously, the conclusions obtained here coincide with those in Sec. II.

IV. NONLINEAR SIMULATION

In this section we analyze how the radial dc electric field influences the energy exchange between the beam and wave. Here a nonconsistent simulation is used since $\omega_p \ll \omega$ has been assumed. We define the total electron efficiency η_t as the ratio of the total energy loss over the initial kinetic energy,

$$\eta_t=\eta_k+\eta_p\ , \tag{21}$$

where

$$\eta_k=\frac{\gamma_0-\gamma}{\gamma_0-1}\ , \tag{22}$$

$$\eta_p=\frac{\gamma_0(1-\xi)\beta_{\perp 0}^2}{\gamma_0-1}\ln\frac{R_0}{R} \tag{23}$$

are, respectively, the conversion efficiencies of the kinetic energy and dc electric potential energy. The beam efficiency is the average value of the test electrons. The initial parameters are normalized as follows:

$$\bar{R}_{\rm in}=kR_{\rm in},\bar{R}_{\rm out}=kR_{\rm out},\quad \bar{R}_0=kR_0\ , \tag{24}$$

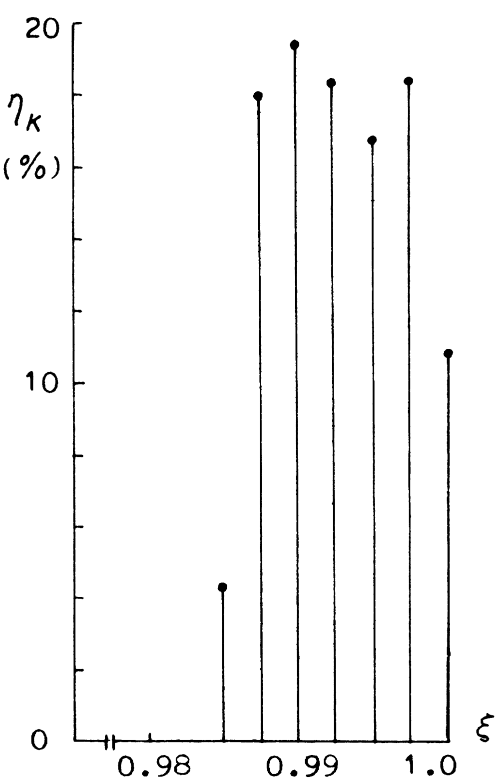

FIG. 2. Influence of ξ on η_k for the $TE_{5,1,1}$ mode, where $\bar{R}_{\rm in}=1.06$, $\bar{R}_{\rm out}=6.45$, $\bar{R}_0=3.2$, $\bar{E}_0=0.085$, $\alpha_1=0.9625$, $\alpha_2=0.025$, $\alpha_3=0.5$, $\gamma_0=1.2759$, $\beta_{\perp 0}=0.616$, and $\beta_{\parallel 0}=0.0791$.

$$\bar{k}_{\parallel}=k_{\parallel}/k,\quad \bar{k}_c=k_c/k,\quad \bar{t}=\omega t\ , \tag{25}$$

$$\bar{B}_0=\frac{\xi\gamma_0\beta_{\perp 0}}{\bar{R}_0},\quad \bar{E}_0=\frac{|e|}{cm_0\omega}E_0\ , \tag{26}$$

$$\alpha_1=\frac{m\omega_{c0}}{\omega}\ , \tag{27}$$

$$\alpha_2=\frac{\omega-k_{\parallel}v_{\parallel 0}-m\omega_{c0}}{\omega}\ , \tag{28}$$

$$\alpha_3=\frac{v_{\parallel 0}v_p}{c^2}\ , \tag{29}$$

where k is the total wave number, E_0 is the amplitude of the rf fields, and v_p is the phase velocity of the wave.

Now we take the $TE_{5,1,1}$ mode as an example. We choose $\bar{R}_{in}=1.06$, $\bar{R}_{out}=6.45$, $\bar{R}_0=3.2$, $\bar{E}_0=0.085$, $\alpha_1=0.9625$, $\alpha_2=0.025$, $\alpha_3=0.5$, $\gamma_0=1.2759$, $\beta_{\perp 0}=0.616$, and $\beta_{\parallel 0}=0.0791$. The influence of ξ on the conversion efficiency of the kinetic energy η_k is shown in Fig. 2. Calculation indicates that for the chosen parameters the

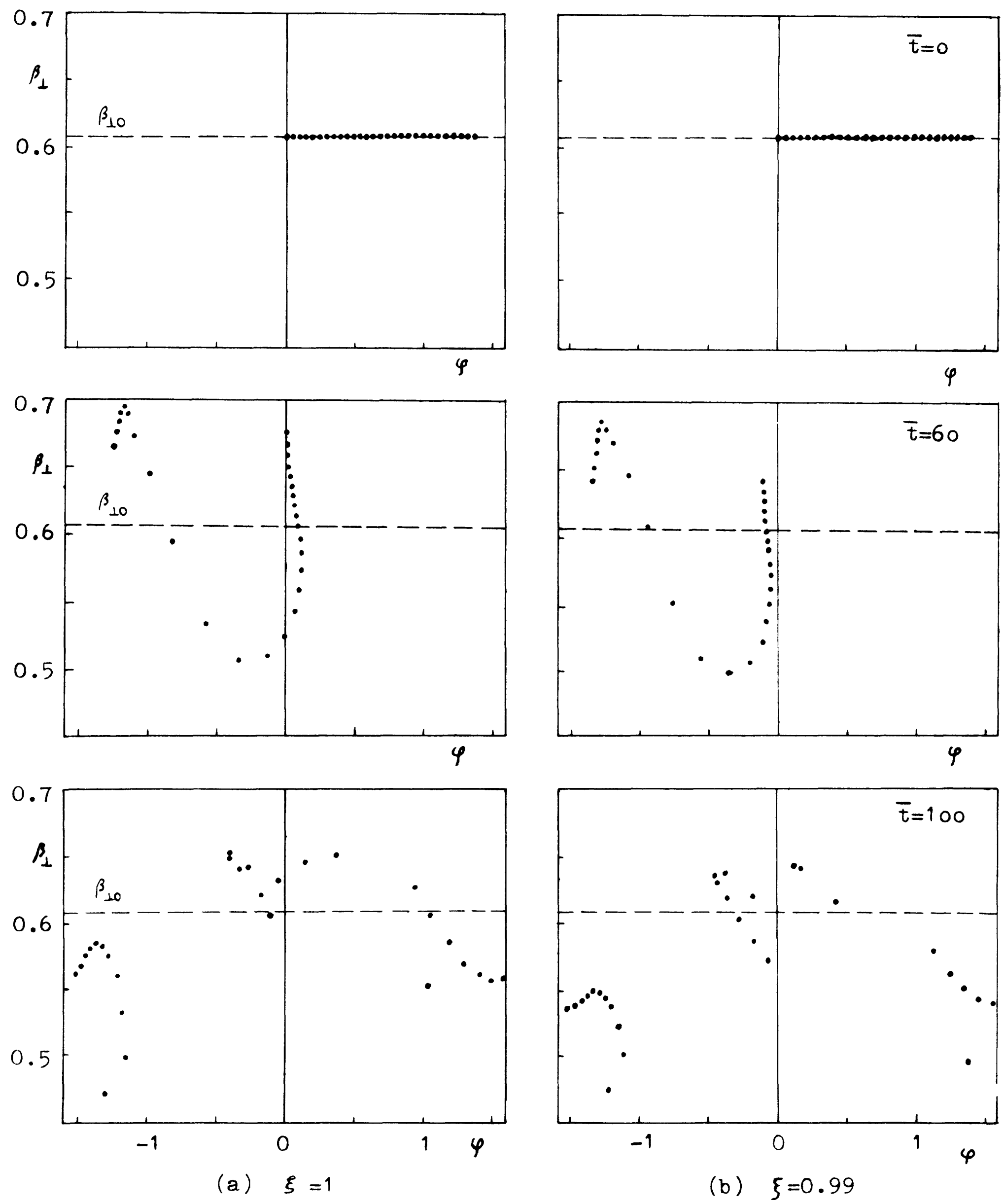

FIG. 3. Azimuthal bunching states of electrons for the $TE_{5,1,1}$ mode: (a) $\xi=1$ and (b) $\xi=0.99$. Here $\bar{R}_{in}=1.06$, $\bar{R}_{out}=6.45$, $\bar{R}_0=3.2$, $\bar{E}_0=0.1023$, $\alpha_1=0.95$, $\alpha_2=0.02$, $\alpha_3=1.0$, $\gamma_0=1.2906$, $\beta_{\perp 0}=0.608$, and $\beta_{\parallel 0}=0.173$. The kinetic energy loss of the beam to the wave for $\xi=0.99$ is more than that for $\xi=1$ (noting that the longitudinal kinetic energy loss is very small in the electron cyclotron maser).

dc electric potential-energy conversion efficiency is quite small ($\ll 1\%$). It is found from Fig. 2 that the radial dc electric field leads to two possibilities: improving or debasing the bunching states, and so increasing or decreasing the conversion efficiency of the kinetic energy; the results depend upon the initial parameters.

In order to demonstrate how the radial dc electric field affects the azimuthal bunching, we compare, in Fig. 3, the evolution of the bunching process under two cases: (a) the electron beam is focused merely by an axial magnetic field ($\xi=1$), and (b) a weak radial dc electric field is used to assist the axial magnetic field to focus the electron beam ($\xi=0.99$). The calculation clearly shows that for the given parameters the bunching state of $\xi=0.99$ is improved before leaving the interaction cavity.

Another example, the $TE_{10,1,1}$ mode, is also given in Fig. 4, where $\bar{R}_{in}=4.52$, $\bar{R}_{out}=11.8$, $\bar{R}_0=9.2$, $\bar{E}_0=0.4$, $\alpha_1=0.935$, $\alpha_2=0.02$, $\alpha_3=1.0$, $\gamma_0=2.1642$, $\beta_{\perp 0}=0.8611$, and $\beta_{\parallel 0}=0.2121$. The conversion efficiency of the kinetic energy may be raised from 2% of $\xi=1$ up to 6.7% of $\xi=0.99$.

Here it should be pointed out that the above-mentioned efficiencies are not optimized for both $\xi=1$ and $0<\xi<1$. Nevertheless, the aforementioned examples demonstrate, at least in principle, the influence of the radial dc electric field on the azimuthal bunching and a mechanism of efficiency enhancement. In other words, if we carefully choose the radial dc electric field the conversion efficiency of the beam's kinetic energy could be enhanced.

V. CONCLUSIONS

In this paper we have analyzed the interaction between a relativistic cold electron beam and the EM wave in terms of qualitative explanation, linear perturbation calculation, and nonlinear simulation, where the equilibrium beam is supported by a combination of the radial dc electric field and the axial magnetic field. Two points can be drawn.

(a) The azimuthal bunching of electrons is affected by both the axial magnetic field through the relativistic effect and the radial dc electric field through the angular momentum effect.

(b) Because of the angular momentum effect, the conversion efficiency of the beam's kinetic energy could be enhanced when a proper radial dc electric field is introduced to assist the axial magnetic field to focus the electron beam.

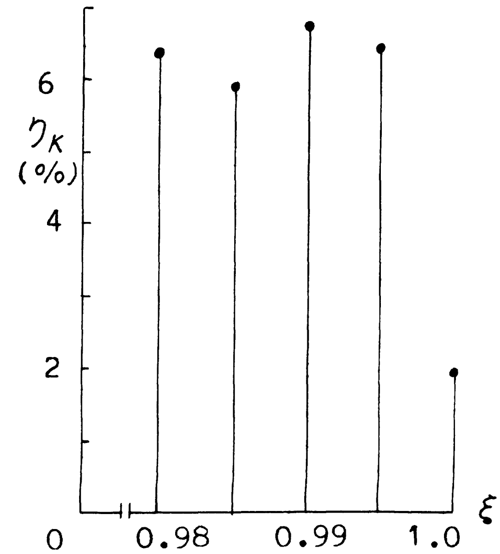

FIG. 4. Influence of ξ on η_k for the $TE_{10,1,1}$ mode, where $\bar{R}_{in}=4.52$, $\bar{R}_{out}=11.8$, $\bar{R}_0=9.2$, $\bar{E}_0=0.4$, $\alpha_1=0.935$, $\alpha_2=0.02$, $\alpha_3=1.0$, $\gamma_0=2.1642$, $\beta_{\perp 0}=0.8611$, and $\beta_{\parallel 0}=0.2121$.

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