Spontaneous emission of atoms coupled to frequency-dependent reservoirs

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We study the spontaneous emission of atoms coupled to frequency-dependent photon-mode reservoirs, and find atomic emission spectra qualitatively different from those of free space. Frequency-dependent photon-mode reservoirs are found in a variety of physical situations (e.g., in waveguides, microcavities, dielectrics, systems with photonic spectral gaps, etc.), and are shown to lead to nonexponential decay and concomitant complicated non-Lorentzian emission line shapes. By studying Autler-Townes spectra in the presence of a frequency-dependent photon-mode density, we present simple examples of dynamical modifications of spontaneous decay properties.

I. INTRODUCTION

It has been known for approximately 30 years[1] that the spontaneous decay of atoms is not fully characterized by intrinsic atomic properties. Indeed, this process depends essentially on the statistical properties of the quantum electromagnetic field surrounding the atom. Since the vacuum electromagnetic field in cavities or waveguides generally differs from that found in the free space, due to the complicated mode structure of the field, so does the spontaneous decay.

In the case when the atom-field coupling is relatively weak and does not depend strongly on the photon frequency in the regime of interest (i.e., close to the atomic transition frequency ω_a), the decay process may be described in the framework of the Wigner-Weisskopf formalism.[2] The atom decays then exponentially at the rate γ_s. The spectrum of emitted photons has a Lorentzian shape of the half width γ_s, centered at the radiatively shifted frequency $\omega_a + \delta\omega_a$. Both the spontaneous-emission rate γ_s and the Lamb shift $\delta\omega_a$ depend on the density of modes in the photon reservoir.

In 1946 Purcell[1] predicted that the spontaneous-emission rate of a cavity-contained atom becomes enhanced in the case of atom–cavity resonance. Atoms in the cavity may therefore radiate spontaneously faster than in free space. Analogously, if atomic transitions are far from any cavity resonance, the spontaneous-emission process will be inhibited. The same kind of inhibition of spontaneous emission takes place if atoms are located in a waveguide and their transition frequency is below the fundamental frequency of the waveguide.[3(a)]

The paper of Kleppner[3(a)] initiated, in fact, a series of studies of the modification of spontaneous emission by cavities and waveguides.[3(b)] First experiments have been done in the microwave regime.[4,5] Recently, modifications of spontaneous emission have been observed in an optical

microcavity (i.e., in a cavity whose size is comparable to that of a wavelength).[6] Macroscopic optical cavities have been used in experiments of Heinzen et al.,[7] who were first to demonstrate cavity-induced modifications of the Lamb shift.[8]

In our recent papers[9,10] we have shown that modifications of spontaneous emission can be induced by dynamical means, i.e., by exposing atoms to a strong driving field. These modifications exhibit themselves through changes in the resonance fluorescence spectrum. Namely, widths, heights, positions, and even shapes of the peaks in the spectrum become dependent on the driving field intensity.

The current literature on decay processes has been devoted, to a great extent, to the study of situations when the Wigner-Weisskopf approach breaks down; i.e., the decay becomes nonexponential. One such situation occurs when spontaneous emission takes place in a cavity of width Γ, where Γ is smaller than spontaneous-emission rate γ_s. In such a case, before the spontaneous emitted photon leaks out of the cavity, it may be reabsorbed by the atom. Eventually, this effect may result in an oscillatory exchange of the energy between atomic and photonic degrees of freedom, which in turn leads to a splitting of the spontaneous-emission spectrum. Such a splitting, termed the vacuum Rabi splitting,[11] has been widely discussed from the theoretical point of view[3(b),12(a)] and recently observed experimentally.[12(b)]

Another case in which nonexponential decay is expected to occur corresponds to the situation when the density of reservoir modes has a thresholdlike behavior, i.e., exhibits a sudden jump or some weaker kind of singular, nonanalytic behavior. If the frequency of the atomic transition lies close to the threshold, the decay usually tends to be algebraic. Such effects were discussed for the first time in the context of bound–free transitions (photodetachment from a negative ion).[13(a)] Recently, we have

indicated that the same effect occurs in spontaneous emission,[14] provided that the density of photon modes exhibits a thresholdlike behavior (as is the case, for example, in a waveguide close to its fundamental frequency). The nonexponential character of the decay leads to strong modifications in the shape of the spontaneous-emission line, which becomes non-Lorentzian and may even exhibit additional peaks or holes.

The purpose of this paper is to present a detailed theory of non-Lorentzian spontaneous-emission spectra and discuss a number of examples in which they occur. Our work is organized as follows. Section II contains a description of a model describing near-threshold spontaneous emission. The model, essentially the same as the one used in Ref. 14, corresponds to the case when an atom is contained in a waveguide. The atomic transition frequency is close to the fundamental frequency of the waveguide. Using exact, analytic solutions we illustrate here such phenomena as threshold shift, critical narrowing, etc.

In Sec. III we discuss the problem of Autler-Townes[15] splitting in the waveguide. A transition between two atomic levels is strongly driven by an external laser field, and spontaneous emission takes place from the upper atomic level to some sidelevel. The frequency of the spontaneous transition is assumed again to lie close to the threshold frequency of a waveguide (or a microcavity). The analytic results presented in Sec. III provide perhaps the simplest possible example of dynamical modifications of spontaneous emission.[9,10]

Section IV is devoted to the study of yet another physical possible situation in which the spontaneous-emission frequency is close to a gap in the photonic spectrum. Photon gaps[16] may occur in spatially periodic dielectric media and effectively correspond to deep "holes" in the density of photon modes. Similar deep minima of the density of photon modes may be found also in homogeneous, linearly polarizable, resonant media which may be described as systems of harmonic oscillators interacting with an electromagnetic field.[17]

Finally, Sec. V contains a short summary of our results.

II. SPONTANEOUS EMISSION NEAR A THRESHOLD IN THE VACUUM RESERVOIR

In this section we shall discuss in detail the model introduced by us in Ref. 14, describing spontaneous emission near threshold. In order to study the interaction of an atom with a frequency-dependent reservoir, we introduce the Hamiltonian

$$H = \omega_a |1\rangle\langle 1| + \int |k| c_k^\dagger c_k \, dk + \int |k| b_k^\dagger b_k \, dk$$
$$+ \int g_c(k)(c_k^\dagger |0\rangle\langle 1| + |1\rangle\langle 0| c_k) dk$$
$$+ \int g_b(k)(b_k^\dagger |0\rangle\langle 1| + |1\rangle\langle 0| b_k) dk . \quad (1)$$

In (1), ω_a denotes the atomic transition frequency, while $|0\rangle$ and $|1\rangle$ are the atomic ground and excited states, respectively. We have included two kinds of photon reservoirs in (1).[9,10] Creation $(c_k^\dagger)$ and annihilation (c_k)

operators correspond to photons associated with the cavity or waveguide resonance. They provide a reservoir, which has a frequency-dependent mode density. They describe also partial losses in the system due to the imperfectness of waveguide walls (or cavity mirrors). The creation $(b_k^\dagger)$ and annihilation (b_k) operators correspond to modes unassociated with the resonance, later referred to as the background modes. These modes describe phenomenologically any other radiative loss mechanisms present, and their density is assumed to depend weakly on frequency.[3(a)]

It should be stressed that the Hamiltonian (1) does not necessarily describe radiative level shifts correctly for two reasons: it does not contain all atomic levels and it is written in the rotating-wave approximation. Results obtained in the context of bound–free transitions[13] indicate, however, that the Hamiltonian (1) would describe the Lamb shift as being quantitatively correct, if a proper renormalization of ω_a has been done. At the same time, dynamics of the decay process are well described by (1).

The full characteristics of photon reservoirs are given by specifying the frequency-dependent couplings $g_b(k), g_c(k)$. Since $|g_b(k)|^2$ is only needed in the neighborhood of the atomic frequency ω_a and characteristic frequency of the waveguide ω_c, we may assume

$$|g_b(k)|^2 = \text{const} = \gamma_b/\pi , \quad (2)$$

so that the background reservoir's response is immediate,

$$\int_0^\infty |g_b(k)|^2 e^{-i(k-\omega_c)(t-t')} dk = \gamma_b \delta(t-t') . \quad (3)$$

The coupling $|g_c(k)|^2$ is assumed to be singly peaked (we neglect, for simplicity, the coupling to other waveguide resonances) and exhibits a threshold behavior. For an ideal waveguide close to its fundamental frequency the coupling $|g_c(k)|^2$ has a form[3(a)]

$$|g_c(k)|^2 \sim \Theta(k-\omega_c)\frac{1}{\sqrt{k-\omega_c}} . \quad (4)$$

Formula (4) indicates that there are no propagating modes in the waveguide for k below ω_c. The density of modes has a weak $(1/\sqrt{k-\omega_c})$ singularity for $k \to \omega_c$ from above. As stressed by Kleppner,[3(a)] in realistic waveguides the singular behavior is always smoothed. In Ref. 14 we used the following smoothed coupling function:

$$|g_c(k)|^2 = \frac{\gamma_c}{\pi} \frac{\sqrt{\epsilon(k-\omega_c)}}{(k-\omega_c)+\epsilon} \Theta(k-\omega_c) , \quad (5)$$

where $\Theta(\cdot)$ in Eqs. (4) and (5) is a unit step function. The density of modes given by Eq. (5) is a continuous function of k. However, it grows extremely fast for k close to, but larger than ω_c. In fact, the function $|g_c(k)|^2$ is nonanalytic and its derivative tends to infinity when $k \to \omega_c$ from above. In this sense, the function (5) approximates the infinite jumplike behavior of ideal waveguides, described by (4). The parameter ϵ has the dimension of frequency and serves as a "smoothing" parameter. On a frequency scale larger than ϵ, the function (5) displays $1/\sqrt{k-\omega_c}$ behavior as in Eq. (4).

It should be noticed that our model is very similar to the one used in the photodetachment problem[13] for bound–free transitions from an ionic p state to an s continuum. We shall use this analogy extensively in the following. Obviously the details of the dynamics of our model do depend quantitatively on the analytic behavior of (5) close to threshold $(k \simeq \omega_c)$. However, different types of threshold singularities corresponding to bound–free transitions between different angular momentum states have been studied in the context of photodetachment.[18] Such transitions are characterized by couplings which behave as $(k - \omega_c)^{l+1/2}$ for $k \geq \omega_c$ (Ref. 19) with $l = 0$, 1, etc. Results of these studies are in fact qualitatively quite independent of l. The same may be expected to occur in near-threshold spontaneous emission. We may safely say that overall qualitative results of the model (5) do not depend essentially on the form of the function $|g_c(k)|^2$, provided that it has the following properties: (i) it grows algebraically or jumps for k close to, but larger than ω_c, and (ii) it saturates or even decreases slowly for k large enough.

In this sense, results described below can also give some insight into the problem of spontaneous emission in the microcavity,[5] in which the density of photon modes behaves as

$$|g_c(k)|^2 \sim (k^2 - \omega_c^2) \tag{6}$$

for k close to, but larger than ω_c.

The Hamiltonian (1) determines the time evolution of our model. We assume that initially the atom is in an excited state $|1\rangle$, while both photon reservoirs are in the vacuum state. Spontaneous decay may then lead to a creation of one and only one photon, due to the assumed rotating-wave approximation. This statement follows from the fact that for the Hamiltonian (1) the total number of atomic and photoionic excitations is a constant of the motion. The solution of the Schrödinger equation which fulfills the above-stated initial conditions can therefore be written in the form

$$|\Psi(t)\rangle = e^{-i\omega_c t} \left[\alpha(t) | 1, \text{vac} \rangle + \int \beta(t,k) | 0, 1k_b \rangle dk \right.$$
$$\left. + \int \gamma(t,k) | 0, 1k_c \rangle dk \right] . \tag{7}$$

The coefficients $\beta(t,k)$ and $\gamma(t,k)$ represent probability amplitudes of emitting one photon of the energy k, belonging to the background or waveguide reservoirs, respectively.

The Schrödinger equation gives a set of coupled equations for the amplitudes α, β, and γ,

$$\dot{\alpha}(t) = -i(\omega_a - \omega_c)\alpha(t) - i \int g_b(k)\beta(t,k)dk$$
$$\qquad - i \int g_c(k)\gamma(t,k)dk ,$$

$$\dot{\beta}(t,k) = -i(k - \omega_c)\beta(t,k) - ig_b^*(k)\alpha(t) , \tag{8}$$

$$\dot{\gamma}(t,k) = -i(k - \omega_c)\gamma(t,k) - ig_c^*(k)\alpha(t) ,$$

with $\alpha(0) = 1$, $\beta(0,k) = \gamma(0,k) = 0$. Equations (8) are easily solved using the Laplace transform method. Denoting the resolvent function

$$\mathcal{H}(z) = z + i(\omega_a - \omega_c) + \int \frac{|g_b(k)|^2}{z + i(k - \omega_c)} dk$$
$$+ \int \frac{|g_c(k)|^2}{z + i(k - \omega_c)} dk , \tag{9}$$

we obtain

$$\tilde{\alpha}(z) = 1/\mathcal{H}(z) . \tag{10}$$

The photons belonging to background and waveguide reservoirs correspond to geometrically different modes. It is therefore reasonable to introduce the spectra of spontaneous emission into the background and waveguide modes separately,

$$W_b(k) = \lim_{t \to \infty} |\beta(t,k)|^2 , \tag{11a}$$

$$W_c(k) = \lim_{t \to \infty} |\gamma(t,k)|^2 . \tag{11b}$$

In the limit $t \to \infty$, only the free evolution pole at $z = -i(k - \omega_c)$ contributes to (11). Therefore

$$W_b(k) = |\tilde{\beta}(z,k)|^2_{z = -i(k - \omega_c)} , \tag{12a}$$

$$W_c(k) = |\tilde{\gamma}(z,k)|^2_{z = -i(k - \omega_c)} . \tag{12b}$$

Explicit calculation yields

$$W_j(k) = |g_j(k)|^2 \left| \frac{1}{k - \omega_a + i\gamma_b + \dfrac{i\gamma_c\sqrt{\epsilon}}{i\sqrt{\epsilon} + \sqrt{k - \omega_c}}} \right|^2 , \tag{13}$$

where $\sqrt{k - \omega_c} = i\sqrt{\omega_c - k}$ for $(k - \omega_c) < 0$ and $j = b, c$. Formula (13) provides an exact analytic result for the spontaneous-emission spectra in our model. Note that the spectrum associated with the cavity (waveguide) resonance vanishes for $k < \omega_c$, due to the threshold behavior of $|g_c(k)|^2$. Obviously the spectrum associated with the background modes extends into the regime of $k < \omega_c$.

In Ref. 14, we have presented some numerical results for the spectra (13) which were calculated for ϵ comparable with γ_c and $\gamma_b \ll \gamma_c$. These results illustrated the following three major features.

(a) For $\omega_a < \omega_c$ most of the energy is dissipated into the background modes. A large (of the order of γ_c or ϵ) radiative red shift of the spectrum is observed. The radiative shift is an analog of a dynamical threshold shift in the photodetachment process[13(a)] which has been recently observed in experiment.[20]

(b) When ω_a is such that the shifted transition frequency is comparable to ω_c, the nonexponential character of the decay becomes dominant and the spectrum becomes non-Lorentzian.

(c) For larger ω_a most of the energy is dissipated into the waveguide reservoir and the line shapes start to resemble Lorentzians again. A nonexponential contribution to the decay is still present, however, and induces new narrow features in the spectrum (such as additional peaks and holes).

In this work, we present results in a regime of small

$\epsilon \ll \gamma_c$. For such a choice, the function $|g_c(k)|^2$ behaves much more singularly in the vicinity of $k \simeq \omega_c$. We may expect that non-Lorentzian features in the spectrum will be even more visible in this case.

In Fig. 1 we have plotted the spontaneous-emission spectra for $\epsilon = 0.01\gamma_c$, $\gamma_b = 0.1\gamma_c$, and for different values of the atom-waveguide detuning $\Delta = \omega_a - \omega_c$. As we see, main characteristics of the spectra are indeed the same as those in Ref. 14. The following points should be stressed, however.

(i) Below the threshold [$\Delta < 0$, Fig. 1(a)] the Lamb shift is negative but its relative magnitude is smaller $\delta\omega_a \simeq \epsilon \ll \gamma_c$.

(ii) For Δ close to zero, the nonexponential character of the decay determines the line shape. The spectra are non-Lorentzian and develop interesting additional narrow holes or cusps at $k \simeq \omega_c$ [Figs. 1(b) and 1(c)]. The non-Lorentzian character of the spectra is much more apparent for small ϵ and persists over a much larger regime of detunings Δ.

(iii) For Δ large and positive [Fig. 1(d)], spectra are Lorentzian again with small additional cusps at $k \simeq \omega_c$. Note, however, that for large Δ the background contribution again becomes dominant due to the slow, but essential decrease of the function (5),

$$|g_c(k)|^2 \sim \Theta(k - \omega_c) \frac{1}{\sqrt{k - \omega_c}}$$

for $k - \omega_c$ large.

Once more we stress that the particular shape and magnitude of the non-Lorentzian features in the spectra do depend on the choice of the model (5). The qualitative picture is model independent, however, because all of the observed effects do have a physical explanation. For example, cusps and holes in the spectrum result from a temporal interference between the exponential and nonexponential contributions to the decay. The narrowness of these structures (which is seen particularly well when the radiatively shifted atomic frequency is close to the threshold frequency ω_c) reflects a critical slowing down of the decay.

The results presented here and in Ref. 14 show how stable the qualitative properties of the spectra are with respect to changes of the coupling function (5). In fact, we do not notice any appreciable differences within the many orders of magnitude in ϵ.

III. AUTLER-TOWNES SPLITTING OF ATOMIC LEVELS IN A WAVEGUIDE

In this section we shall discuss the phenomenon of Autler-Townes splitting[15] in cavities and waveguides. The investigated phenomena provide the simplest example of dynamical (i.e., strong-field-induced) modifications of spontaneous emission.[9,10]

In order to study the Autler-Townes effect, we introduce a model three-level atom having energy levels $|0\rangle$, $|1\rangle$, and $|2\rangle$ of the energies 0, ω_1, and ω_2, respectively. The spontaneous-emission rate from $|1\rangle$ to $|0\rangle$ is assumed to be small and we shall neglect it in the following. The transition $|0\rangle \rightarrow |1\rangle$ is driven by a strong, resonant

laser field of the frequency ω_L. The strength of the driving field is characterized by the Rabi frequency Ω. Spontaneous emission from the upper level $|1\rangle$ may take place, causing a transition to the level $|2\rangle$. The frequency of this transition lies close to the resonance frequency ω_c of a cavity or a waveguide mode. Photons may, however, also be emitted into background modes, unassociated with the cavity resonance.

The Hamiltonian of the system reads

$$H = \omega_1 |1\rangle\langle 1| + \omega_2 |2\rangle\langle 2| + \left[\frac{\Omega}{2} e^{i\omega_L t} |0\rangle\langle 1| + \text{H.c.} \right]$$

$$+ \int |k| (b_k^\dagger b_k + c_k^\dagger c_k) dk$$

$$+ \int \{ [g_b(k) b_k^\dagger + g_c(k) c_k^\dagger] |2\rangle\langle 1| + \text{H.c.} \} dk .$$

$$(14)$$

Before solving the model defined by (14), let us discuss some of its properties in the case of the resonant excitation ($\omega_L = \omega_1$). The model has a constant of the motion (for arbitrary ω_L)

$$N = |0\rangle\langle 0| + |1\rangle\langle 1| + \int (c_k^\dagger c_k + b_k^\dagger b_k) dk . \quad (15)$$

If the system was initially in the ground state $|0\rangle$ of the atom, and the vacuum state of both reservoirs, it may only emit one photon due to the transition $|1\rangle \rightarrow |2\rangle$.

The driving field induces the splitting of the upper atomic level $|1\rangle$ into two levels having the energies close to $\omega_1 \pm (\Omega/2)$. Therefore the frequency of the spontaneously emitted photon will be close to $\omega_1 \pm (\Omega/2) - \omega_2$ rather than the bare transition frequency $\omega_1 - \omega_2$. Obviously, the spectrum of the spontaneous emission will consist of two lines, if the frequencies $\omega_1 \pm (\Omega/2) - \omega_2$ are separated sufficiently. If both frequencies $\omega_1 \pm (\Omega/2) - \omega_2$ lie in the region of high reservoir mode density, both lines will have widths corresponding to this density of modes. As soon as the splitting is large enough, so that either of the frequencies $\omega_1 \pm (\Omega/2) - \omega_2$ shifts outside of the regime of high reservoir mode density, a dramatic narrowing of the corresponding line will occur. Such narrowing indicates the effective closing of some decay channel and a dramatic increase in the lifetime of the corresponding atomic dressed state. We call this effect dynamical suppression of spontaneous emission.[9,10] It should be stressed, however, that close analogs of this effect have been previously discussed in the context of bound–free transitions. One such analogy has been referred to as the confluence of coherence, which may occur in laser-induced autoionization.[21] Exactly the same kind of line narrowing was predicted by Kukliński and Rząźewski,[22] who studied the transition from a bound state into a symmetric autoionizing resonance. In that case, the wings of the resonance must, however, falloff faster than in the case of the usual Lorentzian resonance. These authors termed the effect a strong-field-induced modification of Fermi's golden rule.

In the present work, we will discuss the interplay between the dynamical modifications of spontaneous emis-

sion and the effects caused by the thresholdlike character of the photonic density of modes. In order to do it, we shall consider the case when the coupling constants $|g_{b,c}(k)|^2$ have exactly the same form as that discussed in Sec. II,

$$|g_b(k)|^2 = \text{const} = \gamma_b/\pi \tag{16a}$$

and

$$|g_c(k)|^2 = \frac{\gamma_c}{\pi} \frac{\sqrt{\epsilon(k-\omega_c)}}{(k-\omega_c)+\epsilon}\Theta(k-\omega_c) . \tag{16b}$$

As we have mentioned, if the atom and reservoirs are initially in their ground and vacuum states, respectively, the wave function can be written in the form

$$|\Psi(t)\rangle = e^{-i\omega_c t}\left[\alpha_0(t)\,|\,0,\text{vac}\rangle + \alpha_1(t)\,|\,1,\text{vac}\rangle \right.$$
$$+ \int \beta(t,k)e^{-i\omega_L t}|\,2,1k_b\rangle dk$$
$$\left. + \int \gamma(t,k)e^{-i\omega_L t}|\,2,1k_c\rangle dk \right] . \tag{17}$$

The Schrödinger equation then leads to

$$\dot{\alpha}_0(t) = i\omega_c\alpha_0(t) - i\frac{\Omega}{2}\alpha_1(t) ,$$

$$\dot{\alpha}_1(t) = i(\omega_c + \omega_L - \omega_1)\alpha_1(t) - i\frac{\Omega}{2}\alpha_0(t)$$
$$\qquad - i\int g_b(k)\beta(t,k)dk - i\int g_c(k)\gamma(t,k)dk ,$$
$$\tag{18}$$
$$\dot{\beta}(t,k) = -i(k-\omega_c+\omega_2-\omega_L)\beta(t,k) - g_b^*(k)\alpha_1(t) ,$$

$$\dot{\gamma}(t,k) = -i(k-\omega_c+\omega_2-\omega_L)\gamma(t,k) - ig_c^*(k)\alpha_1(t) .$$

The Lacplace transform technique allows one to solve (18) exactly. We obtain

$$\tilde{\alpha}_0(z) = \frac{1}{z - i\omega_c + (\Omega^2/4)\mathcal{H}^{-1}(z)} , \tag{19a}$$

$$\tilde{\alpha}_1(z) = -i\frac{\Omega}{2}\frac{1}{(z-i\omega_c)\mathcal{H}(z)+\Omega^2/4} , \tag{19b}$$
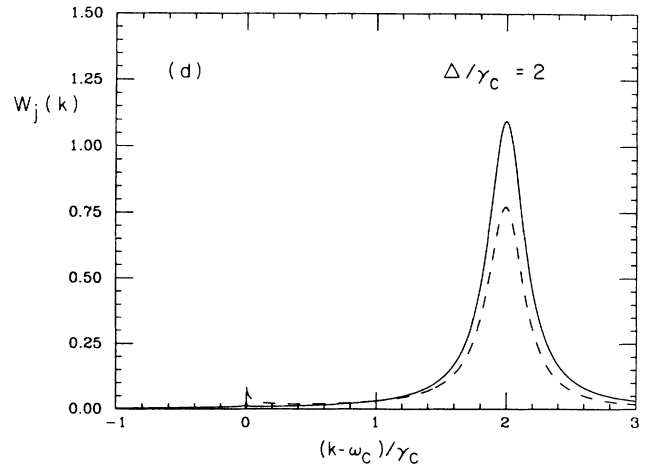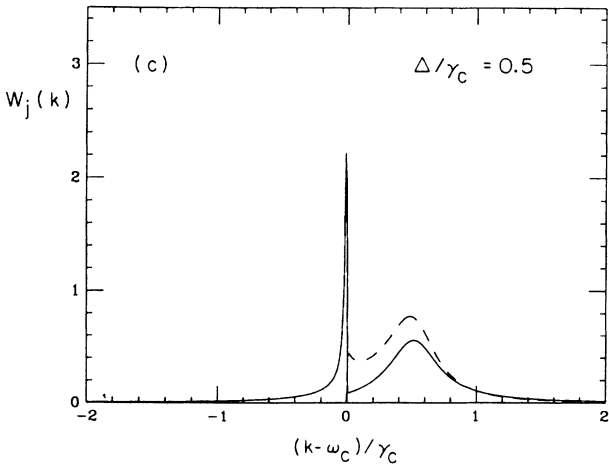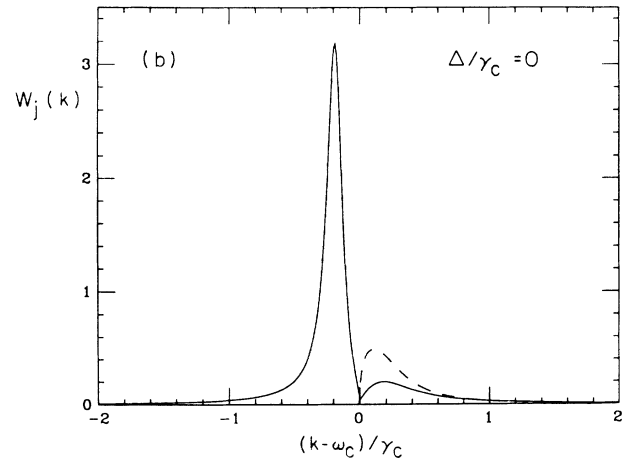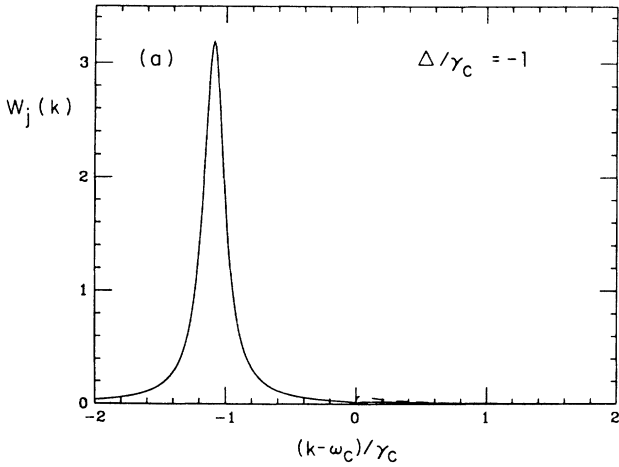

FIG. 1. Spontaneous emission in a waveguide for the atom-cavity detunings $\Delta/\gamma_c = -1$ (a), 0 (b), 0.5 (c), and 2 (d) and $\gamma_b = 0.1\gamma_c$, $\epsilon = 0.01\gamma_c$. Solid lines denote spectra of the background field, and dashed lines correspond to the spectra of the waveguide field.

where

$$\mathcal{H}(z) = z + i(\omega_1 - \omega_L - \omega_c) + \gamma_b$$
$$+ \frac{\gamma_c \sqrt{\epsilon}}{i\sqrt{\epsilon} + \sqrt{iz + \omega_L - \omega_2}} . \tag{20}$$

Denoting the atom-cavity detuning as

$$\Delta = \omega_1 - \omega_2 - \omega_c \tag{21}$$

and the atom-laser detuning as

$$\Delta_L = \omega_1 - \omega_L , \tag{22}$$

we obtain the following result for the spectra defined as in Eq. (11):

$$W_j(k) = \frac{\Omega^2}{4} |g_j(k)|^2 \left| \frac{1}{(z - i\omega_c)\mathcal{H}(z) + \Omega^2/4} \right|^2_{z = -i(k - \omega_c + \omega_2 - \omega_L)} , \tag{23}$$

which reduces to

$$W_j(k) = \frac{\Omega^2}{4} |g_j(k)|^2 \left| \frac{1}{\left[\omega - \Delta + i\gamma_b + \dfrac{i\gamma_c\sqrt{\epsilon}}{i\sqrt{\epsilon} + \sqrt{\omega}}\right](\omega - \Delta + \Delta_L) - \Omega^2/4} \right|^2 , \tag{23a}$$
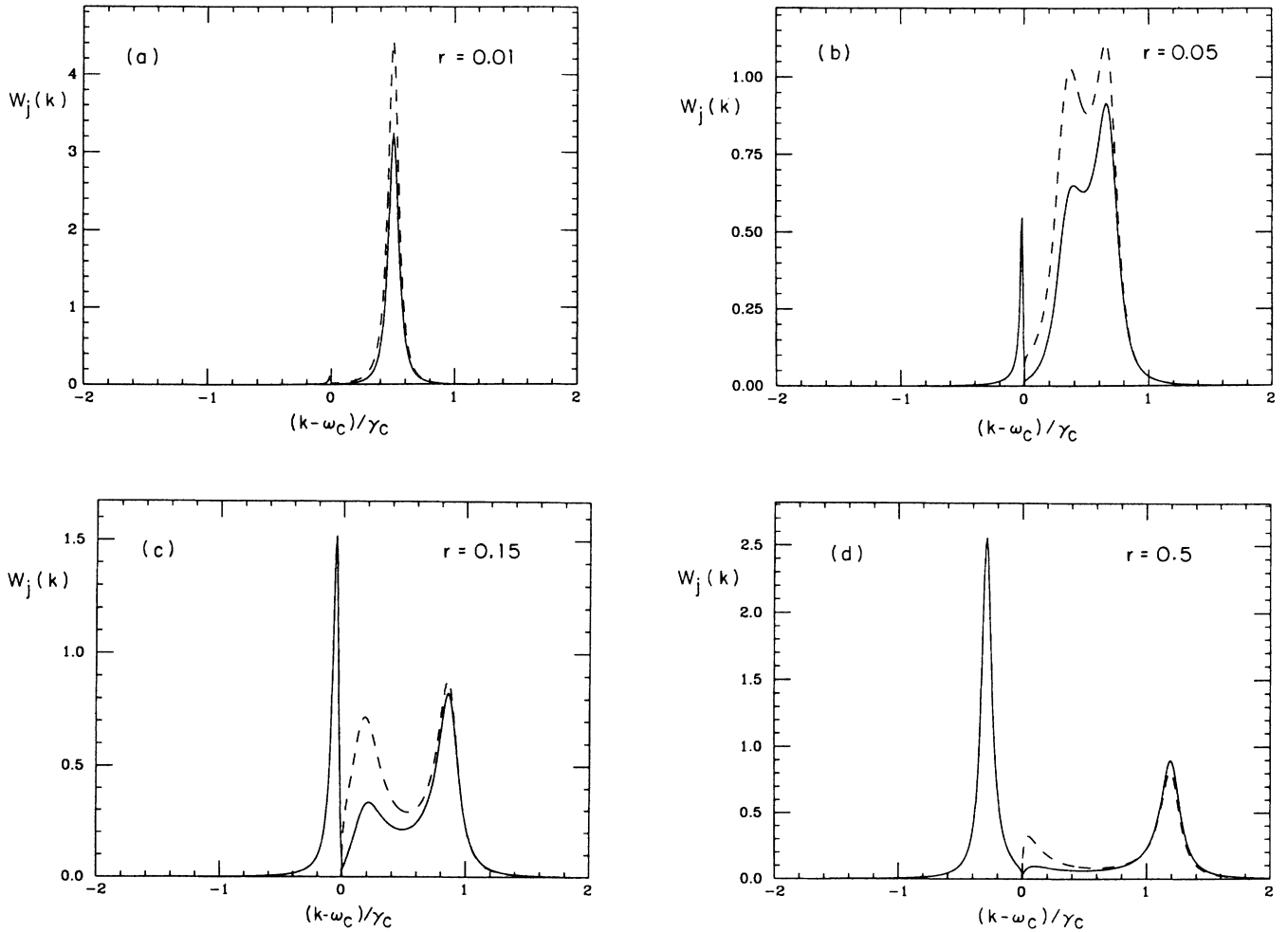

FIG. 2. Spontaneous emission in the presence of a driving field (Autler-Townes spectra) for $r = 0.01$ (a), 0.05 (b), 0.15 (c), and 0.5 (d) and in units of γ_c, $\gamma_b = 0.1$, $\epsilon = 0.01$, $\Delta = 0.5$, and $\Delta_L = 0$. As before, solid and dashed lines denote the background and the waveguide fields, respectively.

with $\omega = k - \omega_c$.

Numerical results are shown in Figs. 2–4. To concentrate on the cavity-induced effects, we present results for the exact atom-field resonance $\Delta_L = 0$ case only, although (23a) gives an analytic expression for arbitrary Δ_L. All parameters are expressed in units of γ_c. The laser-atom coupling is parametrized by

$$r = \frac{\Omega^2}{4\gamma_c^2} \cdot$$

r is proportional to the laser intensity and is dimensionless.

In Fig. 2 the results are presented for fixed atom-cavity detuning, small ϵ, and for different laser intensities. For small $r = 0.01$ [Fig. 2(a)], the spectrum consists of a single line centered at $k - \omega_c \simeq \Delta$. The width of the line is controlled mainly by the intensity factor (power broadening). For $r = 0.05$ the Autler-Townes doublet is already visible. The effective width of the peaks can be estimated as[22]

$$\gamma_{\text{eff}} = \left[\gamma_b + \frac{\gamma_c \sqrt{\epsilon \omega}}{\omega + \epsilon} \right], \qquad (24)$$

where ω is evaluated *at the position of the peak*. Since the density of modes (11b) has a maximum for $\omega \simeq \epsilon = 0.01$, evidently the left peak of the doublet (corresponding to smaller ω) is wider and therefore smaller.

The presence of the threshold at $\omega = 0$ induces a very slow, nonexponential contribution to the decay process. This contribution leads to the appearance of a narrow non-Lorentzian cusp in the background spectrum. The role of the non-Lorentzian features increases even more, as the lower frequency Autler-Townes component moves towards the critical region of $\omega = 0$ [Fig. 2(c)].

Finally, for $r = 0.5$ [Fig. 2(d)] the left peak of the doublet lies entirely below the threshold region. Its effective width is roughly determined by the density of background modes and is smaller than the width of the right peak. The width of the latter is still well estimated by the
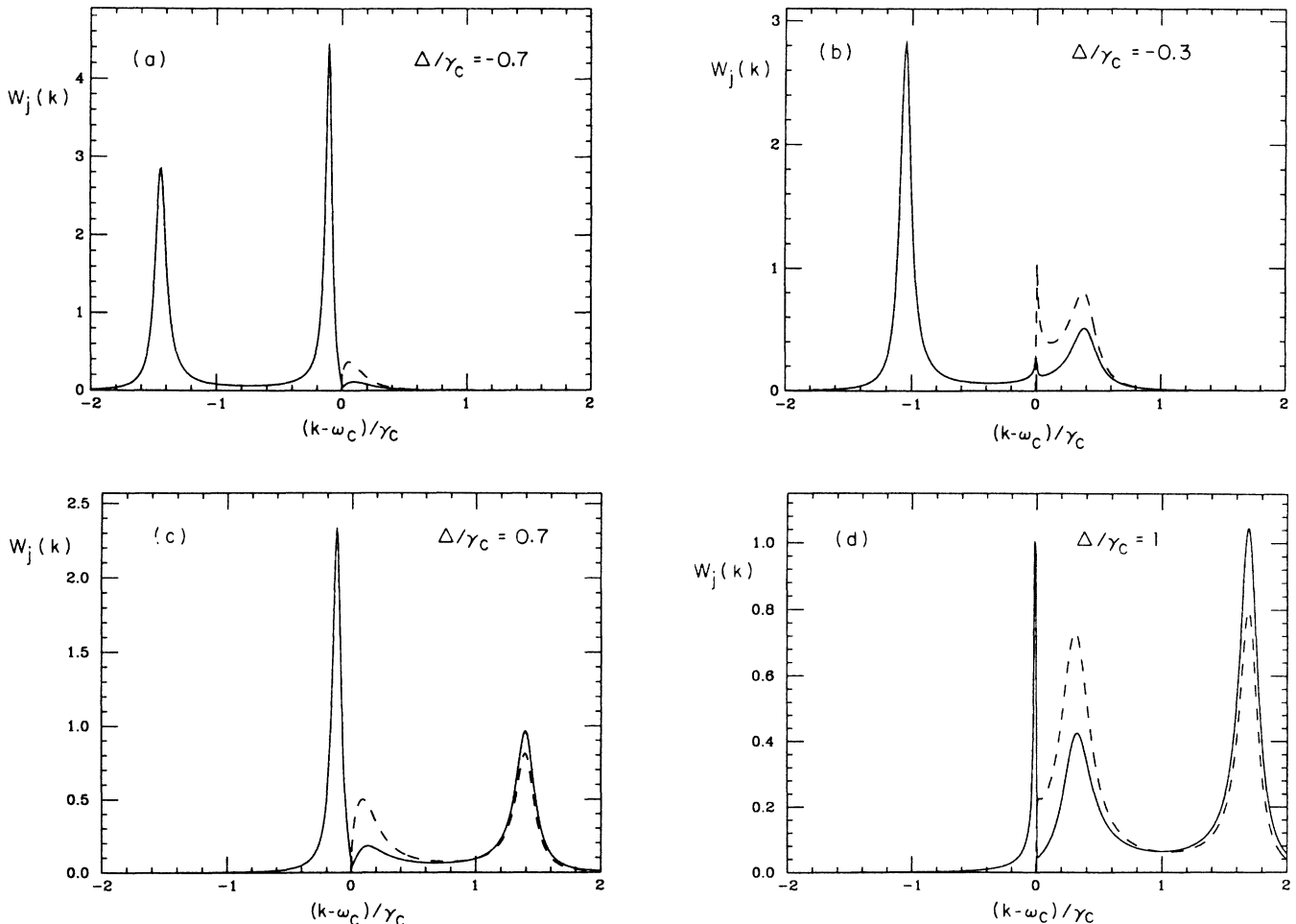

FIG. 3. Same as Fig. 2 but for fixed $r = 0.5$ and $\Delta/\gamma_c = -0.7$ (a), -0.3 (b), 0.7 (c), and 1 (d).

formula (24). The nonexponential part of the decay causes a cusp-shaped hole in the background spectrum.

Another set of data is presented in Fig. 3. This time we have plotted the spectra for fixed $r=0.5$ and for different values of the atom-cavity detuning Δ. In Fig. 3(a) the right peak of the doublet lies below, but close to the threshold. Due to the critical slowing down, it has a narrow, non-Lorentzian shape. For $\Delta=-0.3$ in Fig. 3(b), the different widths of the peaks result from the dynamical modification of the spontaneous emission and can be estimated from Eq. (24). The nonexponential part of the decay induces narrow cusps in both spectra for $\omega\simeq0$. All these effects (critical slowing down, non-Lorentzian line shapes, intensity-dependent widths and shifts) are also seen in Fig. 3(c) and 3(d).

The results for ϵ comparable to γ_c are shown in Fig. 4. Spontaneous-emission spectra in the absence of the laser field have been studied for this case in Ref. 14. For such values of ϵ, large radiative shifts of the atomic levels have been observed. In the present case, radiative shifts may

become intensity dependent and describe the shifts of the dressed atomic states.

In fact, in Fig. 4(a), the bare atomic level $|1\rangle$, which in a zero applied field is Lamb shifted to the vicinity of $\omega=0$, is seen to be dynamically shifted up in frequency by 0.15 units and split into two components at $\omega\simeq-0.2,0.5$ by a weak $r=0.03$ driving field. The splitting is visible only in the background spectrum $W_b(k)$. The widths of the peaks are partially controlled by the local density of modes [Eq. (24)], but some effects of power broadening are still present [compare Figs. 4(a) and (b)].

For larger values of the driving field intensity [Figs. 4(c) and (d)] the widths of the peaks become fully determined by the generalized Fermi rule [Eq. (24)]. The left peak, which lies below the threshold, is therefore narrower and its width is of the order of γ_b. The right peak, which lies above the threshold, is broadened by the waveguide and background modes.

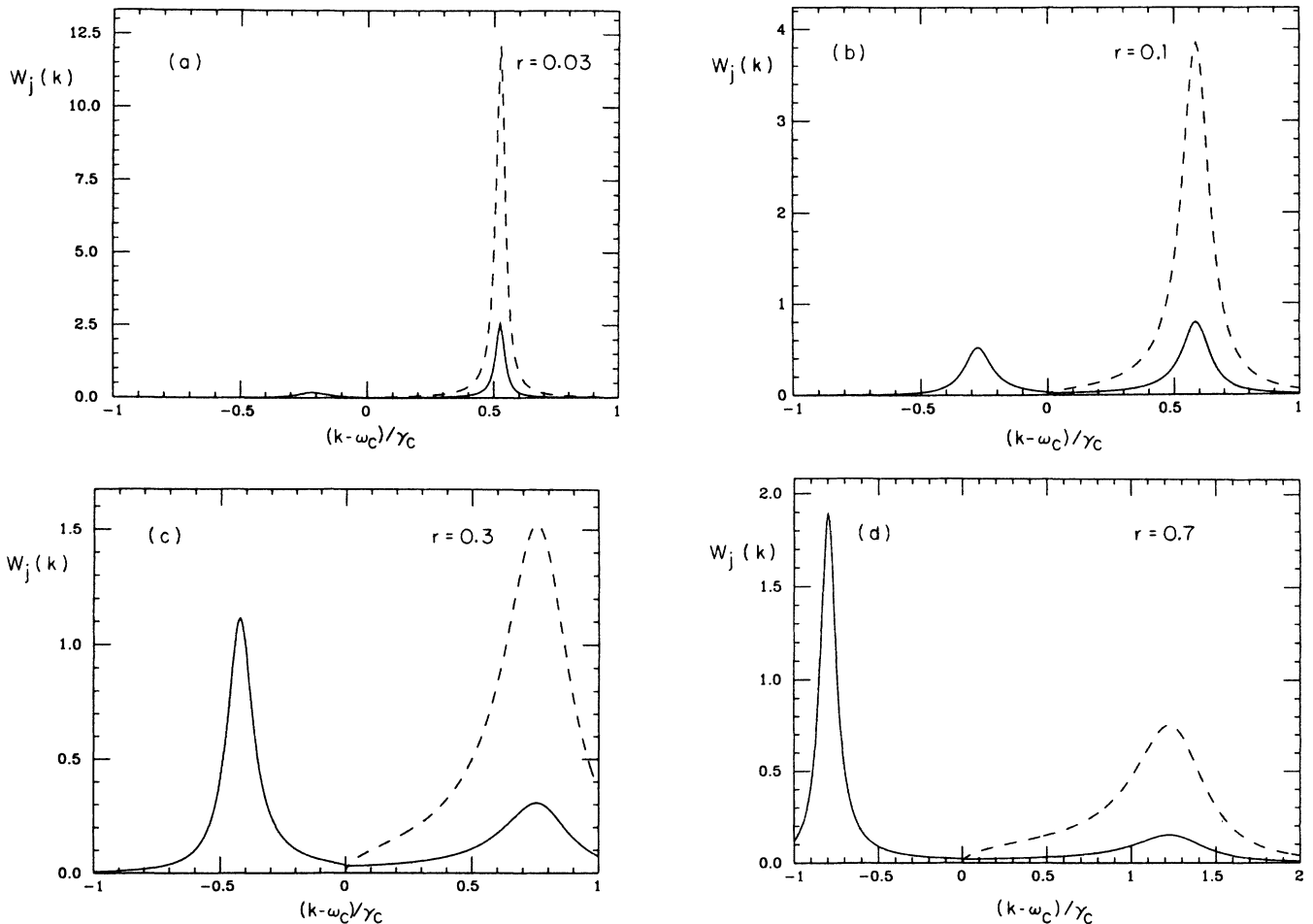The results presented above indicate that Autler-Townes–type splitting in waveguides possesses a rich

FIG. 4. Same as Fig. 2 but for $\epsilon/\gamma_c=1$, fixed detuning $\Delta/\gamma_c=0.5$ and with $r=0.03$ (a), 0.1 (b), 0.3 (c), and 0.7 (d).

phenomenology. Experimental observation of these effects should provide new insights into the physics of the spontaneous-emission process.

IV. SPONTANEOUS EMISSION IN THE PRESENCE OF PHOTON GAPS

In some physical systems, the photonic density of modes may develop deep minima. In spatially periodic

systems, no propagating electromagnetic modes exist in some frequency regimes. This effect, discovered by Ya-blonovitch,[16] may lead to a substantial suppression of spontaneous-emission rates. Quite analogously, in a resonant linear medium consisting of harmonic oscillators, deep holes in the photonic density of modes occur for frequencies close to the oscillator resonance frequency.[17,23] In this section we shall study the spontaneous-emission
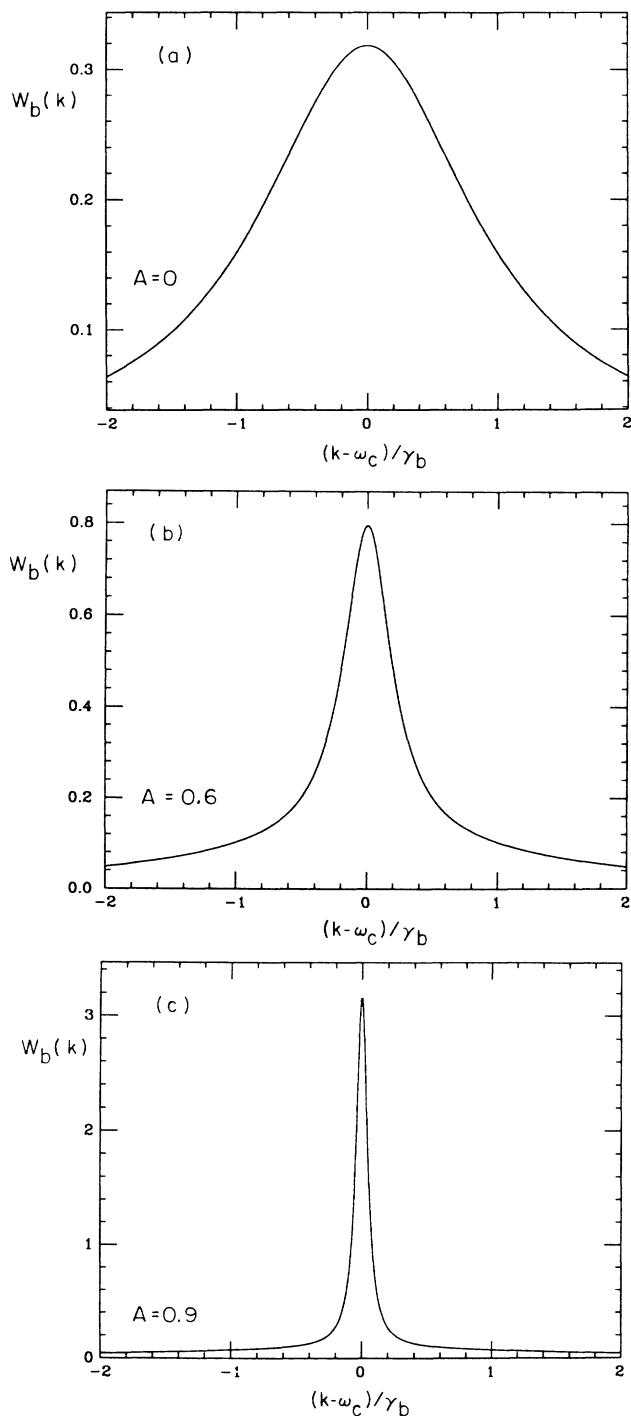

FIG. 5. Spontaneous-emission spectra in the photon gap for gap depths $A=0$ (a), 0.6 (b), and 0.9 (c). Atomic frequency coincides with the center of the gap ($\Delta=0$), $\gamma_b=\tau$.
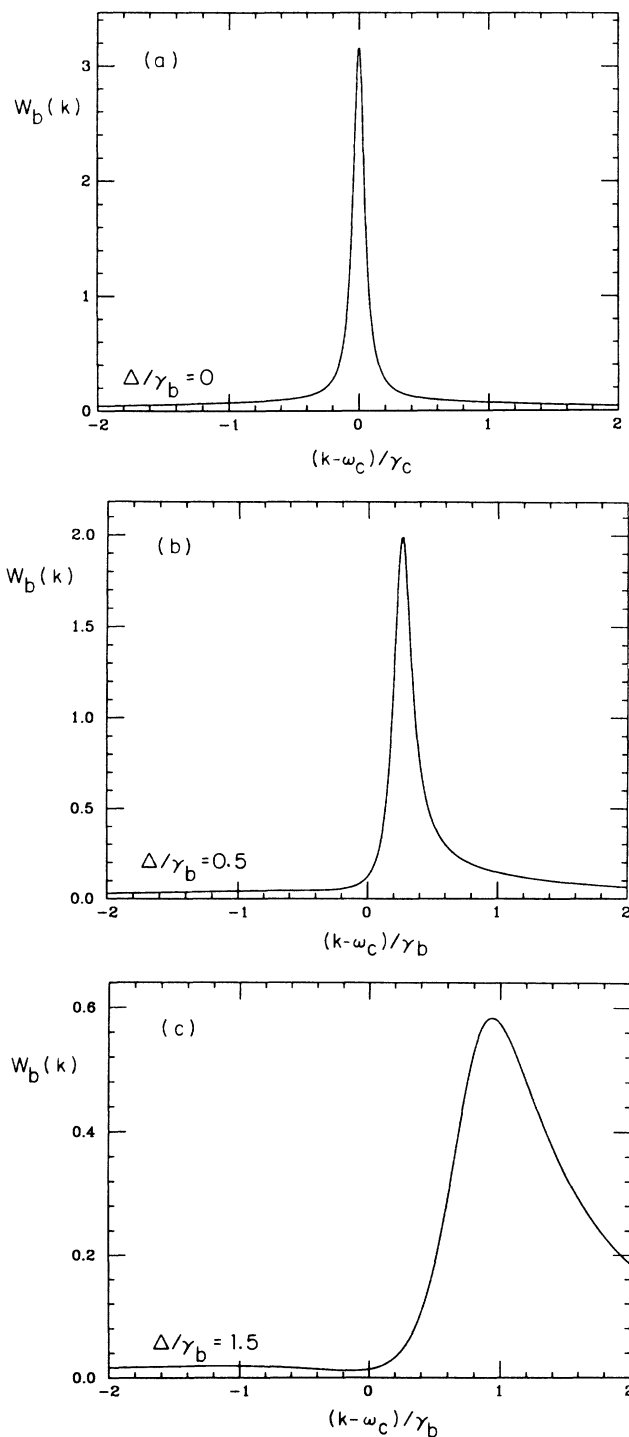

FIG. 6. Same as Fig. 5 but for fixed gap depth $A=0.9$ and atom-gap detunings $\Delta/\gamma_b=0$ (a), 0.5 (b), and 1.5 (c).

process in such cases. We shall assume that an impurity atom is located within a medium, possessing gaps in its photonic density of modes. The transition frequency of the probe atom is assumed to lie close to one such gap. Again, our aim will be to discuss effects which go beyond the validity of the Wigner-Weisshopf approximation.

In the present problem, instead of introducing two separate reservoirs, we shall consider only one, characterized by an appropriate density of modes.

The Hamiltonian will therefore be

$$H = \omega_a \mid 1 \rangle \langle 1 \mid + \int \mid k \mid b_k^\dagger b_k dk$$

$$+ \int g_b(k)(b_k^\dagger \mid 0 \rangle \langle 1 \mid + \mid 1 \rangle \langle 0 \mid b_k) dk \ . \quad (25)$$

It is just the same Hamiltonian as (1), except that we put $g_c(k) = 0$. The model is fully determined by the specification of the coupling $\mid g_b(k) \mid^2$. Since we are only interested here in the qualitative effects, we shall introduce a phenomenological model of $g_b(k)$, which accounts for the presence of a dip in an otherwise uniform photon-ic density of modes. Our model is defined by

$$\mid g_b(k) \mid^2 = \frac{\gamma_b}{\pi} \left[1 - A \frac{\tau^2}{\tau^2 + (k - \omega_c)^2} \right] . \quad (26)$$

The "gap" (or the "hole") is located at the frequency ω_c. It has a Lorentzian shape and the parameter τ describes its width. It should be noted that other shapes (such as, for example, powers of a Lorentzian) which may be more suitable for modeling real solids also lead to analytic, albeit slightly more complicated results. The parameter A changes from 0 to 1 and measures the depth of the gap. No minimum in $\mid g_b(k) \mid^2$ is present for $A = 0$. For $A = 1$ the density of modes (26) is exactly equal to zero at $k = \omega_c$. Our model of the photonic continuum is a close analog of the electronic continuum in autoionization—it corresponds to a Fano profile with the asymmetry parameter $q = 0$.[24]

Assuming that initially the atom was excited and the field was in the vacuum state, the solution of the model is easily obtained using the same techniques as those in Sec. II. As a result, we obtain
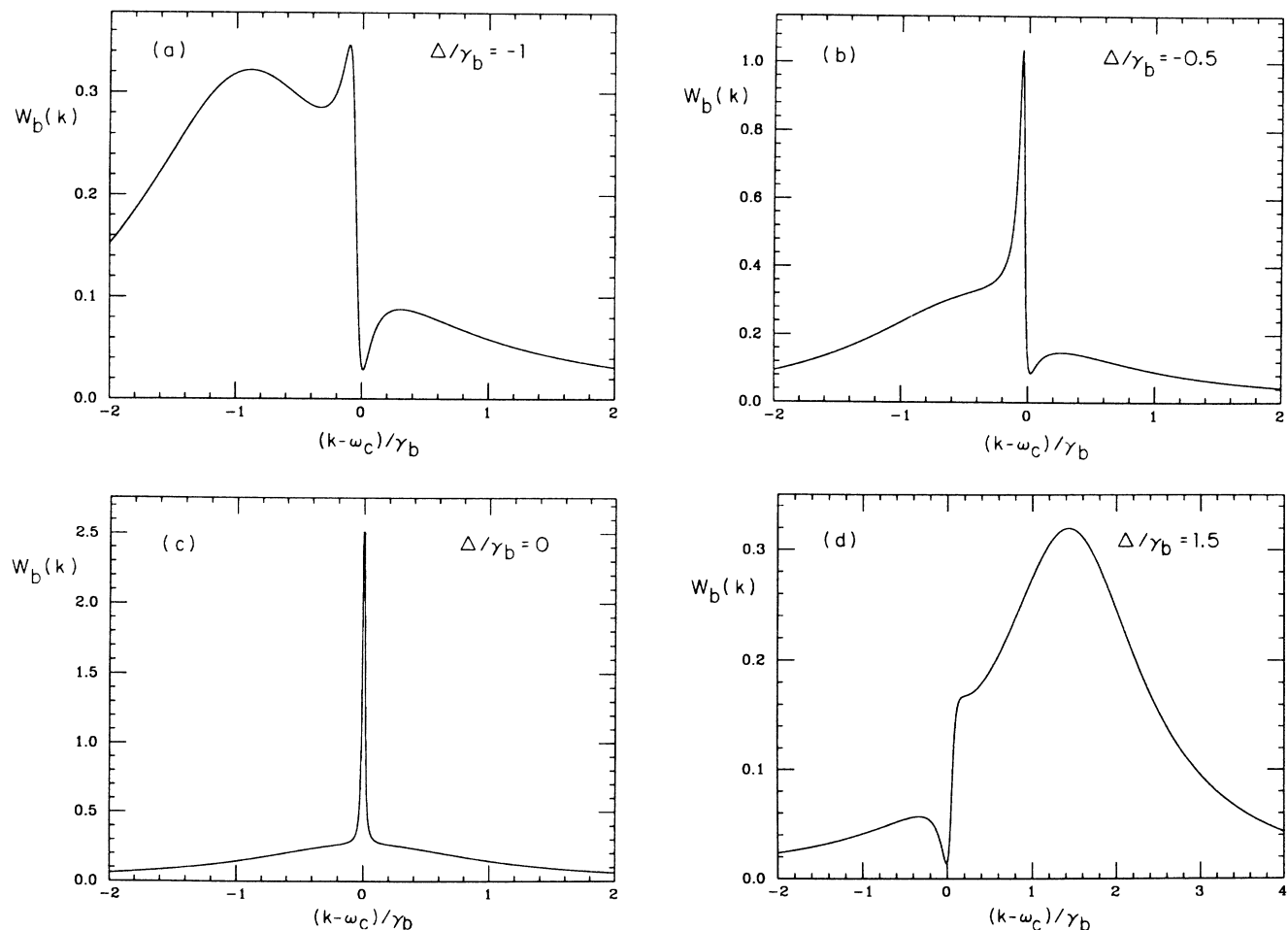

FIG. 7. Spontaneous-emission spectra in the "narrow" photon gap $\tau = 0.1\gamma_b$ for atom-gap detunings $\Delta/\gamma_b = -1$ (a), -0.5 (b), 0 (c), and 1.5 (d), and for gap depth $A = 0.9$.

$$W_b(k) = \frac{\gamma_b}{\pi} \frac{\omega^2 + (1-A)\tau^2}{[\omega(\omega-\Delta)-\gamma_b\tau(1-A)]^2 + [\omega(\gamma_b+\tau)-\tau\Delta]}, \tag{27}$$

where $\omega = k - \omega_c$, while $\Delta = \omega_a - \omega_c$. For our choice of $|g_b(k)|^2$ (26), it is also easy to find the time-dependent solutions. The probability amplitude of the excited atomic state decays as

$$\alpha(t) = A_1 e^{z_1 t} + a_2 e^{z_2 t}, \tag{28}$$

with

$$z_{1,2} = -\frac{\gamma_b+\tau}{2} \pm \tfrac{1}{2}[(\gamma_b-\tau)^2 + 4A\gamma_b\tau]^{1/2} \tag{29}$$

for $\Delta = 0$.

In Fig. 5 we show the resulting spontaneous-emission spectra for $\gamma_b = \tau$, assuming perfect resonance between atomic frequency ω_a and the center of the gap ω_c for different values of the gap's depth A. For $A=0$, the spectrum has a usual Lorentzian shape. For A large, it consists of the sum of two Lorentzians of the widths determined by Eq. (29). An appearance of the narrow component in the spectrum reflects the existence of the photon mode density minimum at $\omega = 0$.

Figure 6 shows the spectrum for different values of the atom-gap detuning. As Δ increases the spectrum broadens, since the resonance shifts away from the regime of small density of photon modes. The spectrum becomes asymmetric and develops a minimum at $\omega \simeq 0$ [see Fig. 6(c)]. Its shape is analogous to that of the Fano profile of an autoionizing resonance.

Similar results are plotted in Fig. 7, for a "narrow gap": $\tau = 0.1\gamma_b$ and $A = 0.9$. The presence of the narrow gap introduces a slow component into the decay process. The spectrum results from a quantum-mechanical temporal interface between the gap-induced and usual exponential terms. Because of this interference, interesting dispersionlike shapes are observed in the spectrum for frequencies close to $\omega = 0$. The shape of these interference features depends on the detuning Δ.

V. CONCLUSIONS

It has been known for many years that the spontaneous-emission process in cavities and waveguides differs from that in free space. In particular, the spontaneous-emission rate, evaluated in the framework of the Wigner-Weisshopf approximation, depends crucially on the density of the photon modes.

If the photonic density of modes changes on a frequency scale comparable to the spontaneous-emission rate (estimated on the basis of the local photon mode density) new phenomena may occur. The Wigner-Weisshopf approach can no longer be used, and the decay necessarily becomes nonexponential. As we have shown in this paper and in Ref. 14, a variety of novel, physical effects are then expected to occur. We have discussed some of these effects here by studying explicitly the processes of spontaneous emission and Autler-Townes splitting in a waveguide near the threshold and spontaneous emission in the presence of the photon gaps.[25]

All these effects may in principle be observable experimentally. The only condition which has to be fulfilled is that the characteristic width of the frequency-dependent reservoir should be comparable to the spontaneous-emission rate. Such a condition can easily be fulfilled in cavities, both in the optical and in the microwave regimes, and should also be realizable in high-Q waveguides. Systems with photon gaps have not yet been observed experimentally. No fundamental reasons exclude, however, the possibility that sufficiently narrow gaps (or holes) in the photonic spectrum will exist to give rise to effects such as those discussed here. In any case, by a proper choice of impurity atoms (with large dipole matrix elements) one should also be able to approach the regime in which the required conditions are fulfilled.

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[25]Calculations analogous to those performed in Sec. III give analytic results for the spontaneous-emission spectra in the presence of a driving field. Similar arguments to those already given easily explain the results.