Squeezing in cooperative resonance fluorescence

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Cooperative effects on squeezing in resonance fluorescence from a finite number N of atoms are examined. Analytical expressions for the spectrum of squeezing and a quantity $Q(T)$ related to the photon-counting statistics of the squeezed states are presented for $N=1$ and 2. These expressions are obtained by invoking the high-field approximation of the relevant master equation. The analytical results are supplemented by the numerical solutions of the exact master equation for $N \leq 10$. It is shown that for intense fields or large detunings, squeezing in resonance fluorescence is predominantly due to the quantum nature of the sidebands of the Mollow triplet. As a manifestation of cooperative effects, squeezing present in the total as well as the sideband radiation increases with the number of atoms. This is further confirmed from the behavior of the photon-statistics function $Q(T)$.

I. INTRODUCTION

Recently there has been a great deal of theoretical[1-27] and experimental interest[28-31] in squeezing of radiation fields. Squeezing is a purely quantum-mechanical effect and is characterized by a field state in which the variance of one of the two noncommuting observables is less than one-half of the absolute value of their commutator. Because of this property of reducing the quantum noise in one quadrature phase, squeezed states have potential applications in optical communication systems[4] and in the detection of gravitational waves.[5,6] Squeezing has been predicted in a number of quantum-mechanical systems such as degenerate parametric oscillators,[2,7-9] degenerate four-wave mixing,[10,11] optical bistability,[12] and free-electron lasers.[13]

Squeezing in resonance fluorescence from a single coherently driven two-level atom has been discussed in some recent papers.[18-24] In fact, this simple system already exhibits other quantum features like photon antibunching[32] and sub-Poissonian statistics[33] both of which have been observed experimentally.[34] It has also been shown[35] that the resonance-fluorescence field radiated by a collection of many motionless two-level atoms exhibits squeezing. However, these studies have assumed that the two-level atoms are independent and have thereby ignored the collective effects. It is expected that inclusion of collective effects will modify the squeezing characteristics of resonance fluorescence considerably.[36] Indeed atomic cooperation is known to affect both the steady-state behavior of the atomic observables[37,38] and the transient behavior characterized by the fluorescent spectrum and intensity-intensity correlations.[39] More recently, it has also been shown that collective effects modify the squeezing behavior of the atomic observables in the steady state[25,26] and also in the transient regime.[27] It is therefore of interest to study in some detail how squeezing characteristics are altered when cooperative emission from many atoms is present.

In this paper, we focus our attention on the time-dependent aspects of squeezing in cooperative resonance fluorescence from a finite number N of atoms. Incidently, squeezing is a phase-sensitive effect, and its detection involves homodyning of the signal with an external intense coherent field. There are two quantities which are of measurable interest in such an experiment. The first is the spectrum of squeezing $S_\theta(\omega)$ (Refs. 24 and 29) which is related to a normal-ordered variance of the signal field. The other quantity of interest is $Q(T)$ introduced by Mandel[19] which determines the photon statistics of the superposed field. The negative values of $S_\theta(\omega)$ and $Q(T)$ over a certain regime of parameters imply squeezing. We obtain in this paper analytical expressions for the spectrum of squeezing and the quantity $Q(T)$ for a two-atom system in the intense-field limit and compare them with those for a single atom. For more than two atoms, we obtain the spectrum and $Q(T)$ from the numerical solution of the relevant master equation. We also study the squeezing behavior of the radiation emitted from a mixture of sidebands of the Mollow triplet. Under the conditions of either intense field or large detunings, the squeezing in resonance fluorescence is found to increase with increasing number of atoms. Moreover, the squeezing behavior is predominantly due to the quantum nature of the sidebands. This is reflected from the fact that squeezing due to sideband radiation alone is of much higher magnitude than that in the total radiation and that it increases with the number of atoms.

In Sec. II we present the basic master equation and its high-field-limit approximation and also define the operators characterizing the sideband radiation. The analytical expressions for the spectrum of squeezing both for the total and sideband radiation for $N=1$ and $N=2$ are presented and discussed along with numerical results for higher values of N in Sec. III. In Sec. IV we discuss the behavior of $Q(T)$ based on analytical and numerical results. Finally, some concluding remarks are added in Sec. V.

II. BASIC EQUATIONS

A. Master equation

We consider a system of closely spaced identical N two-level atoms of transition frequency ω_0 interacting with a coherent field of frequency ω_L. The master equation describing the collective decay of the system[40] is given by

$$d\rho/dt = -i\Omega[S_+ + S_-,\rho] - i\Delta[S_z,\rho] - \gamma(S_+S_-\rho - 2S_-\rho S_+ + \rho S_+S_-) . \quad (2.1)$$

Here ρ is the reduced atomic density, $2\Omega = -2(\mathbf{d}\cdot\mathbf{E})/\hbar$ is the Rabi frequency, $\Delta = \omega_0 - \omega_L$ is the detuning of the atomic transition frequency ω_0 from the laser frequency, while 2γ is the Einstein A coefficient. $S_\pm, S_z$ are the collective polarization and inversion operators obeying the angular momentum commutation relations

$$[S_+,S_-] = 2S_z, \quad [S_z,S_\pm] = \pm S_\pm . \quad (2.2)$$

In deriving Eq. (2.1), the external field has been treated classically while Born and Markov approximations have been used to treat the interaction of atoms with the vacuum modes of the radiation field. Also the equation is written in a frame rotating with the frequency ω_L of the coherent field. Incidently the master equation conserves S^2, the total spin of the N two-level atoms. It has a remarkable feature that it admits an exact steady-state solution which has already revealed some interesting cooperative effects.[37,38]

For discussing the squeezing behavior of the sideband radiation, it is necessary to identify the operators which give rise to these sidebands. For this purpose, we introduce the transformation[41]:

$$S_+ = aR_+ + bR_- + cR_z , \quad (2.3a)$$

$$S_z = -c(R_+ + R_-)/2 + (a+b)R_z , \quad (2.3b)$$

where

$$a = (1+\sqrt{r})/2, \quad b = -(1-\sqrt{r})/2 , \quad (2.4a)$$

$$c = \sqrt{(1-r)}, \quad r = \Delta^2/4\Gamma^2 \quad (0 \le r < 1) , \quad (2.4b)$$

$$\Gamma = [\Omega^2 + (\Delta/2)^2]^{1/2} . \quad (2.4c)$$

The new operators $R_\pm, R_z$ represent a rotation of the old ones and hence satisfy the same commutation relations as the old operators. The transformation (2.3) reduces the interaction Hamiltonian H_0 to a diagonal form:

$$H_0 = \Omega(S_+ + S_-) + \Delta S_z = 2\Gamma R_z . \quad (2.5)$$

Thus under the Hamiltonian H_0, the new operators evolve in time as

$$R_\pm(t) = R_\pm(0)\exp(\pm 2i\Gamma t) , \quad (2.6a)$$

$$R_z(t) = R_z(0) , \quad (2.6b)$$

and hence give rise to the spectrum components at frequencies $\omega_L \pm 2\Gamma$ and ω_L. We may therefore introduce the operators

$$\tilde{S}_\pm = aR_\pm + bR_\mp \quad (2.7)$$

as representing a mixture of the sidebands.

In deriving analytical results for one- and two-atom systems, we use the intense-field limit, where $\Gamma \gg N\gamma$ and a secular approximation to the master equation. We insert the transformation (2.3) in Eq. (2.1) and observe that the Liouville operator on the right breaks up into two parts, viz., the one containing slowly varying terms and the other involving rapidly oscillating terms. The oscillatory terms are neglected, and we arrive at the master equation[41]

$$d\rho/dt = -2i\Gamma[R_z,\rho] - \gamma c^2(R_z^2\rho - 2R_z\rho R_z + \rho R_z^2) - \gamma a^2(R_+R_-\rho - 2R_-\rho R_+ + \rho R_+R_-) - \gamma b^2(R_-R_+\rho - 2R_+\rho R_- + \rho R_-R_+) . \quad (2.8)$$

Under resonance conditions ($r=0$), this equation reduces to the one obtained by Agarwal *et al.*[42] The master equation (2.8) admits a steady-state solution of the form

$$\rho^{SS} = D^{-1}\exp(-\alpha R_z) \quad (2.9)$$

and is found from the steady-state ($d\rho/dt=0$) form of the equation (2.8) while D is obtained by demanding that $\mathrm{Tr}\rho^{SS}=1$. Hence

$$\alpha = \ln(a^2/b^2) \text{ and } D = \mathrm{Tr}[\exp(-\alpha R_z)] . \quad (2.10)$$

The simplified master equation is valid for $\Gamma \gg N\gamma$ and is used for subsequent analytical calculations presented in Secs. III and IV. We note that corrections to the results obtained this way would be of the order of $(N\gamma/\Gamma)^2$. Also the solution (2.9) is used to compare the steady-state averages of the operator products.

B. Equations for one-time atomic-operator averages

For the time-dependent analysis we need the equations of motion for one-time averages $\langle R_+(t)\rangle$ and $\langle R_z(t)\rangle$. The equations of motion for these are derived from the master equation (2.8) and solved. For the one-atom case, we have

$$(d/dt)\langle R_+(t)\rangle = (2i\Gamma - \gamma_1)\langle R_+(t)\rangle , \quad (2.11)$$

$$(d/dt)\langle R_z(t)\rangle = -\gamma_2\langle R_z(t)\rangle - \gamma\sqrt{r} , \quad (2.12)$$

where

$$\gamma_1 = (3-r)\gamma/2, \quad \gamma_2 = (1+r)\gamma . \quad (2.13)$$

The solutions read as

$$\langle R_+(t)\rangle = \langle R_+(0)\rangle\exp[(2i\Gamma - \gamma_1)t] , \quad (2.14)$$

$$\langle R_z(t)\rangle = [\sqrt{r}/(1+r)][\exp(-\gamma_2 t) - 1] + \langle R_z(0)\rangle\exp(-\gamma_2 t) . \quad (2.15)$$

On the other hand, for a two-atom system, the averages

$\langle R_+(t)\rangle$ and $\langle R_z(t)\rangle$ get coupled to higher-order moments. The relevant equations read as

$$(d/dt)\langle R_+(t)\rangle=(2i\Gamma-\gamma_1)\langle R_+(t)\rangle+\gamma\sqrt{r}\,\langle L(t)\rangle\ , \tag{2.16}$$

$$(d/dt)\langle L(t)\rangle=-3\gamma\sqrt{r}\,\langle R_+(t)\rangle+(2i\Gamma-\gamma_3)\langle L(t)\rangle\ , \tag{2.17}$$

$$(d/dt)\langle R_z(t)\rangle=-\gamma_2\langle R_z(t)\rangle-\gamma\sqrt{r}\,\langle M(t)\rangle\ , \tag{2.18}$$

$$(d/dt)\langle M(t)\rangle=4\gamma\sqrt{r}\,\langle R_z(t)\rangle-3\gamma_2\langle M(t)\rangle+8\gamma_2\ , \tag{2.19}$$

where the operators L and M stand for

$$L=R_+R_z+R_zR_+\ ,\quad M=R_+R_-+R_-R_+\ ,\quad \gamma_3=(7+3r)\gamma/2\ . \tag{2.20}$$

These equations can be solved in pairs. In particular, the solutions for $\langle R_+(t)\rangle$ and $\langle R_z(t)\rangle$ are given by

$$\langle R_+(t)\rangle=\frac{(\gamma_3-\beta_1)\langle R_+(0)\rangle+\gamma\sqrt{r}\,\langle L(0)\rangle}{(\beta_2-\beta_1)}\exp[(2i\Gamma-\beta_1)t]+\frac{(\gamma_3-\beta_2)\langle R_+(0)\rangle+\gamma\sqrt{r}\,\langle L(0)\rangle}{(\beta_1-\beta_2)}\exp[(2i\Gamma-\beta_2)t]\ , \tag{2.21}$$

$$\langle R_z(t)\rangle=\frac{[(3\gamma_2-\beta_3)\langle R_z(0)\rangle-\gamma\sqrt{r}\,(\langle M(0)\rangle-8\gamma_2/\beta_3)]}{(\beta_4-\beta_3)}\exp(-\beta_3t)+\frac{[(3\gamma_2-\beta_4)\langle R_z(0)\rangle-\gamma\sqrt{r}\,(\langle M(0)\rangle-8\gamma_2/\beta_4)]}{(\beta_3-\beta_4)}\exp(-\beta_4t)-\frac{8\gamma\gamma_2\sqrt{r}}{\beta_3\beta_4}\ , \tag{2.22}$$

where

$$\beta_{1,2}=[(5+r)/2\pm(1-r+r^2)^{1/2}]\gamma\ , \tag{2.23}$$

$$\beta_3=(3+r)\gamma,\quad \beta_4=(3r+1)\gamma\ . \tag{2.24}$$

These solutions will be used along with the quantum regression theorem to obtain the two-time expectation values needed for computation of spectrum of squeezing and photon statistics discussed in the subsequent sections.

III. SPECTRUM OF SQUEEZING

Since squeezing is a phase-sensitive effect we express the slowly varying part of the radiated field at the detector as

$$E_\theta(t)=[E^{(+)}(t)\exp(i\theta)+E^{(-)}(t)\exp(-i\theta)]/2\ , \tag{3.1}$$

where θ is a phase angle and $E^{(\pm)}$ are the positive and negative parts of the field. The spectrum of squeezing is related to the normal-order variance:[24]

$$\begin{aligned}\Gamma_\theta(t+\tau,t)&=\langle :\Delta E_\theta(t+\tau)\Delta E_\theta(t):\rangle\\&=[\exp(2i\theta)\langle \Delta E^{(+)}(t+\tau)\Delta E^{(+)}(t)\rangle+\exp(-2i\theta)\langle \Delta E^{(-)}(t)\Delta E^{(-)}(t+\tau)\rangle\\&\quad+\langle \Delta E^{(-)}(t+\tau)\Delta E^{(+)}(t)\rangle+\langle \Delta E^{(-)}(t)\Delta E^{(+)}(t+\tau)\rangle]/4\ .\end{aligned} \tag{3.2}$$

Here $\Delta A=A-\langle A\rangle$ and the angular brackets denote the averages with respect to the atomic density operator ρ. In the far-field limit and in the absence of a free field at the detector the radiated field $E^{(+)}(t)$ can be effectively replaced by $\mu S_-(t)$, where μ is a geometrical factor.[40] Hence

$$\begin{aligned}\Gamma_\theta(t+\tau,t)=(|\mu|^2/4)\{&\exp(2i\theta)[\langle S_-(t+\tau)S_-(t)\rangle-\langle S_-(t+\tau)\rangle\langle S_-(t)\rangle]\\&+\exp(-2i\theta)[\langle S_+(t)S_+(t+\tau)\rangle-\langle S_+(t)\rangle\langle S_+(t+\tau)\rangle]\\&+[\langle S_+(t+\tau)S_-(t)\rangle-\langle S_+(t+\tau)\rangle\langle S_-(t)\rangle]\\&+[\langle S_+(t)S_-(t+\tau)\rangle-\langle S_+(t)\rangle\langle S_-(t+\tau)\rangle]\}\ .\end{aligned} \tag{3.3}$$

The steady-state spectrum of squeezing is then defined as[24]

$$S_\theta(\omega)=(\gamma/|\mu|^2)\int_0^\infty[\exp(i\omega\tau)+\exp(-i\omega\tau)]\lim_{t\to\infty}\Gamma_\theta(t+\tau,t)d\tau\ . \tag{3.4}$$

Note the presence of anomalous correlators[43] like $\langle S_-(t+\tau)S_-(t)\rangle$ in the expression (3.3) for $\Gamma_\theta(t+\tau,t)$. The θ

dependence of the squeezed spectrum essentially arises from the anomalous correlators. In the intense-field limit we have

$$\lim_{t\to\infty} \Gamma_\theta(t+\tau,t)=(|\mu|^2/4)([(a^2+b^2)/2+ab\cos(2\theta)] \times\{[\langle R_-(\tau)R_+\rangle+\langle R_+R_-(\tau)\rangle]+\text{H.c.}\} +iab\sin(2\theta)\{[\langle R_-(\tau)R_+\rangle-\langle R_+R_-(\tau)\rangle]-\text{H.c.}\} +[(a^2-b^2)/2]\{[\langle R_+R_-(\tau)\rangle-\langle R_-(\tau)R_+\rangle]+\text{H.c.}\} +2c^2[1+\cos(2\theta)][\langle R_z(\tau)R_z\rangle-\langle R_z(\tau)\rangle\langle R_z\rangle]) , \tag{3.5}$$

where we have used the result that $\langle R_\pm\rangle=0$ in the steady state.

The solutions (2.14) and (2.15) for $N=1$ and (2.21) and (2.22) for $N=2$ for $\langle R_+(t)\rangle$ and $\langle R_z(t)\rangle$ and the quantum regression theorem are used to obtain the two-time expectation values occurring in the expression (3.5). Analytical expression for the spectrum of squeezing then reads as

$N=1$,

$$S_\theta(\omega)=[\gamma(1-r)/8(1+r)]\{[(1-r)-(1+r)\cos(2\theta)]\chi_1(\omega;\gamma_1)-2\sqrt{r}\,\chi_2(\omega;\gamma_1)\sin(2\theta) +[2(1-r)^2/(1+r)][1+\cos(2\theta)]\chi_3(\omega;\gamma_2)\} , \tag{3.6}$$

$N=2$,

$$S_\theta(\omega)=[\gamma(1-r)/2(3r+1)(r+3)] \times\{[(1-r)-(1+r)\cos(2\theta)][A_1\chi_1(\omega;\beta_1)+A_2\chi_1(\omega;\beta_2)]-2\sqrt{r}\,[A_1\chi_2(\omega;\beta_1)+A_2\chi_2(\omega;\beta_2)]\sin(2\theta) +[4(1-r)^2][1+\cos(2\theta)][r\chi_3(\omega;\beta_3)/(r+3)+\chi_3(\omega;\beta_4)/(3r+1)]\} , \tag{3.7}$$

where

$$\chi_1(\omega;\beta)=\beta/[(\omega+2\Gamma)^2+\beta^2]+\beta/[(\omega-2\Gamma)^2+\beta^2] , \tag{3.8}$$

$$\chi_2(\omega;\beta)=(\omega-2\Gamma)/[(\omega-2\Gamma)^2+\beta^2]-(\omega+2\Gamma)/[(\omega+2\Gamma)^2+\beta^2] , \tag{3.9}$$

$$\chi_3(\omega;\beta)=\beta/(\omega^2+\beta^2) , \tag{3.10}$$

$$A_{1,2}=(1+r)\mp(1+r^2)/[(1-r+r^2)]^{1/2} . \tag{3.11}$$

Note here that the spectrum of squeezing $S_\theta(\omega)$ corresponding to the radiation from the sidebands is obtained by simply dropping the last term in the curly brackets of Eqs. (3.6) and (3.7). It is clear that this last term is positive for all values of θ (and r) while the first two terms representing the contribution from the sidebands may take negative values for some values of θ depending on r. It is the balance between these two types of terms that yields negative values of $S_\theta(\omega)$ leading to squeezing in the total field radiated by the atoms. Also $S_\theta(\omega)=S_\theta(-\omega)$, which is a consequence of the fact that the spectrum correlates frequencies from different sides of the reference laser frequency ω_L.

Further the total variance $\langle :(\Delta E_\theta)^2: \rangle$ is given by

$$\langle :(\Delta E_\theta)^2: \rangle=(|\mu|^2/2\pi\gamma)\int_{-\infty}^{\infty} S_\theta(\omega)d\omega . \tag{3.12}$$

Hence we have

$N=1$,

$$\langle :(\Delta E_\theta)^2: \rangle=[|\mu|^2(1-r)/8(1+r)]\{[1-r-(1+r)\cos(2\theta)]+[(1-r)^2/(1+r)][1+\cos(2\theta)]\} , \tag{3.13}$$

$N=2$,

$$\langle :(\Delta E_\theta)^2: \rangle=\{|\mu|^2(1-r)/[(3+r)(3r+1)]\} \times\{[1-r-(1+r)\cos(2\theta)](1+r)+((1-r)^2(3r^2+2r+3)/[(3r+1)(3+r)])[1+\cos(2\theta)]\} . \tag{3.14}$$

Incidently, $\langle :(\Delta E_\theta)^2: \rangle$ can be obtained analytically for arbitrary values of N since the steady-state solution for the master equation (2.1) or (2.8) is available.[37,39] In the high-field limit the following expression for $\langle :(\Delta E_\theta)^2: \rangle$ can be derived:

$$\langle :(\Delta E_\theta)^2: \rangle=(|\mu|^2/4)(\{[1+r-(1-r)\cos(2\theta)]/2\}\langle M\rangle+2\sqrt{r}\,\langle R_z\rangle+2(1-r)[1+\cos(2\theta)](\langle R_z^2\rangle-\langle R_z\rangle^2)) , \tag{3.15}$$

where

$$\langle M\rangle=\sum_{\nu=0}^{N}[2(N-\nu)\nu+N]X^{\nu}\Big/\sum_{\nu=0}^{N}X^{\nu}\,, \tag{3.16}$$

$$\langle R_z\rangle=\sum_{\nu=0}^{N}(\nu-N/2)X^{\nu}\Big/\sum_{\nu=0}^{N}X^{\nu}\,, \tag{3.17}$$

$$\langle R_z^2\rangle=\sum_{\nu=0}^{N}(\nu-N/2)^2X^{\nu}\Big/\sum_{\nu=0}^{N}X^{\nu}\,, \tag{3.18}$$

$$X=[(1-\sqrt{r}\,)/(1+\sqrt{r}\,)]^2\,. \tag{3.19}$$

Note that $\langle:(\Delta E_\theta)^2:\rangle<0$ implies squeezing. Also the contribution from the sidebands to the variance $\langle:(\Delta E_\theta)^2:\rangle$ is represented by the first term in the curly brackets in each of expressions (3.13) and (3.14). Note that this term may take negative values while the second term is positive for all values of θ (and r). Thus squeezing in resonance fluorescence is mainly due to the quantum nature of the sidebands. For subsequent discussion it is convenient to label the fields E_θ and the spectra $S_\theta(\omega)$ corresponding to $\theta=0$, $\theta=\pi/2$, and $\theta=\pi/4$ as E_1, E_2, E_3 and $S_1(\omega)$, $S_2(\omega)$, $S_3(\omega)$, respectively.

Figure 1 shows a plot of the variance $\langle:(\Delta E_1)^2:\rangle$ ($\theta=0$) versus $r=(\Delta/2\Gamma)^2$ for several values of N. The inset in Fig. 1 shows the sideband variance $\langle:(\Delta\tilde{E}_1)^2:\rangle$. Clearly squeezing increases with N for all nonzero values of the parameter r. This is consistent with the previous results of Refs. 25. Also, as expected the magnitude of squeezing is more in the sideband radiation field than in the total field of resonance fluorescence in agreement with the results of Ref. 26. We mention here that $\langle:(\Delta E_{2,3})^2:\rangle$ and $\langle:(\Delta\tilde{E}_{2,3})^2:\rangle$ are positive for all values of r and N as is also seen from Eqs. (3.13) and (3.14).

We have also solved the exact time-dependent master equation (2.1) numerically for some finite values of N. The analytical results for $N=1$ and $N=2$ are consistent with the exact numerical results. Figure 2 shows the behavior of $S_1(\omega)$ versus ω/γ for several values of N and for typical values of $r=0.49$ and $\Gamma=40\gamma$. For this value of r, $S_1(\omega)$ is expected to show squeezing for $N=1$ according to Eq. (3.6). It is seen in Fig. 2 that there is a range of ω/γ over which $S_1(\omega)$ takes negative values for all N. The maximum squeezing is around $\omega=2\Gamma$ for all N which is consistent with the analytical formulas for $N=1$ and 2. The inset in Fig. 2 shows the corresponding spectrum $\tilde{S}_1(\omega)$ for the sidebands. Note that $\tilde{S}_1(\omega)$ is negative over the entire range of ω/γ and peaks at $\omega=2\Gamma$. Also the magnitude of the squeezing increases with N as in the case of $S_1(\omega)$. This is indeed a manifestation of the quantum nature of the sideband radiation. We mention here that $S_2(\omega)$ and $\tilde{S}_2(\omega)$ are positive throughout the range of ω/γ for all $r>0$. The curves in Fig. 3 show the behavior of $S_3(\omega)$ for the same data as in Fig. 2. Note that $S_3(\omega)$ assumes negative values in a region beyond $\omega=2\Gamma$ though the total field shows no squeezing.

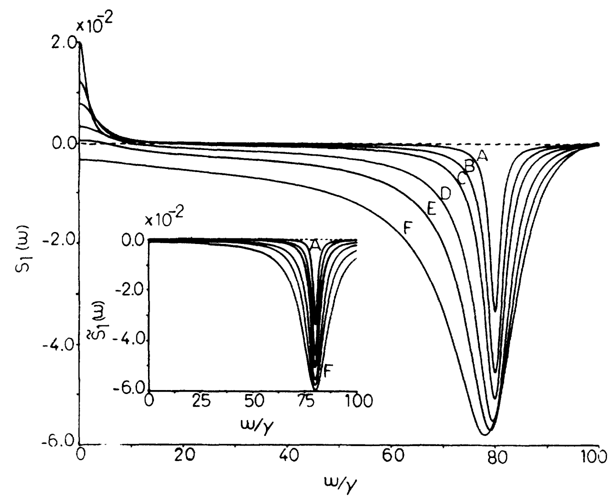

FIG. 2. The spectrum of squeezing $S_1(\omega)$ as a function of ω/γ for $r=0.49$ and $\Gamma=40\gamma$. The inset shows the spectrum of squeezing $\tilde{S}_1(\omega)$ for the sideband emission as a function of ω/γ. Curves A–F refer to $N=1$, 2, 3, 5, 7, and 10, respectively.

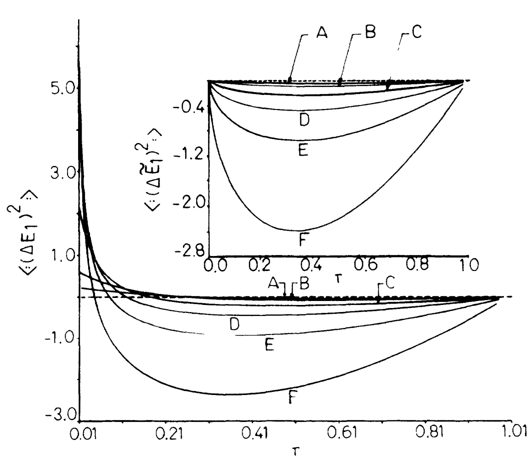

FIG. 1. A plot of the variance $\langle:(\Delta E_1)^2:\rangle$ (in units of $|\mu|^2$) vs the parameter r [$r=(\Delta/2\Gamma)^2$]. The inset in the figure shows the sideband variance $\langle:(\Delta\tilde{E}_1)^2:\rangle$ vs r. Curves A–F correspond to $N=1$, 2, 5, 10, 20, and 50, respectively.

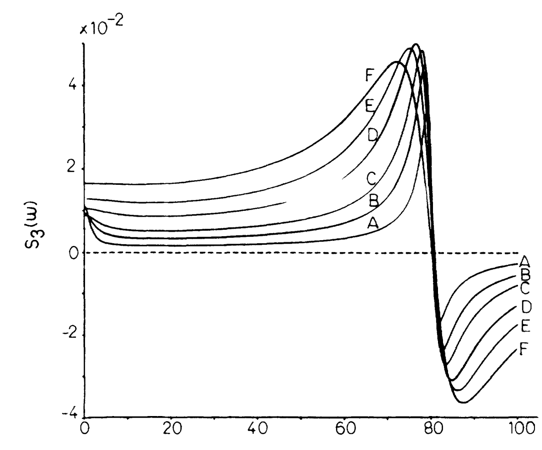

FIG. 3. The spectrum of squeezing $S_3(\omega)$ as a function of ω/γ. Data and labeling as in Fig. 2.

IV. PHOTON-COUNTING STATISTICS

Mandel[19] has suggested that the measurement of the photon-counting distribution $p(n)$ for a superposition of a coherent field E_C and the radiation field E scattered by two-level atoms leads to the detection of squeezing. In particular, the quantity of interest here is $Q(T)$ related to the second factorial moment $\langle n^{[2]}\rangle$ of the photon-counting distribution $p(n)$ by

$$Q(T)=[\langle n^{[2]}\rangle-\langle n\rangle^2]/\langle n\rangle \ , \tag{4.1}$$

where T is the counting time.

The second factorial moment $\langle n^{[2]}\rangle$ of the photon-electron distribution is related to the intensity fluctuations of the light incident on the detector by

$$\langle n^{[2]}\rangle=2q^2\int_0^T d\tau(T-\tau) \times \lim_{t\to\infty}\langle \mathcal{E}^{(-)}(t)\mathcal{E}^{(-)}(t+\tau) \times \mathcal{E}^{(+)}(t+\tau)\mathcal{E}^{(+)}(t)\rangle \ , \tag{4.2}$$

where q is a parameter related to the quantum efficiency of the detector. Thus $Q(T)$ is given by

$$Q(T)=(2q/T)\int_0^T d\tau(T-\tau)\lim_{t\to\infty}[\langle \mathcal{E}^{(-)}(t)\mathcal{E}^{(-)}(t+\tau)\mathcal{E}^{(+)}(t+\tau)\mathcal{E}^{(+)}(t)\rangle-\langle \mathcal{E}^{(-)}(t)\mathcal{E}^{(+)}(t)\rangle^2/\langle \mathcal{E}^{(-)}(t)\mathcal{E}^{(+)}(t)\rangle] \ . \tag{4.3}$$

In these expressions, $\mathcal{E}^{(\pm)}(t)$ stand for the total field

$$\mathcal{E}^{(\pm)}(t)=E^{(\pm)}+E_C^{(\pm)} \ . \tag{4.4}$$

Assuming that the constant coherent field E_C to be sufficiently intense, then to second order in E_C we arrive at the result

$$\begin{aligned}Q(T)=(2q\,|\mu|^2/T)\int_0^T d\tau(T-\tau)&\times\{\exp(2i\theta)(\langle S_-(\tau)S_-(0)\rangle-\langle S_-\rangle^2)+\exp(-2i\theta)(\langle S_+(0)S_+(\tau)\rangle-\langle S_+\rangle^2)\\&+\langle S_+(0)S_-(\tau)\rangle-2\langle S_+\rangle\langle S_-\rangle+\langle S_+(\tau)S_-(0)\rangle\} \ ,\end{aligned} \tag{4.5}$$

where θ is the phase of the coherent field.

The expression for $Q(T)$ can be evaluated from the solution of Eq. (2.1) or Eq. (2.8) and the quantum regression theorem. The sub-Poissonian nature of the counting distribution implies the squeezing. Thus $Q(T)<0$ implies that the radiation field is squeezed.

The expression (4.5) can be evaluated in the intense-field limit by expressing $S_\pm$ in terms of $R_\pm$ and R_z according to the transformation equation (2.3). The resulting two-time expectation values like $\langle R_\pm(\tau)R_\mp\rangle$ and $\langle R_z(\tau)R_z\rangle$ are evaluated as before from the one-time expectation values $\langle R_\pm(\tau)\rangle$ and $\langle R_z(\tau)\rangle$ by means of quantum regression theorem. The final analytical expressions for $Q(T)$ read as follows:

$N=1$,

$$\begin{aligned}Q(T)=[q(1-r)|\mu|^2/T(1+r)]\{&[1-r-(1+r)\cos(2\theta)]F(T;\gamma_1)-2\sqrt{r}\sin(2\theta)G(T;\gamma_1)\\&+[(1-r)^2/(1+r)][1+\cos(2\theta)]H(T;\gamma_2)\} \ ,\end{aligned} \tag{4.6}$$

$N=2$,

$$\begin{aligned}Q(t)=&[4q(1-r)|\mu|^2/T(3r+1)(r+3)]\\&\times\{[1-r-(1+r)\cos(2\theta)][A_1F(T;\beta_1)+A_2F(T;\beta_2)]-2\sqrt{r}[A_1G(T;\beta_1)+A_2G(T;\beta_2)]\sin(2\theta)\\&+2(1-r)^2[rH(T;\beta_3)/(r+3)+H(T;\beta_4)/(3r+1)][1+\cos(2\theta)]\} \ .\end{aligned} \tag{4.7}$$

Here the functions F,G,H have the form

$$F(t;\beta)=\frac{\exp(-\beta t)[(\beta^2-4\Gamma^2)\cos(2\Gamma t)-4\beta\Gamma\sin(2\Gamma t)]+\beta t(\beta^2+4\Gamma^2)-(\beta^2-4\Gamma^2)}{(\beta^2+4\Gamma^2)^2} \ , \tag{4.8}$$

$$G(t;\beta)=\frac{\exp(-\beta t)[4\beta\Gamma\cos(2\Gamma t)+(\beta^2-4\Gamma^2)\sin(2\Gamma t)]+2\Gamma t(\beta^2+4\Gamma^2)-4\beta\Gamma}{(\beta^2+4\Gamma^2)^2} \ , \tag{4.9}$$

$$H(t;\beta)=[\exp(-\beta t)+\beta t-1]/\beta^2 \ . \tag{4.10}$$

It may be added here that $Q(T)$ corresponding to the side-band radiation is obtained from (4.6) and (4.7) by omitting the last term in the curly brackets. For subsequent discussion we write $Q(T)$ corresponding to $\theta=0$, $\theta=\pi/2$, and $\theta=\pi/4$ as Q_1, Q_2, and Q_3, respectively. Note that for small values of T ($2\Gamma T\ll 1$).

$$Q(T)=4qT\langle :(\Delta E_\theta)^2:\rangle \ , \tag{4.11}$$

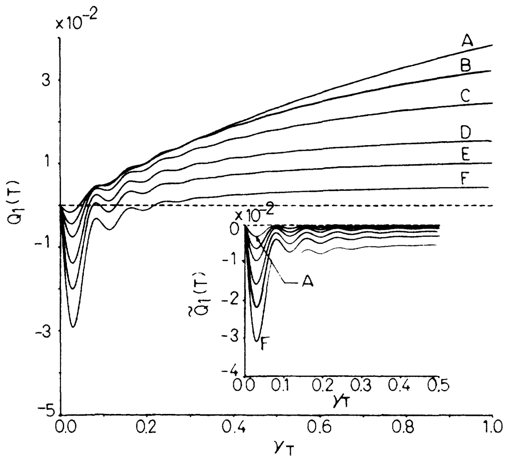

FIG. 4. A plot of $Q_1(T)$ (in units of $q\,|\mu|^2$) vs γT for $r=0.49$ and $\Gamma=40\gamma$. The inset in the figure shows plot of $\tilde{Q}_1(T)$ vs γT. Curves A–F refer to $N=1$, 2, 3, 5, 7, and 10, respectively.

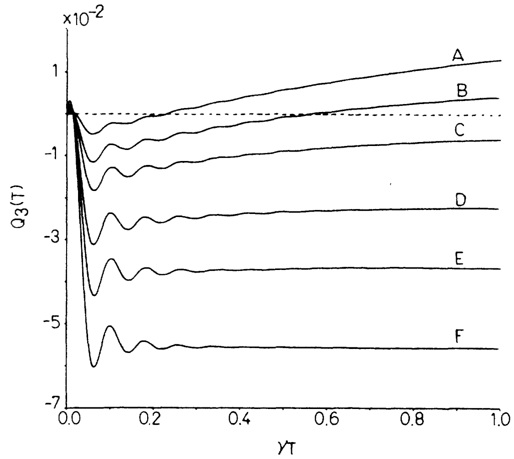

FIG. 5. The behavior of $Q_3(T)$ as a function of γT. Data and labeling as in Fig. 4.

where $\langle :(\Delta E_\theta)^2: \rangle$ is given by (3.13) and (3.14) for $N=1$ and 2, respectively. Thus for small values of T, $Q_1<0$. Also, it can be shown that $Q_1(T)>0$ for $N=1$ and 2 as $T\rightarrow\infty$. However, $\tilde{Q}_1(T)$ remains negative over the entire range of T.

Figure 4 shows the variation of $Q_1(T)$ with γT for $r=0.49$ and $\Gamma=40\gamma$. The curves in this figure are obtained from the numerical solution of the high-field master equation (2.8). The behavior of $Q_1(T)$ for $N=1$ and 2 predicted analytically is in agreement with numerical results. It is important to note here that with increasing N the region of time T over which $Q_1(T)$ remains negative is extended. The inset in Fig. 4 shows the corresponding quantity $\tilde{Q}_1(T)$ for the sideband radiation. Here $\tilde{Q}_1(T)$ is negative for all values of N over the entire range of T. The increasing negative values assumed by $Q_1(T)$ as N is increased confirms further that squeezing increases with N. Finally, in Fig. 5 we show the variation of $Q_3(T)$ with γT for the same data as in Fig. 4. Note that $Q_3(T)$ takes positive values initially over a small interval of time before becoming negative. For $N=1$ and 2 it assumes positive values subsequently in agreement with analytical results. However, for $N>3$, $Q_3(T)$ continues to remain negative. The behavior of $\tilde{Q}_3(T)$ is similar except that after initial positive values, it continues to remain negative over the entire range of T for all values of N. We may also add here that both $Q_2(T)$ and $\tilde{Q}_2(T)$ are positive for all values of T.

V. CONCLUSIONS

In the present paper we have examined collective effects on squeezing in resonance fluorescence in the intense-field limit. The analytical results for one and two atoms for the spectrum of squeezing as well as the numerical results for higher number N of atoms show that squeezing increases with N. Moreover, the squeezing arises essentially from the quantum nature of the sidebands. Further analysis on the basis of photon-counting statistics of the fluorescent radiation also confirms this.

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