Quantum analysis of intensity fluctuations in the nondegenerate parametric oscillator

A. S. Lane and M. D. Reid
Physics Department, University of Waikato, Hamilton, New Zealand

D. F. Walls
Physics Department, University of Auckland, Auckland, New Zealand

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A quantum analysis of the intensity fluctuations is given for the nondegenerate parametric oscillator both above and below threshold. Theoretical analysis and experiments by Heidmann, Horowicz, Reynaud, Giacobino, and Fabre [Phys. Rev. Lett. **59**, 2555 (1987)] have shown that a noise suppression below the vacuum level is possible in the difference of the intensities of the signal and idler modes. The effect of nonequal cavity decay rates and intracavity losses is shown to modify significantly the fluctuation spectra for the difference in the intensity of the signal and idler beams. The results of exact nonlinear solutions for the internal cavity modes are discussed. We demonstrate that a very sensitive absorption detector may be made by operating the oscillator near threshold.

I. INTRODUCTION

The parametric amplifier has long been a prototype system for the production of light with quantum fluctuations less than those of coherent light. It was first pointed out by Takahashi[1] that the parametric amplifier will amplify one quadrature of the signal radiation and attenuate the other quadrature. The quantum fluctuations associated with the quadrature that is attenuated are reduced below the level of vacuum fluctuations. In order to increase the gain, the parametric medium may be placed inside an optical cavity where it is coherently pumped and becomes a parametric oscillator. An early quantum analysis of the parametric oscillator was given by Graham and Haken.[2] A fully nonlinear quantum analysis of the degenerate parametric oscillator was given by Drummond, McNeil, and Walls.[3] Milburn and Walls[4] calculated the squeezing in the internal cavity mode below and above threshold. A calculation of the squeezing spectrum in the output field for the degenerate parametric oscillator below threshold was given by Collett and Gardiner[5] and Yurke.[6] Above threshold, the squeezing spectrum for the degenerate parametric oscillator has been calculated by Collett and Walls,[7] and Savage and Walls.[8] Experimentally, squeezing amounting to a noise reduction of 62% below the vacuum level has been achieved in a near degenerate parametric oscillator operating below threshold.[9]

The quantum statistics of the nondegenerate parametric oscillator contain some interesting features not found in the degenerate device, particularly above threshold. In the nondegenerate parametric amplifier, one pump photon is destroyed and a signal and idler photon are simultaneously created. The simultaneous creation of the photon pairs has been demonstrated experimentally by Burnham and Weinberg[10] and Friberg *et al.*[11] Experiments which use the correlation between the signal and idler photons to generate a photon number state have been done by Jakeman and co-workers.[12,13] Since the photons are created in pairs, the fluctuations in the difference of the signal and idler photocurrents will be greatly reduced. This property is exploited by Reynaud *et al.*,[14] who show that perfect noise suppression is possible in the difference between the intensities for the signal and idler beams in the output of a parametric oscillator above threshold.

Heidmann *et al.*[15] have recently reported an experimental measurement on the reduction of fluctuations below the vacuum level in the difference intensity of the signal and idler modes of a parametric oscillator. An earlier experiment measuring fluctuations in the difference of the quadrature phases of signal and idler beams in four-wave mixing has been performed by Levenson *et al.*[16]

It was first shown by Graham and Haken[2] that the phases of the signal and idler modes in a nondegenerate parametric amplifier are not stable but diffuse. In our analysis we include this phase diffusion, unlike the work of Reynaud *et al.*,[14] which assumed that the phases were stable. We show that the results of Reynaud *et al.*[14] for the fluctuations in the difference of intensities of the two modes remain valid. The squeezing, however, is sensitive to the phase diffusion, and calculations on the squeezing are presented elsewhere.[17]

In our calculations we include the possibility of different cavity decay rates for the signal and idler modes. For the case of equal cavity decay rates of the signal and idler modes, the pump fluctuations do not enter and there is complete noise suppression in the intensity difference between the two modes. For different cavity decay rates, while one still gets perfect noise suppression around zero frequency, the spectral behavior may differ markedly, as pump fluctuations now affect the system. We calculate the noise spectrum for the intensity difference both above and below threshold.

We also include the effect of intracavity losses in our analysis. The basic effect of intracavity losses is to reduce the correlation between the signal and idler modes so that the noise suppression is no longer perfect. This may be made use of in the design of an absorption detector where one places an absorber at the signal frequency inside the optical cavity. The presence of the absorber is marked by an increase in the noise, an effect which may be enhanced by operating the device close to threshold.

II. INTENSITY FLUCTUATIONS ABOVE THRESHOLD

The nondegenerate parametric oscillator consists of three interacting field modes: a pump, signal, and idler within an optical cavity which is coherently driven at the pump frequency. The Hamiltonian describing this interaction is

$$\begin{aligned}
H &= \sum_{j=0}^{3} H_j ,\\
H_0 &= \sum_{i=0}^{2} \hbar\omega_i a_i^\dagger a_i ,\\
H_1 &= i\hbar(\varepsilon e^{-i\omega_0 t} a_0^\dagger - a_0 \varepsilon^* e^{i\omega_0 t}) ,\\
H_2 &= i\hbar\chi(a_0 a_1^\dagger a_2^\dagger - a_0^\dagger a_1 a_2) ,\\
H_3 &= \sum_{i=0}^{2} a_i \Gamma_i^\dagger + \text{H.c.} ,
\end{aligned} \tag{2.1}$$

where a_0, a_1, and a_2 are the boson annihilation operators for the pump, signal, and idler modes, respectively. We assume that resonance $\omega_0=\omega_1+\omega_2$. The pump mode is driven by a coherent driving field with amplitude ε and frequency ω_0. χ is the strength of the parametric interaction. Γ_0, Γ_1, and Γ_2 are the heat bath operators for the cavity damping of the pump, signal, and idler modes, respectively.

One may derive a master equation for the density operator ρ of the three coupled modes in the form[18]

$$\dot\rho = \frac{1}{i\hbar}[H_0+H_1+H_2,\rho] + \sum_{i=0}^{2} \kappa_i([a_i\rho,a_i^\dagger]+[a_i,\rho a_i^\dagger]) , \tag{2.2}$$

where κ_j are the cavity mode damping rates. The mode spacing has been assumed to be large compared to the cavity linewidths and we have taken the bath to be at zero temperature.

We may transform this operator master equation into a Fokker-Planck equation using the generalized P representation,[19]

$$\rho(t) = \int \frac{P(\{\alpha\},\{\alpha^\dagger\},t)\,|\{\alpha\}\rangle\langle\{\alpha^\dagger\}^*|\,d\{\alpha\}d\{\alpha^\dagger\}}{\langle\{\alpha^\dagger\}^*|\{\alpha\}\rangle} , \tag{2.3}$$

where $\{\alpha\}=(\alpha_0,\alpha_1,\alpha_2)$ and $\{\alpha^\dagger\}$ are independent complex variables. The resulting Fokker-Planck equation has the form[18]

$$\frac{\partial P}{\partial t} = \left[\frac{\partial}{\partial\alpha_0}(\kappa_0\alpha_0-\varepsilon-\chi\alpha_1\alpha_2)+\frac{\partial}{\partial\alpha_1}(\kappa_1\alpha_1-\chi\alpha_2^\dagger\alpha_0) + \frac{\partial}{\partial\alpha_2}(\kappa_2\alpha_2-\chi\alpha_1^\dagger\alpha_0)+\frac{\partial^2}{\partial\alpha_1\partial\alpha_2}\chi\alpha_0+\text{c.c.}\right]P . \tag{2.4}$$

This Fokker-Planck equation is equivalent to the following set of stochastic differential equations:

$$\begin{aligned}
\dot\alpha_0 &= -\kappa_0\alpha_0+\varepsilon-\chi\alpha_1\alpha_2 ,\\
\dot\alpha_1 &= -\kappa_1\alpha_1+\chi\alpha_0\alpha_2^\dagger+R_1(t) ,\\
\dot\alpha_2 &= -\kappa_2\alpha_2+\chi\alpha_0\alpha_1^\dagger+R_2(t) .
\end{aligned} \tag{2.5}$$

The noise terms R_1 and R_2 have the following correlations:

$$\begin{aligned}
\langle R_1(t)R_2(t')\rangle &= \chi\langle\alpha_0\rangle\delta(t-t') ,\\
\langle R_1^\dagger(t)R_2^\dagger(t')\rangle &= \chi\langle\alpha_0^\dagger\rangle\delta(t-t') ,
\end{aligned} \tag{2.6}$$

with all other correlations being zero.

The equations of motion are parallel to those of Graham and Haken,[2] who derived the operator form of the equations. The equations may be solved by linearization techniques. However, care must be taken above threshold because one of the eigenvalues is zero in that region. This is best seen by moving into the amplitude-phase representation.[2,20,21] Define r,ϕ_j,μ_j,ψ_j by

$$\alpha_j(t) = r_j[1+\mu_j(t)]e^{-i[\phi_j+\psi_j(t)]} , \tag{2.7}$$

where r_j,ϕ_j are the steady-state solutions. Solving for the steady state, we find, below threshold,

$$\begin{aligned}
r_1 &= r_2 = 0 ,\\
r_0 &= \frac{|\varepsilon|}{\kappa_0},\quad \phi_0=-\phi_P ,
\end{aligned} \tag{2.8}$$

where $\varepsilon=|\varepsilon|e^{i\phi_P}$ and ϕ_P is the phase of the coherent pump. The threshold value of the pump ε is $\varepsilon_{\text{thr}}=(\kappa_0\sqrt{\kappa_1\kappa_2})/\chi$. Above threshold,

$$\begin{aligned}
r_0 &= \frac{\sqrt{\kappa_1\kappa_2}}{\chi},\quad \phi_0=-\phi_P\\
r_1,r_2 &= \frac{c}{\sqrt{\kappa_{1,2}}},\quad \phi_1+\phi_2=\phi_0
\end{aligned} \tag{2.9}$$

where

$$c = \frac{\sqrt{\kappa_0\kappa_1\kappa_2}}{\chi}(E-1)^{1/2},\quad E=\frac{|\varepsilon|}{\varepsilon_{\text{thr}}} .$$

The interesting point in the above-threshold solution is that only the sum of the phases $\phi_1+\phi_2$ is defined. No steady state exists for the phase difference. The phase difference (with zero eigenvalue for its equation) is free to wander with noise fluctuations.

We linearize the equations of motion about their steady-state values. We write the linearized equations in terms of the variables

$$\begin{aligned}
\Phi &= \psi_1+\psi_2 ,\\
\psi &= \frac{r_1}{r_2}\psi_1-\frac{r_2}{r_1}\psi_2 ,\\
\mu_s &= \mu_1+\mu_2 ,\\
\mu_d &= \mu_1-\mu_2 .
\end{aligned} \tag{2.10}$$

The equations read

$$\begin{aligned}
\dot{\mu}_s &= -(\kappa_1-\kappa_2)\mu_d+(\kappa_1+\kappa_2)\mu_0+R_s(t)\ ,\\
\dot{\mu}_d &= -(\kappa_1+\kappa_2)\mu_d+(\kappa_1-\kappa_2)\mu_0+R_d(t)\ ,\\
\dot{\mu}_0 &= -\kappa_0\mu_0-\kappa_0(E-1)\mu_s\ ,\\
\dot{\Phi} &= -(\kappa_1+\kappa_2)(\Phi-\psi_0)+R_\phi(t)\ ,\\
\dot{\psi} &= R_\psi(t)\ ,\\
\dot{\psi}_0 &= -\kappa_0(\psi_0+(E-1)\Phi)\ ,
\end{aligned}\tag{2.11}$$

where the noise terms have the following correlations:

$$\begin{aligned}
\langle R_s(t)R_s(t')\rangle &= -\langle R_d(t)R_d(t')\rangle=\frac{\kappa_1\kappa_2}{c^2}\delta(t-t')\ ,\\
\langle R_s(t)R_d(t)\rangle &= \langle R_d(t)R_s(t')\rangle=0\ ,\\
\langle R_\psi(t)R_\psi(t')\rangle &= -\langle R_\Phi(t)R_\Phi(t')\rangle=\frac{\kappa_1\kappa_2}{c^2}\delta(t-t')\ ,\\
\langle R_\psi(t)R_\Phi(t')\rangle &= \langle R_\Phi(t)R_\psi(t')\rangle\\
&= \tfrac{1}{2}(\kappa_1-\kappa_2)\frac{\sqrt{\kappa_1\kappa_2}}{c^2}\delta(t-t')\ .
\end{aligned}\tag{2.12}$$

The Lienard-Chipart stability criterion[22] shows that the μ system is stable. We also find that the Φ,ψ_0 system is stable. The Ψ variable, however, has a zero eigenvalue and moves about randomly under the influence of the stochastic force.

We wish to calculate the spectrum of fluctuations in the intensity difference between the signal and idler modes. We assume a single ported cavity where the output fields from the cavity at the signal and idler frequencies, $b_1(t)$ and $b_2(t)$, are related to the cavity modes by the boundary conditions[5]

$$b_j(t)=\sqrt{2\kappa_j}a_j(t)+b_{jin}(t)\ .\tag{2.13}$$

The b_{jin} specify the fields that are input to the cavity boundary. We are required to calculate two-time correlation functions of the form

$$\langle b_j^\dagger(\tau)b_j(\tau),b_k^\dagger(0)b_k(0)\rangle\ ,$$

where $\langle a,b\rangle=\langle ab\rangle-\langle a\rangle\langle b\rangle$. Using the boundary condition (2.13) for a vacuum input, we proceed as follows:

$$\begin{aligned}
\langle b_j^\dagger(\tau)b_j(\tau),b_k^\dagger(0)b_k(0)\rangle &= 4\kappa_j\kappa_k[\langle a_k^\dagger(0)a_j^\dagger(\tau)a_j(\tau)a_k(0)\rangle-\langle a_j^\dagger(\tau)a_j(\tau)\rangle\langle a_k^\dagger(0)a_k(0)\rangle]+2\sqrt{\kappa_j\kappa_k}\langle a_j^\dagger(\tau)a_k(0)\rangle\delta(\tau)\delta_{jk}\\
&= 4\kappa_j\kappa_k\langle \alpha_j^\dagger(\tau)\alpha_j(\tau),\alpha_k^\dagger(0)\alpha_k(0)\rangle+2\kappa_j\langle \alpha_j^\dagger(0)\alpha_j(0)\rangle\delta(\tau)\delta_{jk}\\
&= 16\kappa_j\kappa_k r_j^2r_k^2\langle\mu_j(\tau),\mu_k(0)\rangle+2\kappa_j r_j^2\delta_{jk}\delta(\tau)\ .
\end{aligned}\tag{2.14}$$

The spectrum of the variance in the output intensity difference $\hat{I}_d=b_1^\dagger b_1-b_2^\dagger b_2$ is then (: : denotes normal ordering)

$$\begin{aligned}
S_d(\omega) &= \int_{-\infty}^{\infty}d\tau\, e^{-i\omega\tau}\langle\hat{I}_d(\tau),\hat{I}_d(0)\rangle=\langle b_1^\dagger b_1\rangle+\langle b_2^\dagger b_2\rangle+\int_{-\infty}^{\infty}d\tau\, e^{-i\omega\tau}\langle:\hat{I}_d(\tau),\hat{I}_d(0):\rangle\\
&= 16\int_{-\infty}^{\infty}d\tau\, e^{-i\omega\tau}\langle\kappa_1r_1^2\mu_1(\tau)-\kappa_2r_2^2\mu_2(\tau),\kappa_1r_1^2\mu_1(0)-\kappa_2r_2^2\mu_2(0)\rangle+2(\kappa_1r_1^2+\kappa_2r_2^2)\ .
\end{aligned}\tag{2.15}$$

We proceed by transforming to frequency space,

$$\mu_j(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}d\tau\, e^{-i\omega\tau}\mu_j(\tau)\ .\tag{2.16}$$

The noise correlations

$$\langle R_j(t)R_k(t')\rangle=D_{jk}\delta(t-t')\tag{2.17}$$

become

$$\langle R_j(\omega)R_k(\omega')\rangle=D_{jk}\delta(\omega+\omega')\ ,\tag{2.18}$$

and the cumulants $\langle\mu_j(\tau),\mu_k(0)\rangle$ have the stationary spectrum given by the $\langle\mu_j(\omega),\mu_k(\omega')\rangle$ cumulants with the $\delta(\omega+\omega')$ omitted. That is, for a stationary state,

$$\delta(\omega+\omega')\int d\tau\, e^{-i\omega\tau}\langle\mu_j(\tau),\mu_k(0)\rangle=\langle\mu_j(\omega),\mu_k(\omega')\rangle\ .\tag{2.19}$$

So,

$$S_d(\omega)=2(\kappa_1r_1^2+\kappa_2r_2^2)+\frac{16}{\delta(\omega+\omega')}\langle\kappa_1r_1^2\mu_1(\omega)-\kappa_2r_2^2\mu_2(\omega),\ \kappa_1r_1^2\mu_1(\omega')-\kappa_2r_2^2\mu_2(\omega')\rangle\ .\tag{2.20}$$

Above threshold $\kappa_jr_j^2=c^2$, so that

$$S_d(\omega)=4c^2\left[1+4c^2\frac{\langle\mu_d(\omega),\mu_d(\omega')\rangle}{\delta(\omega+\omega')}\right]=4c^2\bar{S}_d(\omega)\ .\tag{2.21}$$

We define $\bar{S}_d(\omega)$ as the normalized spectrum. The vacuum or shot-noise level is $\bar{S}_d(\omega)=1$ and perfect noise suppression is $\bar{S}_d(\omega)=0$. The solution of this system in frequency space is an algebraic problem,

$$\begin{aligned}
0 &= -i\omega\mu_s-2\delta\mu_d+2\kappa\mu_0+R_s\ ,\\
0 &= -(2\kappa+i\omega)\mu_d+2\delta\mu_0+R_d\ ,\\
0 &= -(\kappa_0+i\omega)\mu_0-\kappa_0(E-1)\mu_s\ ,
\end{aligned}\tag{2.22}$$

where

$$\kappa=\tfrac{1}{2}(\kappa_1+\kappa_2),\quad \delta=\tfrac{1}{2}(\kappa_1-\kappa_2)\ ,$$

with solution

$$\mu_d(\omega)=\frac{R_d(\omega)+2\delta\mu_0(\omega)}{2\kappa+i\omega}\ ,$$
$$\mu_s(\omega)=\frac{-(\kappa_0+i\omega)}{\kappa_0(E-1)}\mu_0(\omega)\ , \quad (2.23)$$
$$\mu_0(\omega)=\frac{\left[-R_s(\omega)+\dfrac{2\delta}{2\kappa+i\omega}R_d(\omega)\right]}{\left[2\kappa+\dfrac{i\omega(\kappa_0+i\omega)}{\kappa_0(E-1)}-\dfrac{4\delta^2}{2\kappa+i\omega}\right]}\ .$$

From this solution and the correlations of the noise terms given by Eqs. (2.12) and (2.17), we obtain

$$S_d(\omega)=4c^2\frac{\omega'^2+4\delta'^2(1+B)}{\omega'^2+4}\ , \quad (2.24)$$

where

$$B=\frac{\varepsilon\{\omega'^2[4+\kappa_0'(E+1)]-\varepsilon\}}{[\varepsilon-\omega'^2(2+\kappa_0')]^2+\omega'^2(2\kappa_0'E-\omega'^2)^2}\ ,$$

$$\varepsilon=4(1-\delta'^2)\kappa_0'(E-1)\ ,$$

and we have scaled the dampings and frequency by κ, that is,

$$\omega'=\omega/\kappa,\quad \kappa_0'=\kappa_0/\kappa,\quad \delta'=\delta/\kappa\ .$$

The case of equal signal and idler decay rates $\delta=0$ has been well discussed by Reynaud *et al.*[14] and is plotted in Fig. 1. The spectrum is a simple inverted Lorentzian with full width at half maximum (FWHM) 4κ and maximum noise suppression at zero frequency. Thus

$$\bar{S}_d(\omega)=1-\frac{4}{4+\omega'^2}\ . \quad (2.25)$$

As pointed out by Reynaud *et al.*, the fluctuations in the signal-idler intensity difference are reduced below the

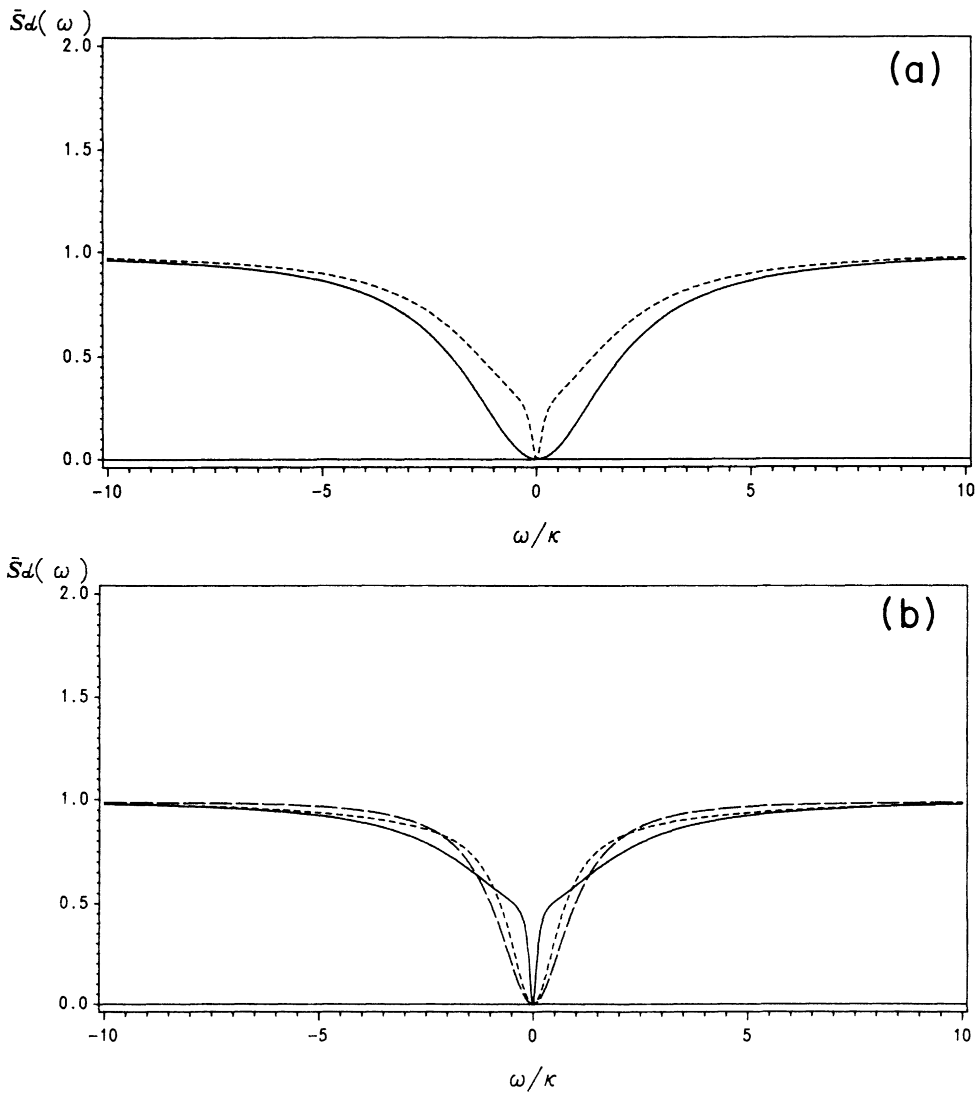

FIG. 1. (a) Plot of $\bar{S}_d(\omega)$ above threshold for high pump cavity damping. $E=1.1$, $\kappa_0'=10$, $\delta'=0$ (——), $\delta'=0.5$ (— — —). (b) Plot of $\bar{S}_d(\omega)$ above threshold for high pump cavity damping. $\delta'=0.6$, $\kappa_0'=10$, $E=1.1$ (——), $E=2$ (----), $E=5$ (— — —).

shot- or quantum-noise level and are independent of the pump fluctuations (and hence independent of pump decay rate κ_0' and pump power E). It is readily seen from Eqs. (2.10) that, where $\kappa_1=\kappa_2$, the signal-idler intensity-difference variable μ_d decouples from the equations for the signal-idler intensity sum μ_s and pump fluctuations μ_0, and the simple solution (2.25) follows directly. Thus, although the individual signal and idler intensities show significant fluctuations that are sensitive to pump parameters, the intensities are correlated so that the signal-idler intensity-difference fluctuations are reduced according to (2.25).

McNeil and Gardiner[18] and Graham[23] have obtained nonlinear solutions for the intensity correlations in the internal cavity modes. Quantum mechanics predicts the general Cauchy-Schwartz inequality[23]

$$|\langle a_1^\dagger a_1 a_2^\dagger a_2\rangle| \le (\langle a_1^{\dagger 2} a_1^2\rangle\langle a_2^{\dagger 2} a_2^2\rangle)^{1/2} , \tag{2.26}$$

which simplifies, with symmetry arguments, to

$$|\langle a_1^\dagger a_1 a_2^\dagger a_2\rangle| \le \langle a_1^{\dagger 2} a_1^2\rangle + \langle a_1^\dagger a_1\rangle . \tag{2.27}$$

The equality would imply maximum correlation between signal and idler modes. This is equivalent to zero fluctuation in the difference intensity, since

$$\langle (a_1^\dagger a_1 - a_2^\dagger a_2)^2\rangle = 2(\langle a_1^{\dagger 2} a_1^2\rangle + \langle a_1^\dagger a_1\rangle - \langle a_1^\dagger a_1 a_2^\dagger a_2\rangle) \ge 0 , \tag{2.28}$$

and implies that one can infer with total precision at a distance the intensity of the signal by measuring the intensity of the idler.[23] Classical theory predicts the weaker inequality

$$|\langle a_1^\dagger a_1 a_2^\dagger a_2\rangle| \le \langle a_1^{\dagger 2} a_1^2\rangle ,$$

so that the fields with $\langle a_1^\dagger a_1 a_2^\dagger a_2\rangle > \langle a_1^{\dagger 2} a_1^2\rangle$ are quantum.[18] This is equivalent to

$$\langle (a_1^\dagger a_1 - a_2^\dagger a_2)^2\rangle < 2\langle a_1^\dagger a_1\rangle ,$$

that is, the intensity-difference fluctuations reduced below the shot-noise level of $2\langle a_1^\dagger a_1\rangle$ [or $\bar{S}_d(\omega)<1$ in the spectral variance]. Graham derived the following correlation identity for the nondegenerate parametric oscillator (for $\kappa_1=\kappa_2$):

$$\langle a_1^\dagger a_1 a_2^\dagger a_2\rangle = \langle a_1^{\dagger 2} a_1^2\rangle + \tfrac{1}{2}\langle a_1^\dagger a_1\rangle . \tag{2.29}$$

This implies that $\langle (a_1^\dagger a_1 - a_2^\dagger a_2)^2\rangle = \langle a_1^\dagger a_1\rangle$, that is, fluctuations in the difference intensity are at half the shot-noise level. The identity refers to the correlation between the internal cavity fields. Though above that allowed classically, the correlation is not maximum because of the vacuum fluctuations imposed by the loss of photons on a time scale $(2\kappa)^{-1}$ through the cavity mirror. The result of Reynaud *et al.*, however, for the spectrum of fluctuations, indicates perfect correlation to be possible between the external fields at zero frequency (for a single ported cavity). This is assuming no additional losses. The zero-frequency result refers to very long detection times where signal and idler photons that are generated in the same parametric process have certainly escaped to the field external to the cavity.[14] The fluctuations occurring on a time scale $(2\kappa)^{-1}$ due to the cavity loss will affect the higher-frequency components of the spectrum.

Figures 1–4 plot the intensity fluctuation $\bar{S}_d(\omega)$ for differing signal and idler and pump decay rates. The perfect noise suppression in the external signal and idler difference intensity is obtained at zero frequency, regardless of the relative size of decay rates, since the zero-frequency result is for detection times that are long compared to both signal and idler cavity escape times. However, with nonequal signal and idler decay rates it is clear,

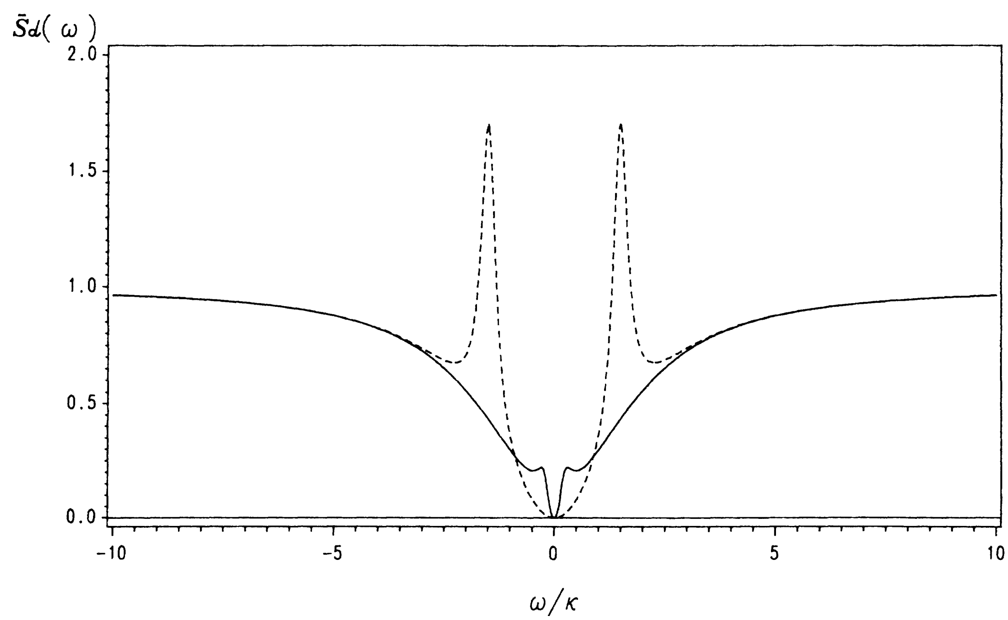

FIG. 2. Plot of $\bar{S}_d(\omega)$ above threshold for low pump cavity damping. $\kappa_0'=0.3$, $\kappa_2/\kappa_1=2$, $E=1.1$ (——), $E=5$ (— — —).

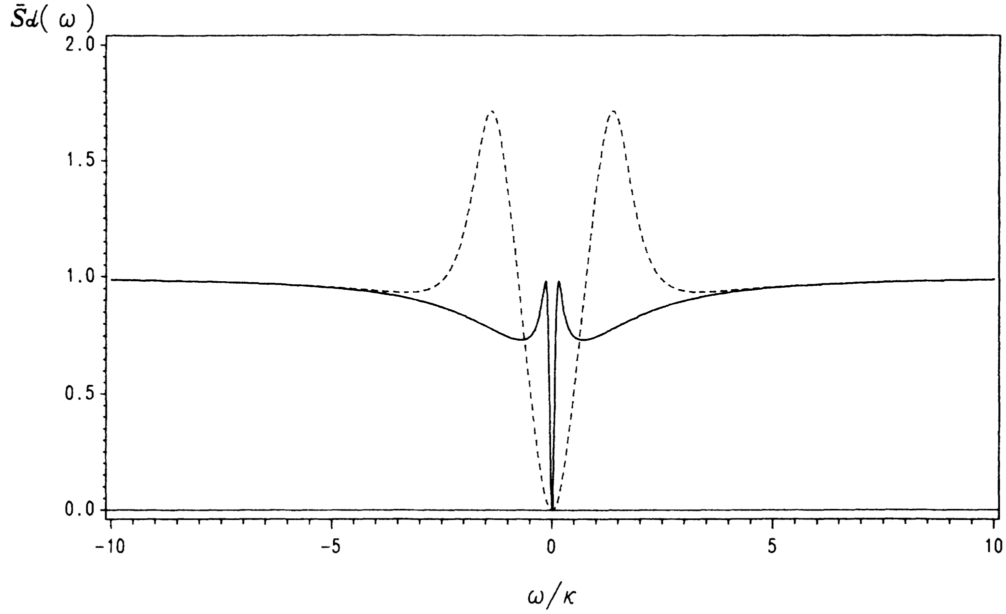

FIG. 3. Plot of $\bar{S}_d(\omega)$ above threshold for low pump cavity damping. $\kappa_0'=0.182$, $\kappa_2/\kappa_1=10$, $E=1.1$ (——), $E=10$ (----).

from Eqs. (2.10), that the fluctuations in the difference intensity couple to the fluctuations in the pump and signal-idler intensity sum. We see from (2.10) that the noise in the signal-idler intensity sum fluctuation μ_s is increased above the vacuum-noise level. The bandwidth of the $\bar{S}_d(\omega)$ noise reduction is now sensitive to the ratios of decay rates of the cavity modes and to the pump power. For some parameter values, relatively large fluctuations due to coupling of μ_d with the pump and μ_s give increased noise levels in $\bar{S}_d(\omega)$ above the shot-noise limit, and within the bandwidth indicated by the average signal-idler decay rate.

To facilitate discussions, we rewrite the linearized equations (2.10) for the amplitudes μ_j in terms of the intensity fluctuations directly. We define

$$\delta I_j = \alpha_j^\dagger \alpha_j - r_j^2 \,,$$

$$\delta I_s = \kappa_1 \delta I_1 + \kappa_2 \delta I_2 \,,$$

$$\delta I_d = \kappa_1 \delta I_1 - \kappa_2 \delta I_2$$

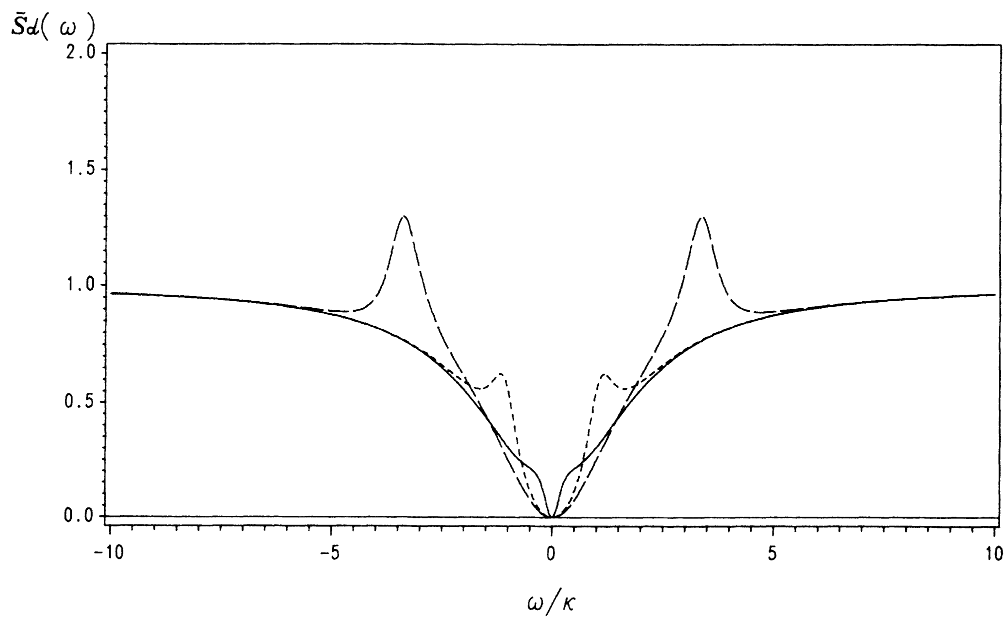

FIG. 4. Plot of $\bar{S}_d(\omega)$ above threshold for comparable pump, signal, and idler cavity dampings. $\kappa_0'=0.67$, $\kappa_2/\kappa_1=2$, $E=1.1$ (——), $E=2$ (----), $E=10$ (— — —).

to obtain the equations

$$\delta\dot{I}_0 = -\kappa_0 \delta I_0 - \delta I_s \;,$$
$$\delta\dot{I}_s = -2\delta\delta I_d + 2\kappa\kappa_0(E-1)\delta I_0 + F_s(t) \;, \quad (2.30)$$
$$\delta\dot{I}_d = -2\kappa\delta I_d + 2\delta\kappa_0(E-1)\delta I_0 + F_d(t) \;,$$

where the nonzero noise correlations are

$$\langle F_s(t)F_s(t')\rangle = -\langle F_d(t)F_d(t')\rangle = 4\kappa_1\kappa_2 c^2 \delta(t-t') \;.$$

We discuss firstly the limit where the pump cavity decay rate κ_0 is much larger than the signal and idler decay rates κ_1 and κ_2. Then the pump may be adiabatically eliminated to give the following intensity equations for signal and idler:

$$\delta\dot{I}_s = -2\delta\delta I_d - 2\kappa(E-1)\delta I_s + F_s(t) \;,$$
$$\delta\dot{I}_d = -2\kappa\delta I_d - 2\delta(E-1)\delta I_s + F_d(t) \;. \quad (2.31)$$

Thus the solution (2.24) for $S_d(\omega)$ in the limit of $\kappa_0' \to \infty$ can be written in the form

$$\bar{S}_d(\omega) = 1 - \kappa_1\kappa_2 \{ C_{\delta,E} L_{\lambda+} + D_{\delta,E} L_{\lambda-} \} \;, \quad (2.32)$$

where $L_{\lambda+} = -2\lambda_+/(\lambda_+^2 + \omega^2)$ and $L_{\lambda-} = -2\lambda_-/(\lambda_-^2 + \omega^2)$, and $C_{\delta,E}$ and $D_{\delta,E}$ are coefficient functions of δ and E. λ_+ and λ_- are the eigenvalues of the drift part of Eqs. (2.31),

$$\lambda_\pm = -\kappa E \pm \kappa[(E-2)^2 + 4(E-1)\delta'^2]^{1/2} \;. \quad (2.33)$$

Thus the spectrum is the sum of two Lorentzians both centered at $\omega=0$ and with widths $2\lambda_+$ and $2\lambda_-$, respectively. For the case of δ'^2 quite small, such that $4(E-1)\delta'^2 << (E-2)^2$, the eigenvalues become $\lambda_+ \to -2\kappa(E-1)$ and $\lambda_- \to -2\kappa$ for $E<2$. Thus, near to threshold $E \geq 1$, we see [Fig. 1(a)] a narrow Lorentzian component $L_{\lambda+}$ with width $4\kappa(E-1)$ and a broader component with width 4κ. The height (or depth) of the narrow component increases with increasing δ'. Figure 1(a) shows that for $E=1.1$, $\delta'>0.5$, the bandwidth of significant noise reduction is much smaller than 4κ. On moving further above threshold, the narrow line $L_{\lambda+}$ broadens [Fig. 1(b)] significantly. The effect of sufficiently large δ' is still to reduce the bandwidth of effective noise suppression.

One may also consider the limit where the pump decay rate is much smaller than the sum of the signal and idler decay rates, i.e., $\kappa_0 << 2\kappa$ ($\kappa_0' \to 0$). In this case the intensity-difference variable δI_d may be adiabatically eliminated to give

$$\delta I_d(t) = \frac{\delta\kappa_0}{\kappa}(E-1)\delta I_0 + \frac{F_d(t)}{2\kappa} \quad (2.34)$$

and a coupled $\delta I_0, \delta I_s$ subsystem

$$\delta\dot{I}_0 = -\kappa_0\delta I_0 - \delta I_s \;,$$
$$\delta\dot{I}_s = \left[-\frac{2\delta^2}{\kappa}\kappa_0(E-1) + 2\kappa\kappa_0(E-1) \right]\delta I_0 + F_s(t) - \frac{\delta}{\kappa}F_d(t) \;. \quad (2.35)$$

The final solution for the spectrum [Eq. (2.24) with $\kappa_0' \to 0$] will thus be of the form $\bar{S}_d(\omega) = \delta^2 + L$ where the eigenvalues $\lambda_{1,2}$ of the $\delta I_0, \delta I_s$ subsystem are

$$\lambda_{1,2} = -\frac{\kappa_0}{2} \pm \frac{\kappa_0}{2}\left[1 - \frac{8\kappa}{\kappa_0}(E-1) + 8(E-1)\frac{\delta^2}{\kappa} \right]^{1/2} \;, \quad (2.36)$$

and L symbolizes the Lorentzian components associated with the eigenvalues λ_1 and λ_2. Thus the solution is a flat floor of noise δ^2 with noisy sidepeaks, for nonzero δ, due to the coupling of signal and idler intensity-difference fluctuations with the pump and intensity-sum fluctuations. The flat perfect noise suppression for $\delta=0$ is due to the correlation of signal and idler intensities over the (now infinite) bandwidth 4κ. With κ/κ_0 large, the eigenvalues $\lambda_{1,2}$ are complex for sufficient E and hence we see noisy sidepeaks. The peak separation increases with pump power E. Figure 2 shows the true noise spectrum (2.24) in the limit of κ_0' very small. We note the initial sharp increase in the height of the noise peaks with increased E. The central frequencies between the noisy peaks have reduced fluctuations with perfect reduction at zero frequency. In essence then, for reasonable pump powers E, the bandwidth of noise reduction is reduced below 4κ, especially at lower intensities near threshold.

An interesting parameter regime of possible experimental interest is where the signal (or idler) cavity decay rate is much greater than that of the pump or idler. This corresponds to large δ and small κ_0' in the solution (2.24). Figure 3 shows such correlation spectra, demonstrating the small κ_0' ("good pump") features discussed above in the limit of larger δ. Comparison of Figs. 2 and 3 shows that the effect of increasing δ' is to increase the base noise level δ^2 that is prominent at outer frequencies over the bandwidth 4κ. There is little noise reduction for higher frequencies beyond the sidepeaks.

Figure 4 shows the appearance of the intensity-sum fluctuation sidepeaks in the noise spectrum in parameter regimes of moderate κ_0'. The features discussed above are apparent. The noisy sidepeaks are more apparent at high intensities and low κ_0' values.

III. INTENSITY FLUCTUATIONS BELOW THRESHOLD

Below threshold, we may neglect pump depletion and set $\alpha_0 = \langle \alpha_0 \rangle = \varepsilon/\kappa_0$, and so

$$\dot{\alpha}_1 = -\kappa_1\alpha_1 + d\alpha_2^\dagger + R_1 \;,$$
$$\dot{\alpha}_2 = -\kappa_2\alpha_2 + d\alpha_1^\dagger + R_2 \;, \quad (3.1)$$

plus the conjugate equations where $d = \chi\varepsilon/\kappa_0$.

The equations form two subsystems $(\alpha_1, \alpha_2^\dagger)$ and $(\alpha_2, \alpha_1^\dagger)$, each having eigenvalues

$$\lambda_1 = -(\kappa+\zeta) \;,$$
$$\lambda_2 = -(\kappa-\zeta) \;, \quad (3.2)$$

where $|\zeta|^2=|d|^2+\delta^2=E^2(\kappa^2-\delta^2)+\delta^2(E=\varepsilon/\varepsilon_{\text{thr}})$ is the effective below-threshold driving term. Now $0\le\zeta^2<\kappa^2$ with the maximum occurring at threshold and also at $\kappa_2/\kappa_1\to 0$ or ∞; so the system is stable.

The equations are easily solved in frequency space with

$$\alpha_j(\omega)=\frac{1}{\sqrt{2\pi}}\int d\tau\, e^{-i\omega\tau}\alpha_j(\tau)\ .$$

The solutions are

$$\begin{aligned}\alpha_1(\omega)&=\frac{(\kappa_2+i\omega)R_1(\omega)+dR_2^\dagger(\omega)}{(\kappa_1+i\omega)(\kappa_2+i\omega)-|d|^2}\ ,\\ \alpha_2^\dagger(\omega)&=\frac{d^*R_1(\omega)+(\kappa_1+i\omega)R_2^\dagger(\omega)}{(\kappa_1+i\omega)(\kappa_2+i\omega)-|d|^2}\ ,\end{aligned}\tag{3.3}$$

plus symmetric equations for the $(\alpha_1^\dagger,\alpha_2)$ pair. The nonzero correlations are

$$\begin{aligned}\langle R_1(\omega)R_2(\omega')\rangle&=d\delta(\omega+\omega')\ ,\\ \langle R_1^\dagger(\omega)R_2^\dagger(\omega')\rangle&=d^*\delta(\omega+\omega')\ .\end{aligned}\tag{3.4}$$

We wish to calculate intensity correlations $\langle\alpha_j^\dagger(t)\alpha_j(t),\alpha_k^\dagger(t')\alpha_k t')\rangle$. However, the normal linearization procedure which relates amplitude and intensity fluctuations by a linear factor

$$\begin{aligned}I_j(t)&=[r_j+\rho_j(t)]^2=r_j^2+2r_j\rho_j(t)+\rho_j^2(t)\\ &\approx I_j^{\text{ss}}+2r_j\rho_j(t)\ ,\end{aligned}\tag{3.5}$$

where $I_j^{\text{ss}}=r_j^2$ cannot be used if $r_j=0$. Instead, the four-time correlation is broken down into two-time correlations using the Gaussian properties of the Wiener noise processes for which

$$\begin{aligned}&\langle x_1(t_1)x_2(t_2)x_3(t_3)x_4(t_4)\rangle\\ &\quad=\langle x_1(t_1)x_2(t_2)\rangle\langle x_3(t_3)x_4(t_4)\rangle\\ &\qquad+\langle x_1(t_2)x_3(t_3)\rangle\langle x_2(t_2)x_4(t_4)\rangle\\ &\qquad+\langle x_1(t_1)x_4(t_4)\rangle\langle x_2(t_2)x_3(t_4)\rangle\ .\end{aligned}\tag{3.6}$$

The spectrum of the intensity correlations

$$S_{jk}(\omega)=\int_{-\infty}^{\infty}e^{-i\omega\tau}\langle I_j(\tau),I_k(0)\rangle\tag{3.7}$$

is then given by the convolution integrals of the frequency space correlations.

$$\begin{aligned}S_{jk}(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}d\omega'[&C_{\alpha_j^+\alpha_k^+}(\omega')C_{\alpha_j\alpha_k}(\omega-\omega')\\ &+C_{\alpha_j^+\alpha_k}(\omega')C_{\alpha_j\alpha_k^+}(\omega-\omega')]\ ,\end{aligned}\tag{3.8}$$

where we define $C_{xy}(\omega)$ by $\langle x(\omega)y(\omega')\rangle=C_{xy}(\omega)\delta(\omega+\omega')$. Thus the spectrum of the intensity-difference fluctuations

$$\begin{aligned}S_d(\omega)=\int d\tau\, e^{-i\omega\tau}[&2\delta(\tau)\langle\kappa_1I_1(0)+\kappa_2I_2(0)\rangle\\ &+4\langle\kappa_1I_1(\tau)-\kappa_2I_2(\tau),\kappa_1I_1(0)\\ &-\kappa_2I_2(0)\rangle]\end{aligned}\tag{3.9}$$

are given by

$$\begin{aligned}S_d(\omega)=\frac{1}{\pi}\int d\omega'\{&\kappa_1C_{\alpha_1^\dagger\alpha_1}(\omega')+\kappa_2C_{\alpha_2^\dagger\alpha_2}(\omega')+2\kappa_1^2[C_{\alpha_1^\dagger\alpha_1^\dagger}(\omega')C_{\alpha_1\alpha_1}(\omega-\omega')+C_{\alpha_1^\dagger\alpha_1}(\omega')C_{\alpha_1\alpha_1^\dagger}(\omega-\omega)]\\ &+2\kappa_2^2[C_{\alpha_2^\dagger\alpha_2^\dagger}(\omega')C_{\alpha_2\alpha_2}(\omega-\omega')+C_{\alpha_2^\dagger\alpha_2}(\omega')C_{\alpha_2\alpha_2^\dagger}(\omega-\omega')]\\ &-2\kappa_1\kappa_2[C_{\alpha_1^\dagger\alpha_2^\dagger}(\omega')C_{\alpha_1\alpha_2}(\omega-\omega')\\ &+C_{\alpha_1^\dagger\alpha_2}(\omega')C_{\alpha_1\alpha_2^\dagger}(\omega-\omega')+C_{\alpha_2^\dagger\alpha_1^\dagger}(\omega')C_{\alpha_2\alpha_1}(\omega-\omega')+C_{\alpha_2^\dagger\alpha_1}(\omega')C_{\alpha_2\alpha_1^\dagger}(\omega-\omega')]\}\ .\end{aligned}\tag{3.10}$$

The correlations $C_{\alpha_j\alpha_k}$ may be calculated from Eqs. (3.3) and (3.4). The resultant expression for the spectrum is

$$\begin{aligned}S_d(\omega)=\frac{4}{\pi}|d|^2\kappa_1\kappa_2\int_{-\infty}^{\infty}d\omega'&\{[\kappa_1(\omega')\kappa_2(\omega')-|d|^2][\kappa_1^*(\omega')\kappa_2(\omega')-|d|^2]\}^{-1}\\ &\times\left[1+\frac{4\kappa_1\kappa_2|d|^2-\text{Re}\{[|d|^2+\kappa_1^*(\omega')\kappa_2(\omega')](|d|^2+\kappa_1^*(\omega-\omega')\kappa_2(\omega-\omega')\}}{[\kappa_1(\omega-\omega')\kappa_2(\omega-\omega')-|d|^2][\kappa_1^*(\omega-\omega')\kappa_2^*(\omega-\omega')-|d|^2]}\right]\ ,\end{aligned}\tag{3.11}$$

where $\kappa_j(\omega)=\kappa_j+i\omega$.

This integral can be solved by contour integration by rewriting

$$|d|^2-\kappa_1(\omega)\kappa_2(\omega)=[\omega-i(\kappa+\zeta)][\omega-i(\kappa-\zeta)]\ .\tag{3.12}$$

The poles do not touch each other or the real axis, provided that $0<|\zeta|<\kappa$. The result written in scaled form $(p'=p/\kappa)$ is

$$\bar{S}_d(\omega)=\frac{S_d(\omega)}{S_0}=\frac{\omega'^2}{\omega'^2+4}\left[1+\frac{4\delta'^2}{\omega'^2+4\zeta'^2}(1+4B)\right]\ ,\tag{3.13}$$

where

$$B=\frac{\omega'^2(3-2\zeta'^2)-4(2\zeta'^4-7\zeta'^2+1)}{[\omega'^2+4(1-\zeta')^2][\omega'^2+4(1+\zeta')^2]}\ ,$$

$$S_0=\frac{2\kappa E^2(1-\delta'^2)^2}{(1-\zeta'^2)},$$

$$\zeta'^2=E^2+\delta'^2(1-E^2),$$

and $\delta'=(1-\kappa_2/\kappa_1)/(1+\kappa_2/\kappa_1)$ is the scaled difference in the signal and idler damping rates, and $E=|\varepsilon|/\varepsilon_{\rm thr}$ is the scaled driving intensity. For equal damping $\kappa_1=\kappa_2$,

$$\frac{S_d(\omega)}{S_0}=\frac{\omega^2}{\omega^2+4\kappa^2}, \tag{3.14}$$

just as was obtained in the above-threshold calculation.

For $\kappa_1\neq\kappa_2$ ($\delta\neq0$), the effect of the other combinations of the eigenvalues given by Eq. (3.2) comes into play. As the effective driving term ζ increases to the threshold value of κ, the $\kappa-\zeta$ eigenvalue drops to zero and the central dip is pinched, while a broader curve is formed further away from $\omega=0$ (see Fig. 5). In the limit of $\kappa_2/\kappa_1\to0$ or ∞ or $|\varepsilon|\to\varepsilon_{\rm thr}$ the width of the central dip falls to zero.

$$\lim_{\zeta\to\kappa}\frac{S_d(\omega)}{S_0}=\begin{cases}\dfrac{\omega^2+4\delta^2}{\omega^2+4\kappa^2}, & \omega\neq0\\ 0, & \omega=0.\end{cases} \tag{3.15}$$

This expression is also that obtained in this limit approaching from the above-threshold side. In this limit the squeezing minimum is limited by the difference in cavity damping $|\delta'|=\frac{1}{2}|\kappa_1-\kappa_2|/\kappa$ (apart from a point discontinuity at $\omega=0$).

Below threshold, the scaled spectrum behavior is not influenced by the cavity damping of the pump (which serves only to change the pump threshold). This is in contrast to the above-threshold result, whose behavior could be significantly altered by reducing the pump damping below that of the other modes.

IV. EFFECT OF LOSSES

We shall consider the effect of losses in the intracavity medium. We shall denote the total losses (cavity plus internal) by κ_j and the cavity losses by γ_j ($\gamma_j\leq\kappa_j$). The internal equations remain unchanged, but the relationship between the external modes $\hat{I}_j(t)$ and the internal ones is given by the γ_j.

The spectrum of the difference current is

$$S_d(\omega)=\int d\tau\, e^{-i\omega\tau}\langle\hat{I}_1(\tau)-\hat{I}_2(\tau),\hat{I}_1(0)-\hat{I}_2(0)\rangle_{\rm ss}\,. \tag{4.1}$$

The relationship between the external intensities $\hat{I}_i$ and the internal intensities I_i in the P representation is given by the input and/or output formalism of Collett and Gardiner[5] [see Eqs. (2.13) and (2.14)],

$$\langle\hat{I}_j(\tau),\hat{I}_k(0)\rangle=4\gamma_j\gamma_k\langle I_j(\tau),I_k(0)\rangle+2\delta_{jk}\delta(\tau)\gamma_j\langle I_j(0)\rangle\,. \tag{4.2}$$

This may be written in the form

$$S_d(\omega)=S_0+4\int d\tau\, e^{-i\omega\tau}\langle I_d(\tau),I_d(0)\rangle_{\rm ss}\,, \tag{4.3}$$

where

$$S_0=2\Gamma_1\kappa_1\langle I_1\rangle_{\rm ss}+2\Gamma_2\kappa_2\langle I_2\rangle_{\rm ss}\,,$$

$$I_d(\tau)=\Gamma_1\kappa_1I_1(t)-\Gamma_2\kappa_2I_2(t)\,,$$

and $\Gamma_j=\gamma_j/\kappa_j$. We consider the above-threshold regime where the usual linearization procedure gives

$$I_j(t)=r_j^2[1+\mu_j(t)]^2\approx r_j^2[1+2\mu_j(t)],\quad r_j^2=\frac{c^2}{\kappa_j}\,,\quad j\leftarrow1,2\,. \tag{4.4}$$

This gives

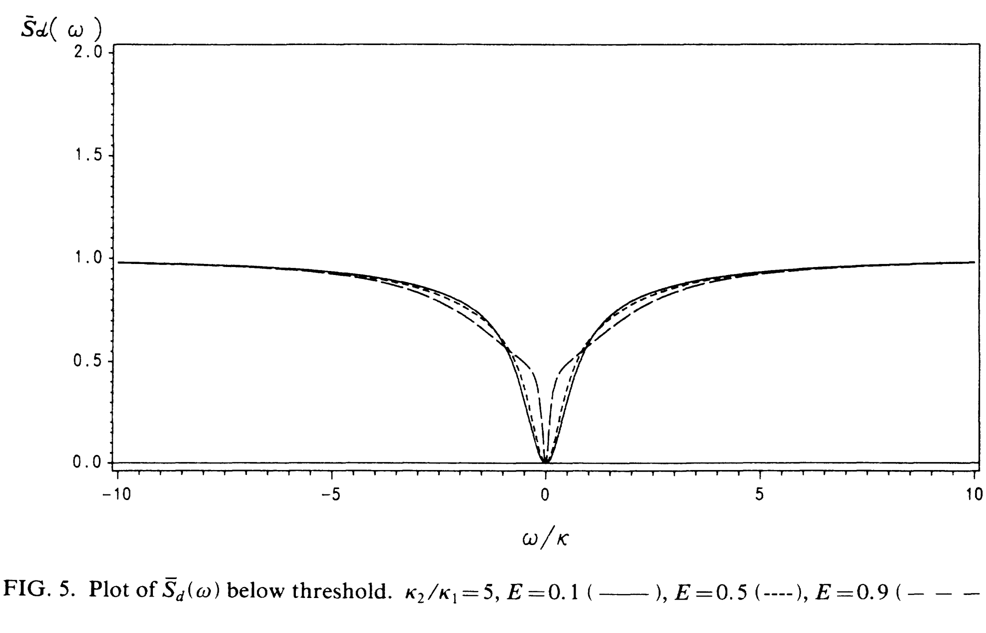

FIG. 5. Plot of $\bar{S}_d(\omega)$ below threshold. $\kappa_2/\kappa_1=5$, $E=0.1$ (——), $E=0.5$ (----), $E=0.9$ (— — —).

$$S_0=2c^2(\Gamma_1+\Gamma_2)\ ,$$
$$I_d(t)=c^2\{2[\Gamma_1\mu_1(t)-\Gamma_2\mu_2(t)]+\Gamma_1-\Gamma_2\}\ . \tag{4.5}$$

Rewriting,

$$\mu_{1,2}=\tfrac{1}{2}(\mu_s+\mu_d)\ . \tag{4.6}$$

We may write the difference current as

$$I_d(t)=c^2\{(\Gamma_1+\Gamma_2)\mu_d(t)+(\Gamma_1-\Gamma_2)[\mu_s(t)+1]\}\ . \tag{4.7}$$

In the steady state

$$\langle x(\omega),y(\omega)\rangle_{ss}=\delta(\omega+\omega')\int d\tau\, e^{-i\omega\tau}\langle x(\tau),y(0)\rangle_{ss}\ , \tag{4.8}$$

which enables us to write

$$\bar{S}_d(\omega)=\frac{S_d(\omega)}{S_0}=1+\Gamma 4c^2\left[\frac{\langle\mu_d(\omega)+g\mu_s(\omega),\mu_d(\omega')+g\mu_s(\omega')\rangle}{\delta(\omega+\omega')}\right]\ , \tag{4.9}$$

where

$$S_0=4c^2\Gamma\ ,$$
$$\Gamma=\tfrac{1}{2}(\Gamma_1+\Gamma_2)=\frac{1}{2}\left[\frac{\gamma_1}{\kappa_1}+\frac{\gamma_2}{\kappa_2}\right]\ ,$$
$$g=\frac{\tfrac{1}{2}(\Gamma_1-\Gamma_2)}{\Gamma}\ .$$

We note that the effect of losses is to scale down the spectral change in fluctuations by the factor $\Gamma\le 1$ and introduce the extra terms $g\mu_s(\omega)$.

Using the solutions given by Eqs. (2.23) for μ_s, μ_d, and μ_0, we find, for the intensity spectrum,

$$\frac{S_d(\omega)}{S_0}=\frac{\omega^2+4\kappa^2(1-\Gamma)+4\delta^2\Gamma}{\omega^2+4\kappa^2}+\frac{\Gamma Y(\omega)}{|Z(\omega)|^2}\ , \tag{4.10}$$

where

$$Y(\omega)=\frac{4\delta^2\varepsilon\{\omega^2[4\kappa+\kappa_0(E+1)]-\varepsilon\}}{\omega^2+4\kappa^2}+g4\delta\varepsilon\omega^2+g^2 4(\kappa^2-\delta^2)[4(\kappa^2-\delta^2)+\omega^2](\kappa_0^2+\omega^2)\ ,$$

$$|Z(\omega)|^2=[\varepsilon-\omega^2(2\kappa+\kappa_0)]^2+\omega^2(2\kappa\kappa_0E-\omega^2)^2\ ,$$

with

$$\varepsilon=4(\kappa^2-\delta^2)\kappa_0(E-1)\ ,$$
$$g=\frac{\frac{1}{2}\left[\frac{\gamma_1}{\kappa_1}-\frac{\gamma_2}{\kappa_2}\right]}{\Gamma}\ ,$$
$$\Gamma=\frac{1}{2}\left[\frac{\gamma_1}{\kappa_1}+\frac{\gamma_2}{\kappa_2}\right]\ ,$$

and $S_0=4c^2\Gamma$. Let us first consider the case where the dampings are symmetric between the signal and idler, that is, $\gamma_1=\gamma_2=\gamma$, $\kappa_1=\kappa_2=\kappa\Longrightarrow\delta=0$, $g=0$. Then,

$$\frac{S_d(\omega)}{S_0}=\frac{\omega^2+4\kappa^2(1-\Gamma)}{\omega^2+4\kappa^2}\ . \tag{4.11}$$

At $\omega=0$, $S_d(0)=S_0(1-\Gamma)=S_0(1-\gamma/\kappa)$ with $\gamma\le\kappa$. Thus the effect of losses is to reduce the correlation between the modes, and there is no longer perfect suppression of the noise (Fig. 6).

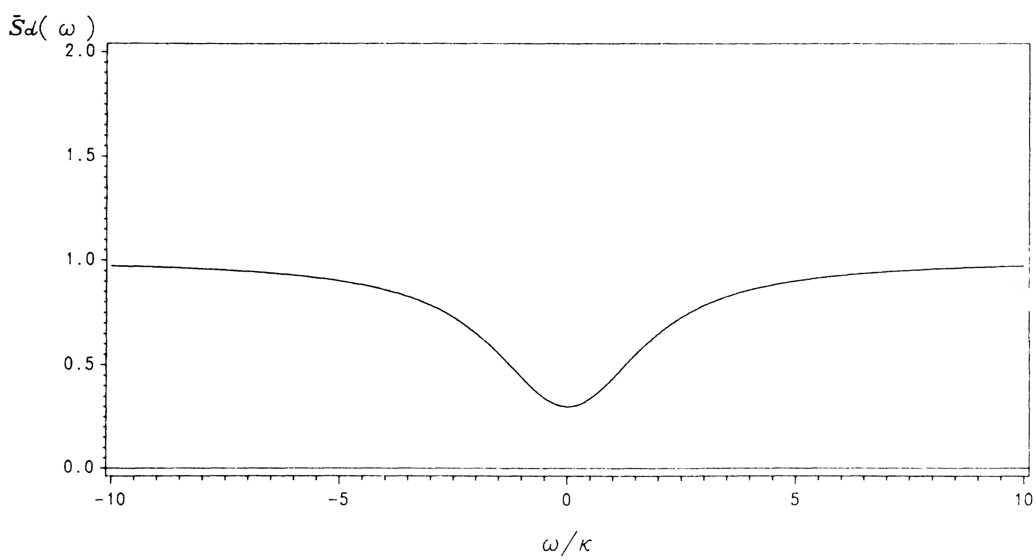

FIG. 6. Effect of intracavity absorption, plot of $\bar{S}_d(\omega)$. $\gamma_1=\gamma_2=\gamma$, $\kappa_1=\kappa_2=\kappa$, $\kappa_0'=1$, $\gamma/\kappa=0.7$.

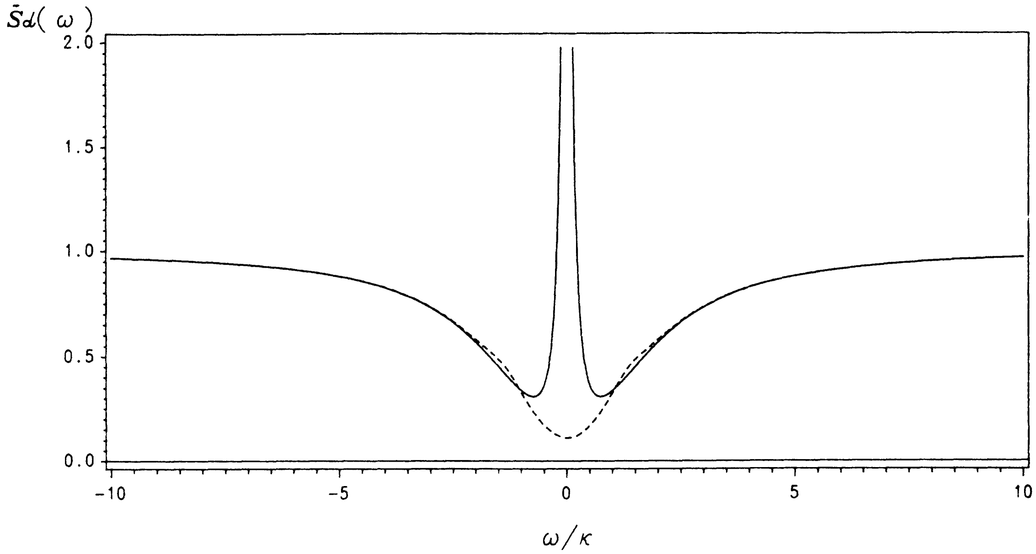

FIG. 7. Effect of asymmetrical intracavity absorption; plot of $\bar{S}_d(\omega)$. $\kappa_0'=1$, $\gamma_1/\kappa_1=1$, $\gamma_2/\kappa_2=0.8$, $\kappa_2/\kappa_1=1.25$, $E=1.05$ (——), $E=2$ (— — —).

We now consider the case where $\kappa_1 \neq \kappa_2$. This could be the case if an absorber at the idler frequency, say, was introduced into the cavity. In this case, at $\omega=0$,

$$\frac{S_d(0)}{S_0}=1-\Gamma+\frac{g^2\Gamma}{(E-1)^2} . \tag{4.12}$$

There is an extra noise term $g^2\Gamma/(E-1)^2$, which is sensitive to the difference $(\gamma_1/\kappa_1)-(\gamma_2/\kappa_2)$. For equal cavity damping rates $\gamma_1=\gamma_2$, this may be a very sensitive detection method for absorption. The sensitivity is enhanced in near-threshold operation by the $(E-1)^{-2}$ factor, as is evident in Fig. 7. In the region of threshold there are critical fluctuations present which become manifest if there is an imbalance in the absorption in the two arms. The level of absorption may be measured by inserting a calibrated variable absorber in the beam. The absorption is then varied until a null in the noise is obtained. This indicates that the absorption in both arms is equal. Let us denote $\eta_1=\kappa_1-\gamma_1$ as the absorption to be measured, and $\eta_2=\kappa_2-\gamma_2$ as the known (variable) absorber. Then the small difference $\varepsilon=\eta_1-\eta_2$ (we take $\gamma_1=\gamma_2=\gamma$) between the absorber η_1 and the variable absorber η_2 provides a signal amplified by the $E-1$ factor, which is easily detectable above a low-background-noise level. The power level with ε nonzero is

$$\begin{aligned}
\bar{S}_d(0)_{\varepsilon\neq 0}&=B+\frac{1}{(E-1)^2}A ,\\
B&=1-\frac{1}{2}\left[\frac{1}{1+\bar{\eta}}+\frac{1}{(1+\bar{\eta})[1+\bar{\varepsilon}/(1+\bar{\eta})]}\right] ,\\
A&=\frac{\bar{\varepsilon}^2}{4}\frac{(1+\bar{\eta}+\bar{\varepsilon})}{(1+\bar{\eta})^6[1+\bar{\varepsilon}/(1+\bar{\eta})]^3} ,
\end{aligned} \tag{4.13}$$

where $\bar{\eta}=\eta_1/\gamma$, $\bar{\varepsilon}=\varepsilon/\gamma$. This power level has a contribution that is not due to the absorption difference ε. This "background level" is

$$\bar{S}(0)_{\varepsilon=0}=1-\frac{1}{1+\bar{\eta}} , \tag{4.14}$$

and where $\eta_1 \ll \gamma$; this background noise is well below the noise level $\bar{S}_d=1$ and is, in principle, zero. This low-background level plus the amplification of the signal brought about by the $E-1$ factor gives us a very high signal-to-noise ratio $(\bar{S}_d(0)_{\varepsilon\neq 0}/\bar{S}_d(0)_{\varepsilon=0})$, where $E-1$ is small.

V. DISCUSSION

A detailed study of the intensity fluctuations in a nondegenerate parametric oscillator is given. In the absence of losses and for equal cavity damping in the signal and idler modes, there has been shown to be a complete cancellation of noise in the spectrum of the difference current at zero frequency.[14] In existing experiments these ideal conditions are not necessarily attained. We have therefore calculated the characteristics of the output of the parametric oscillator when losses are present and the signal and idler have different cavity decay rates. The effect of different cavity decay rates is to narrow and change the shape of the spectrum of the difference current. The exact details will depend on the decay rate of the pump mode in the cavity. Both the above- and below-threshold behavior of the oscillator is calculated.

In the case of intracavity losses the quantum correlation between the modes is partially destroyed. There is no longer a perfect suppression of the quantum noise at zero frequency. This property may be made use of as an absorption detector for absorption at levels below that of the vacuum noise. The sensitivity of such a detector is enhanced when the oscillator is operated close to threshold.

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