Correlated-emission laser: Phase noise quenching via coherent pumping and the effect of atomic motion

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(Received 4 June 1987)

We show that in a three-level system, if the atoms are initially prepared in a coherent superposition of the two upper states, as in Hanle experiments, the spontaneous-emission events from these two states to the lowest one can, under certain conditions, be strongly correlated, with a vanishing diffusion constant for the relative phase. The conditions under which correlated spontaneous emission occurs in a Hanle laser remain essentially unchanged if atomic motion is included.

I. INTRODUCTION

The idea of correlated spontaneous emission[1] has generated a great deal of interest because of its potential applications to problems involving high-precision interferometry where measurements of ultrasmall phase shifts are important. The typical examples include laser gyroscopes[2] and laser gravity wave detectors.[3] It is also interesting from a spectroscopic viewpoint because of its capability of producing continuous-wave quantum beats. The central idea is to eliminate the quantum noise, caused by independent spontaneous-emission events, from the relative phase of two laser modes generated from three-level atoms inside a doubly resonant cavity.

In the Ref. 1 it was shown that if one prepares the three-level atoms in a coherent superposition of the two upper levels a and b by a strong external resonant signal, then the spontaneous-emission events from these two levels are, indeed, strongly correlated. In a subsequent, more detailed, work[4] on the quantum-beat laser, it was shown that if one considers a strong classical driving field that coherently couples the upper two levels then, under certain detuning and power conditions, one may achieve a zero diffusion coefficient for the relative phase of the two laser signals.

In recent works (Ref. 5, henceforth referred to as I, and Ref. 6, henceforth referred to as II) we have generalized the linear theory outlined in Ref. 4 and worked out the nonlinear theory of the correlated-emission quantum-beat laser (I) and micromaser (II). In particular, we have shown that correlated-emission laser (CEL) operation persists, i.e., noise quenching occurs in a steady state above threshold and this operation is stable.

In the present work we take a different approach. As was pointed out in Ref. 1, CEL operation might also be expected if one does not have a strong microwave field coherently coupling the upper two levels as in the CEL quantum-beam laser but, instead, prepares an initial coherent superposition of these two levels via, e.g., the Hanle effect.[7] The purpose of the present paper is to bring the theory of the CEL Hanle laser to the same level of sophistication as that for the quantum-beat laser. In particular, we want to find the physical conditions under which CEL operation occurs. We address another important issue as well. In the above theoretical works, only homogeneously broadened systems were considered and, consequently, the effects of atomic motion have not been dealt with. Here, following the lines of Ref. 8, we develop a quantum theory of the CEL Hanle laser which includes the effect of atomic motion (Doppler broadening) and we show that the CEL effect persists in inhomogeneously broadened systems.

The paper is organized as follows. In Sec. II we present a Hamiltonian model of the system and develop a linear theory of the Hanle laser. We show that, in contrast with the quantum-beam laser, this is essentially an initial-condition problem. From the requirement of the vanishing diffusion constant we find the conditions for pumping, coupling, and detuning under which CEL operation is possible. In Sec. III we derive the diffusion constant for the relative phase and show that the condition for CEL operation remains essentially unchanged.

II. PHASE NOISE QUENCHING IN A THREE-LEVEL SYSTEM WITH COHERENT PUMPING

We consider the model Hamiltonian for the problem to be

$$H=\hbar(H_0+V)\ , \tag{1}$$

where

$$H_0=\sum_{i=a,b,c}\omega_i\,|\,i\,\rangle\langle\,i\,|+\nu_1 a_1^\dagger a_1+\nu_2 a_2^\dagger a_2+\nu_3 a_3^\dagger a_3 \tag{2}$$

and

$$V=g_1 a_1\,|\,a\,\rangle\langle\,c\,|+g_2 a_2\,|\,b\,\rangle\langle\,c\,|+g_3 a_3\,|\,a\,\rangle\langle\,b\,|+\text{H.c.} \tag{3}$$

Here $|\,a\,\rangle$ and $|\,b\,\rangle$ are the upper levels (e.g., Zeeman sublevels of a degenerate upper state) and $|\,c\,\rangle$ is the ground state. g_1, g_2, and g_3 are the coupling constants for the transitions $|\,a\,\rangle\rightarrow|\,c\,\rangle$, $|\,b\,\rangle\rightarrow|\,c\,\rangle$, and $|\,a\,\rangle\rightarrow|\,b\,\rangle$, respectively. a_1, a_2, and a_3 ($a_1^\dagger$, $a_2^\dagger$, and $a_3^\dagger$) denote absorption (emission) operators for photons in the

modes 1, 2, and 3. Note that without an applied magnetic field $\omega_a \sim \omega_b$, and thus the fields emitted by the atom in the $|a\rangle$-$|c\rangle$ and $|b\rangle$-$|c\rangle$ transitions will differ in polarization, but their frequencies are essentially the same ($\nu_1=\nu_2$). If one goes into the interaction picture, it is straightforward to show that the interaction Hamiltonian, in an obvious matrix representation, is given by

$$V=\begin{bmatrix} 0 & g_3 a_3 e^{i\Delta_3 t} & g_1 a_1 e^{i\Delta_1 t} \\ g_3 a_3^\dagger e^{-i\Delta_3 t} & 0 & g_2 a_2 e^{i\Delta_2 t} \\ g_1 a_1^\dagger e^{-i\Delta_1 t} & g_2 a_2^\dagger e^{-i\Delta_2 t} & 0 \end{bmatrix} , \tag{4}$$

where

$$\begin{aligned} \Delta_1&=\omega_a-\omega_c-\nu_1 , \\ \Delta_2&=\omega_b-\omega_c-\nu_2 , \\ \Delta_3&=\omega_a-\omega_b-\nu \end{aligned} \tag{5}$$

stand for the detunings of the corresponding transitions.

The interaction Hamiltonian, as given by Eq. (4), represents our starting point. When compared to previous theoretical treatments of the CEL it is easy to see that Eq. (4) is a slight generalization of the model elaborated upon in most detail in I, inasmuch as all these fields are treated on an equal footing, in a quantum-mechanical manner, and with perturbation theory.

The density operator ρ of the coupled system (three-level atoms plus three-mode field) satisfies the following Liouville equation:

$$\dot{\rho}=-i[V,\rho] , \tag{6}$$

where the bracket stands for the commutator, and V is given by (4). The reduced density operator ρ_f, for the field only, is defined as the trace of ρ over the atoms

$$\rho_f=\mathrm{Tr}_a\rho \tag{7}$$

Now, starting from Liouville's equation, we perform a second-order expansion in the coupling constants and take the trace over the atomic states, leading to

$$\dot{\rho}_f=-r\int_0^\infty d\tau\,\gamma e^{-\gamma\tau}\int_t^{t+\tau}dt'\int_t^{t'}dt''\mathrm{Tr}_a[V(t'),[V(t''),\rho_a(t)\rho_f(t)]]+\mathcal{L} , \tag{8}$$

where $\mathcal{L}$ represents losses. In the derivation of this equation from Eq. (6), we have, as usual,[9] assumed the following.

(i) The Markov approximation in the second-order term, i.e., at the time t of injection of an excited atom into the laser cavity, the density operator can be written as $\rho=\rho_a(t)\rho_f(t)$.

(ii) The coarse-time-graining approximation. Here γ is the decay constant of the atom (assumed to be equal for all three states) and r is the pump rate. The coarse-time-graining approximation is valid if both γ and $r \gg \gamma_c$ where γ_c is the cavity decay rate.

Although, in Eq. (8), we have a loss term (proportional to γ_c) due to the presence of a doubly resonant cavity tuned to the two lasing modes, we do not have to specify its explicit expression, since it does not influence the relative phase-diffusion constant. In the rest of the paper we drop it from the equations. We also notice that in Eq. (8) the first-order term is missing, which is due to the fact that its trace is zero. Only even-order terms appear in the expansion.

The key point in the solution of Eq. (8), i.e., in finding an explicit equation for ρ_f, is in the initial condition. We assume that the atoms have initially been prepared in a coherent superposition of the two upper states, as in Hanle-effect experiments,[7] namely,

$$\rho_a(t)=\tfrac{1}{2}(e^{i\phi}|a\rangle+|b\rangle)(e^{-i\phi}\langle a|+\langle b|) , \tag{9}$$

where ϕ is some fixed relative phase between the two states. It is now straightforward to show that the resulting master equation for the field-density operator ρ is (with $\rho_f=\rho$, i.e., in the following we drop the subscript f from the notation)

$$\begin{aligned} \dot{\rho}=&\tfrac{1}{2}\alpha_{33}(a_3^\dagger a_3\rho+\rho a_3 a_3^\dagger-a_3\rho a_3^\dagger-a_3^\dagger\rho a_3) \\ &+\tfrac{1}{2}\alpha_{11}(\rho a_1 a_1^\dagger-a_1^\dagger\rho a_1)+\tfrac{1}{2}\alpha_{22}(\rho a_2 a_2^\dagger-a_2^\dagger\rho a_2) \\ &+\tfrac{1}{2}\alpha_{12}(\rho a_2 a_1^\dagger-a_1^\dagger\rho a_2)e^{i\phi} \\ &+\tfrac{1}{2}\alpha_{21}(\rho a_1 a_2^\dagger-a_2^\dagger\rho a_1)e^{-i\phi}+\mathrm{H.c.} , \end{aligned} \tag{10}$$

with

$$\begin{aligned} \alpha_{11}&=-\frac{rg_1^2}{\gamma(\gamma+i\Delta_1)} , \quad \alpha_{22}=-\frac{rg_2^2}{\gamma(\gamma+i\Delta_2)} , \\ \alpha_{12}&=-\frac{rg_1g_2}{(\gamma+i\Delta_1)[\gamma+i(\Delta_1-\Delta_2)]} , \\ \alpha_{21}&=-\frac{rg_1g_2}{(\gamma+i\Delta_2)[\gamma+i(\Delta_2-\Delta_1)]} , \\ \alpha_{33}&=-\frac{rg_3^2}{\gamma(\gamma+i\Delta_3)} . \end{aligned} \tag{11}$$

We can now convert this Liouville equation into a Fokker-Planck equation by introducing a coherent-state representation for a_1 and a_2 and the diagonal or P representation for ρ.[9] A similar Liouville equation has been converted into a Fokker-Planck equation in Ref. 4. Here, instead of repeating the derivation, we simply borrow the results. If we define the coherent state as

$$a_i|v_1,v_2\rangle=v_i|v_1,v_2\rangle, \quad i=1,2 , \tag{12}$$

where v_i is an arbitrary complex number and we represent v_i as

$$v_i=\rho_i e^{i\theta_i}, \quad i=1,2 \tag{13}$$

then we can introduce the relative phase between the two laser fields 1 and 2 as $\theta=\theta_1-\theta_2$. The Fokker-Planck equation in terms of ρ_i, $\mu=\frac{1}{2}(\theta_1+\theta_2)$, and θ will contain a term which describes diffusion of the relative phase θ as

$$\dot{P}=D(\theta)\frac{\partial^2 p}{\partial\theta^2} . \tag{14}$$

Here

$$D(\theta)=\frac{1}{8}\left[\frac{\alpha_{11}}{\rho_1^2}+\frac{\alpha_{22}}{\rho_2^2}-\frac{\alpha_{12}}{\rho_1\rho_2}e^{-i\psi}-\frac{\alpha_{21}}{\rho_1\rho_2}+e^{i\psi}\right]+\text{c.c.} , \tag{15}$$

with $\psi=\theta-\phi$.

We are interested in finding conditions under which the diffusion constant $D(\theta)$ vanishes. A possible set of conditions is the following (with $\rho_1=\rho_2=\rho$):

$$g_1=g_2\ (\equiv g),\quad \Delta_1=\Delta_2\ (\equiv\Delta) . \tag{16}$$

Under these conditions

$$D(\theta)=\frac{rg^2}{2(\gamma^2+\Delta^2)\rho^2}(1-\cos\psi) . \tag{17}$$

When $\psi=0$ the vanishing of the diffusion constant takes place. It can be shown, as was done in Ref. 1 for the quantum-beat laser, that the relative phase (i.e., ψ) locks to zero in the present case as well.

As a conclusion, the present calculation shows that CEL operation can also be obtained via coherent pumping as in a Hanle laser, i.e., if one prepares the atoms in a coherent superposition of the two upper states. The only condition is that the detuning on the a-c transition must be equal to the detuning on the b-c transition; otherwise, the initial coherence is destroyed on a time scale γ^{-1} much shorter than the time scale γ_c^{-1} governing the time evolution of the photon field in the cavity. In particular, $\Delta=0$ satisfies Eq. (17), in contrast with the quantum-beat laser (I). Also, this condition is independent of g_3, i.e., again in contrast with the quantum-beam laser, no specific coherence-maintaining mechanism is necessary in the Hanle laser to obtain CEL operation. Therefore, in order to simplify the treatment of Sec. III, in the following we set $g_3=0$.

III. ATOMIC MOTION AND RECOIL

In previous theoretical treatments of the CEL effect, only systems with homogeneous broadening have been considered. In order to investigate the possibility of CEL operation in inhomogeneously broadened systems, we extend the model of Sec. II to allow for atomic motion, along the lines of Ref. 8. We consider a moving, three-level atom coupled by a dipole interaction to two standing-wave modes $\cos(K_i z)$ $(i=1,2)$ in the cavity. Since the electric field has a spatial dependence only on z we may consider one-dimensional atomic motion. The Hamiltonian of the problem can be written as [cf. Eqs. (1)–(3)]

$$H=\hbar(H_0+V) , \tag{18}$$

where

$$H_0=\sum_{i=a,b,c}\omega_i\,|\,i\,\rangle\langle\,i\,|+\nu_1 a_1^\dagger a_1+\nu_2 a_2^\dagger a_2+\frac{\hbar\hat{k}^2}{2m} , \tag{19}$$

and

$$V=g_1 a_1\,|\,a\,\rangle\langle\,c\,|\cos(K_1 z)+g_2 a_2\,|\,b\,\rangle\langle\,c\,|\cos(K_2 z)+\text{H.c.} \tag{20}$$

Here $(\hbar\hat{k}^2)/2m$ is the kinetic energy operator of the c.m. motion of the atom [$\hat{k}=-i(\partial/\partial z)$ in the coordinate representation] and does not commute with the spatial part of the interaction. The theory of Sec. II can be recovered if we replace $\cos(K_i z)$ in (20) by its (quadratic) spatial average $1/\sqrt{2}$ and neglect the kinetic energy in (19). Also, in (20) we have set $g_3=0$.

To express the motional parts of H explicitly, we introduce a complete set of momentum eigenfunctions $|\,k\,\rangle$ satisfying periodic boundary conditions in the interval $0\le z\le L$ (L is the length of the cavity),

$$\hat{k}\,|\,k\,\rangle=k\,|\,k\,\rangle,\quad |\,k\,\rangle\langle\,k\,|=1 . \tag{21}$$

With the help of the above representation of the unity operator we can write the kinetic energy and the mode eigenfunctions as

$$\frac{\hbar\hat{k}^2}{2m}=\sum_k\frac{\hbar k^2}{2m}\,|\,k\,\rangle\langle\,k\,| ,$$
$$\cos(K_i z)=\tfrac{1}{2}\sum_k(\,|\,k+K_i\,\rangle\langle\,k\,|+|\,k-K_i\,\rangle\langle\,k\,|\,) , \tag{22}$$
$$i=1,2 .$$

If we now transform Eq. (20) into the interaction picture with respect to H_0, it is straightforward to show that the interaction Hamiltonian, in an obvious matrix representation, is given by

$$V=\frac{1}{2}\begin{bmatrix} 0 & 0 & g_1 a_2 e^{i\Delta_1 t}\hat{F}_1 \\ 0 & 0 & g_2 a_2 e^{i\Delta_2 t}\hat{F}_2 \\ g_1 a_1^\dagger e^{-i\Delta_1 t}\hat{F}_1 & g_2 a_2^\dagger e^{-i\Delta_2 t}\hat{F}_2 & 0 \end{bmatrix} . \tag{23}$$

Here we have introduced the notation

$$\hat{F}_i=\sum_k(\,|\,k+K_i\,\rangle\langle\,k\,|\,e^{i\Lambda(k+K_i,k)t}+|\,k-K_i\,\rangle\langle\,k\,|\,e^{i\Lambda(k-K_i,k)t}),\quad i=1,2 \tag{24}$$

with

$$\Lambda(k\pm K_i,k)=\frac{\hbar(k\pm K_i)^2}{2m}-\frac{\hbar k^2}{2m}\ . \tag{25}$$

Otherwise, the notation is the same as that in Eq. (4). In Eq. (23) we have also used the fact that $\hat{F}_i$ is self-adjoint.

The density operator of the complete system (three-level atoms in motion plus two-mode field) satisfies, in the interaction picture, a Liouville equation similar to Eq. (6) with V given by Eq. (23). Next we introduce the reduced density operator ρ_f for the field only, as in Eq. (7), but now the trace over atomic states includes both internal (a, b, and c) and external (c.m. momentum) states of the atom.

In order to obtain the coarse-time-grained equation of motion for the field-density operator we again perform a second-order expansion in the coupling constants. Upon taking the trace over atomic states we can find the contribution to the field-density operator due to one atom injected into a coherent superposition of upper states $|a\rangle$ and $|b\rangle$ and into a momentum state $|k\rangle$. We find the total time rate of change of the field caused by an ensemble of such atoms by multiplying the one-atom expression with the pump rate r_k of these states and summing over the momentum states $|k\rangle$. In doing this we assume that the atoms are moving in a Maxwellian distribution at some temperature T. We may then write

$$\begin{aligned}\dot{\rho}&=\sum_k r_k\left[(-i)^2\int_0^\infty d\tau e^{-\gamma\tau}\int_t^{t+\tau}dt'\int_t^{t'}dt''[V(t'),[V(t''),\rho_a(t)\rho_f(t)]]\right]\\&=r\frac{1}{\pi^{1/2}\kappa}\int_{-\infty}^{\infty}dk\exp(-k^2/\kappa^2)\left[(-i)^2\int_0^\infty d\tau e^{-i\gamma\tau}\int_t^{t+\tau}dt'\int_t^{t'}dt''[V(t'),[V(t''),\rho_a(t)\rho_f(t)]]\right]\ .\end{aligned} \tag{26}$$

Here r is the total number of atoms excited into the coherent superposition of $|a\rangle$ and $|b\rangle$ per unit time and κ is the parameter of the Maxwellian distribution,

$$\kappa=mv/\hbar=(2mk_BT)^{1/2}/\hbar\ . \tag{27}$$

In solving Eq. (26) we assume the following initial condition:

$$\rho_a(t)=\tfrac{1}{2}(e^{i\phi}|a\rangle+|b\rangle)(e^{-i\phi}\langle a|+\langle b|)|k\rangle\langle k|\ , \tag{28}$$

corresponding to an atom prepared initially in a coherent superposition of internal states $|a\rangle$ and $|b\rangle$ and in an arbitrary momentum state. This initial condition is an obvious extension of (9) for the case when atomic motion is included.

It is now straightforward to show that the resulting master equation for the field-density operator ρ_f (with $\rho_f=\rho$, i.e., in the following we drop the subscripts f from the notation) is

$$\begin{aligned}\dot{\rho}=\frac{1}{\pi^{1/2}\kappa}\int_{-\infty}^{\infty}dk\exp(-k^2/\kappa^2)[&\tfrac{1}{2}\alpha_{11}(k)(\rho a_1a_1^\dagger-a_1^\dagger\rho a_1)+\tfrac{1}{2}\alpha_{22}(k)(\rho a_2a_2^\dagger-a_2^\dagger\rho a_2)\\&+\tfrac{1}{2}\alpha_{12}(k)(\rho a_2a_1^\dagger-a_1^\dagger\rho a_2)e^{i\phi}+\tfrac{1}{2}\alpha_{21}(k)(\rho a_1a_2^\dagger-a_2^\dagger\rho a_1)e^{-i\phi}+\text{H.c.}]\ ,\end{aligned} \tag{29}$$

with

$$\begin{aligned}\alpha_{11}(k)&=-\frac{1}{4}\frac{rg_1^2}{\gamma}\left[\frac{1}{\gamma+i\Delta_1^+(k)}+\frac{1}{\gamma+i\Delta_1^-(k)}\right],\\\alpha_{22}(k)&=-\frac{1}{4}\frac{rg_2^2}{\gamma}\left[\frac{1}{\gamma+i\Delta_2^+(k)}+\frac{1}{\gamma+i\Delta_2^-(k)}\right],\\\alpha_{12}(k)&=-\frac{1}{4}\frac{rg_1g_2}{\gamma+i(\Delta_1-\Delta_2)}\left[\frac{1}{\gamma+i\Delta_1^+(k)}+\frac{1}{\gamma+i\Delta_1^-(k)}\right],\\\alpha_{21}(k)&=-\frac{1}{4}\frac{rg_1g_2}{\gamma+i(\Delta_2-\Delta_1)}\left[\frac{1}{\gamma+i\Delta_2^+(k)}+\frac{1}{\gamma+i\Delta_2^-(k)}\right].\end{aligned} \tag{30}$$

Here

$$\Delta_{1,2}^{\pm}(k)=\Delta_{1,2}\mp\frac{\hbar kK_{1,2}}{m}-\frac{\hbar K_{1,2}^2}{2m} \tag{31}$$

is the detuning on the corresponding transition with the Doppler-shifted frequency and including recoil [cf. Eq. (5)]. The integrals in (29) involving $\alpha_{ij}(k)$ and the velocity distribution can be expressed in terms of the plasma dispersion

function Z (see Ref. 8), but for our purposes we do not need the explicit expressions.

In a manner similar to the one described in Sec. II we can convert the Liouville equation (29) into a Fokker-Planck equation. The term corresponding to the diffusion of the relative phase θ will be of the form given by Eq. (14) with

$$D(\theta)=\frac{1}{\pi^{1/2}\kappa}\int_{-\infty}^{\infty}dk\,\exp(-k^2/\kappa^2)\left[\frac{1}{8}\left[\frac{\alpha_{11}(k)}{\rho_1^2}+\frac{\alpha_{22}(k)}{\rho_2^2}-\frac{\alpha_{12}(k)}{\rho_1\rho_2}e^{-i\psi}-\frac{\alpha_{21}(k)}{\rho_1\rho_2}e^{i\psi}\right]+\text{c.c.}\right]. \tag{32}$$

We, again, are interested in finding conditions under which the diffusion constant $D(\theta)$ vanishes. A possible set of conditions is

$$g_1=g_2(\equiv g),\quad \Delta_1=\Delta_2(\equiv\Delta),\quad K_1=K_2(\equiv K)\,. \tag{33}$$

Under these conditions

$$D(\theta)=\frac{rg^2}{8\rho^2}\int_{-\infty}^{\infty}dk\,\exp(-k^2/\kappa^2)\left[\frac{1}{\gamma^2+\Delta^+(k)^2}+\frac{1}{\gamma^2+\Delta^-(k)^2}\right](1-\cos\psi)\,. \tag{34}$$

When $\psi=0$ the vanishing of the diffusion constant takes place. It can quite generally be shown that the relative phase ψ indeed locks to zero, i.e., the diffusion constant vanishes.

As a conclusion, the present calculation shows that CEL operation can be obtained in an inhomogeneously broadened Hanle laser. Furthermore, the first two of the conditions of Eqs. (33) are identical with the CEL operating conditions of a homogeneously broadened system, Eq. (16). The last of the conditions imposes a much stricter constraint than just the detuning conditions of Eq. (16). The actual laser frequencies have to be equal. Otherwise, the two modes experience different Doppler shifts and phase coherence will be destroyed. Thus the two modes in question can only differ in polarization.

IV. SUMMARY AND DISCUSSION

We have developed a quantum theory of the Hanle laser and found physical conditions for CEL operation. Namely, we have shown that the diffusion constant for the relative phase vanishes in the homogeneously broadened case if the detunings and coupling constants on the two lasing transitions are equal. Furthermore, no specific coherence-maintaining mechanism, other than the initial preparation of the upper lasing levels in a coherent superposition (coherent pumping via the Hanle effect), was found necessary to achieve quenching of the quantum noise from the relative phase.

The above conclusions are, essentially, maintained in the inhomogeneously broadened systems (inclusion of atomic motion) as well. The only difference is that, besides the equality of the detunings and coupling constants, one has to require the actual equality of the laser frequencies. Thus, in an inhomogeneously broadened Hanle laser, CEL operation is possible if the two modes differ in polarization only. In another publication[10] we show that inclusion of the atomic motion leads to some spectacular dynamical effects in the quantum-beat laser.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge numerous discussions on different parts of the problem with J. Gea-Banacloche, J. Krause, L. Pedrotti, and W. Schleich. This work was supported by the U.S. Office of Naval Research.

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