Theory of the detection of the field surrounding half-dressed sources

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(Received 22 October 1987; revised manuscript received 16 February 1988)

Half-dressed sources are defined as sources deprived partially or totally of the cloud of virtual quanta which surrounds them in the ground state of the total system. Two models of a half-dressed point source S are considered, the first in the framework of the theory of massive scalar fields and the second in quantum electrodynamics (QED). In both cases the detector is modeled by a second fully dressed source T of the same field, which is also bound to an oscillation center by harmonic forces. It is shown that when S at time $t=0$ is suddenly coupled to or decoupled from the field, the detector T, which is initially at rest, is set in motion after a time $t=R_0/c$, where R_0 is the S-T distance. Neglecting the reaction back on the field due to the oscillatory motion of T, the amplitude of oscillation for $t=\infty$ is obtained as a function of R_0. Thus the time-varying virtual field of S is shown to be capable of exerting a force which excites the model detector. For the QED case, this force is related to the properties of the energy density of the virtual field. This energy density displays a singularity at $r=ct$, and the mathematical nature of this singularity is studied in detail. In this way it is shown that the energy density of the time-dependent virtual field is rather different from that of a pulse of radiation emitted by a source during energy-conserving processes. The differences are discussed in detail, as well as the limitations of the model.

I. INTRODUCTION

The sources of a quantum-mechanical field are known to be surrounded by a cloud of virtual quanta even in the ground state of the whole system.[1] In these conditions one usually refers to the system in terms of a dressed source, in the vicinity of which the quantum fluctuations of the field are different from their zero-point values in the absence of the source.[2,3] In elementary-particle theory one may envisage situations where the cloud of virtual particles, following a traumatic event, is suddenly shaken off the source, leaving the latter partially deprived of its original cloud. In these cases one speaks of a half-dressed source, an extreme (and idealized) situation being the bare source when the stripping is total. These are obviously nonequilibrium situations, and one expects that processes should take place to regenerate the normal cloud of virtual particles around the source. A semiquantitative analysis of the regeneration process has been performed in a QED framework for a free electron, and extended to high-energy QCD (quantum chromodynamics) cases where gluon exchange is the dominant source-field interaction mechanism.[4] A complementary situation arises when an initially fully dressed source of a field is suddenly annihilated, leaving the virtual particles, belonging originally to the cloud dressing the source, without the physical support of the latter. This second type of situation has been analyzed for simple cases. Switching off of a fixed hadronic source in a Klein-Gordon field is a problem in quantum mesodynamics (QMD), and it is known to lead to the release of mesons originally belonging to the cloud of the hadron.[5] The process of this release has been discussed in terms of global quantities, such as the number of mesons being released and the total energy delivered to the meson field. The released mesons have been interpreted as real particles.

In both QED and QMD cases, a complete quantitative analysis of the detailed time development of the virtual cloud is still missing, although preliminary results of such an analysis have been published recently.[6] This is in sharp contrast with the situation related to the emission of real photons in a spontaneous-decay process of a QED source; in this case, in fact, detailed calculations of the real electromagnetic field surrounding an atom during the emission act do exist.[7] On the other hand, the desirability of a similarly quantitative approach for the time-dependent virtual cloud is also high, in view of the growing interest in this sort of problems.[8] In general, one would like to answer the following questions.

(i) How does one characterize the amount of virtual-particle cloud at a given point in space during the time development of the system?

(ii) How is it possible to obtain a space-time description of the transformation of the virtual quanta into radiation after decoupling a source from the field?

(iii) Should one also expect emission of real radiation during regeneration of the virtual-particle cloud of an initially bare source?

(iv) Is it possible to think of an experiment to detect the time evolution of the virtual field in a nonequilibrium situation of the kind described above?

The answers to the questions above are very much interrelated. It is instructive, however, to outline briefly the discussion of each of them separately. An answer to question (i) has been given in a series of papers, in terms of the total energy density of the meson or of the photon field for QMD and for QED models, respectively.[2,3] A preliminary answer to questions (ii) and (iii) has been provided by a theory presented in another recent paper,[6] where the time dependence of the energy density in the dressing-undressing process of an isolated source has been obtained for simple QMD and QED models. In all cases considered the dressing-undressing process of an isolated source was shown to take place within an expanding sphere of radius $r=ct$ centered at the source. Moreover, the process was shown to yield asymptotically at any point in space the equilibrium configuration of the virtual energy density corresponding to the ground state of the coupled source-field system for dressing, and to the bare zero-point vacuum for undressing. The energy density distribution in all cases studied was shown to possess a singularity at $r=ct$, but the nature of this singularity was not investigated.

The main aim of this paper is to provide an answer to question (iv). In fact, here we will limit ourselves to describing a gedanken experiment, which follows the introduction of a model consisting of a source S whose coupling with the field (scalar or electromagnetic) can be suddenly switched on or off, and of a detector consisting of a second source T, constantly coupled to the field by forces of the same nature as for S, placed at a distance r_0 from S. In addition, T is also bound to an oscillation center by harmonic forces as in Fig. 1, and the time development of the field induced by changes in S is monitored by the oscillatory motion of T, which is assumed to be initially at rest. In a certain sense this model may be considered as an elaboration of a model recently discussed by Drummond for QED.[9] Within the framework of this very idealized model, in Secs. II and III we shall be able to give a quantitative discussion as to the virtual or real nature of the time-dependent field which is being detected by T. In an effort to shed some light on the peculiar features of the motion of T in the QED case, in Sec. IV we will take up again the problem of the energy density at $r=ct$ of the virtual photon cloud, and we will discuss in some detail the structure of the expanding singularity as well as some of the integral properties of the electromagnetic energy density. This shall give us a chance to discuss in Sec. V energy conservation and the integral transform of the photon field, and to provide partial answers to questions (ii) and (iii) above, at least in the QED case. Finally, we will summarize our results in Sec. VI.

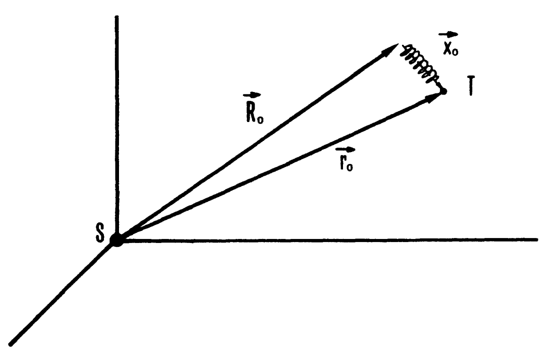

FIG. 1. Source-detector configuration in space. The coupling of the field with source S at the origin can be turned on and off. The spring represents harmonic forces binding detector T to oscillation center $\mathbf{R}_0$.

II. DETECTION OF HALF-DRESSED STATES IN QMD

A. General procedure

Consider two rigid meson sources S and T of densities $\rho_1(\mathbf{r})$ and $\rho_2(\mathbf{r})$, respectively, with S fixed at the origin and T, of mass m_0, bound by a local harmonic force of constant K to point $\mathbf{R}_0$. The actual position of T under the action of the local harmonic force is $\mathbf{r}_0$, with $\mathbf{x}_0=\mathbf{r}_0-\mathbf{R}_0$, as in Fig. 1. We take the Hamiltonian of the system to be[5]

$$\begin{aligned} H &= H_F+H'+H_T\ , \\ H_F &= \tfrac{1}{2}\int \{\dot\phi^2(\mathbf{r})+c^2[\nabla\phi(\mathbf{r})]^2+\mu^2c^2\phi^2(\mathbf{r})\}d^3r \\ &= \sum_{\mathbf{k}} \hbar\omega_k a^\dagger_{\mathbf{k}} a_{\mathbf{k}}\ , \\ H' &= -g\int \rho(\mathbf{r})\phi(\mathbf{r})d^3r \\ &= -g\sum_{\mathbf{k}}\left[\frac{\hbar}{2\omega_k V}\right]^{1/2}(\rho^*_{\mathbf{k}}a_{\mathbf{k}}+\rho_{\mathbf{k}}a^\dagger_{\mathbf{k}})\ , \\ \omega_k &= (c^2k^2+\mu^2c^2)^{1/2},\quad \rho_{\mathbf{k}}=\int\rho(\mathbf{r})e^{-i\mathbf{k}\cdot\mathbf{r}}d^3r\ , \\ \rho(\mathbf{r}) &= \rho_1(\mathbf{r})+\rho_2(\mathbf{r})\ ,\quad H_T=\frac{1}{2m_0}\mathbf{p}_0^2+\tfrac{1}{2}K\mathbf{x}_0^2\ . \end{aligned} \tag{2.1}$$

$\phi(\mathbf{r})$ in (2.1) is the scalar field amplitude, which can be expressed in terms of Bose creation and destruction operators as

$$\phi(\mathbf{r})=\sum_{\mathbf{k}}\left[\frac{\hbar}{2\omega_k V}\right]^{1/2}(a_{\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{r}}+a^\dagger_{\mathbf{k}}e^{-i\mathbf{k}\cdot\mathbf{r}})\ , \tag{2.2}$$

V being the quantization volume of the field. Moreover, c is the velocity of light, $\mu=mc/\hbar$ is the inverse Compton wavelength of the meson of mass m, g is the meson-source coupling strength, and $\mathbf{p}_0$ is the kinetic momentum of source T. One also has

$$\rho_{\mathbf{k}}=\rho_{1\mathbf{k}}+\rho_{2\mathbf{k}},\quad \rho_{i\mathbf{k}}=\int \rho_i(\mathbf{r})e^{-i\mathbf{k}\cdot\mathbf{r}}d^3r\ ,$$
$$H'=H_1'+H_2'\ , \tag{2.3}$$
$$H_i'=-g\sum_{\mathbf{k}}\left[\frac{\hbar}{2\omega_k V}\right]^{1/2}(\rho_{i\mathbf{k}}^* a_{\mathbf{k}}+\rho_{i\mathbf{k}}a_{\mathbf{k}}^\dagger)\ .$$

Specializing to unit point sources, one has

$$\rho_1(\mathbf{r})=\delta(\mathbf{r}),\quad \rho_2(\mathbf{r})=\delta(\mathbf{r}-\mathbf{r}_0),\quad \rho_{1\mathbf{k}}=1\ ,$$
$$\rho_{2\mathbf{k}}=e^{-i\mathbf{k}\cdot\mathbf{r}_0},\quad H_1'=-g\sum_{\mathbf{k}}\left[\frac{\hbar}{2\omega_k V}\right]^{1/2}(a_{\mathbf{k}}+a_{\mathbf{k}}^\dagger)\ ,$$
$$\begin{aligned}H_2'&=-g\sum_{\mathbf{k}}\left[\frac{\hbar}{2\omega_k V}\right]^{1/2}(a_{\mathbf{k}}e^{-i\mathbf{k}\cdot\mathbf{r}_0}+a_{\mathbf{k}}^\dagger e^{i\mathbf{k}\cdot\mathbf{r}_0})\\ &\simeq -g\sum_{\mathbf{k}}\left[\frac{\hbar}{2\omega_k V}\right]^{1/2}(a_{\mathbf{k}}e^{-i\mathbf{k}\cdot\mathbf{R}_0}+a_{\mathbf{k}}^\dagger e^{i\mathbf{k}\cdot\mathbf{R}_0})\\ &\quad+ig\mathbf{x}_0\cdot\sum_{\mathbf{k}}\left[\frac{\hbar}{2\omega_k V}\right]^{1/2}\mathbf{k}(a_{\mathbf{k}}e^{-i\mathbf{k}\cdot\mathbf{R}_0}-a_{\mathbf{k}}^\dagger e^{i\mathbf{k}\cdot\mathbf{R}_0})\ ,\end{aligned} \tag{2.4}$$

where the "dipole" approximation

$$e^{\pm i\mathbf{k}\cdot\mathbf{r}_0}\sim e^{\pm i\mathbf{k}\cdot\mathbf{R}_0}(1\pm i\mathbf{k}\cdot\mathbf{x}_0) \tag{2.5}$$

has been used. One should expect (2.5) to be fairly good if the distance between the two sources is much larger than the Compton radius of the meson, since in this case S and T can exchange essentially virtual mesons of long wavelength. In order that (2.5) be valid, we shall assume that the displacement of T from its equilibrium position is small compared with the wavelength of the mesons.

Substitution of (2.4) and (2.3) into (2.1) leads to

$$H=H_0+H_T+H_{\mathrm{TF}}\ ,$$
$$\begin{aligned}H_0&=\sum_{\mathbf{k}}\hbar\omega_k a_{\mathbf{k}}^\dagger a_{\mathbf{k}}-g\sum_{\mathbf{k}}\left[\frac{\hbar}{2\omega_k V}\right]^{1/2}(a_{\mathbf{k}}+a_{\mathbf{k}}^\dagger)\\ &\quad-g\sum_{\mathbf{k}}\left[\frac{\hbar}{2\omega_k V}\right]^{1/2}(a_{\mathbf{k}}e^{-i\mathbf{k}\cdot\mathbf{R}_0}+a_{\mathbf{k}}^\dagger e^{i\mathbf{k}\cdot\mathbf{R}_0})\ ,\end{aligned} \tag{2.6}$$
$$H_{\mathrm{TF}}=ig\mathbf{x}_0\cdot\sum_{\mathbf{k}}\left[\frac{\hbar}{2\omega_k V}\right]^{1/2}\mathbf{k}(a_{\mathbf{k}}e^{-i\mathbf{k}\cdot\mathbf{R}_0}-a_{\mathbf{k}}^\dagger e^{i\mathbf{k}\cdot\mathbf{R}_0})\ .$$

In view of (2.5), one has $\mathbf{k}\cdot\mathbf{x}_0<<1$ and H_{TF} can be considered as small with respect to H_0. In other words, the displacement due to the oscillations of source T may be considered to have a small influence on the field generated by the two sources considered fixed at the origin and at $\mathbf{R}_0$ respectively. We shall neglect this influence, which is equivalent to neglecting the reaction of T on the field due to its oscillations. Moreover, it is evident that H_{TF} in (2.6) can also be expressed as

$$H_{TF}=\mathbf{x}_0\cdot\mathbf{F}_0,\quad \mathbf{F}_0=\dot{\mathbf{p}}_0=\frac{i}{\hbar}[H,p]\ . \tag{2.7}$$

Thus operator

$$\mathbf{F}_0=ig\sum_{\mathbf{k}}\left[\frac{\hbar}{2\omega_k V}\right]^{1/2}\mathbf{k}(a_{\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{R}_0}-a_{\mathbf{k}}^\dagger e^{-i\mathbf{k}\cdot\mathbf{R}_0}) \tag{2.8}$$

plays the role of the force operator acting on the test body T at point $\mathbf{R}_0$.

Thus our program will be the following. First we obtain the time development of the field from H_0 alone, starting from an appropriate initial configuration of the system which corresponds to a half-dressed initial state of S. The time-dependent field obtained in this way will be used to evaluate a time-dependent quantum average of the force operator $\mathbf{F}_0$ acting on the oscillator degrees of freedom of the test body T, thereby yielding an effective Hamiltonian for the latter. This is eventually studied to discuss the influence of the time-dependent dressing of source S on the motion of test body T.

As for the initial conditions, we shall always assume that test body T is completely dressed, and consider the two cases in which source S is completely bare at $t=0$ (case A) and the complementary case in which S is completely dressed but it becomes suddenly decoupled from the field at $t=0$ (case B). In a more pictorial way, the two cases can be described as those of source S "appearing" at $t=0$ (case A) and of source S "being removed" at $t=0$ (case B). The energy density of the field has been obtained previously for both cases when S is isolated.[6] Since the procedure is similar when T also is present, we shall not discuss it in detail, and we only outline its application to the situation of interest here.

B. Force acting on the oscillator

Case A. The field Hamiltonian is given by H_0 as in (2.6). The bare vacuum $|0\rangle$ is the ground state of

$$H_F=\sum_{\mathbf{k}}\hbar\omega_k a_{\mathbf{k}}^\dagger a_{\mathbf{k}}\ . \tag{2.9}$$

We take as the state of the field at $t=0^+$ that obtained by dressing the test body alone, or $T_0|0\rangle$, with

$$\begin{aligned}T_0&=\exp(-n_0/2)\exp\left[\sum_{\mathbf{k}}\chi_{0\mathbf{k}}a_{\mathbf{k}}^\dagger\right]\\ &\quad\times\exp\left[-\sum_{\mathbf{k}}\chi_{0\mathbf{k}}^* a_{\mathbf{k}}\right]\ ,\\ n_0&=g^2\sum_{\mathbf{k}}\frac{1}{2\hbar\omega_k V},\quad \chi_{0\mathbf{k}}=\frac{1}{(2\hbar\omega_k^3 V)^{1/2}}ge^{-i\mathbf{k}\cdot\mathbf{R}_0}\ .\end{aligned} \tag{2.10}$$

This state evolves at time t into

$$\begin{aligned}|t\rangle&=\exp\left[-\frac{i}{\hbar}H_0 t\right]T_0|0\rangle\\ &=\exp\left[\frac{i}{\hbar}At\right]T\exp\left[-\frac{i}{\hbar}H_F t\right]T^{-1}T_0|0\rangle\ ,\end{aligned} \tag{2.11}$$

where

$$\begin{aligned}T&=\exp(-n/2)\exp\left[\sum_{\mathbf{k}}\chi_{\mathbf{k}}a_{\mathbf{k}}^\dagger\right]\exp\left[\sum_{\mathbf{k}}\chi_{\mathbf{k}}^* a_{\mathbf{k}}\right]\ ,\\ A&=g^2\sum_{\mathbf{k}}\frac{|1+e^{-i\mathbf{k}\cdot\mathbf{R}_0}|^2}{2\omega_k^2 V}\ ,\\ n&=g^2\sum_{\mathbf{k}}\frac{|1+e^{-i\mathbf{k}\cdot\mathbf{R}_0}|^2}{2\hbar\omega_k^3 V}\ ,\\ \chi_{\mathbf{k}}&=\frac{1}{(2\hbar\omega_k^3 V)^{1/2}}g(1+e^{-i\mathbf{k}\cdot\mathbf{R}_0})\ .\end{aligned} \tag{2.12}$$

Using (2.10) and (2.12) one obtains in a straightforward way

$$T_0^{-1}T\exp\left[\frac{i}{\hbar}H_Ft\right]T^{-1}\mathbf{F}_0T\exp\left[-\frac{i}{\hbar}H_Ft\right]T^{-1}T_0$$
$$=ig\sum_{\mathbf{k}}\left[\frac{\hbar}{2\omega_kV}\right]^{1/2}\mathbf{k}\{[(a_{\mathbf{k}}+\mathcal{X}_{0\mathbf{k}}-\mathcal{X}_{\mathbf{k}})e^{-i\omega_kt}+\mathcal{X}_{\mathbf{k}}]e^{i\mathbf{k}\cdot\mathbf{R}_0}-\text{H.c.}\} \quad (2.13)$$

and

$$\langle t\,|\,\mathbf{F}_0\,|\,t\rangle=-\frac{1}{V}g^2\text{Im}\left[\sum_{\mathbf{k}}\frac{1}{\omega_k^2}\mathbf{k}e^{i\mathbf{k}\cdot\mathbf{R}_0}(1-e^{-i\omega_kt})\right]. \quad (2.14)$$

Rather than working with a vector, it is more convenient to project $\mathbf{F}_0$ in the $\mathbf{R}_0$ direction, obtaining

$$\langle t\,|\,F_{0R}\,|\,t\rangle=-\frac{1}{V}g^2\text{Im}\left[\sum_{\mathbf{k}}\frac{1}{\omega_k^2}\mathbf{k}\cdot\hat{\mathbf{R}}_0e^{i\mathbf{k}\cdot\mathbf{R}_0}\times(1-e^{-i\omega_kt})\right]. \quad (2.15)$$

Changing the sum into an integral and performing the latter in a fashion parallel to that described in a previous paper[6] finally yields

$$\langle t\,|\,F_{0R}\,|\,t\rangle=-\frac{1}{4\pi c^2}g^2\frac{\partial}{\partial R_0}\frac{1}{R_0}\frac{\partial}{\partial R_0}\int_0^t\mathcal{F}(t',R_0)dt',$$
$$\mathcal{F}(t,R_0)=cJ_0[\mu c(t^2-R_0^2/c^2)^{1/2}]\Theta(t-R_0/c), \quad (2.16)$$

where J_0 is the Bessel function of integer order and $\Theta(x)$ is the usual Heaviside function.

Case B. The field Hamiltonian for $t>0$ is obtained from (2.6) by eliminating the interaction between S and the field, and it is

$$H_0=\sum_{\mathbf{k}}\hbar\omega_ka_{\mathbf{k}}^\dagger a_{\mathbf{k}}-g\sum_{\mathbf{k}}\left[\frac{\hbar}{2\omega_kV}\right]^{1/2}(a_{\mathbf{k}}e^{-i\mathbf{k}\cdot\mathbf{R}_0}+a_{\mathbf{k}}^\dagger e^{i\mathbf{k}\cdot\mathbf{R}_0}) \quad (2.17)$$

The state of the field at $t=0^+$ is $T\,|\,0\rangle$, where T is the same as in (2.12). This state at time t evolves into

$$|\,t\rangle=e^{-(i/\hbar)H_0t}T\,|\,0\rangle$$
$$=\exp\left[\frac{i}{\hbar}A_0t\right]\exp\left[-\frac{i}{\hbar}H_Ft\right]T_0^{-1}T\,|\,0\rangle, \quad (2.18)$$

with H_0 given by (2.17) and

$$A_0=g^2\sum_{\mathbf{k}}\frac{1}{2\omega_k^3V}. \quad (2.19)$$

In a way similar to that leading to (2.13) one has

$$T^{-1}T_0\exp\left[\frac{i}{\hbar}H_Ft\right]T_0^{-1}\mathbf{F}_0T_0\exp\left[-\frac{i}{\hbar}H_Ft\right]T_0^{-1}T$$
$$=ig\sum_{\mathbf{k}}\left[\frac{\hbar}{2\omega_kV}\right]^{1/2}\mathbf{k}\{[(a_{\mathbf{k}}+\mathcal{X}_{\mathbf{k}}-\mathcal{X}_{0\mathbf{k}})e^{-i\omega_kt}+\mathcal{X}_{0\mathbf{k}}]e^{i\mathbf{k}\cdot\mathbf{R}_0}-\text{H.c.}\} \quad (2.20)$$

and, for the radial component of the force,

$$\langle t\,|\,F_{0R}\,|\,t\rangle=-\frac{1}{V}g^2\text{Im}\left[\sum_{\mathbf{k}}\frac{1}{\omega_k^2}\mathbf{k}\cdot\hat{R}_0e^{i\mathbf{k}\cdot\mathbf{R}_0}e^{-i\omega_kt}\right]. \quad (2.21)$$

Changing the sum into an integral leads to

$$\langle t\,|\,F_{0R}\,|\,t\rangle=-\frac{1}{4\pi c^2}g^2\frac{\partial}{\partial R_0}\frac{1}{R_0}\frac{\partial}{\partial R_0}\times\left[\frac{1}{\mu}e^{-\mu R_0}-\int_0^t\mathcal{F}(t',R_0)dt'\right], \quad (2.22)$$

where $\mathcal{F}(t,R_0)$ is the same as in (2.16).

Unfortunately we have not been able to evaluate explicitly the integral in $\mathcal{F}(t,R_0)$. A few qualitative considerations, however, are useful to understand the physical meaning of results (2.16) and (2.22). When the bare source "appears" at $t=0$, the force (2.16) acting on the test oscillator T remains zero until $t=R_0/c$. After this time it oscillates until it settles to the asymptotic value obtained by using[5]

$$\int_0^\infty\mathcal{F}(t',R_0)dt'=\frac{1}{\mu}e^{-\mu R_0}. \quad (2.23)$$

This is the well-known derivative of the Yukawa potential (case A)

$$\langle\infty\,|\,F_{0R}\,|\,\infty\rangle=\frac{\partial}{\partial R_0}\left[\frac{1}{4\pi c^2}g^2\frac{1}{R_0}e^{-\mu R_0}\right]. \quad (2.24)$$

On the other hand, when the source is removed at $t=0$, the force acting on the test oscillator remains of the static Yukawa type until $t=R_0/c$, due to the vanishing of the integral in (2.22) for $t<R_0/c$. After this time, the force oscillates and it settles to zero asymptotically (case B),

$$\langle\infty\,|\,F_{0R}\,|\,\infty\rangle=0, \quad (2.25)$$

in view of (2.23).

This behavior is an interesting example of causality built in the relativistic formulation of the meson detector problem.

C. Dynamics of the detector oscillator

We assume for simplicity that the motion of the detector oscillator can only take place along the direction of $\mathbf{R}_0$, and we also put $\langle t\,|\,F_{0R}\,|\,t\rangle=\langle F_{0R}\rangle$. According to the program outlined in Sec. II A, we neglect the reaction due to oscillatory motion on the field, and we study the effective oscillator Hamiltonian

$$H_{\rm eff}=H_T+H_{TF},\quad H_T=\frac{1}{2m_0}\mathbf{p}_0^2+\tfrac{1}{2}K\mathbf{x}_0^2\,,$$
$$H_{TF}=-x_0\langle F_{0R}\rangle \tag{2.26}$$

which is a time-dependent Hamiltonian, in view of (2.16) and (2.22).

We shall imagine that the detector oscillator's mass m_0 is so large that a quantum description would reveal essentially an effectively classical behavior. This enables us to use an entirely classical and well-known approach for forced oscillations.[10] In these conditions, and assuming the oscillator to be at rest in equilibrium at $t=0$, in the case of an essentially bare source S (case A) we obtain

$$x_0(t)=\mathrm{Im}\left[e^{i\omega_0 t}\int_0^t\frac{1}{\omega_0 m_0}\langle F_{0R}\rangle e^{-i\omega t'}dt'\right], \tag{2.27}$$

where $\omega_0=\sqrt{K/m_0}$ is the natural frequency of the test oscillator. Integrating by parts and using (2.16) yields

$$x_0(t)=\mathrm{Im}\left[-\frac{i}{\omega_0^2 m_0}\frac{1}{4\pi c^2}g^2\frac{\partial}{\partial R_0}\frac{1}{R_0}\frac{\partial}{\partial R_0}\int_{R_0/c}^t \mathcal{F}(t',R_0)dt' + \frac{i}{\omega_0^2 m_0}e^{i\omega_0 t}\frac{1}{4\pi c}g^2\frac{\partial}{\partial R_0}\frac{1}{R_0}\frac{\partial}{\partial R_0}\int_{R_0/c}^t e^{-i\omega t'}J_0[\mu c(t'^2-R_0^2/c^2)^{1/2}]dt'\right]. \tag{2.28}$$

Clearly $x_0(t)$ vanishes for $t<R_0/c$ as expected.

Ignorance of the explicit form of $\mathcal{F}(t,R_0)$ prevents us from obtaining the detailed behavior of the detector amplitude for short times. For large t, however, we may use (2.23), and (2.28) tends asymptotically to

$$x_0(t)=-\frac{1}{\omega_0^2 m_0}\frac{1}{4\pi c^2}\frac{\partial}{\partial R_0}\frac{1}{R_0}\frac{\partial}{\partial R_0}\frac{1}{\mu}e^{-\mu R_0} +\mathrm{Im}\left[\frac{i}{\omega_0^2 m_0}e^{i\omega_0 t}\frac{g^2}{4\pi c}\frac{\partial}{\partial R_0}\frac{1}{R_0}\frac{\partial}{\partial R_0}\int_{R_0/c}^\infty e^{-i\omega_0 t'}J_0[\mu c(t'^2-R_0^2/c^2)^{1/2}]dt'\right]\quad(t\to\infty)\,. \tag{2.29}$$

Moreover,[11]

$$\int_{R_0/c}^\infty e^{-i\omega_0 t}J_0[\mu c(t'^2-R_0^2/c^2)^{1/2}]dt'=\frac{1}{c}\frac{1}{(\mu^2-\omega_0^2/c^2)^{1/2}}e^{-R_0(\mu^2-\omega_0^2/c^2)^{1/2}}, \tag{2.30}$$

which can be substituted into (2.29) to obtain

$$x_0(t)=-\frac{1}{\omega_0^2 m_0}\frac{1}{4\pi c^2}g^2\frac{1}{R_0}\left[\mu+\frac{1}{R_0}\right]e^{-\mu R_0} +\mathrm{Im}\left[\frac{i}{\omega_0^2 m_0}e^{i\omega_0 t}\frac{1}{4\pi c^2}g^2\frac{1}{R_0}\left[(\mu^2-\omega_0^2/c^2)^{1/2}+\frac{1}{R_0}\right]e^{-R_0(\mu^2-\omega_0^2/c^2)^{1/2}}\right]\quad(t\to\infty)\,. \tag{2.31}$$

It is easy to convince oneself that the first of the two terms on the rhs (right-hand side) of (2.31) is the shift of the oscillation center of detector T due to the static asymptotic part (2.24) of $\langle F_{0R}\rangle$, whereas the second term represents oscillations which survive for large t. This second part can be written explicitly as

$$\begin{aligned}&\frac{1}{\omega_0^2 m_0}\frac{1}{4\pi c^2}g^2\frac{1}{R_0}\left[(\mu^2-\omega_0^2/c^2)^{1/2}+\frac{1}{R_0}\right]e^{-R_0(\mu^2-\omega_0^2/c^2)^{1/2}}\cos(\omega_0 t)\quad(\omega_0<\mu c)\,,\\ &\frac{1}{\omega_0^2 m_0}\frac{1}{4\pi c^2}g^2\frac{1}{R_0}\left[\frac{1}{R_0}\cos[\omega_0 t-R_0(\omega_0^2/c^2-\mu^2)^{1/2}]-(\omega_0^2/c^2-\mu^2)^{1/2}\sin[\omega_0 t-R_0(\omega_0^2/c^2-\mu^2)^{1/2}\right]\quad(\omega_0>\mu c)\,.\end{aligned} \tag{2.32}$$

We see that the amplitude with which the detector is left to oscillate is severely reduced by the presence of the exponential for $\omega_0<\mu/c$ at a relatively large source-detector distance R_0. On the other hand, this exponential damping of the oscillation amplitude is absent for $\omega_0>\mu/c$, in which case it is substituted by an R_0^{-1} behavior at large S-T distances.

This behavior can be qualitatively understood in terms of Fig. 2, which displays the stop band in the dispersion relation of the meson medium between frequencies 0 and μc. This stop band is obtained from the meson spectrum in (2.1), and its physical origin is related to the minimum energy $\hbar\mu c$ which is necessary in order to create a meson from the vacuum. It is obvious that the propagation of virtual mesons, capable of resonantly exciting the oscilltor, from the source out to the oscillator location, is hindered if ω_0 happens to be located within this stop band, because these mesons are "reflected" by the vacuum, very much like phonons trying to propagate in the forbidden gap between the acoustic and the optical branches in an

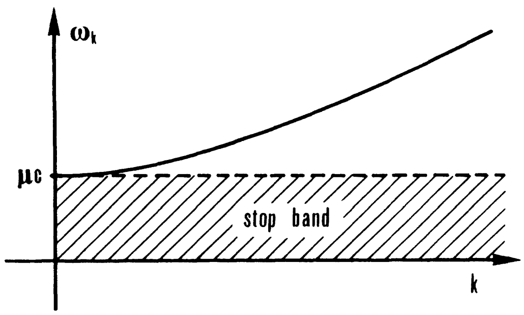

FIG. 2. Dispersion relation of the meson field in arbitrary units. μ^{-1} is the meson Compton radius. The stop band between 0 and μc originates from the minimum energy $\hbar\mu c$ required to create a meson from vacuum.

insulating crystal. On the other hand, if $\omega_0 > \mu c$, some of the virtual mesons which propagate freely from S to T shall be able to resonate with the test oscillator, and this explains the absence of the damping exponential in the second part of (2.32).

In conclusion, we have shown that the simple detector model discussed is capable of detecting the regeneration of the virtual meson cloud which develops around an initially bare source. A similar analysis, which we do not report here, can be performed for the case of the source S disappearing at $t=0$. Finally, we like to mention that a quantum treatment of the detector oscillator T does not seem to add much to the concepts that we have discussed for the classical treatment.

III. DETECTION OF HALF-DRESSED STATES IN QED

A. General procedure

Here we consider two sources of ground-state static polarizability α^S and α^T, such as two neutral atoms or molecules; more generally, we may assume electrical anisotropy with static polarizability tensors α^S_{mn} and α^T_{mn}. The geometrical arrangement is the same as in Fig. 1, with S at the origin and T (of mass m_0) bound by a local harmonic force of constant K to point $\mathbf{R}_0$. As for the source-field coupling, we will take the simplest Craig-Power (CP) form, which is quadratic in the field components and which does not involve atomic operators.[12] Thus the Hamiltonian of the system is

$$\begin{aligned} &H = H_F + H' + H_T + H_M \ , \\ &H_F = \frac{1}{8\pi}\int [\mathbf{E}_\perp^2(\mathbf{r}) + \mathbf{B}^2(\mathbf{r})] d^3r \ , \\ &H' = -\tfrac{1}{2}\alpha^S_{mn} E_{\perp m}(0) E_{\perp n}(0) - \tfrac{1}{2}\alpha^T_{mn} E_{\perp m}(\mathbf{r}_0) E_{\perp n}(\mathbf{r}_0) \ , \\ &H_T = \frac{1}{2m_0}\mathbf{p}_0^2 + \tfrac{1}{2}K\mathbf{x}_0^2, \quad H_M = \sum_{i,l} E^i_l \,|\, l \rangle\langle l \,| \quad (i = T,S) \ , \end{aligned} \tag{3.1}$$

where H_M has been added here for completeness, l labeling the internal molecular eigenstates, since in reality we will take both S and T to be permanently in their lowest possible eigenstate with $l=0$.

Some care must be used in interpreting the quantities appearing in (3.1). In fact, if one is to assign a well-defined physical meaning to this Hamiltonian in the framework of molecular QED, one must remember that the CP Hamiltonian is derived from a multipolar one (in dipole approximation) where the physical meaning of $\mathbf{E}_\perp$ is really the transverse electric displacement, and not the transverse electric field.[13] It can be shown that in the course of the unitary transformation leading from the multipolar to the CP form, this interpretation of $\mathbf{E}_\perp$ does not change, and it is well known that the transverse displacement coincides with the total electric field outside the source; this ensures gauge invariance of our $\mathbf{E}_\perp$. We hope to discuss this aspect of the CP Hamiltonian in a forthcoming paper. Moreover, if in this scheme one wishes to maintain the physical meaning of α^i as the ground-state electric polarizability of the sources, one should remember that the influence of the sources on the field modes resonant with any of the internal frequencies is badly misrepresented by (3.1), which consequently can be considered as a fair approximation of the true Hamiltonian only for the low-frequency, or long-wavelength, virtual photons. In turn, these are the only photons likely to reach regions far away from each source in view of their relatively long lifetime (the "radiation zone").[3] Thus the validity of the conclusions derived on the basis of (3.1) shall be limited to situations where the distance between S and T is large enough to place each source in the radiation zone of the other.

On the other hand, one may also legitimately pretend that (3.1) is a model Hamiltonian which describes an abstract source-field system in QED, much in the same sense as (2.1) for a scalar field in QMD, which should possess internal mathematical consistency, and which should be valid for any intersource distance (at least in our scheme of pointlike sources).

We are now ready to outline our approach. We expand $\mathbf{E}_\perp(r)$ and $\mathbf{B}(r)$ as

$$\begin{aligned} &\mathbf{E}_\perp(\mathbf{r}) = i\sum_{\mathbf{k},j}\left[\frac{2\pi\hbar\omega_k}{V}\right]^{1/2}(\mathbf{e}_{\mathbf{k}j}a_{\mathbf{k}j}e^{i\mathbf{k}\cdot\mathbf{r}} - \mathbf{e}^*_{\mathbf{k}i}a^\dagger_{\mathbf{k}_i}e^{-i\mathbf{k}\cdot\mathbf{r}}) \ , \\ &\mathbf{B}(\mathbf{r}) = -i\sum_{\mathbf{k},j}\left[\frac{2\pi\hbar\omega_k}{V}\right]^{1/2}(\mathbf{b}_{\mathbf{k}i}a_{\mathbf{k}j}e^{i\mathbf{k}\cdot\mathbf{r}} - \mathbf{b}^*_{\mathbf{k}j}a^\dagger_{\mathbf{k}j}e^{-i\mathbf{k}\cdot\mathbf{r}}) \ , \end{aligned} \tag{3.2}$$

$$\omega_k = ck, \quad \mathbf{b}_{\mathbf{k}j} = \hat{\mathbf{k}}\times\mathbf{e}_{\mathbf{k}j} \ ,$$

where ω_k is the frequency of the field modes, $\mathbf{e}_{\mathbf{k}j}$ being polarization vectors and $\hat{\mathbf{k}}$ the unit vector in the direction of $\mathbf{k}$. The creation and destruction operators $a^\dagger_{\mathbf{k}j}$ and $a_{\mathbf{k}j}$ are usual Bose operators pertaining to the $\mathbf{k}j$ photons, and V is the quantization volume.

Introducing (3.2) into (3.1) one has

$$H_F = \sum_{\mathbf{k},i}\hbar\omega_k a^\dagger_{\mathbf{k}j}a_{\mathbf{k}j} \tag{3.3}$$

apart from zero-point terms (ZPT). Moreover,

$$E_{\perp m}(0)E_{\perp n}(0)=-\frac{2\pi\hbar}{V}\sum_{\mathbf{k}_1,\mathbf{k}_2,j_1,j_2}\sqrt{\omega_{k_1}\omega_{k_2}}[(\mathbf{e}_{\mathbf{k}_1j_1})_m(\mathbf{e}_{\mathbf{k}_2j_2})_n a_{\mathbf{k}_1j_1}a_{\mathbf{k}_2j_2}-(\mathbf{e}_{\mathbf{k}_ij_1})_m(\mathbf{e}^*_{\mathbf{k}_2j_2})_n a_{\mathbf{k}_1j_1}a^\dagger_{\mathbf{k}_2j_2}+\text{H.c.}] \tag{3.4}$$

and

$$\begin{aligned}E_{\perp m}(\mathbf{r}_0)E_{\perp n}(\mathbf{r}_0)=&-\frac{2\pi\hbar}{V}\sum_{\mathbf{k}_1\mathbf{k}_2,j_1j_2}\sqrt{\omega_{k_1}\omega_{k_2}}[(\mathbf{e}_{\mathbf{k}_1j_1})_m(\mathbf{e}_{\mathbf{k}_2j_2})_n a_{\mathbf{k}_1j_1}a_{\mathbf{k}_2j_2}e^{i(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{r}_0}\\
&-(\mathbf{e}_{\mathbf{k}_1j_1})_m(\mathbf{e}^*_{\mathbf{k}_2j_2})_n a_{\mathbf{k}_1j_1}a^\dagger_{\mathbf{k}_2j_2}e^{i(\mathbf{k}_1-\mathbf{k}_2)\cdot\mathbf{r}_0}+\text{H.c.}]\\
\simeq&\, E_{\perp m}(\mathbf{R}_0)E_{\perp n}(\mathbf{R}_0)\\
&-i\frac{2\pi\hbar}{V}\mathbf{x}_0\cdot\sum_{\mathbf{k}_1,\mathbf{k}_2,j_1,j_2}\sqrt{\omega_{k_1}\omega_{k_2}}[(\mathbf{k}_1+\mathbf{k}_2)(\mathbf{e}_{\mathbf{k}_1j_1})_m(\mathbf{e}_{\mathbf{k}_2j_2})_n a_{\mathbf{k}_1j_1}a_{\mathbf{k}_2j_2}e^{i(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{R}_0}\\
&-(\mathbf{k}_1-\mathbf{k}_2)(\mathbf{e}_{\mathbf{k}_1j_1})_m(\mathbf{e}^*_{\mathbf{k}_2j_2})_n a_{\mathbf{k}_1j_1}a^\dagger_{\mathbf{k}_2j_2}e^{i(\mathbf{k}_1-\mathbf{k}_2)\cdot\mathbf{R}_0}-\text{H.c.}]\ ,\end{aligned} \tag{3.5}$$

where we have approximated

$$e^{\pm i(\mathbf{k}_1\pm\mathbf{k}_2)\cdot\mathbf{r}_0}\sim e^{\pm i(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{R}_0}[1\pm i(\mathbf{k}_1\pm\mathbf{k}_2)\cdot\mathbf{x}_0]\ . \tag{3.6}$$

This is a "dipole" approximation of the same nature as (2.5), and it involves the assumption that the displacement of the T oscillator should be small in comparison with the wavelength of the radiation exchanged between S and T. This is certainly compatible with the restrictions on the applicability of (3.1) to physical systems discussed at the beginning of this section, because distant atoms or molecules are likely to exchange mostly long-wavelength photons. We shall come to this point again later on in the course of this paper. In view of (3.5), it is convenient to rearrange terms in (3.1) to obtain

$$\begin{aligned}H&=H_M+H_0+H_{TF}+H_T\ ,\\
H_0&=H_F-\tfrac{1}{2}\alpha^S_{mn}E_{\perp m}(0)E_{\perp n}(0)\\
&\quad-\tfrac{1}{2}\alpha^T_{mn}E_{\perp m}(\mathbf{R}_0)E_{\perp n}(\mathbf{R}_0)\ ,\\
H_{TF}&=-\tfrac{1}{2}\alpha^T_{mn}[E_{\perp m}(\mathbf{r}_0)E_{\perp n}(\mathbf{r}_0)-E_{\perp m}(\mathbf{R}_0)E_{\perp n}(\mathbf{R}_0)]\ .\end{aligned} \tag{3.7}$$

Like in the meson case, condition (3.6) implies $H_{TF}\ll H_0$, and it becomes plausible to neglect approximately the effects on the field due to the harmonic displacement of source T.

Our program here runs parallel to that outlined in Sec. II A. Thus we first evaluate the field using H_0 alone, thereby neglecting the reaction of the oscillations of detector T. Successively we feed this result into H_{TF}; consequently H_T+H_{TF} becomes an effective Hamiltonian for the detector, which we use to obtain its dynamics under the influence of the time-dependent dressing of source S. The initial conditions for the field are the same as those considered in Sec. II A, namely, T with its full dressing cloud, and S completely bare at $t=0$ (case A) or S suddenly decoupled from its fully developed cloud at $t=0$ (case B). A special subsection shall be dedicated to a discussion of the energy density, which is necessary here to obtain much more information about the energy density than in a previous paper.[6]

B. Dynamics of the field

From (3.3) and (3.5), the field Hamiltonian for case A is

$$\begin{aligned}H_0=&\sum_{\mathbf{k},j}\hbar\omega_k a^\dagger_{\mathbf{k}j}a_{\mathbf{k}j}+\tfrac{1}{2}\alpha^S_{mn}\frac{2\pi\hbar}{V}\sum_{\mathbf{k}_1,\mathbf{k}_2,j_1,j_2}\sqrt{\omega_{k_1}\omega_{k_2}}[(\mathbf{e}_{\mathbf{k}_1j_1})_m(\mathbf{e}_{\mathbf{k}_2j_2})_n a_{\mathbf{k}_1j_1}a_{\mathbf{k}_2j_2}(1+\beta_{mn}e^{i(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{R}_0})\\
&-(\mathbf{e}_{\mathbf{k}_1j_1})_m(\mathbf{e}_{\mathbf{k}_2j_2})_n a_{\mathbf{k}_1j_1}a^\dagger_{\mathbf{k}_2j_2}(1+\beta_{mn}e^{i(\mathbf{k}_1-\mathbf{k}_2)\cdot\mathbf{R}_0})+\text{H.c.}]\ ,\end{aligned} \tag{3.8}$$

where we have introduced $\beta_{mn}=\alpha^T_{mn}/\alpha^S_{mn}$.

In contrast with the meson case in Sec. II, here we have found it convenient to develop our theory in the Heisenberg representation. The solution of the relevant Heisenberg equations, up to terms linear in α (i.e., quadratic in e) is

$$\begin{aligned}a_{\mathbf{k}j}(t)=&\,a_{\mathbf{k}i}(0)e^{-i\omega_k t}+\tfrac{1}{2}\alpha^S_{mn}\frac{2\pi}{V}\sum_{\mathbf{k}',j'}\sqrt{\omega_k\omega_{k'}}\{[(\mathbf{e}_{\mathbf{k}'j'})_m(\mathbf{e}^*_{\mathbf{k}j})_n+(\mathbf{e}^*_{\mathbf{k}j})_m(\mathbf{e}_{\mathbf{k}'j'})_n](1+\beta_{mn}e^{i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{R}_0})e^{-i\omega_k t}F_{kk'}(t)a_{\mathbf{k}'j'}(0)\\
&-[(\mathbf{e}^*_{\mathbf{k}'j'})_m(\mathbf{e}^*_{\mathbf{k}j})_n+(\mathbf{e}^*_{\mathbf{k}j})_m(\mathbf{e}^*_{\mathbf{k}'j'})_n][1+\beta_{mn}e^{-i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{R}_0}]\\
&\times e^{-i\omega_k t}G_{kk'}(t)a^\dagger_{\mathbf{k}'j'}(0)\}\ ,\end{aligned} \tag{3.9}$$

where

$$F_{kk'}(t)=\frac{e^{i(\omega_k-\omega_{k'})t}}{\omega_k-\omega_{k'}},\quad G_{kk'}(t)=\frac{e^{i(\omega_k+\omega_{k'})t}}{\omega_k+\omega_{k'}}\ . \tag{3.10}$$

Remember that we are considering case A, with the S source completely bare at $t=0$, in the presence of the complete virtual cloud dressing source T, which we evaluate by perturbation theory at $O(\alpha)$. The corresponding state is

$$|0'\rangle=|0\rangle+(E_0-H_F-H_M)^{-1}\tfrac{1}{2}\alpha_{mn}^T\frac{2\pi\hbar}{V}\sum_{\mathbf{k}'',\mathbf{k}''',j'',j'''}\sqrt{\omega_{k''}\omega_{k'''}}(\mathbf{e}^*_{\mathbf{k}''j''})_m(\mathbf{e}^*_{\mathbf{k}'''j'''})_n a^\dagger_{\mathbf{k}''j''}a^\dagger_{\mathbf{k}'''j'''}e^{-i(\mathbf{k}''+\mathbf{k}''')\cdot\mathbf{R}_0}|0\rangle\ , \tag{3.11}$$

where $|0\rangle$ is the ground state of H_F+H_M of energy E_0 (bare vacuum). From (3.9) and (3.11) we obtain, neglecting $O(\alpha^2)$,

$$\begin{aligned}\langle 0'|a_{\mathbf{k}_1j_1}(t)a_{\mathbf{k}_2j_2}(t)|0'\rangle&=[\langle 0'|a^\dagger_{\mathbf{k}_1j_1}(t)a^\dagger_{\mathbf{k}_2j_2}(t)|0'\rangle]^*\\&=-\frac{\pi}{V}\sqrt{\omega_{k_1}\omega_{k_2}}[(\mathbf{e}^*_{\mathbf{k}_1j_1})_m(\mathbf{e}^*_{\mathbf{k}_2j_2})_n+(\mathbf{e}^*_{\mathbf{k}_2j_2})_m(\mathbf{e}^*_{\mathbf{k}_1j_1})_n]e^{-i(\omega_{k_1}+\omega_{k_2})t}\\&\quad\times\left[\alpha^T_{mn}e^{-i(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{R}_0}\frac{1}{\omega_{k_1}+\omega_{k_2}}+\alpha^S_{mn}(1+\beta_{mn}e^{-i(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{R}_0})G_{k_2k_1}(t)\right]\ .\end{aligned} \tag{3.12}$$

Moreover, we find at the same order in α,

$$\langle 0'|a^\dagger_{\mathbf{k}_1j_1}(t)a_{\mathbf{k}_2j_2}(t)|0'\rangle=0,\quad \langle 0'|a_{\mathbf{k}_1j_1}(t)a^\dagger_{\mathbf{k}_2j_2}(t)|0'\rangle=\delta_{\mathbf{k}_1\mathbf{k}_2}\delta_{j_1j_2}\equiv Z\ . \tag{3.13}$$

For case B, with S suddenly decoupled from its complete cloud at $t=0$, the field Hamiltonian is obtained from (3.8) by putting $\beta_{mn}=0$, and the initial state, dressed by both S and T, is

$$\begin{aligned}|0'\rangle=|0\rangle+(E_0-H_F-H_M)^{-1}\tfrac{1}{2}\alpha^S_{mn}\frac{2\pi\hbar}{V}\sum_{\mathbf{k}'',\mathbf{k}''',j'',j'''}\sqrt{\omega_{k''}\omega_{k'''}}\\\times(\mathbf{e}^*_{\mathbf{k}''j''})_m(\mathbf{e}^*_{\mathbf{k}'''j'''})_n(1+\beta_{mn}e^{-i(\mathbf{k}''+\mathbf{k}''')\cdot\mathbf{R}_0})a^\dagger_{\mathbf{k}''j''}a^\dagger_{\mathbf{k}'''j'''}|0\rangle\ .\end{aligned} \tag{3.14}$$

The calculations are developed along the same lines as for case A. The final result at order α is

$$\begin{aligned}\langle 0'|a_{\mathbf{k}_1j_1}(t)a_{\mathbf{k}_2j_2}(t)|0'\rangle&=[\langle 0'|a^\dagger_{\mathbf{k}_1j_1}(t)a^\dagger_{\mathbf{k}_2j_2}(t)|0'\rangle]^*\\&=-\frac{\pi}{V}\sqrt{\omega_{k_1}\omega_{k_2}}[(\mathbf{e}^*_{\mathbf{k}_1j_1})_m(\mathbf{e}^*_{\mathbf{k}_2j_2})_n+(\mathbf{e}^*_{\mathbf{k}_2j_2})_m(\mathbf{e}^*_{\mathbf{k}_1j_1})_n]e^{-i(\omega_{k_1}+\omega_{k_2})t}\\&\quad\times\left[\alpha^S_{mn}(1+\beta_{mn}e^{-i(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{R}_0})\frac{1}{\omega_{k_1}+\omega_{k_2}}+\alpha^T_{mn}e^{-i(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{R}_0}G_{k_1k_2}(t)\right]\end{aligned} \tag{3.15}$$

and, at the same order in α,

$$\langle 0'|a^\dagger_{\mathbf{k}_1j_1}(t)a_{\mathbf{k}_2j_2}(t)|0'\rangle=0,\quad \langle 0'|a_{\mathbf{k}_1j_1}(t)a^\dagger_{\mathbf{k}_2j_2}(t)|0'\rangle=\delta_{\mathbf{k}_1\mathbf{k}_2}\delta_{j_1j_2}\equiv Z \tag{3.16}$$

The Z appearing in the second part of (3.13) and of (3.16) are space independent and time independent. They cannot contribute any net force acting on the oscillator degrees of freedom of T, nor any space dependence to the energy density, and we shall completely disregard them in the future.

C. Coupling of the detector oscillator to the electromagnetic field

H_{TF} in (3.7) is obtained in second quantization using (3.5). Then, considering first case A as usual, from (3.12) and (3.13) we obtain, after some algebra,

$$\begin{aligned}\langle 0'|H_{TF}|0'\rangle=-\alpha^S_{mn}\alpha^T_{pq}\frac{\pi^2c\hbar}{V^2}\mathbf{x}_0\cdot\nabla\sum_{\mathbf{k},\mathbf{k}'}\{[(\delta_{pm}-\hat{k}_p\hat{k}_m)(\delta_{qn}-\hat{k}'_q\hat{k}'_n)+(\delta_{pn}-\hat{k}_p\hat{k}_n)(\delta_{qm}-\hat{k}'_q\hat{k}'_m)]\\\times e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{R}_0}\frac{kk'}{k+k'}(1-e^{-i(k+k')ct})+\text{c.c.}\}\ ,\end{aligned} \tag{3.17}$$

where ∇ involves differentiation with respect to $\mathbf{R}_0$, and where polarization sums have been performed as usual.[14]

Transforming sums into integrals in (3.17), performing angular integrations, and defining

$$f(k,k',R_0)=2j_0(kR_0)j_0(k'R_0) -2\left[j_0(kR_0)\frac{1}{k'R_0}j_1(k'R_0) +j_0(k'R_0)\frac{1}{kR_0}j_1(kR_0)\right] +6\frac{1}{kR_0}j_1(kR_0)\frac{1}{k'R_0}j_1(k'R_0)\ , \tag{3.18}$$

$$\int_0^\infty\int_0^\infty f(k,k',R_0)\frac{k^3k'^3}{k+k'}e^{-i(k+k')ct}dk\,dk'\equiv I(t,R_0)\ , \tag{3.19}$$

we obtain

$$\langle 0'\,|\,H_{TF}\,|\,0'\rangle=-\alpha^S\alpha^T\frac{c\hbar}{\pi^2}\mathbf{x}_0\cdot\nabla\mathrm{Re}[I(0,R_0)-I(t,R_0)]\ . \tag{3.20}$$

Introducing

$$\frac{1}{k+k'}=\int_0^\infty e^{-(k+k')\eta}d\eta \tag{3.21}$$

to decouple k and k' integrations in (3.19), and after a lengthy procedure involving Bessel function integrations[15] and the properties of the hypergeometric[16] function ${}_2F_1$, we obtain

$$I(t,R_0)=16\int_0^\infty F(z)d\eta,\quad F(z)=\frac{3z^4-2z^2R_0^2+3R_0^4}{(z^2+R_0^2)^6}\ ,$$
$$z=\eta+ict\ . \tag{3.22}$$

In order to evaluate the real part of the integral in (3.22), we continue $F(z)$ into the complex $\zeta=\eta+ic\tau$ plane, thereby obtaining

$$F(\zeta)=\frac{3\zeta^4-2\zeta^2R_0^2+3R_0^4}{(\zeta^2+R_0^2)^6}\ , \tag{3.23}$$

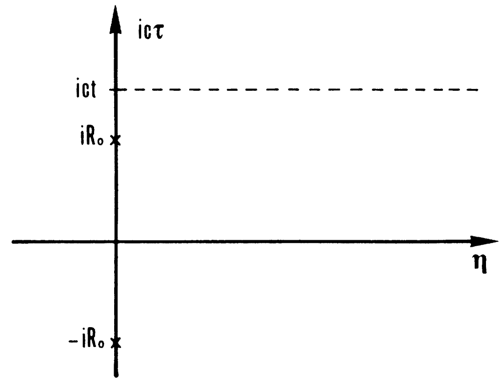

FIG. 3. The broken line is the integration path for $F(\zeta)$ in the $\zeta=\eta+ic\tau$ plane. $\pm iR_0$ are the two sixth-order poles of $F(\zeta)$. The path is always parallel to the η axis, but the integration technique is different for the cases $ct\geq R_0$ and $ct<R_0$.

which has two poles of the sixth order at $\zeta=\pm iR_0$, as shown in Fig. 3, and which vanishes as $|\zeta|^{-8}$ for $|\zeta|\to\infty$. In this plane, $I(t,R_0)$ is obtained by contour integration along a path parallel to the positive η axis, as shown by the dashed line in Fig. 3 for $R_0>ct$. Evaluation of $\mathrm{Re}I(t,R_0)$ yields

$$\mathrm{Re}I(t,R_0)=\frac{23\pi}{4}\frac{1}{R_0^7}\Theta(R_0-ct)-\frac{23\pi}{4}\frac{1}{R_0^6}\delta(R_0-ct) +\frac{5\pi}{2}\frac{1}{R_0^5}\delta'(R_0-ct)-\frac{7\pi}{12}\frac{1}{R_0^4}\delta''(R_0-ct) +\frac{\pi}{12}\frac{1}{R_0^3}\delta'''(R_0-ct)-\frac{\pi}{60}\frac{1}{R_0^2}\delta^{\mathrm{iv}}(R_0-ct)\ , \tag{3.24}$$

where derivatives of the δ function are with respect to R_0-ct. Since $R_0\neq 0$,

$$\mathrm{Re}I(0,R_0)=\frac{23\pi}{4}\frac{1}{R_0^7}\ , \tag{3.25}$$

and from (3.20) we have (case A)

$$\langle 0'\,|\,H_{TF}\,|\,0'\rangle=-\frac{1}{4\pi}\alpha^S\alpha^T c\hbar\mathbf{x}_0\cdot\nabla\left[\frac{23}{R_0^7}[1-\Theta(R_0-ct)]+\frac{23}{R_0^6}\delta(R_0-ct)-\frac{10}{R_0^5}\delta'(R_0-ct)+\frac{7}{3R_0^4}\delta''(R_0-ct) -\frac{1}{3R_0^3}\delta'''(R_0-ct)+\frac{1}{15R_0^2}\delta^{\mathrm{iv}}(R_0-ct)\right]\ . \tag{3.26}$$

For case B, when source S is suddenly decoupled from the field at $t=0$, one follows the same development, starting from expressions (3.15) and (3.16). Here we shall only report the final result (case B)

$$\langle 0' \mid H_{TF} \mid 0' \rangle = -\frac{1}{4\pi}\alpha^S\alpha^T c\hbar \mathbf{x}_0\cdot\nabla \left[\frac{23}{R_0^7}\Theta(R_0-ct) - \frac{23}{R_0^6}\delta(R_0-ct) + \frac{10}{R_0^5}\delta'(R_0-ct) - \frac{7}{3R_0^4}\delta''(R_0-ct) + \frac{1}{3R_0^3}\delta'''(R_0-ct) - \frac{1}{15R_0^2}\delta^{\mathrm{iv}}(R_0-ct) \right] . \tag{3.27}$$

D. Dynamics of the detector oscillator

Like in the meson case of Sec. II, we take m_0 large enough to permit a classical treatment of the dynamics of T, and assume only radial displacement x_0 of the oscillator. Furthermore here we restrict our considerations to case A, since case B follows rather trivially.

We put $\langle 0' \mid H_{TF} \mid 0' \rangle = -x_0 \langle F_{0R} \rangle$, where

$$\langle F_{0R} \rangle = \frac{1}{4\pi}\alpha^S\alpha^T c\hbar \left[-\frac{161}{R_0^8}[1-\Theta(R_0-ct)] - \frac{161}{R_0^7}\delta(R_0-ct) - \frac{83}{R_0^6}\delta'(R_0-ct) - \frac{58}{3R_0^5}\delta''(R_0-ct) - \frac{10}{3R_0^4}\delta'''(R_0-ct) - \frac{7}{15R_0^3}\delta^{\mathrm{iv}}(R_0-ct) - \frac{1}{15R_0^2}\delta^{\mathrm{v}}(R_0-ct) \right] . \tag{3.28}$$

It should be noted that in (3.28) all the δ-function derivatives are now with respect to ct. Thus the effective Hamiltonian for detector T, which is analogous to (2.26) in the QMD case, is

$$H_{\mathrm{eff}} = H_T + H_{\mathrm{TF}} \ , \quad H_T = \frac{1}{2m_0}p_0^2 + \frac{1}{2}Kx_0^2 \ , \quad H_{TF} = -x_0\langle F_{0R} \rangle \ , \tag{3.29}$$

where $\langle F_{0R} \rangle$ is explicitly given by (3.28), and plays the role of an effective time-dependent force acting on T. Thus we can apply (2.27) to obtain the final amplitude of oscillation of T under the action of the time-dependent field created by S during the dressing event.

Using the properties of the δ function[17] and substituting (3.28) into (2.27) yields

$$x_0(t) = -\frac{1}{4\pi}\alpha^S\alpha^T\frac{\hbar}{m_0\omega_0}\left[\frac{161}{R_0^8}\frac{c}{\omega_0}\{1-\cos[\omega_0(t-R_0/c)]\} + \left[\frac{161}{R_0^7} - \frac{58}{3R_0^5}\frac{\omega_0^2}{c^2} + \frac{7}{15R_0^3}\frac{\omega_0^4}{c^4} \right]\sin[\omega_0(t-R_0/c)] - \left[\frac{83}{R_0^6}\frac{\omega_0}{c} - \frac{10}{3R_0^4}\frac{\omega_0^3}{c^3} + \frac{1}{15R_0^2}\frac{\omega_0^5}{c^5} \right]\cos[\omega_0(t-R_0/c)] \right]\Theta(ct-R_0) \ . \tag{3.30}$$

It should be noted that $x_0(t=R_0/c+\epsilon) \neq x_0(t=R_0/c-\epsilon)$, where ϵ is an infinitesimal. This is due to the complicated nature of the $R_0=ct$ singularity of $\langle F_{0R} \rangle$ in (3.28), and in particular to the presence of the odd derivatives of the δ function. This feature implies a sudden displacement with infinite velocity of the oscillator detector at $t=R_0/c$, and it is clearly at variance with physical intuition. This might have been expected, because our treatment of the oscillatory motion of T is nonrelativistic; moreover, it is also to be connected with the assumption of sudden appearance of a point source (S) at $t=0$, which is rather unrealistic, too. In this sense we deem that this feature of the solution should not be the cause of particular worry, but should rather be taken as an indication of the limitations of our model.

Contrary to the meson case, the absence of a stop band in the photon spectrum eliminates the exponential dependence of the oscillation amplitude of the detector on the S-T distance, although again $x_0(t)$ vanishes for $t<R_0/c$. The R_0 dependence of $x_0(t)$, however, introduces some rather interesting features, since it is evident from (3.30) that for $R_0<c/\omega_0$ the oscillation amplitude is directly proportional to the static R_0^{-8} van der Waals force, which is a consequence of the virtual nature of the expanding photon cloud. At the opposite end of the scale, for $R_0>c/\omega_0$, the oscillation amplitude becomes proportional to R_0^{-2}, which is similar to the behavior of a detector under the action of the e.m. field emitted by a normal source, such as an atom decaying from an excited state to the ground state via real processes.[18] It should also be noted that the distance c/ω_0 at which the apparent nature of the expanding virtual photon cloud changes, as seen by the detector, depends only on the physical parameters of the detector itself.

In summary, we have shown that in principle it is possible to detect the growth of the virtual photon cloud which develops around an initially bare and neutral source, such as an atom or a molecule. Like in the QMD case discussed in Sec. II, a quantum treatment of the oscillatory motion of T does not seem to add much to the concepts discussed here.

IV. THE ELECTROMAGNETIC ENERGY DENSITY OF HALF-DRESSED STATES

As mentioned in Sec. I, the time-dependent energy density of a half-dressed source has been investigated recently both for the meson case and the electromagnetic case.[6]

In that work, however, only the behavior of the energy densities at any point $r \neq ct$ was reported, since the attention was not focused on the singularity appearing at $r=ct$. On the other hand, the singular behavior of the e.m. energy density is also important for the dynamics of the detector oscillator in Sec. III, since the force acting on T is proportional to the matrix elements of H_{TF} as given by (3.7), which in turn, for electrically isotropic sources and in our dipole approximation, is proportional to the gradient of this electromagnetic energy density. We remark that this is not the case for the scalar field of Sec. II, where force (2.8) is proportional to the gradient of the field amplitude rather than to the gradient of the meson energy density.

These considerations lead us to investigate in more detail the electromagnetic energy density of a half-dressed source, including the $r=ct$ singularity. Thus we take up the analogous of case A for one isotropic source at the origin, which at $t=0$ is completely bare, and we wish to obtain the energy density at any point $\mathbf{r}$ as a function of $t>0$. The energy density operator for the field $\mathbf{E}_\perp$ is averaged on the initial bare vacuum $|0\rangle$. The time-dependent quantum averages of the two-particle operators (4.1) can be easily obtained form (3.12) and (3.13) by using real polarization vectors for simplicity, by putting $\alpha^T=\beta=0$ and by using $\alpha^S_{mn}=\alpha\delta_{mn}$. Performing the polarization sums according to the usual rules, transforming sums over $\mathbf{k}$ into integrals, and performing the angular parts of the latter yields

$$\langle 0|\mathbf{E}_\perp^2(\mathbf{r})|0\rangle = \frac{\alpha\hbar c}{\pi^2}\int_0^\infty\int_0^\infty f(k_1,k_2,r)\frac{k_1^3k_2^3}{k_1+k_2}\times(1-e^{-i(k_1+k_2)ct})\times dk_1dk_2+\text{c.c.}\ , \quad (4.1)$$

where $f(k_1,k_2,r)$ has been defined previously. Thereby, using (3.19), we obtain

$$\langle 0|\mathbf{E}_\perp^2(\mathbf{r})|0\rangle = \frac{\alpha\hbar c}{\pi^2}[I(0,r)-I(t,r)+\text{c.c.}]\ . \quad (4.2)$$

Use of (3.24) yields finally

$$\langle 0|\mathcal{H}_{\text{el}}(\mathbf{r})|0\rangle = \frac{1}{(4\pi)^2}\alpha\hbar c\left\{\frac{23}{r^7}[1-\Theta(r-ct)]+\frac{23}{r^6}\delta(r-ct) -\frac{10}{r^5}\delta'(r-ct)+\frac{7}{3r^4}\delta''(r-ct)-\frac{1}{3r^3}\delta'''(r-ct)+\frac{1}{15r^2}\delta^{\text{iv}}(r-ct)\right\}\ . \quad (4.3)$$

As for the energy density of field $\mathbf{B}$, an analogous procedure yields

$$\langle 0|\mathbf{B}^2(\mathbf{r})|0\rangle = -2\frac{\alpha\hbar c}{\pi^2}[L(0,r)-L(t,r)+\text{c.c.}]\ , \quad (4.4)$$

where

$$L(t,r)=64r^2\int_0^\infty G(z)d\eta,\quad G(z)=\frac{z^2}{(z^2+r^2)^6}\ ,\quad z=\eta+ict\ . \quad (4.5)$$

Following the procedure adopted in Sec. III C, we define in the complex $\zeta=\eta+ic\tau$ plane

$$G(\zeta)=\frac{\zeta^2}{(\zeta^2+r^2)^6} \quad (4.6)$$

and integrate along the same contour as in Fig. 3 for $F(\zeta)$ with poles at $\zeta=\pm ir$ instead of $\pm iR_0$.

The integration procedure gives

$$\text{Re}L(t,r)=\frac{7\pi}{8}\frac{1}{r^7}\Theta(r-ct)-\frac{7\pi}{8}\frac{1}{r^6}\delta(r-ct)+\frac{\pi}{4}\frac{1}{r^5}\delta'(r-ct)+\frac{\pi}{24}\frac{1}{r^4}\delta''(r-ct)-\frac{\pi}{24}\frac{1}{r^3}\delta'''(r-ct)+\frac{\pi}{120}\frac{1}{r^2}\delta^{\text{iv}}(r-ct)\ . \quad (4.7)$$

Substitution of (4.7) into (4.4) yields

$$\langle 0|\mathcal{H}_{\text{mag}}(\mathbf{r})|0\rangle = -\frac{1}{(4\pi)^2}\alpha\hbar c\left\{\frac{7}{r^7}[1-\Theta(r-ct)]+\frac{7}{r^6}\delta(r-ct)-\frac{2}{r^5}\delta'(r-ct)-\frac{1}{3r^4}\delta''(r-ct)+\frac{1}{3r^3}\delta'''(r-ct)-\frac{1}{15r^2}\delta^{\text{iv}}(r-ct)\right\}\ . \quad (4.8)$$

A few remarks are in order.

First, the finite parts of (4.3) and (4.8) coincide with those previously obtained by Persico and Power.[6] The infinite part, which develops on the surface of a sphere of radius $r=ct$, has a rather complicated behavior. The last term in δ^{iv}, however, is equal in both expressions; this is similar to the behavior of a real electromagnetic wave emitted by a point source, whose energy density decreases like r^{-2} from the source, and in which the "electric" and "magnetic" parts of the energy density are equal. In agreement with the remarks at the end of Sec. III, this seems to confirm that the energy density acquires gradually some of the characters of a real pulse at large distances from the source.

Second, it should be noted that in the energy density problem discussed here, the pointlike, static nature of the source does not provide a finite scale length for this gradual change, whereas in the detection problem discussed in Sec. III this finite scale length is provided by the detector itself in the form of the quantity c/ω_0. This is related to an apparently inconsistent aspect of results (4.3) and (4.8) for the energy densities, namely, that the sharpness of the singularity is incompatible with the dipole approximations in (2.5) and (3.6) for finite values of the detector amplitude x_0. This difficulty, however, does not exist for a source of finite dimensions, where preliminary calculations have shown that the singularity is smeared out over a region of linear dimensions comparable with those of the source, in these circumstances in fact the only requirement for the validity of dipole approximation is that x_0 should be smaller than the linear dimensions of the source, a condition which is not very restrictive. Thus the (admittedly rather artificial) remedy in the present situations of pointlike sources is to make $x_0(t)$ in (3.30) infinitesimal by letting the oscillator mass m_0 and force constant K diverge simultaneously, in such a way that K/m_0 (i.e., the oscillator frequency ω_0) is constant. In this way the consistency of dipole approximation with result (3.30) is legitimated by a limiting procedure.

Finally, we also remark that the energy densities for the complementary problem, in which the source is suddenly decoupled from the field at $t=0$, are easily obtained from (4.3) and (4.8) by substituting $\Theta(r-ct)$ for $1-\Theta(r-ct)$ and by changing the sign of the δ parts.

V. INTEGRAL PROPERTIES OF THE FIELD OF A HALF-DRESSED ELECTROMAGNETIC SOURCE

We now wish to discuss some integral properties of the virtual electromagnetic energy density of a single half-dressed source.

(i) First we consider a fully dressed ground-state source of polarizability α_{mn}, whose Hamiltonian in the CP model is given by

$$H=H_0+H_M,\quad H_M=\sum_l E_l\,|\,l\,\rangle\langle\,l\,|\,,\quad H_0=H_F+H_{MF},H_F=\sum_{\mathbf{k},j}\hbar\omega_k a^\dagger_{\mathbf{k}j}a_{\mathbf{k}j}\ ,$$
$$H_{MF}=\tfrac{1}{2}\alpha_{mn}\frac{2\pi\hbar}{V}\sum_{\mathbf{k}_1,\mathbf{k}_2,j_1,j_2}\sqrt{\omega_{k_1}\omega_{k_2}}[(\mathbf{e}_{\mathbf{k}_1j_1})_m(\mathbf{e}_{\mathbf{k}_2j_2})_n a_{\mathbf{k}_1j_1}a_{\mathbf{k}_2j_2}-(\mathbf{e}_{\mathbf{k}_1j_1})_m(\mathbf{e}^*_{\mathbf{k}_2j_2})_n a_{\mathbf{k}_1j_1}a^\dagger_{\mathbf{k}_2j_2}+\text{H.c.}]\ . \tag{5.1}$$

H_0 in (5.1) can be obtained from (3.8) by putting $\beta_{mn}=0$, and H_M is the bare source Hamiltonian, with all eigenstates idle, except for the ground state of energy E_0. The ground state of the system, up to terms linear in α, is

$$|\,0'\rangle=|\,0\rangle+(E_0-H_F-H_M)^{-1}\tfrac{1}{2}\alpha_{mn}\frac{2\pi\hbar}{V}\times\sum_{\mathbf{k},\mathbf{k}',j,j'}\sqrt{\omega_k\omega_{k'}}(\mathbf{e}^*_{\mathbf{k}_j})_m(\mathbf{e}^*_{\mathbf{k}'j'})_n\times a^\dagger_{\mathbf{k}j}a^\dagger_{\mathbf{k}'j'}\,|\,0\rangle\ . \tag{5.2}$$

Using (5.2) we can immediately obtain, up to terms linear in α,

$$\langle 0'\,|\,H_F\,|\,0'\rangle=0\ . \tag{5.3}$$

This is a surprising result because it seems contrary to our Heisenberg representation results (4.3) and (4.8) in the limit $t=\infty$, when the virtual cloud is totally regenerated. These results, translated into the Schrödinger representation, yield

$$\langle 0'\,|\,\mathcal{H}_{\text{el}}(\mathbf{r})\,|\,0'\rangle=\lim_{t\to\infty}\langle 0\,|\,\mathcal{H}_{\text{el}}(\mathbf{r},t)\,|\,0\rangle=\frac{1}{(4\pi)^2}\alpha\hbar c\frac{23}{r^7}\ ,$$
$$\langle 0'\,|\,\mathcal{H}_{\text{mag}}(\mathbf{r})\,|\,0'\rangle=\lim_{t\to\infty}\langle 0\,|\,\mathcal{H}_{\text{mag}}(\mathbf{r},t)\,|\,0\rangle=-\frac{1}{(4\pi)^2}\alpha\hbar c\frac{7}{r^7}\ . \tag{5.4}$$

This implies

$$\langle 0'\,|\,(\mathcal{H}_{\text{el}}+\mathcal{H}_{\text{mag}})\,|\,0'\rangle\equiv\langle 0'\,|\,\mathcal{H}_F(\mathbf{r})\,|\,0'\rangle=\frac{\alpha\hbar c}{\pi^2}\frac{1}{r^7}\ , \tag{5.5}$$

which would seem at variance with (5.3) when integrated over all space. It is however possible to show that this apparent paradox *in the integral* of (5.5) over all space originates entirely from an illegitimate exchange of the η and r integrations, where η is the convergence factor introduced in (3.21). In fact it is possible to obtain the total field energy, without performing the mentioned exchange, in the form

$$4\pi\int_0^\infty \langle 0'|\mathcal{H}_F(r)|0'\rangle r^2 dr = \frac{16}{\pi^2}\alpha\hbar c\int_0^\infty d\eta\int_0^\infty dr\frac{3(\eta^2+r^2)^2-16\eta^2r^2}{(\eta^2+r^2)^6}r^2 . \tag{5.6}$$

When integrations are performed in the order shown in (5.6) the integral vanishes. On the other hand, if the order of integrations is inverted, one obtains

$$\frac{4}{\pi}\alpha\hbar c\int_0^\infty r^{-5}dr ,$$

which diverges. The noninterchangeability of the η and r integrations is obviously related to the singularity of the integrand at the origin in the (η,r) plane. Consequently we may say that (5.5) is the correct field energy density for the CP Hamiltonian, but that the total energy density cannot be obtained by integrating it over all space, in the absence of a prescription for dealing with the singularity at the origin. The same argument can be shown to apply at any time t for the energy of the field of a source which is initially bare, that is,

$$4\pi\int_0^\infty \langle 0|\mathcal{H}_F(r,t)|0\rangle r^2 dr = 0 , \tag{5.7}$$

up to terms linear in α.

(ii) We shall now discuss some aspects of energy conservation during the regeneration process of the virtual photon cloud. The total energy density at time t is obtained as the sum of (4.3) and (4.8). It is qualitatively represented by the continuous line in Fig. 4 everywhere except at $r=0$, and analytically we divide it as

$$\langle 0|\mathcal{H}_F(r,t)|0\rangle = \langle \mathcal{H}_F\rangle_{r<ct} + \langle \mathcal{H}_F\rangle_{r=ct} , \tag{5.8}$$

where

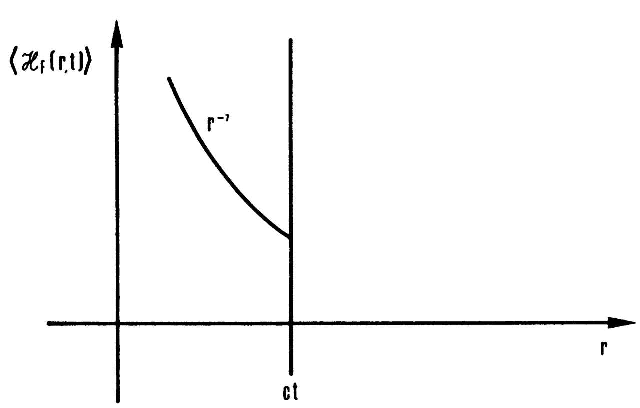

FIG. 4. Energy density (in arbitrary units) at time t around a source which is suddenly coupled to the electromagnetic field at time 0. The vertical line at $r=ct$ represents a singularity. The continuous line for $r<ct$ follows the r^{-7} law.

$$\langle \mathcal{H}_F\rangle_{r<ct} = \frac{\alpha\hbar c}{\pi^2}\frac{1}{r^7}\Theta(ct-r) \quad (r\neq 0) ,$$

$$\langle \mathcal{H}_F\rangle_{r=ct} = \frac{1}{(4\pi)^2}\alpha\hbar c\left[\frac{16}{r^6}\delta(r-ct)-\frac{8}{r^5}\delta'(r-ct) + \frac{8}{3r^4}\delta''(r-ct)-\frac{2}{3r^3}\delta'''(r-ct) + \frac{2}{15r^2}\delta^{\text{iv}}(r-ct)\right] . \tag{5.9}$$

From (5.7) we have

$$\int_0^\infty \langle \mathcal{H}_F\rangle_{r<ct} r^2 dr = -\int_0^\infty \langle \mathcal{H}_F\rangle_{r=ct} r^2 dr . \tag{5.10}$$

Equality (5.10) is valid for any t, and in particular for $t'<t$. Consequently, also

$$\int_0^\infty (\langle \mathcal{H}_F\rangle_{r<ct} - \langle \mathcal{H}_F\rangle_{r<ct'})r^2 dr = -\int_0^\infty (\langle \mathcal{H}_F\rangle_{r=ct} - \langle \mathcal{H}_F\rangle_{r=ct'})r^2 dr \tag{5.11}$$

is valid, where the lhs represents the energy of the virtual cloud deposited in the region $ct'<r<ct$, and the rhs represents the energy lost by the singularity in the time interval between t' and t.

Thus (5.11) shows clearly that all the energy contained in the singularity moving at speed c is gradually deposited in space and directly transformed into the final ground-state virtual cloud surrounding the source. Direct integration of (5.9) also yields the energy contained in the singularity at time t, in the form

$$4\pi\int_0^\infty \langle \mathcal{H}_F\rangle_{r=ct} r^2 dr = \frac{1}{\pi}\alpha\hbar c\frac{1}{(ct)^4} . \tag{5.12}$$

This behavior of the energy pulse is in sharp contrast with the behavior of a pulse of real radiation, of the kind emitted by a source in an energy-conserving process. In the latter case, in fact, the energy in the pulse does not change with time. The t^{-4} dependence in (5.12) follows from the fact that the energy in the singularity is gradually lost, because it is transformed into the energy of the static virtual cloud.

The case complementary to that considered above has also been studied, when a source is suddenly decoupled from the field at $t=0$. It can be shown that in this case a sphere of radius ct expands outward from the source. The energy density vanishes within this sphere, whereas outside it has the normal ground-state value, since information about disappearance of the source has not had time to reach those points. The singularity of the energy density at $r=ct$ has exactly the same form as in the second part of (5.9), but with the opposite sign. Thus the singularity sweeps out, during its motion, all the static virtual field, leaving the bare vacuum behind, and in this way its negative energy content decreases to zero. It is perhaps worth recalling that in all our calculations we have discarded the uniform and infinite background of the zero-point energy density of the electromagnetic field.

(iii) We now write the total energy density in (5.8) as

$$\langle \mathcal{H}_F \rangle = \langle \mathcal{H}_F \rangle_0 + \langle \mathcal{H}_F \rangle_t \ , \quad \langle \mathcal{H}_F \rangle_0 = \frac{\alpha \hbar c}{\pi^2} \frac{1}{r^7} \ ,$$
$$\langle \mathcal{H}_F \rangle_t = \langle \mathcal{H}_F \rangle - \langle \mathcal{H}_F \rangle_0 \ . \tag{5.13}$$

The explicit form of the time-dependent part can be obtained from (5.9) for the case of the initially bare source as

$$\langle \mathcal{H}_F \rangle_t = \frac{1}{(4\pi)^2} \alpha \hbar c \left[-\frac{16}{r^7} \Theta(r-ct) + \frac{16}{r^6} \delta(r-ct) - \frac{8}{r^5} \delta'(r-ct) + \frac{8}{3r^4} \delta''(r-ct) - \frac{2}{3r^3} \delta'''(r-ct) + \frac{2}{15r^2} \delta^{\text{iv}}(r-ct) \right] . \tag{5.14}$$

The Fourier transform of $\langle \mathcal{H}_F \rangle_t$ with respect to r is obtained after some algebra as

$$\mathcal{F}[\langle \mathcal{H}_F \rangle_t] = \frac{1}{(2\pi)^{3/2}} \int \langle \mathcal{H}_F \rangle_t e^{-i\mathbf{k}\cdot\mathbf{r}} d^3 r = \left[\frac{2}{\pi} \right]^{1/2} \frac{1}{k} \int_0^\infty \langle \mathcal{H}_F \rangle_t r \sin(kr)\, dr = \frac{1}{32\pi} \frac{\alpha \hbar c}{(2\pi)^{3/2}} k^4 \mathrm{Ci}(ckt) \ , \tag{5.15}$$

where the cosine-integral function is

$$\mathrm{Ci}(z) = -\int_z^\infty \frac{\cos z'}{z'} dz' \ .$$

The violent oscillations in the wings of (5.15) (i.e., for large k at a given t) are very much at variance with Feinberg's prediction that regeneration time for a component of wave vector k should decrease with increasing k.[4] The origin of the discrepancy is in the singularity at $r = ct$, which is not taken into account in Feinberg's scheme. In fact, if one discards the δ parts in (5.14) and takes only the Fourier transform (FT) of $-r^{-7}\Theta(r-ct)$, one finds

$$\mathcal{F}[-r^{-7}\Theta(r-ct)] = -\left[\frac{1}{5(ct)^5} - \frac{k^2}{60(ct)^3} + \frac{k^4}{120ct} \right] \sin(ckt) - \left[\frac{k}{20(ct)^4} - \frac{k^3}{120(ct)^2} \right] \cos(ckt) + \frac{k^5}{120} \mathrm{Ci}(ckt) \ , \tag{5.16}$$

and, in the appropriate limits,

$$\mathcal{F}[-r^{-7}\Theta(r-ct)] \sim -\frac{1}{5(ct)^5} \quad [k \ll (ct)^{-1}] \ ,$$
$$\mathcal{F}[-r^{-7}\Theta(r-ct)] \sim \frac{1}{(ct)^5} \frac{\cos(ckt)}{ckt} \quad [k \gg (ct)^{-1}] \ , \tag{5.17}$$

which displays quite clearly the decrease for large t of the FT of the time-dependent part of the energy density, as well as earlier regeneration of large-k components, in agreement with Feinberg's suggestion.

VI. SUMMARY AND CONCLUSIONS

We have considered two models of half-dressed states. The first is within the domain of QMD, and consists of a relativistic scalar field linearly coupled to a source S with no internal degrees of freedom. The ground state of this system consists of the source surrounded by a cloud of virtual mesons. The second model is concerned with QED, and consists of a ground-state pointlike molecule S quadratically coupled in a CP fashion to the electromagnetic field. Also the ground state of this system consists of the ground-state source surrounded by a cloud of virtual photons. These ground-state configurations pertain to fully dressed sources. We obtain half-dressed sources by assuming that for times $t<0$, S is decoupled from the field, which is assumed to be empty of mesons or of photons, respectively, and that the source-field coupling is suddenly switched on at $t=0$. Consequently, at $t=0$ the source is completely bare and deprived of its ground-state virtual cloud. This initial configuration is certainly an abstraction of the same sort as pointlike charges or masses, since perfectly bare sources are not observable; it is, however, representative of a nonequilibrium situation for the virtual field, and we have exploited it to investigate the processes which regenerate the full dress of the source. In a previous paper[6] such a regeneration was studied by following the time development of the energy density of the virtual field in space, which led to the conclusion that the regeneration proceeded within a sphere of radius ct, the field remaining in its bare vacuum state outside the sphere. Also the opposite situation, in which S is fully dressed at times $t<0$ and is suddenly decoupled from the field at $t=0$, was considered. The result was that the energy density of the virtual field decreased to zero within an expanding sphere of radius ct, retaining its normal behavior in the region outside this sphere, whose points could not be reached by the information of the "disappearance" of the source.

In this paper our interest has been focused on the possibility of detecting the time evolution of the virtual field of a half-dressed source in a gedanken experiment. We have modeled a detector of the virtual field by a second source T, coupled to the field in the same way as the first, as well as to a center of oscillation by elastic forces of a different nature. The time-dependent virtual field of the half-dressed source S induces a force on T, and as a consequence the detector is set in motion. We have taken the amplitude of oscillations of T as a measure of the sensitivity of the model detector. We have treated the oscillatory degrees of freedom of the detector classically and we have neglected, in the limit of small oscillations, the reaction of the oscillatory motion of T back onto the field. We have thus been able to show that some energy is transferred (permanently within our approximations) from the virtual field of the half-dressed source to the detector. In the meson case the force has been shown to be proportional to the gradient of the field, whereas in the

QED case the force turns out to be proportional to the gradient of the energy density of the field (for electrically isotropic sources). The force has been evaluated in the Schrödinger representation in the first case and in the Heisenberg representation in the second. The effects of the force on the detector has been evaluated in both cases by well-known Fourier transform techniques.

In the QMD case the sensitivity of the detector, as measured by its oscillation amplitude, depends essentially on its natural frequency ω_0. If $\omega_0 < mc^2/\hbar$, where m is the rest mass of the mesons, its natural frequency falls inside the forbidden gap of the meson spectrum and, because of this, it depends exponentially on the distance R_0 from the source. If instead $\omega_0 > mc^2/\hbar$, oscillations can be excited by energy-conserving processes and the exponential R_0 dependence turns into a power dependence. For S-T distances large enough, the oscillation amplitude of T changes from a R_0^{-2} to a R_0^{-1} dependence, yielding a relatively high sensitivity. We note that the distance at which this change takes place is given by c/ω_0, which plays the role of a length scale in this model, since the source is pointlike.

In QED, photons have no mass, and correspondingly there is no gap in the photon spectrum. Consequently the sensitivity of the detector varies according to a power-law R_0^{-n} with the S-T distance. We find that n changes from 8 (for $R_0 < c/\omega_0$) to 2 (for $R_0 > c/\omega_0$). Thus the oscillation amplitude of the detector is a direct measure of the van der Waals static force at a short S-T distance, while at large distances it is similar to the typical behavior of a detector under the action of a field originated by real emission processes.

In order to have a better insight in the detection process, we have investigated the time-dependent energy density of the virtual field in the QED case only, since in the QMD case the field energy density is not directly relevant for the detection process, which is related to the field amplitude. This part is concerned with the details of the structure of the singularity in the electromagnetic energy density at $r = ct$, which were not considered in previous work on the same subject,[6] but which are important for the detection process. For the case of an initially bare source, the energy density immediately attains the final value at points inside the sphere $r = ct$, whereas it vanishes in the outside region. The singularity at $r = ct$ is a complicated one, which can be expressed in terms of δ-function derivatives weighed by r^{-n} factors, with n an integer from 2 to 6. This behavior of the energy density mirrors the highly singular behavior of the effective force acting on the model detector as a function of time. In particular, at large distances from the source ($r > c/\omega_0$) the electric and magnetic parts of the energy density become equal, which again is similar to the behavior of a free electromagnetic wave (in Gauss units).

The field energy contained in the singularity at the surface of the sphere of radius $r = ct$ has rather interesting features. It amounts to

$$\pm \frac{1}{\pi} \alpha \hbar c \frac{1}{(ct)^4} ,$$

where the $+$ refers to the initially bare source and the $-$ to a source suddenly decoupled from the field at $t = 0$. Thus in the first case the energy content of the singularity is positive and it decreases with time (in contrast with a pulse of radiation emitted by a source in an energy-conserving process), while in the second case it is negative (against the positive infinite zero-point background that we have neglected throughout this paper) and it tends to zero at $t \to \infty$. Its rate of change

$$\mp \frac{4}{\pi} \alpha \hbar c^2 \frac{1}{(ct)^5}$$

corresponds exactly to the rate $4\pi r^2 c \langle 0 | \mathcal{H}_F(r,t) | 0 \rangle$ at which the virtual field energy increases (at $r = ct - \epsilon$ for an initially bare source, ϵ being an infinitesimal) or decreases (at $r = ct + \epsilon$ for a source suddenly decoupled at $t = 0$). Thus in the first case the virtual field can be visualized as arising entirely from the positive-energy singularity, which deposits it in the amount appropriate to each point in space during its outward motion. In the second case the virtual field is gradually annihilated by the negative-energy singularity, which at the asymptotic end of the process ($t = \infty$) disappears. Also this behavior is very different from that expected in a normal case of a pulse of radiation which is emitted by an atom or a molecule in a real process, e.g., of spontaneous emission. In the latter case in fact, the energy is emitted on top of the positive infinite zero-point background, and one should not expect regions where the other contributions to the energy density are negative. In spite of this, the time-dependent virtual field or a half-dressed source is capable of exciting a classical model detector, as we have shown. We are thus led to suggest that in an unstable system such as the positronium one should in principle be able to observe, apart from the γ radiation coming from the relativistic part of the process, also effects connected with the undressing of the positronium atom. Observability of this effect in practice would require a careful analysis in view of the complicated nature of positronium; such an analysis is clearly out of the scope of this paper, which is concerned with a general investigation of obviously oversimplified models of half-dressed sources. The above discussion on the difference between radiation emitted by normal energy-conserving processes and that emitted by radical perturbation of ground-state sources also shows that some caution is in order when discussing conversion of virtual particles into real particles during sudden events,[5] since integrated quantities such as total energy, particle number, and similar quantities may well have to be supplemented by a space-time analysis of the kind presented in this paper before definite conclusions concerning their physical nature can be drawn. This point of view is also supported by a space Fourier analysis of the time-dependent part of the energy density, which illustrates quite clearly the importance of the $r = ct$ singularity in determining the properties of the energy density in k space.

Thus the quantitative study of our QED and QMD models leads us to conclude that the virtual quanta released by ground-state sources following a traumatic event can be detected, although their physical properties are in many ways dissimilar from those of real quanta normally emitted in an energy-conserving process. It is fair, however, to remark that these conclusions have been obtained on the basis of very idealized models of source-field interactions, and that it is not immediately clear to what extent the simplifications introduced may limit the validity of the results we have obtained. In particular, it would seem desirable to investigate the consequences of the assumption of pointlike sources on the structure of the energy density of the field in the neighborhood of $r=ct$, since it is likely that the singularity does not survive as such for sources of finite dimensions, but it gets smeared out in a region of dimensions comparable to the dimensions of the source. Preliminary calculations seem to indicate that this is indeed the case; this would be very convenient for our purposes, since it would make the behavior of the energy density in the neighborhood of $r=ct$ compatible with the dipole approximations (2.5) and (3.6) by introducing a cutoff at wavelengths of the order of the dimensions of the source. Also the neglect of the internal degrees of freedom of the sources is likely to have an adverse influence on the validity of our conclusions in the near zone of more realistic sources, where the contributions of modes of the field of short wavelength, capable of exciting internal resonances, is dominant with respect to the contribution of long-wavelength radiation. Moreover, our neglect of the radiation of the detector oscillator reacting back on the field spoils to some extent overall energy conservation, which is certainly embarrassing and undesirable. Finally, the assumption about the abruptness of "creation" and "annihilation" of the sources is certainly a very rough idealization, which might make one suspicious about extension of the conclusions of the present work to some particular processes such as positronium annihilation; in fact, one may legitimately speculate about the influence of the details of the process leading to the annihilation or to the creation of a source on the nature of the virtual radiation which is released in the same process. An advantage of our model, however, is that the $t=0$ state may be taken to be half dressed rather than completely bare. This permits us to circumvent the conceptual difficulties connected with the nonobservable nature of the bare source configuration. The initially half-dressed configuration in fact is certainly more realistic than the bare one, since it is representative of a case where the initial state has been prepared in a finite time interval, short with respect to the propagation time of the virtual pulse to the detector. We hope that it is easily understandable, however, that without the simplifications introduced here, it would have been extremely difficult to adopt a quantitative approach as we have done, and that because of this the present work yields a definite advantage over previous semiquantitative or global treatments on the nature of half-dressed sources. We wish to emphasize that the model discussed here does not pretend to represent adequately the properties of the radiation field in all possible cases of half-dressed sources.

ACKNOWLEDGMENTS

The authors are indebted to E. A. Power and A. O. Barut for discussions on the subject of this paper and to P. D. Drummond for communication of unpublished results. They acknowledge with thanks the receipt of a NATO collaborative Grant No. RG85/0112, during which this work was carried out. Partial support by Comitato Regionale Ricerche Nucleari e Struttura della Materia and by Gruppo Nazionale Struttura della Materia (CNR, Italy) is also acknowledged.

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