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Frequency locking of two transverse optical modes of a laser

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The interaction of nearly degenerate TEM_{01} and TEM_{10} modes of a laser is investigated. When the difference between the oscillation frequencies of the two modes is decreased, a transition occurs from a nearly sinusoidal "mode-beating" intensity modulation via periodic and irregular pulsing to frequency-locked steady-state emission in a TEM_{01}^{*} hybrid mode. The formation of this hybrid mode by frequency locking is a simple example of the recently predicted emergence of stable transverse patterns in nonlinear optical resonators, brought about by frequency locking of near-degenerate, competing transverse modes.

Recently, instabilities forming or changing stable transverse radiation patterns were predicted for passive[1] and active[2,3] nonlinear optical resonators. We report on a simple experiment in which a stable spatial pattern is formed by "cooperative frequency locking"[2] of the nearly degenerate TEM_{01} and TEM_{10} modes of a helium-neon laser.

Although there have been early observations of frequency locking of transverse modes in multiple-transverse-[4] and multiple-longitudinal-mode lasers,[5] this state of operation seems to have been regarded as rather atypical up to now. Instead, particularly for the case that transverse laser modes are only weakly coupled by an inhomogeneously broadened gain medium, it is more common to find that the laser-output intensity contains oscillating "mode-beat" components whose frequencies can be related in a simple manner to frequency differences between pairs of transverse modes.[6] A very different situation arises if the frequencies of different oscillating transverse modes lie within the homogeneous linewidth of the gain medium, and if the modes have sufficient spatial overlap to be strongly coupled by competition for the available inversion: Recent experiments with lasers oscillating in three or more transverse modes have shown that under these conditions, mode interaction can lead to quasiperiodic, intermittent, and chaotic emission.[7–10]

The experiment reported here demonstrates how the nonlinear interaction of *two* nearly degenerate transverse modes TEM_{01} and TEM_{10} in a *single*-longitudinal-mode He-Ne laser changes the spectral composition and shape of the mode-beat signal if the oscillation frequency difference of the two modes *is tuned towards zero.* It is observed that under these conditions, the time dependence of the laser-output intensity changes qualitatively from nearly sinusoidal mode beating via regular and irregular pulsing of the mode intensities to a state where the optical phases of the two modes are locked to a constant difference of $\pm\pi/2$ to yield a stationary TEM_{01}^{*} hybrid mode[11] pattern. It may be noted that the common theoretical models on mode interaction in inhomogeneously broadened gas lasers[12,13] do not permit a suitable description of this case since here, the transverse field distribution of different modes is not equal but orthogonal. However, both chaotic two-transverse-mode lasing[14] and, particularly interesting, the formation of stable stationary spatial patterns that arise from the coexistence of frequency-locked transverse modes have been treated recently.[2,3]

In order to relate the theoretical model of Refs. 2 and 3 more closely to our experimental situation, the coupled-mode equations given in Ref. 2 were reformulated to describe the modal geometry discussed here. The resulting equations describe a homogeneously broadened traveling-wave laser with circular-symmetric pumping that oscillates in the two first-order transverse modes. The equations were solved numerically to study the temporal evolution of the amplitudes and optical phases of the modes for different parameter sets. In particular, the two cases of a "bad-cavity" laser ($\gamma_{\parallel}=\gamma_{\perp}=\kappa/3$, where $\gamma_{\parallel}$ is the population decay rate, $\gamma_{\perp}$ is the polarization decay rate, and κ is the cavity field decay rate) and of a laser with slow inversion relaxation ($\gamma_{\parallel}=\gamma_{\perp}/100=\kappa/10$) were studied for conditions of symmetrical tuning and weak pumping (pump parameter two times larger than threshold value). The main results of the calculations may be summarized as follows.

(a) For both parameter combinations, there exist critical values for the difference $|\delta|$ of the empty-cavity mode eigenfrequencies below which both modes oscillate with equal amplitude and frequency, maintaining a constant phase difference close to $\pm\pi/2$ (stable steady-state TEM_{01}^{*} operation). For $|\delta|$ above the critical value, the modes are not phase locked and show temporal variations of frequency and amplitude due to their mutual interaction.

(b) For the bad-cavity parameters, decreasing $|\delta|$ to the critical value results in a continuous transition from synchronous oscillation of the mode amplitudes at $2\nu_B$ (ν_B is the mode beat frequency) to synchronous regular pulsing; during each pulse, the phase difference between the modes changes by $\pm\pi$.

(c) For the "slow-inversion" parameters, approaching the frequency-locked state by decreasing $|\delta|$ leads to a period doubling of the mode-amplitude modulation at $2\nu_B$. In this bifurcation, the synchronicity of the mode-amplitude variations is lost. A further small decrease of δ results in irregular, unsynchronized pulsing of the mode amplitudes. The period-doubled and chaotic solutions coexist for a certain interval of $|\delta|$ with stable steady-state solutions that correspond to TEM_{01}^{*} operation.

Small deviations from symmetry in the coupled-mode model considered, e.g., by asymmetric tuning or inhomogeneous pumping, did not qualitatively change the results of the calculations.

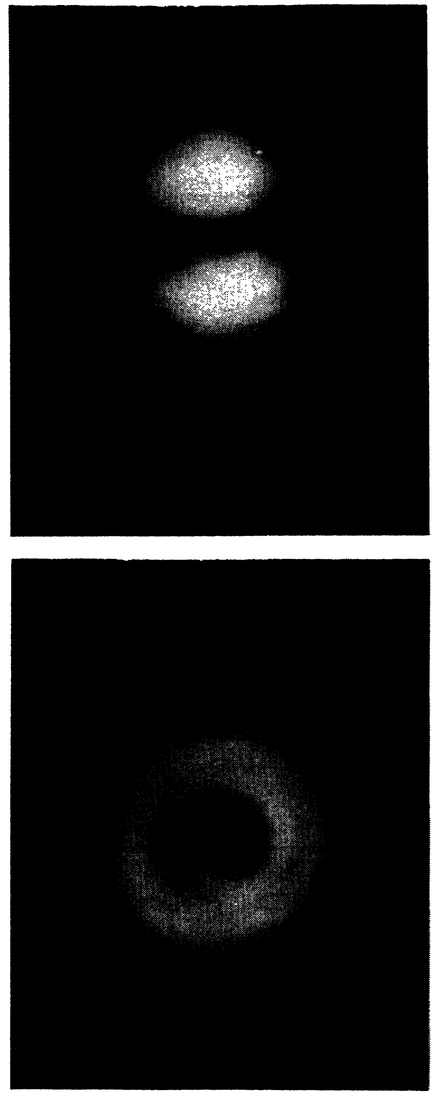
The experimental setup consisted of a 22-cm long commercial He-Ne laser tube terminated by Brewster windows (Melles-Griot type 05-LHB-290). The linear, near-confocal laser resonator had a length of 25 cm and used mirrors with a radius of curvature of 60 cm and a reflectivity of 99.2% at $\lambda = 633$ nm. The astigmatism of the ~1.5-mm thick Brewster windows was calculated[15] to lead to a difference in resonance frequency for the TEM_{01} and TEM_{10} modes of $|\delta| = 0.6$ MHz for an orientation of the nodal planes of the modes parallel and perpendicular, respectively, to the plane of incidence of the Brewster windows. The mode eigenfrequencies can, however, become degenerate ($\delta = 0$) if the nodal planes of the modes are rotated by 45° from this position. One output beam of the laser was used to determine visually the time-averaged transverse-mode pattern of the laser. A section of the other output beam illuminated a fast photodetector to allow for generation of transverse-mode-beating signal.[6]

With a discharge current about four times larger than the threshold current, the laser operated in a mixture of TEM_{00}, TEM_{01}, and TEM_{10} modes that were linearly polarized along a common axis defined by the Brewster windows. For the experiments described below a resonator length was chosen at which only one longitudinal mode was oscillating (oscillation in two longitudinal modes did not seem to significantly affect the observed behavior).

The transverse-mode structure of the laser and the frequency separation $|\delta|$ was controlled by a dot of absorbing material (approximate size, 50 μm) on the surface of an intracavity Brewster plate which could be positioned precisely with respect to the resonator axis by means of an x-y translation stage. Suitable positioning of the absorbing dot suppressed oscillation in the TEM_{00} mode. Small translations of some tens of micrometers also reproducibly permitted us to define the losses and, together with the effect of the slightly noncentral aperture of the laser tube bore, the orientation of the modal planes of the first-order transverse modes. In this way δ could be varied through zero while maintaining the time-averaged intensities of both modes approximately equal.

Figure 1 shows the output-intensity pattern of the laser for oscillation in a single first-order transverse mode [Fig. 1(a)] and for approximately equally strong TEM_{01} and TEM_{10} modes [Fig. 1(b)]. For the recording of the data shown below, the dot absorber was first positioned to obtain the "doughnut" pattern of Fig. 1(b) which usually resulted in a mode-beat frequency of $\nu_B = 1-2$ MHz. The dot was then moved in small steps of a few μm along an approximately straight line in a direction that led to a decrease of ν_B but left the visual shape of the intensity pattern unchanged.

The radio-frequency (rf) intensity spectra obtained for a decreasing fundamental mode-beat frequency ν_B are shown in Fig. 2. It can be seen that the decrease of ν_B first leads to an increase in the amplitude of harmonics



(a)

(b)

FIG. 1. Photographs of the transverse intensity distribution of the helium-neon laser for the cases (a) of single first-order transverse-mode oscillation and (b) oscillation in the two orthogonal first-order transverse modes. The stripes visible at the bottom of the patterns result from interference caused by spurious reflection off the back side of the outcoupling mirror.

[Fig. 2 (b)] and, then, to a period-doubling bifurcation [Fig. 2(c)]; after at least one more observable period doubling (not shown for brevity), the spectrum becomes broad and quite featureless [Fig. 2(d)]. Finally, there is a sudden transition to stationary (frequency-locked) two-mode operation [Fig. 2(e)]. It was not possible to determine the value of $|\delta|$ below which frequency locking occurs; it can only be noted that the frequency-locked state could be reached reproducibly and that it remained stable for virtually infinite periods (several hours) in spite of drifts and noise in the experimental setup.

Figure 3 shows the time dependence of the detector signal under conditions similar to those of Fig. 2. In agreement with what can be inferred from the spectra shown in Fig. 2, it turns out that for decreasing ν_B, the mode-beat

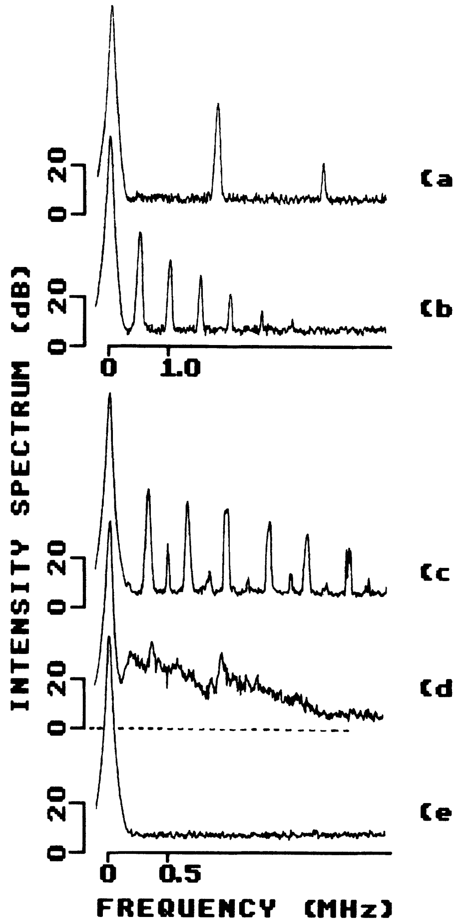


FIG. 2. rf intensity spectra of the laser emission when the laser simultaneously oscillates in the two first-order transverse modes. Spectra (a) - (e) are recorded when the mode-beating frequency is successively reduced towards frequency-locked, stationary operation (e) (see text). Please note the different frequency scales as indicated.

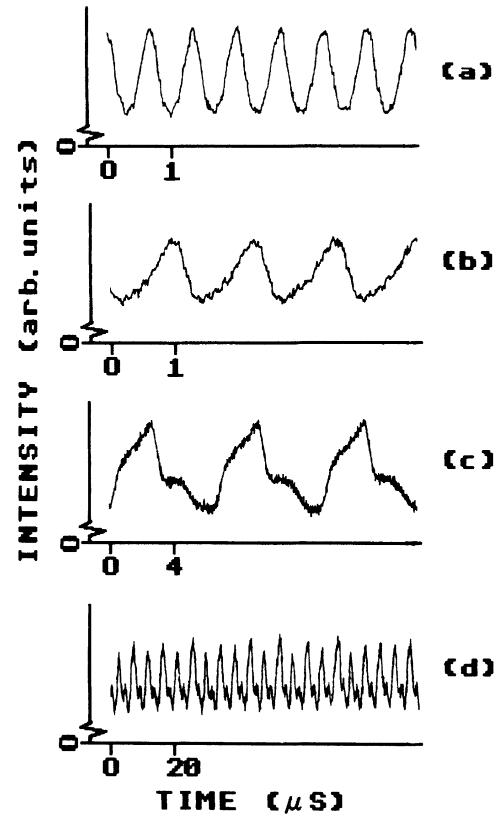


FIG. 3. Time dependence of the laser emission when the laser simultaneously oscillates in the two first-order transverse modes, showing a transition from mode-beating-type intensity modulation (a) via regular self-pulsing (b), (c) to "chaotic" pulsing (d) when the mode-beating frequency is successively reduced. Please note the different time scales as indicated.

signal evolves from a nearly sinusoidal shape [Fig. 3(a)] into a pulse train [Fig. 3(b) and 3(c)] which becomes irregular with respect to the pulse heights [Fig. 3(d)] in the region where the corresponding spectrum is broad. Different pulse shapes of the detector signal could be observed for the case that the detector was scanned across the transverse-beam profile of the laser and for the case that the integral laser power was focused on the active area of the photodetector. In all cases, however, the basic spectral and pulse-shape features remained unchanged.

An interpretation of the experimental findings may start from the observation (see Fig. 2) that the harmonic content of the mode-beat signal strongly increases if its fundamental frequency ν_B is reduced: Under the condition that ν_B becomes comparable to or smaller than the homogeneous linewidth of the gain medium, the two oscillating modes share a large part of the inhomogeneously broadened population inversion. On the one hand, this situation is known to lead to an increase of the effective mode frequency separation ("mode pushing"[12]); in fact, the observed maximum mode-beat frequency ($\nu_B \sim 1$-2 MHz) is significantly larger than the calculated empty-cavity frequency spacing of $|\delta| = 0.6$ MHz. On the other hand, interference of the modal fields in the nonlinear gain medium also implies that gain and refractive index of the active medium become time dependent. More concretely, the theoretical model mentioned above predicts that the intensities and the optical phases of the modes are modulated with $n\nu_B$ (n even), while odd harmonics of ν_B are not excited as a result of the orthogonality of the modal field distributions. Difference-frequency mixing of the various spectral components at the photodetector leads to the observed harmonics $n\nu_B$ ($n = 2,3,4\ldots$) of the intermode-beat signal. It is reasonable to suppose that the formation of harmonics is most effective for those harmonics where $n\nu_B$ is small as compared to the relaxation rates of the gain medium and the linewidth of the resonator.

The time dependence of the detector signal (Fig. 3) reflects the intensity variations of a part of the mode interference pattern as the phase difference between the modal fields evolves according to both their average oscillation frequency difference and the imposed phase modulation. The results of the numerical studies suggest to relate the distinct sawtoothlike asymmetries of the pulses that develop for $\nu_B \to 0$ to the effect of a phase-locking "force" that tends to keep the system close to states of a certain constant phase difference between the modes [the two low slope regions per period in Fig. 3(c)] while other phase differences are more unstable (high slope regions). This picture, however, is not helpful in establishing a simple dynamical model for the observed period-doubling transition to irregular, "chaotic" pulsing [Fig. 3(d)].

It may be noted that the theoretical model considered above differs from the experimental situation in that it neglects inhomogeneous broadening, the standing-wave structure of the field and the supposedly complicated relaxation dynamics of the lasing transition. Nonetheless, the main observed phenomena, particularly the self pulsing, the locking, and the chaotic dynamics close to the locking point are also found in the model.

It is perhaps the most interesting aspect of the experiment that it reveals one possible route towards the formation of a stable, stationary transverse structure formed by coexisting, frequency-locked transverse cavity modes [see Figs. 1(b) and 2(e)]. For other modal geometries[2,3] the calculated transverse patterns seem to reflect the interplay of self-focusing and local-gain saturation in the laser medium. In the case considered here, an intensity distribution of circular symmetry is formed that consists of the two first-order transverse modes locked to a phase difference of $\pm\pi/2$; this field configuration obviously makes best use of the population inversion that is available for the two modes in the gain medium.

Finally, it should be mentioned that like the patterns discussed in Ref. 2, the case of TEM_{01}^{*} operation also corresponds to a state of broken spatial symmetry since the equiphase surface of the optical field of the TEM_{01}^{*} configuration must be thought to wind around the optical axis in an either left- or right-handed spiral, corresponding to the two possible phase-locking angles of the constituent modes of $+\pi/2$ or $-\pi/2$. Transitions between these two stable states will leave the intensity pattern unchanged; however, they should be easily detectable interferometrically.[16]

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[1]L. A. Lugiato and R. Lefever, Phys. Rev. Lett. **58**, 2209 (1987).

[2]L. A. Lugiato, C. Oldano, and L. M. Narducci, J. Opt. Soc. Am. B **5**, 879 (1988).

[3]L. A. Lugiato, G. -L. Oppo, M. A. Pernigo, J. R. Tredicce, L. M. Narducci, and D. K. Bandy, Opt. Commun. **68**, 63 (1988).

[4]D. H. Auston, IEEE J. Quantum Electron. **QE-4**, 471 (1968).

[5]Y. Watanabe, T. Fujioka, and M. Kobayashi, IEEE J. Quantum Electron. **QE-4**, 880 (1968).

[6]J. P. Goldsborough, Appl. Opt. **3**, 267 (1964); T. Uchida, *ibid.* **4**, 129 (1964).

[7]R. Hauck, F. Hollinger, and H. Weber, Opt. Commun. **47**, 141 (1983).

[8]C. O. Weiss, A. Godone, and A. Olafsson, Phys. Rev. A **28**, 892 (1983); see also N. J. Halas, S. -N. Liu, and N. B. Abraham, Phys. Rev. A **28**, 2915 (1983).

[9]D. J. Biswas and R. G. Harrison, Phys. Rev. A **32**, 3835 (1985); D. J. Biswas, Vas Dev, and U. K. Chatterjee, Phys. Rev. A **38**, 555 (1988).

[10]W. Klische and C. O. Weiss (unpublished).

[11]W. W. Rigrod, Appl. Phys. Lett **2**, 51 (1963).

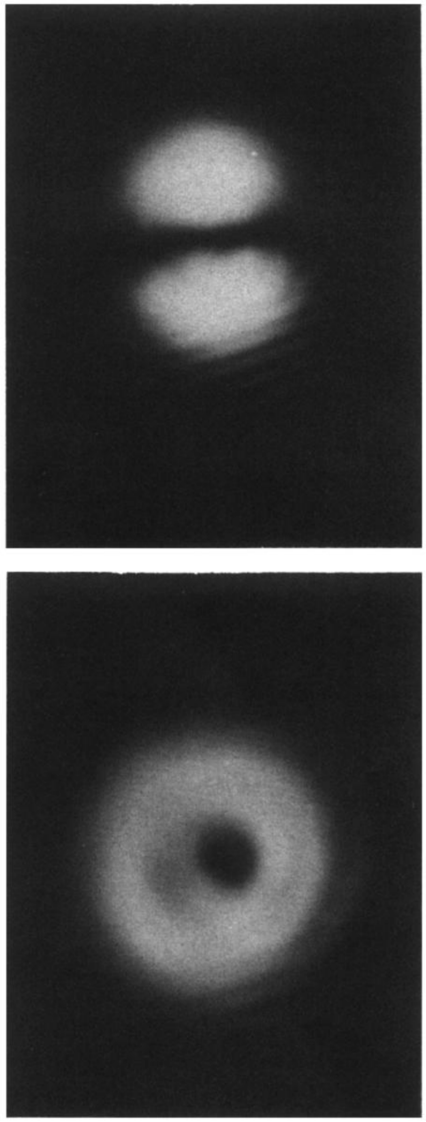
[12]W. R. Bennet, Jr., Phys. Rev. **126**, 580 (1962).

[13]W. E. Lamb, Jr., Phys. Rev. **134**, A1429 (1964).

[14]M. L. Shih and P. W. Milonni, Opt. Commun. **49**, 155 (1984).

[15]D. C. Hanna, IEEE J. Quantum Electron. **QE-5**, 483 (1969).

[16]Chr. Tamm (unpublished).



(a)

(b)

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