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Quantum beats for the measurement of topological phases in Rabi oscillations of two-level atoms

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We demonstrate how the topological phase associated with Rabi flopping in two-level atoms in optical fields can be measured by quantum-beat experiments. We use the Zeeman coherences in $j=0$ to $j=1$ transition which is excited by both broad-band field and circularly polarized pulses.

Recently, many papers have been devoted to the geometrical or topological phases associated with either adiabatic[1] or cyclic evolution.[2] The predictions of the theory have been verified by several experiments.[3,4] It is well known[5] that the dynamical evolution of a two-level atom in presence of external electromagnetic field is like the evolution of a spin in a magnetic field. Thus the ideas of Berry[1] or Aharonov and Anandan[2] are also applicable to the wave functions of two-level atoms in an electromagnetic field. The following question arises: How does one measure the topological phases associated with Rabi oscillations of two-level atoms? In case of light beams such phases are usually measured by an interference experiment.[3] In this Rapid Communication we propose a method based on the ideas of coherent transients in three-level systems. We show how the geometrical phase associated with Rabi oscillations can be measured using quantum beats[6] in a suitably prepared three-level system.

Our proposal consists of the following: consider $j=0$ to $j=1$ transition in an atomic system, as shown schematically in Fig. 1. We will consider fields polarized such that only $|j=0,m=0\rangle \rightarrow |j=1,m=\pm 1\rangle$ transitions are allowed. Consider the excitation of the system by a broad-band field which is linearly polarized. We will see that this broad-band field creates coherence between two Zeeman levels $|j=1,m=\pm 1\rangle$ but no coherence between excited levels and ground state. This coherence ρ_{12} is responsible for the production of quantum beats. We next consider the two-level system consisting of states $|1\rangle$ and $|3\rangle$. Transitions in this two-level system can be induced by a left-circularly polarized light which does not couple to the level $|2\rangle$. Thus, the left-circularly polarized light will lead to Rabi oscillations between the levels $|1\rangle$ and $|3\rangle$. We can now apply a series of left-circularly polarized fields to produce a cyclic evolution and to have a circuit in the appropriate space. This will result in the geometrical phases for the state $|1\rangle$ and $|3\rangle$ but no geometrical phase for the state $|2\rangle$. Thus, the quantum-beat amplitude, which is given by ρ_{12}, can monitor the geometrical phase of the state $|1\rangle$. Thus, our proposal involves the following steps: (a) prepare the system $j=0 \leftrightarrow j=1$ by exciting it with a broad-band linearly polarized field, (b) monitor the beat amplitude, (c) use left-circularly polarized pulses to selectively have the cyclic evolution between two states, and (d) observe the phase changes in the beat amplitude. These phase changes would be a measure of the geometrical phases induced by the cyclic evolution in step (c). We will see that one advantage of using the incoherent excitation in step (a) is that step (c) does not introduce any dynamical phase.

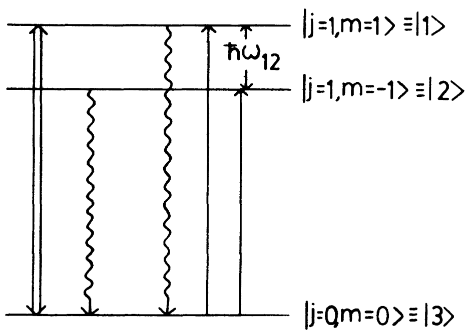

FIG. 1. Schematic diagram of the three-level system considered in the text. Single arrows represent the excitation of the system by broad-band incoherent light. The double arrow represents the cyclic evolution of the two-level system (states $|1\rangle$ and $|3\rangle$) in presence of left-circularly-polarized pulses. The wavy lines give the spontaneous emission from a coherently prepared system. The interference between the two spontaneous-emission amplitudes gives rise to quantum beats.

We next discuss the mathematical basis for the above proposal. Consider the interaction of the two-level system[7] with the electromagnetic field. Let the state of this two-level system be designated as $|1\rangle$ and $|3\rangle$. The interaction Hamiltonian can be expressed as

$$H=\hbar g|\varepsilon|(A_{13}e^{i\theta}+e^{-i\theta}A_{31}), \quad A_{ij}=|i\rangle\langle j| , \tag{1}$$

where $|\varepsilon|$ is the amplitude of the field, θ its phase, and g denotes the coupling constant. Using (1), the wave function evolves as

$$\begin{aligned}\psi_1(t)&=\cos(g|\varepsilon|t)\psi_1(0)-ie^{i\theta}\psi_3(0)\sin(g|\varepsilon|t) ,\\ \psi_3(t)&=\cos(g|\varepsilon|t)\psi_3(0)-ie^{-i\theta}\psi_1(0)\sin(g|\varepsilon|t) .\end{aligned} \tag{2}$$

The evolution is obviously cyclic, i.e.,

$$\psi(\tau)=e^{i\pi}\psi(0) \quad \text{if } g|\varepsilon|\tau=\pi . \tag{3}$$

The dynamical phase will depend on the initial state. If we choose $\psi(0)$ as

$$\psi(0)=(1+|\mu|^2)^{-1/2}(|1\rangle+|\mu|e^{i\chi}|3\rangle)\,, \tag{4}$$

then the dynamical phase δ is found to be

$$\delta=\frac{2\pi|\mu|}{(1+|\mu|^2)}\cos(\theta+\chi)\,. \tag{5}$$

Hence, the geometrical phase β will be

$$\beta=\pi+\frac{2\pi|\mu|}{(1+|\mu|^2)}\cos(\theta+\chi)\,. \tag{6}$$

The dynamical phase will be zero if the two-level atom starts either in the state $|1\rangle$ or $|3\rangle$. Dynamical phase will also be zero if χ were a random quantity. Equation (6) shows how geometrical phases arise in the usual Rabi oscillation of atoms.

Consider another example of cyclic evolution. Let us apply a pulse with phase θ_1 such that $g|\varepsilon|\tau=\pi/2$ and then a second pulse with phase θ_2 and $g|\varepsilon|\tau=\pi/2$. In this case it is clear from (2) that

$$\begin{aligned}\psi_1\left(\frac{\pi}{g|\varepsilon|}\right)&=e^{i(\pi+\theta_2-\theta_1)}\psi_1(0)\,,\\ \psi_3\left(\frac{\pi}{g|\varepsilon|}\right)&=e^{i(\pi+\theta_1-\theta_2)}\psi_3(0)\,.\end{aligned} \tag{7}$$

If we assume that the initial state corresponds to either $\psi_3(0)=0$ or $\psi_1(0)=0$, then the dynamical phases are zero and the geometrical phase associated with the two initial conditions will be

$$\begin{aligned}\beta_1&=\pi+\theta_2-\theta_1 \quad \text{if } \psi_3(0)=0\,,\\ \beta_2&=\pi+\theta_1-\theta_2 \quad \text{if } \psi_1(0)=0\,.\end{aligned} \tag{8}$$

The Bloch vector with components $2\mathrm{Re}\langle A_{13}\rangle$, $2\mathrm{Im}\langle A_{13}\rangle$, $\langle A_{11}-A_{33}\rangle$ moves on a sphere with a radius determined by the initial population in the states $|1\rangle$ and $|3\rangle$. Specifically, for the case $\psi_3(0)=0$, the Bloch vector, at any time, makes an angle $2g|\varepsilon|t$ with the z axis and its projection in xy plane makes an angle $(3\pi/2)-\theta_1$ with the x axis. At the end of the first pulse $[g|\varepsilon|t=(\pi/2)]$ the Bloch vector is pointing downwards. During the second pulse, the Bloch vector moves on the sphere making an angle $\pi-2g|\varepsilon|t$ with the z axis and its projection in xy plane makes an angle $[(\pi/2)-\theta_2]$ with the x axis. It is clear that the Bloch vector follows a closed circuit on the sphere during the above evolution. The solid angle subtended by this circuit is $2\pi+2\theta_2-2\theta_1$ and, hence, the geometrical phase is half of this solid angle.[8]

We next discuss how the geometrical phases can be seen in a quantum-beat experiment. Let the three-level system (Fig. 1) be prepared by an initial broad-band x-polarized excitation ε_B. The field ε_B is taken as a Gaussian δ-correlated process $[\langle\varepsilon_B(t)\varepsilon_B^*(t')\rangle=2I_B\delta(t-t')]$ with zero mean. The evolution of the density matrix of the system during the preparation stage can be studied in terms of the master equation obtained by eliminating the degree of freedom associated with broad-band field.[9] It can be shown that the evolution is described by

$$\begin{aligned}\dot\rho=&-I_0[A_{13}+A_{23},[A_{31}+A_{32},\rho]]-I_0[A_{31}+A_{32},[A_{13}+A_{23},\rho]]\\ &-i[(\omega_{13}-\omega_L)A_{11}+(\omega_{23}-\omega_L)A_{22},\rho]-2\gamma(A_{11}\rho-A_{31}\rho A_{13}-A_{32}\rho A_{23}+\rho A_{11})\,,\end{aligned} \tag{9}$$

where I_0 is proportional to the square of the radial matrix element and I_B and 2γ is the rate of spontaneous emission from $|1\rangle$ and $|2\rangle$. The steady-state solution of (9) can be shown to be

$$\begin{aligned}&\rho_{13}=\rho_{31}=\rho_{23}=\rho_{32}=0\,,\\ &\mathrm{Im}\rho_{12}=-\frac{\omega_{12}}{2(\gamma+I_0)}\mathrm{Re}\rho_{12}\,,\\ &\mathrm{Re}\rho_{12}=\frac{4\gamma I_0(\gamma+I_0)}{(\gamma+3I_0)}\Bigg/\left[\omega_{12}^2+\frac{4\gamma(\gamma+4I_0)(\gamma+I_0)}{(\gamma+3I_0)}\right].\end{aligned} \tag{10}$$

Thus, initially only coherence between the two Zeeman levels exists.[10] This coherence depends on parameters such as magnetic field. There is, of course, the population in all three levels. We do not give explicit expression for these. Thus, as far as the levels $|1\rangle$ and $|3\rangle$ are concerned, we have an incoherent superposition of these. *This incoherent superposition makes the dynamical phase zero* and thus the *total-phase change in cyclic evolution is of geometric origin.* For the case when one applies a single pulse $g|\varepsilon|\tau=\pi$ with phase θ, the beat amplitude will be

$$\begin{aligned}\rho_{12}=\langle\psi_1\psi_2^*\rangle=&\cos(g|\varepsilon|t)\langle\psi_1(0)\psi_2^*(0)\rangle-ie^{i\theta}\\ &\times\sin(g|\varepsilon|t)\langle\psi_3(0)\psi_2^*(0)\rangle\\ =&\cos(g|\varepsilon|t)\langle\psi_1(0)\psi_2^*(0)\rangle=-\rho_{12}(0)\,,\end{aligned} \tag{11}$$

since $\rho_{32}(0)=\langle\psi_3\psi_2^*\rangle$ is zero. Thus, the phase of the beat amplitude changes by π, which is precisely the geometric phase[11] associated with cyclic evolution. Thus quantum beats can be directly used to measure the geometric phases associated with the Rabi flopping in two-level atoms. Note that the above phase change is in addition to the phase change $\omega_{12}\pi/g|\varepsilon|$ due to free evolution of the system.

For the second example of cyclic evolution [Eq. (7)], the beat amplitude can be shown to change by

$$\rho_{12}\left(\frac{\pi}{g|\varepsilon|}\right)=e^{i(\pi+\theta_2-\theta_1)}\rho_{12}(0)\,. \tag{12}$$

Thus, the phase of the beat amplitude will change by an amount equal to the geometrical phase $\beta_1\equiv\pi+\theta_2-\theta_1$. One can thus monitor the change in the phase of the beat amplitude for different phase settings of the laser fields.

As mentioned previously this change is half the solid angle subtended by the closed circuit traced by the Bloch vector during its evolution.

Thus, in conclusion we have shown how the geometrical phases associated with the Rabi flopping of atoms in optical fields can be monitored by quantum-beat spectroscopy. We have also shown how the dynamical phases can be made zero so that the measurement directly gives geometrical phases.

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[10]It is not necessary to consider the steady-state preparation (10). One can consider excitation with a broad-band pulse of finite duration. Even then $\rho_{12}=\rho_{23}=0$.

[11]The geometrical phase associated with the coherence ρ_{12} can also be monitored by other methods such as Hanle measurements.