Relativistic excitation of envelope solitons in electron-positron plasmas of the pulsar magnetosphere

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The nonlinear propagation of an external magnetic-field-aligned circularly polarized electromagnetic wave in electron-positron plasma is investigated. Relativistic and the time-derivative ponderomotive nonlinearities are considered. A new kind of envelope soliton in the magnetoactive plasma is obtained. A large-amplitude localized electromagnetic radiation may exist in the pulsar magnetosphere.

Recently, there has been much interest[1–3] in the problem of wave propagation in electron-positron plasma. It is due to its application in the fields of pulsar radiation and solid-state plasma whose positive and negative particles are of the same (or similar) mass. Pulsars are regarded as the rotating magnetized neutron stars with a strong magnetic field ($\sim 10^{12}$ G). Theoretical models[4,5] had been developed to predict the production of electron-positron plasma. In such plasma, waves which are quite different from those of the ion-electron plasma can exist. The reason is that when both the charge and the mass are of similar magnitude for the two species, some of the natural separation of time and space scales associated with high- and low-frequency motions no longer exist. In fact, the separation of time and space scales may also originate in electron-positron plasma, exclusively from nonlinear effects such as wave modulation and turbulence.

In this report we reconsider the nonlinear propagation of field-aligned (parallel to the external magnetic field $B_0\hat{\mathbf{z}}$), circularly polarized electromagnetic wave in an electron-positron plasma. The previous investigations[6–8] are thus generalized to include the effects of time derivative poderomotive force. The field amplitude is large enough that particles acquire relativistic velocities in the field. The external magnetic field causes substantial change in the interaction, especially; relativistic effects come into play at a weaker high-frequency field. A weakly nonlinear treatment of the problem shows that a new kind of envelope solitons may exist in the magnetoactive plasma. A large-amplitude localized electromagnetic wave also can exist in the pulsar magnetosphere.

In our earlier paper,[6] it has been shown that the nonlinear interaction of the right-hand circularly polarized electromagnetic wave

$$\mathbf{E} = = E(\hat{\mathbf{x}} + i\hat{\mathbf{y}})\exp(ikz - i\omega t) + \text{c.c.} \ , \tag{1}$$

where slow plasma motion along the ambient field $B_0\hat{\mathbf{z}}$ gives rise to a slowly varying wave electric field envelope which may be localized. When the frequency of the modulation is much smaller than the carrier wave (viz., $\partial_t \ll \omega$) within the WKB approximation the evolution of the wave electric field envelope is governed by the nonlinear Schrödinger equation[9]

$$i(\partial_t + v_g\partial_z)E + \tfrac{1}{2}v_g'\partial_z^2 E - \Delta E = 0 \ , \tag{2}$$

where $v_g = \partial\omega/\partial k$ and $v_g' = \partial^2\omega/\partial k^2$ represent the group velocity and the group dispersion, respectively. The frequency ω and the wave number k are related by the cold plasma dispersion relation

$$\frac{k^2c^2}{\omega^2} = 1 - \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega(\omega\sqrt{1+v_\alpha^2} - \omega_{c_\alpha})} \ , \tag{3}$$

where the quantity $|\omega\sqrt{1+v_\alpha^2} - \omega_{c_\alpha}|/k$, has been assumed to be much larger than the thermal velocities, c is the speed of light, $\omega_{p_\alpha}^2 = 4\pi e_\alpha^2 n_\alpha/m$, $\omega_{c_\alpha} = (e_\alpha B_0/mc)$, m is the particle rest mass, e_α the charge, and n_α the particle number density of species α (e for electron and p for positron):

$$v_\alpha^2 = \frac{p_{\alpha_x}^2 + p_{\alpha_y}^2}{m^2c^2} = \frac{e^2|E|^2(1+v_\alpha^2)}{m^2c^2(\omega\sqrt{1+v_\alpha^2} - \omega_{c_\alpha})^2} \tag{4}$$

is the parameter describing the importance of relativistic effects on the wave propagation.[10]

From the wave equation

$$\left[\frac{1}{c^2}\partial_t^2 - \partial_z^2\right]\mathbf{E} = \frac{4\pi}{c^2}\partial_t \mathbf{J}$$

(where $\mathbf{J}$ is the nonlinear current) one can easily derive the nonlinear frequency shift Δ which is given by

$$\Delta = -\frac{1}{2\omega}\left[\omega^2 - \kappa^2c^2 - \sum_\alpha \frac{\omega\omega_p^2(1+N_\alpha)}{\omega\sqrt{1+v_\alpha^2} - \omega_{c_\alpha}}\right] \ . \tag{5}$$

Here $n_\alpha = n_0 + \delta n_\alpha$ and $N_\alpha = \delta n_\alpha/n_0$ with n_0 the equilibrium density and δn_α the small perturbation due to radiation.

We assume that modulation frequency is much smaller than the particle gyrofrequency; then the slow motion of the plasma can be described by the magnetohydrodynamics (MHD) equations:

$$\partial_t N_\alpha + \partial_z V_{\partial_z} = 0 \,,$$

$$m\partial_t V_{\partial_z} = -e_\alpha \partial_z \Phi - \frac{T_\alpha}{n_\alpha}\nabla n_\alpha + \frac{f}{n_\alpha} \,, \tag{6}$$

$$\partial_z^2 \Phi = 4\pi e n_0 (N_e - N_p) \,.$$

Here Φ is the slow ambipolar potential driven by the ponderomotive force f, which incorporates the relativistic motion of the particles,

$$f = -\frac{1}{16\pi}\sum_\alpha \frac{\omega_{p_\alpha}^2}{\omega(\omega\sqrt{1+v_\alpha^2}-\omega_{c_\alpha})} \times \left[\partial_z - \frac{k\omega_{c\alpha}}{\omega(\omega\sqrt{1+v_\alpha^2}-\omega_{c_\alpha})}\partial_t\right]|E|^2 \,. \tag{7}$$

An expression for the ponderomotive force f was derived in Ref. 9.

The system of equations (6) is equivalent to the following equation:

$$\partial_t^2 N_\alpha - V_{th_\alpha}^2(1-N\alpha)\partial_z^2 N_\alpha + V_{\mathrm{th}\alpha}^2(\partial_z N_\alpha)^2 = \frac{e_\alpha}{|e|}\omega_p^2(N_e-N_p) - \frac{1}{n_0 m}\partial_z(1-N_\alpha)f \,, \tag{8}$$

where $V_{\mathrm{th}\alpha}^2 = T_\alpha/m$. Equation (8) describes the slow plasma density perturbation due to the time derivative ponderomotive force and the generated ambipolar field.

We thus have derived a system of coupled equations, namely, (2), (5), and (8), which describes the nonlinear evolution of a circularly polarized electromagnetic wave in which field particles attain relativistic velocities in a magnetoactive electron-positron plasma. It is to be noted that the time derivative ponderomotive force is more significant in the case of the magnetoactive plasma.

We now consider the nonlinear propagation of the field-aligned circularly polarized wave in the weak relativistic limit (i.e., $v_\alpha^2 \ll 1$). In this particular case we find that

$$\Delta = \frac{1}{2\omega}\sum_\alpha \frac{\omega\omega_p^2}{\omega-\omega_{c_\alpha}}(N_\alpha - R_\alpha|E|^2) \,,$$

$$R_\alpha = \frac{e^2\omega}{2m^2c^2(\omega-\omega_{c_\alpha})^3} \tag{9}$$

$$f = -\frac{\omega_p^2}{8\pi(\omega^2-\omega_c^2)}\left[\partial_z - \frac{2k\omega_c^2}{\omega(\omega^2-\omega_c^2)}\partial_t\right]|E| \,,$$

$$\omega_c = \frac{|e|B_0}{mc} \,. \tag{10}$$

The first term in Eq. (9) represents the ponderomotive nonlinearity, while the second term the relativistic nonlinearity.

Let us consider a linear phase shift of the wave due to the interaction with the plasma. In other words we represent the complex field amplitude as

$$E = |E|e^{-i\theta t + i\kappa z} \,, \tag{11}$$

where θ,κ are constants. Then it can be shown[7] that

$$|E|^2 = f(\xi), \quad \xi = z - V_0 t, \quad V_0 = \frac{kc^2}{\omega}\left[1+\frac{\kappa}{k}\right] \,. \tag{12}$$

In the case of electron-positron plasma, we see that the ponderomotive force is charge independent even in the magnetoactive plasma. Therefore the charge separation occurs only in the higher order of nonlinearity due to the term $\partial_z(N_\alpha f)$ which is in the order of $|E|^4$ nonlinearity. In this particular case of weakly nonlinear plasma we may neglect the charge separation. Then from Eqs. (8)–(10) we find that

$$N = \frac{e^2}{2m^2(\omega^2-\omega_c^2)(v_0^2-V_{\mathrm{th}}^2)}\left[1+\frac{2kv_0\omega_c^2}{\omega(\omega^2-\omega_c^2)}\right]|E|^2 \,, \tag{13}$$

and

$$\Delta = -Q|E|^2 \,, \tag{14}$$

with

$$Q = \frac{e^2\omega_p^2\omega}{2m^2c^2(\omega^2-\omega_c^2)^2}\left[\frac{1}{2}\frac{(\omega-\omega_c)^4+(\omega+\omega_c)^4}{(\omega^2-\omega_c^2)^2} - \frac{c^2}{v_b^2-V_{\mathrm{th}}^2}\left[1+\frac{2kv_0\omega_c^2}{\omega(\omega^2-\omega_c^2)}\right]\right] \,. \tag{15}$$

Here, we have considered $T_e = T_p = T$ and $N_e = N_p = N$.

The evolution of the wave is then governed by the usual nonlinear Schrödinger equation

$$i(\partial_t + v_g\partial_z)E + \tfrac{1}{2}v_g'\partial_z^2 E + Q|E|^2 E = 0 \,, \tag{16}$$

which, for $v_g' Q > 0$, has the solution

$$|E| = E_m \operatorname{sech}(k_0\xi) \,, \tag{17}$$

where $E_m = (A/Q)^{1/2}$ where $A = \kappa(v_g + \frac{1}{2}v_g'\kappa) - \theta$ is the amplitude of the soliton and $k_0 = E_m(Q/v_g')^{1/2} = (A/v_g')^{1/2}$ is the inverse pulse width of the soliton.

In the pulsar magnetosphere we have $\omega_c^2 \gg \omega^2$. Then

$$Q \cong \frac{e^2\omega_p^2\omega}{2m^2c^2\omega_c^4}\left[1+\frac{c^2}{V_{\mathrm{th}}^2-v_0^2}\left[1-\frac{2kv_0}{\omega}\right]\right] \,. \tag{18}$$

For $v_0^2 < V_{\mathrm{th}}^2 < \omega^2/k^2$, Q is positive, which admits the subthermal envelope solitons as the final state of wave modulation in the pulsar magnetosphere. The pulse width is independent of the magnetic field, while the amplitude of the envelope is proportional to the ambient field as B_0^2. Let us calculate the maximum amplitude of the localized field (soliton) in the pulsar environment.

For the soliton's speed much smaller than the wave phase velocity (i.e., $v_0 < \omega/k$) and for $(V_{\rm th}^2 - v_0^2/c^2 \ll 1$ we may write

$$Q = \frac{e^2\omega_p^2\omega}{2m^2\omega_c^4(V_{\rm th}^2 - v_0^2)}, \tag{19}$$

which gives

$$\frac{eE_m}{m\omega c} = \eta\frac{\omega_c^2}{\omega\omega_p}\left[\frac{V_{\rm th}^2 - v_0^2}{c^2}\right]^{1/2}, \quad \eta = \left[\frac{2A}{\omega}\right]^{1/2}. \tag{20}$$

In the case of $v_0^2 \ll v_{\rm th}^2$ we have

$$E_m = \eta'\frac{B_0^2}{n_0}\sqrt{T}, \quad \eta' = \frac{\omega}{4\pi ec}\eta. \tag{21}$$

It shows that a large-amplitude localized field may exist in the pulsar magnetosphere, where the ambient magnetic field is super strong and the plasma density is relatively low.

To summarize, we have investigated the nonlinear propagation of intense electromagnetic wave, in whose fields particles attain relativistic velocities in a magnetoactive electron-positron plasma. Such a situation prevails in the pulsar magnetosphere. We have incorporated the effects of ponderomotive force-driven density fluctuations to study the problem. It is a generalization of our previous works to include such effects as the time derivative ponderomotive force (which is significant for the magnetoactive plasma), relativistic mass modulation nonlinearity, and the charge separation due to the higher-order nonlinearity. A general system is derived to include all such effects. A weakly nonlinear treatment of the system predicts the existence of large-amplitude subthermal solitons in the pulsar magnetosphere. The result obtained may get its application in the explanation of microstructure of pulsar radio emission and other related phenomena.

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