PHYSICAL REVIEW A VOLUME 38, NUMBER 11 DECEMBER 1, 1988

Dye-laser fluctuations: Comparison of colored loss-noise and white gain-noise models

M. Aguado, E. Hernández-García, and M. San Miguel
Departament de Física, Universitat de les Illes Balears, E-07071 Palma de Mallorca, Spain
(Received 14 March 1988)

A single-mode dye-laser model incorporating pump white noise through gain-parameter fluctuations is analyzed. It includes a fluctuating saturation term. Intensity fluctuations, a first-order-like transition, and intensity correlation functions are calculated and discussed. It is found that effects previously claimed to be a consequence of the presence of colored noise in the context of loss-noise models can be also explained by white gain noise, except for the existence of two time regimes in the early decay of intensity correlation functions.

I. INTRODUCTION

Dye-laser light exhibits anomalous statistical properties. These properties cannot be described by conventional laser theory based on the Langevin equation obtained in the good-cavity limit for the amplitude of the electrical field. This equation incorporates spontaneous-emission noise. Experimental work on dye-laser fluctuations has been recently reviewed by Roy, Yu, and Zhu.[1] Theoretical studies on this problem have also been reviewed.[2] Following the suggestions of Kaminishi *et al.*[3(a)] and of Short, Mandel, and Roy,[3(b)] the current theoretical model of a dye laser includes pump fluctuations with a finite correlation time (colored noise). Pump noise seems to be responsible for the observed anomalous statistical properties. At a formal level the current model can be obtained replacing the loss parameter by a stochastic quantity. In fact, experimental results[4] on a He:Ne laser in which noise is added in a controllable fashion to the loss parameter seem to mimic some of the statistical properties of a dye laser. There are two basic qualitative facts that were reported to support the need of introducing colored noise as opposed to white pump-noise modeling. One is the existence of a first-order-like transition in the most probable intensity value.[5,6] A second one is a very slow initial decay of the intensity correlation function.[7,8,9] Colored noise effects in the normalized intensity fluctuations[1,2] and transient statistics[1,2,10] have also been investigated. More recently, experimental evidence has been reported which identifies the pump laser as the source of noise.[11] This result indicates that a source of colored gain noise is the appropriate one in dye-laser modeling. Still, the theoretical model[1,2,5–10] used in Ref. 11 is the one which can be formally obtained replacing the loss parameter by a colored Gaussian noise.

A natural alternative to the current model used in theoretical studies is the explicit consideration of fluctuations in the gain parameter.[12] This leads to fluctuations in the nonlinear saturation term of the equation for the amplitude of the electrical field.[13] In this paper we study in detail such a model with gain-noise fluctuations. We restrict ourselves to white gain noise. We find that this model describes correctly the anomalous intensity fluctuations. More important is that we find that a white gain-noise model also predicts a first-order-like transition for the most probable intensity value. In addition, it includes parameter ranges for which the intensity correlation function has an initial slow decay. It is therefore shown that effects previously claimed to be due to colored pump noise can be described by a white gain-noise model. However, the initial decay of the intensity correlation function does not show two separated time scales as typically appear for colored noise models. As a consequence, the experimental evidence of noise with a finite correlation time is only of qualitative significance in the modeling of dye-laser fluctuations when considering the initial decay of the intensity correlation function. More detailed experiments on the early time decay of the intensity correlation function would be desirable to discriminate between white and colored gain-noise models.

The paper is organized as follows. The gain-noise model and its relation with more standard models is discussed in Sec. II. Sections III–V contain, respectively, the predictions of the model for the intensity fluctuations, a phase-transition analogy for the most probable intensity, and intensity correlation functions.

II. GAIN-NOISE MODEL

Starting from the semiclassical Maxwell-Bloch equations for single-mode operation and after adiabatic elimination, in the good-cavity limit, of polarization and population inversion variables one arrives at the laser equation for the amplitude of dimensionless electric field $\bar{E}=\bar{E}_1+i\bar{E}_2$.[14] For the case on resonance,

$$\partial_{\bar{t}}\bar{E}=-\kappa\bar{E}+\Gamma\frac{\bar{E}}{1+|\bar{E}|^2}\ , \tag{2.1}$$

κ is the loss parameter or decay constant for the electric field and Γ the gain parameter,

$$\Gamma=\frac{g^2\sigma}{\gamma}\ , \tag{2.2}$$

where g is the matter-radiation coupling constant, γ the decay rate for the polarization, and σ the equilibrium inversion. The standard Langevin equation for the laser follows from (2.1) expanding the nonlinear term and adding a Gaussian white noise $\bar{q}(\bar{t})=\bar{q}_R(\bar{t})+i\bar{q}_I(\bar{t})$ which models spontaneous-emission noise:

$$\partial_{\bar{t}}\bar{E}=(\Gamma-\kappa)\bar{E}-\Gamma\,|\,\bar{E}\,|^2\bar{E}+\bar{q}(\bar{t})\ , \tag{2.3}$$

$$\langle \bar{q}_i(\bar{t})\bar{q}_j(\bar{t}')\rangle = D\delta(\bar{t}-\bar{t}')\delta_{ij},\ \ i,j=R,I\ . \tag{2.4}$$

The model commonly used[1-10] to study dye-laser fluctuations includes a multiplicative noise term in (2.3):

$$\partial_{\bar{t}}\bar{E}=(\Gamma-\kappa)\bar{E}-\Gamma\,|\,\bar{E}\,|^2\bar{E}+\bar{p}(\bar{t})\bar{E}+\bar{q}(\bar{t})\ . \tag{2.5}$$

It is typically assumed that $\bar{p}(\bar{t})$ is an Ornstein-Uhlenbeck noise, that is, a Gaussian noise with zero mean and correlation

$$\langle \bar{p}_i(\bar{t})\bar{p}_j(\bar{t}')\rangle = \frac{Q}{2\tau}\delta_{ij}e^{-|\bar{t}-\bar{t}'|/\tau},\ \ i,j=R,I \tag{2.6}$$

where Q is the noise intensity and τ its correlation time. From a completely phenomenological point of view the fluctuating term $\bar{p}(\bar{t})\bar{E}$ in (2.5) can be understood as arising from fluctuations of the loss parameter κ. In this sense we will refer to (2.5) as the loss-noise model. Equation (2.5) has also been used to describe experiments on gas lasers with a fluctuating loss parameter.[4] A different justification of (2.5) can be given: analyzing the adiabatic elimination procedure from Maxwell-Bloch equations with fluctuating forces for the electric field, polarization, and population inversion one arrives at[15]

$$\partial_{\bar{t}}\bar{E}=-\kappa\bar{E}+\Gamma\frac{\bar{E}}{1+|\,\bar{E}\,|^2}+\bar{\xi}(\bar{t})\frac{\bar{E}}{1+|\,\bar{E}\,|^2}+\bar{q}(\bar{t})\ , \tag{2.7}$$

$\bar{q}(\bar{t})$ is the spontaneous-emission noise in (2.4), and it appears as a combination of the original fluctuating forces associated with the electric field and polarization in the Maxwell-Bloch equations. The random force $\bar{\xi}(\bar{t})$ is proportional to the noise term associated with the population inversion. Its physical meaning is pump noise, and it models in (2.7) fluctuations of the gain parameter Γ originated in σ fluctuations [see (2.2)]. If the nonlinear terms containing $\bar{E}/(1+|\,\bar{E}\,|^2)$ in (2.7) are expanded to third order in $\bar{E}$, an equation of the form (2.5) is formally recovered when the fluctuating saturation term $\bar{\xi}(\bar{t})\,|\,\bar{E}\,|^2\bar{E}$ is neglected.

In this paper we analyze the gain-noise model (2.7) where $\bar{\xi}(\bar{t})=\bar{\xi}_R(\bar{t})+i\bar{\xi}_I(\bar{t})$ is taken to be a Gaussian white noise with correlaton

$$\langle \bar{\xi}_i(\bar{t})\bar{\xi}_j(\bar{t}')\rangle = Q\delta_{ij}\delta(\bar{t}-\bar{t}'),\ \ i,j=R,I\ . \tag{2.8}$$

As mentioned in the Introduction we will see that (2.7) can describe correctly important experimental features which have been attributed to colored-noise effects in the context of the loss-noise model (2.5). We further note that an expansion of the fluctuating nonlinear term in (2.7) introduces a fictitious boundary in the problem which would restrict the possible values of the intensity. As a consequence, the effects of fluctuations in the saturation cannot be consistently studied when using that expansion.[16]

Equation (2.7) is better analyzed when written in terms of the intensity $\bar{I}=|\,\bar{E}\,|^2$ and phase variables. With the same procedure as for the ordinary laser equation[17] and for the loss-noise model,[6] it is possible to obtain a stochastically equivalent model to (2.7) in which the intensity is decoupled from the phase variable:

$$\partial_{\bar{t}}\bar{I}=2\bar{I}\left[-\kappa+\frac{\Gamma}{1+\bar{I}}\right]+\frac{2\bar{I}\sqrt{Q}}{1+\bar{I}}\bar{\xi}_R(\bar{t}) +D+(D\bar{I})^{1/2}\bar{q}_R(\bar{t})\ , \tag{2.9}$$

$$\partial_{\bar{t}}\varphi=\frac{\sqrt{Q}}{1+\bar{I}}\bar{\xi}_I(\bar{t})-\left[\frac{D}{\bar{I}}\right]^{1/2}\bar{q}_I(\bar{t})\ , \tag{2.10}$$

where, for $i,j=R,I$,

$$\langle \bar{\xi}_i(\bar{t})\bar{\xi}_j(\bar{t}')\rangle = \delta(\bar{t}-\bar{t}')\delta_{ij}\ , \tag{2.11}$$

$$\langle \bar{q}_i(\bar{t})\bar{q}_j(\bar{t}')\rangle = \delta(\bar{t}-\bar{t}')\delta_{ij}\ . \tag{2.12}$$

In the following we will only consider the relevant intensity variable. Redefining the intensity variable, time scale, and introducing new parameters α,α_1,α_2,

$$I=(\Gamma/Q)\bar{I},\ \ t=2Q\bar{t}\ ,\qquad \alpha_1=\frac{\Gamma}{Q},\ \ \alpha_2=\frac{\kappa}{Q},\ \ \alpha=\alpha_1-\alpha_2\ , \tag{2.13}$$

we obtain

$$\partial_t I=I\left[-\alpha_2+\frac{\alpha_1}{1+I/\alpha_1}\right]+\frac{I}{1+I/\alpha_1}\xi_R(t) +\frac{\alpha_1 D}{2Q}+\left[\frac{\alpha_1 D}{Q}I\right]^{1/2}q_R(t)\ , \tag{2.14}$$

$$\langle \xi_R(t)\xi_R(t')\rangle = \langle q_R(t)q_R(t')\rangle = 2\delta(t-t')\ . \tag{2.15}$$

The Fokker-Planck equation for the intensity probability distribution associated with (2.14) and (2.15) is given in the Stratonovich interpretation by

$$\partial_t P(I,t)=LP(I,t) \equiv -\frac{\partial}{\partial I}v(I)P(I,t)+\frac{\partial^2}{\partial I^2}D(I)P(I,t)\ , \tag{2.16}$$

$$v(I)=\frac{D\alpha_1}{Q}+I\left[-\alpha_2+\frac{\alpha_1}{1+I/\alpha_1}\right]+\frac{I}{(1+I/\alpha_1)^3}\ , \tag{2.17}$$

$$D(I)=\frac{D\alpha_1}{Q}I+\frac{I^2}{(1+I/\alpha_1)^2}\ . \tag{2.18}$$

The stationary solution of (2.16)–(2.18) is explicity given in the Appendix. In the remainder of this paper we will mostly consider situations above threshold where spontaneous-emission noise is known to have a very small influence.[1,2] We will therefore take the limit $D\to 0$ in (2.16)–(2.18). In this limit the model has two independent parameters (α_1,α_2) while the corresponding white-noise version of the loss-noise model has only one independent parameter. The latter is obtained from (2.14) taking the limit $\alpha_1\to\infty$ with $\alpha=\alpha_1-\alpha_2$ fixed.[18] It is only in this limit that effects of saturation fluctuations can be neglected. We will see below that situations in which (2.15) cannot be reduced to the loss-noise model include cases of small laser intensity with relatively important losses.

III. INTENSITY FLUCTUATIONS

The stationary solution of (2.16)–(2.18) for $D=0$ is given by

$$P_{st}(I)=N\left[I^{\alpha-1}+\frac{I^{\alpha}}{\alpha_1}\right]\times\exp\left[\left[1-\frac{2(\alpha_1-\alpha)}{\alpha_1}\right]I-\frac{\alpha_1-\alpha}{2\alpha_1^2}I^2\right], \quad \alpha>0, \tag{3.1}$$

where the normalization constant N is

$$N^{-1}=\left[\frac{\alpha_1-\alpha}{\alpha_1^2}\right]^{-\alpha/2}\Gamma(\alpha)\exp\left[\frac{\alpha_1-\alpha}{4}\right]\times\left[D_{-\alpha}[(\alpha_1-\alpha)^{1/2}]+\frac{\alpha}{(\alpha_1-\alpha)^{1/2}}D_{-\alpha-1}[(\alpha_1-\alpha)^{1/2}]\right], \tag{3.2}$$

$\Gamma(x)$ is a gamma function and $D_\nu[x]$ is a parabolic cylinder function.[19] No stationary solution exists for $\alpha<0$ in the limit $D=0$ that we consider here.

The moments of the stationary distribution calculated from (3.1) are

$$\langle I^n\rangle=\frac{\alpha_1^n}{(\alpha_1-\alpha)^{n/2}}\frac{\Gamma(\alpha+n)}{\Gamma(\alpha)}\frac{D_{-\alpha-n}[(\alpha_1-\alpha)^{1/2}]+\frac{\alpha+n}{(\alpha_1-\alpha)^{1/2}}D_{-\alpha-n-1}[(\alpha_1-\alpha)^{1/2}]}{D_{-\alpha}[(\alpha_1-\alpha)^{1/2}]+\frac{\alpha}{(\alpha_1-\alpha)^{1/2}}D_{-\alpha-1}[(\alpha_1-\alpha)^{1/2}]}. \tag{3.3}$$

In particular,

$$\langle I\rangle=\frac{\alpha}{1-\frac{\alpha}{\alpha_1}}. \tag{3.4}$$

We note that this value of $\langle I\rangle$ coincides with the one which follows from a deterministic analysis [$Q=D=0$ in (2.14)], but this is not the case if we expand the fluctuating nonlinear term in (2.14).[12] From (3.4) we see that the limit $\alpha_1\to\infty$, with α fixed, in which (2.14) reduces to the loss-noise model can be understood as a limit of small laser intensity [$I\to\alpha$ and $\bar{I}$ in (2.13) going to zero]. However, small laser intensity also corresponds to the limit $\alpha\to0$, with α_1 fixed, in which the loss-noise and gain-noise models are significantly different. They are also obviously different for parameters for which saturation effects are important and the laser intensity is large, as, for example, $\alpha\to\alpha_1$ with α_1 fixed. Lines of constant $\langle I\rangle$ in the plane α,α_1 are shown in Fig. 2.

Intensity fluctuations

$$\lambda(0)=(\langle I^2\rangle-\langle I\rangle^2)/\langle I\rangle^2 \tag{3.5}$$

computed from (3.3) are shown in Fig. 1. For comparison we have also included the law $\lambda(0)=\langle I\rangle^{-1}$ which follows from the loss-noise model in the white-noise limit. It is important to note that to obtain this law it is enough to use a linearization approximation to (2.14). Linearizing around the deterministic steady state $I_0=\alpha/[1-(\alpha/\alpha_1)]$, the Langevin equation for $\delta I=I-I_0$ becomes

$$\partial_t(\delta I)=-\alpha\frac{\alpha_1-\alpha}{\alpha_1}\delta I+\alpha\xi_R(t), \tag{3.6}$$

so that the exact result $\langle I\rangle=I_0$ is obviously reobtained and $\lambda(0)=(\alpha_1-\alpha)/\alpha\alpha_1=\langle I\rangle^{-1}$. This inverse-mean-intensity law is known to be in good agreement with experiments above threshold.[1] Our results in Fig. 1 indicate that in this regime $\lambda(0)$ is rather insensitive to the value of α_1 and that both gain-noise and loss-noise models give a good description. The divergence of $\lambda(0)$ as $\langle I\rangle\to0$ is a consequence of having neglected spontaneous-emission noise, which becomes important when going below threshold. This is seen in Fig. 1 where we also show $\lambda(0)$, computed taking into account

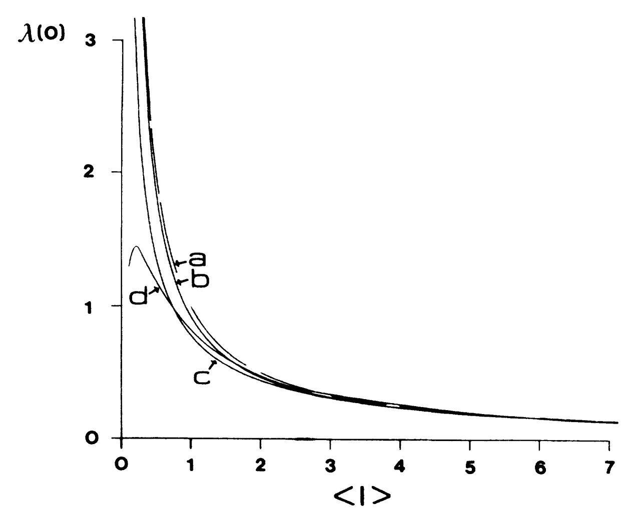

FIG. 1. Dashed line a corresponds to $\lambda(0)=\langle I\rangle^{-1}$. Curve b is for $\alpha_1=8$, $D=0$ and curve c for $\alpha_1=0.4$, $D=0$. Curve d, $\alpha_1=8$ and $(D/Q)=0.02$.

spontaneous-emission noise as obtained from the stationary solution of (2.16) given in the Appendix.

IV. FIRST-ORDER-LIKE TRANSITION

The stationary intensity distribution (3.1) diverges at $I=0$ when $\alpha<1$, while $P_{st}(I=0)=0$ for $\alpha>1$. When $\alpha>1$, P_{st} has a well-developed maximum at $I_{max}\neq 0$. The important point is that the emergence of the maximum at a nonzero value does not occur continuously. Rather, a relative maximum occurs when increasing $\langle I\rangle$ while still $P_{st}(I=0)=\infty$: The extrema of $P_{st}(I)$ obey a cubic equation whose numerical solution indicates the existence of three different regions in the parameter space according to the number of positive real roots (see Fig. 2). In region I ($\alpha>1$) a single maximum at $I\neq 0$ exists. In region II, $P_{st}(I=0)\to\infty$ and a relative maximum and minimum exist. In region III, $P_{st}(I)$ decreases monotonically with the intensity. The mean intensity grows when decreasing α_1 at α fixed or when increasing α at α_1 fixed. In the first case and for $\alpha<1$ a relative maximum appears but the divergence at $I=0$ always dominates. Figure 3(a) shows the changes in $P_{st}(I)$ for this case. Points 8 and 9 are close to the limit in which gain-noise and loss-noise models coincide. Figure 3(b) shows the changes in $P_{st}(I)$ when increasing $\langle I\rangle$ at α_1 fixed. The most probable intensity I_{MP} changes discontinuously at $\alpha=1$ (point 4) regardless of the value of α_1 if $2.53\geq\alpha_1>1$. This change becomes continuous at the same point $\alpha=1$ for $\alpha_1\geq 2.53$. Beyond this value of α_1 the qualitative behavior is the same than in the white noise limit of the loss-noise model. In the context of a loss-noise model the analogy of the discontinuous change of I_{MP} with a first-order transition has been pointed out.[5] In the loss-noise model the discontinuity only occurs for colored noise [$\tau\neq 0$ in (2.6)]. The experimental finding of this discontinuity and of the presence of relative extrema in $P_{st}(I)$ was interpreted as evidence of the existence of colored noise fluctuations. We see here that both effects can be consistently described within a gain-noise model which only includes white noise.[20]

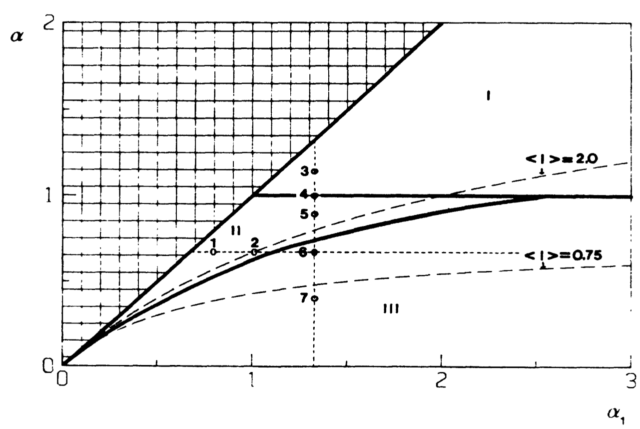

FIG. 2. Parameter plane α,α_1. Unphysical domain, $\alpha>\alpha_1$. Dashed lines are constant-$\langle I\rangle$ lines with $\langle I\rangle\to\alpha$ as $\alpha_1\to\infty$. Regions I, II, and III, correspond to different shapes of $P_{st}(I)$, as explained in the text. Intensity probability distributions corresponding to points 1–7 are shown in Fig. 3.

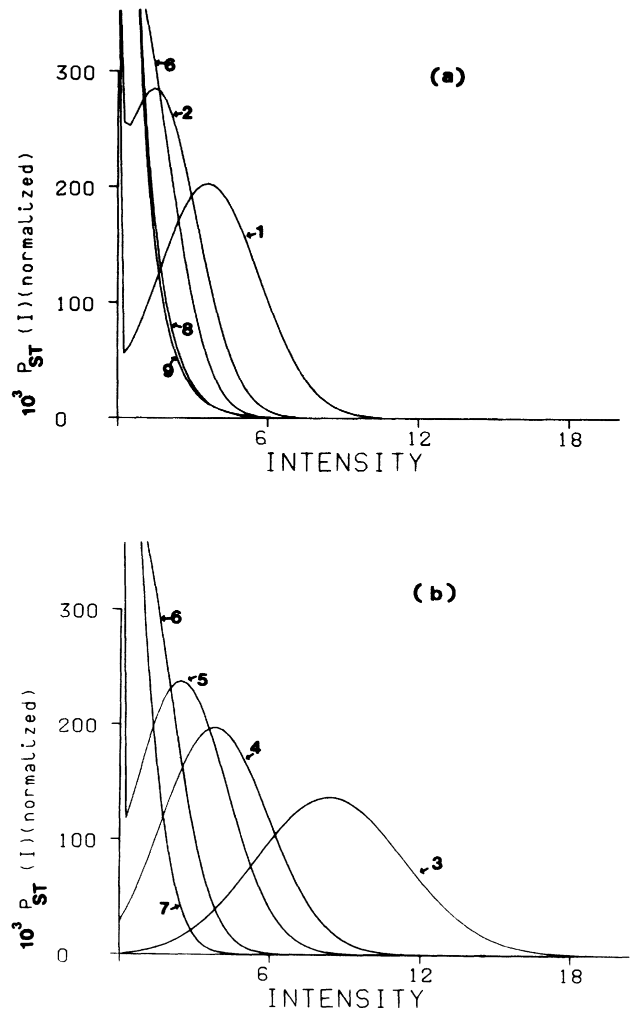

FIG. 3. Stationary intensity distributions corresponding to points 1–7 in Fig. 2. Distributions 8 and 9 are for values of $\alpha_1=10, 10^4$, respectively, not included in Fig. 2. Distributions 1, 2, 6, 8, and 9 are for $\alpha=0.66$ fixed and $\langle I\rangle=3.77$, 1.94, 1.31, 0.71, and 0.66, respectively. Distributions 3, 4, 5, 6, and 7 are for $\alpha_1=1.33$ fixed and $\langle I\rangle=8.50$, 4.03, 2.78, 1.31, and 0.57.

V. INTENSITY CORRELATION FUNCTIONS

In this section we consider the behavior of the normalized steady-state intensity correlation function $\lambda(s)$ defined as

$$\lambda(s)=\frac{\langle I(t+s)I(t)\rangle-\langle I\rangle^2}{\langle I\rangle^2}\ . \tag{5.1}$$

The simplest approximation to calculate $\lambda(s)$ is to use the linearization (3.6). This gives for $\lambda(s)$ a single decaying exponential with time constant $\alpha_1/\alpha(\alpha_1-\alpha)$. Generally speaking $\lambda(s)$ will be a superposition of decaying exponentials. A systematic calculation of such exponentials can be carried out, for example, through a continued fraction expansion.[17,21] To lowest order in that expansion, $\lambda(s)$ is still given by a single exponential whose decay rate includes static nonlinear effects:[21]

$$\lambda(s)=\lambda(0)e^{-\gamma_0 s}, \tag{5.2}$$

where

$$\gamma_0=-\frac{\langle (L^{\dagger}\delta I)\delta I\rangle}{\langle(\delta I)^2\rangle},\quad \delta I=I-\langle I\rangle, \tag{5.3}$$

and $L^{\dagger}$ is the adjoint of the Fokker-Planck operator in (2.16). It is known that the approximation (5.2) is reliable for small time intervals s, and it is equivalent to a decoupling ansatz in the hierarchy of equations for $\lambda(s)$.[21] A description of the early time decay of the correlation function in terms of an effective eigenvalue λ_{eff} proportional to the initial slope of $\lambda(s)$ has recently been proposed:[9]

$$\lambda_{\mathrm{eff}}=-\frac{1}{\lambda(0)}\frac{d}{ds}\lambda(s)\Bigg|_{s\to 0^+}. \tag{5.4}$$

The general result[9] that $\lambda_{\mathrm{eff}}=\gamma_0$ indicates that the approximation (5.2) gives the exact results for the initial slope. In our case the explicit form for λ_{eff} is[9]

$$\lambda_{\mathrm{eff}}=\left\langle\left[\frac{I}{1+I/\alpha_1}\right]^2\right\rangle\Big/(\langle I^2\rangle-\langle I\rangle^2). \tag{5.5}$$

Formulas (5.2) and (5.3) apply for Markov processes obeying a Fokker-Planck equation. When a colored noise is present the process becomes non-Markovian. It has been proved that when only colored noise is present λ_{eff} defined by (5.4) is strictly zero.[9] This is the case of the colored loss-noise model when spontaneous noise is neglected. As a consequence a small initial slope of the correlation function has been interpreted as a signature of colored noise.[7–9] To check this idea we have calculated λ_{eff} as given in (5.5) using the stationary distribution (3.1) of the white gain-noise model. Results are shown in Fig. 4. We find that for α close to α_1 and both not too large, λ_{eff} takes very small values, much smaller than the ones for the white noise limit of the loss-noise model. This implies that the effect attributed to colored noise within the loss-noise model can also be satisfactorily explained in the context of a white gain-noise model. It is also interesting to note that the domain of parameters in which λ_{eff} is very small includes the domain in which $P_{\mathrm{st}}(I)$ has a relative maximum, which has also been explained as a colored noise effect in the loss-noise model.

A different question is the time interval s for which the early time approximation (5.2) remains meaningful. In such time domain $\lambda(s)$ is well characterized by its initial slope. Information on this point is contained in Fig. 5 where the correlation function obtained by linearization and from (5.2) is compared with a direct simulation[22] for different values of the parameter α,α_1. Figure 5(a) corresponds to a situation with a large value of the mean intensity $\langle I\rangle$. In this situation the correlation function has a rather fast decay which is well approximated even by linear theory. In Fig. 5(b) we show three cases with intermediate values of $\langle I\rangle$. Each of these cases belongs to one of the three regions of parameter space in Fig. 2. We find that with similar values of $\langle I\rangle$ and $\lambda(0)$, the correlation function can decay rather differently. In the three cases linearization is not a good approximation because of important nonlinear effects. For the large-gain parameter (case 1) the decay is rather fast, and the approximation in terms of the effective eigenvalue gives good results in a rather short time interval. Decreasing the gain parameter with $\langle I\rangle$ essentially fixed, the correlation function decays more and more slowly, giving a large time interval in which (5.2) is a good approximation. Finally Fig. 5(c) corresponds to a situation with a smaller value of $\langle I\rangle$ in which we find an even slower decay of the correlation function with a large time interval of validity of (5.2). In summary, it is seen that whenever λ_{eff} is small, the single-exponential approximation (5.2) gives for this model a good characterization of $\lambda(s)$ for a very large time interval.

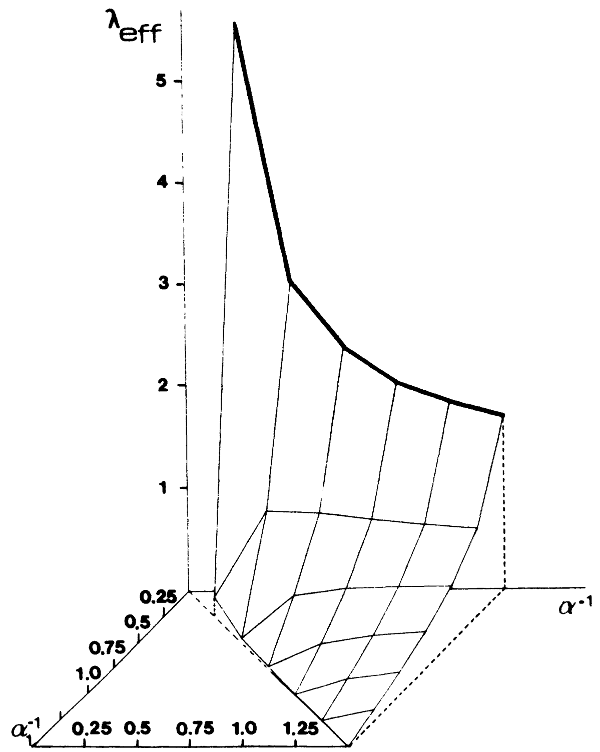

FIG. 4. Effective eigenvalue λ_{eff} as a function of the parameters α and α_1. The thick line corresponds to the white loss-noise model ($\alpha_1\to\infty$).

Still, the question remains of how similar are the correlation functions of our white gain-noise model and the colored loss-noise model when both have very small values of λ_{eff}. An answer to this point is given in Fig. 6, where we have compared a correlation function for the colored loss-noise model with several correlation functions of the white gain-noise model. All these correlation functions have an initial value close to $\lambda(0)=0.60$, and they have been obtained by simulation. For the colored loss-noise model λ_{eff} is strictly zero. This is reflected in an initial plateau which lasts a time of the order of the correlation time of the noise. But beyond this plateau a second time regime occurs. A good representation of the overall decay is given by a combination of two exponentials.[8] This is different from the behavior of the white gain-noise model for which when λ_{eff} is small a single exponential dominates. Therefore the only qualitative difference that remains after the analysis of this paper between colored loss-noise and white gain-noise models is the existence of two time scales in the early-time decay of the correlation function when colored noise is present. In connection with this point we note that the calculation of the power spectra by Yu *et al.*[11] is based on a linearization approximation of an equation which can be formally identified with a colored loss-noise model. Their result involves a product of two Lorentzians, one of which has a width associated with the correlation time of the noise. This Lorentzian is directly connected with the initial pla-

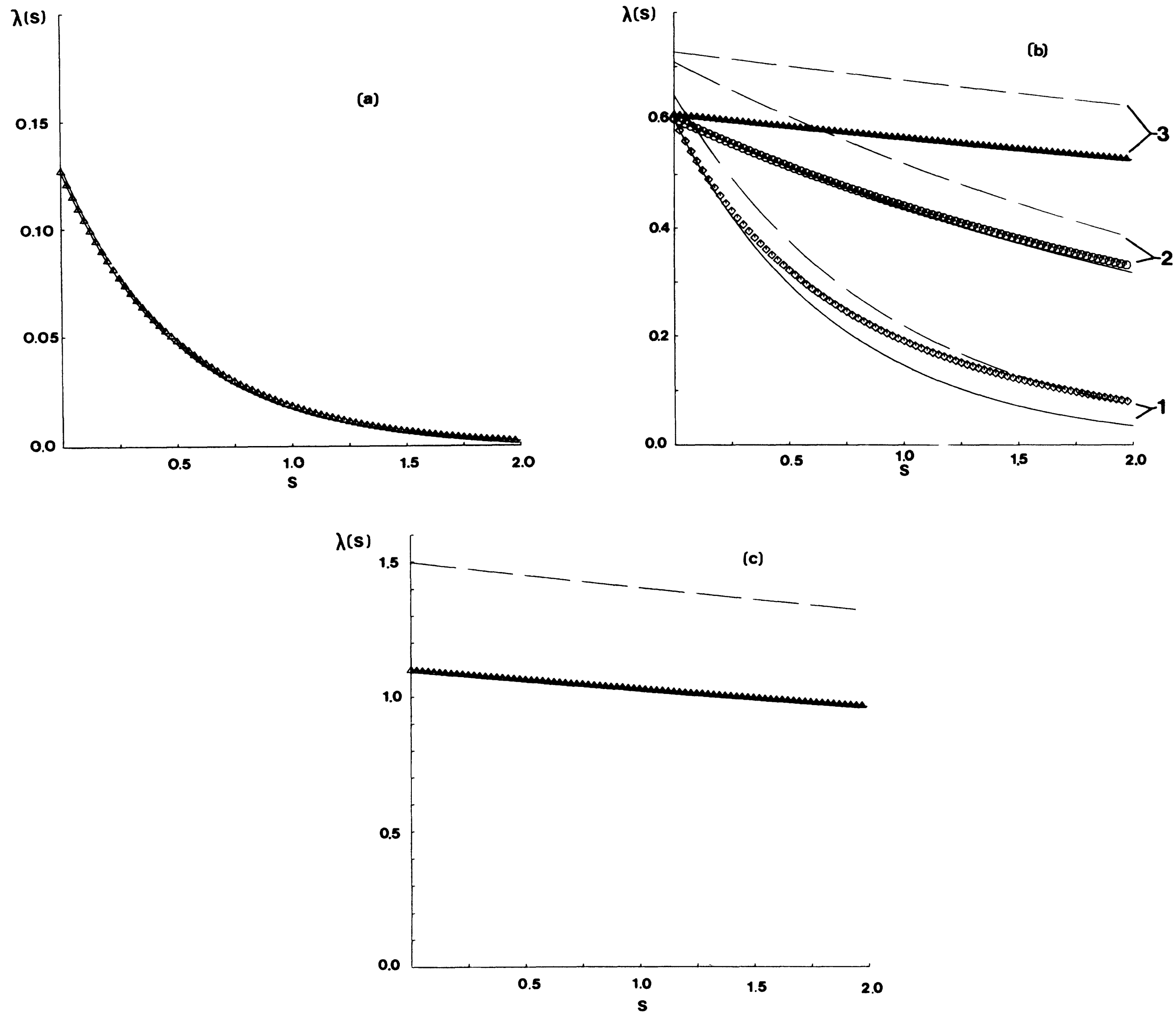

FIG. 5. Intensity correlation functions for the gain-noise model [(2.14) with $D=0$]. [Δ, simulation; – –, linearization, —, effective eigenvalue approximation (5.2).] (a) $\alpha=3.92$, $\alpha_1=8$ (Region I in Fig. 2), $\langle I\rangle=6.71$, $\lambda_{\text{eff}}=2.01$. (b) curve 1: $\alpha=1.30$, $\alpha_1=8$ (region I, $\langle I\rangle=1.55$, $\lambda_{\text{eff}}=1.42$. Curve 2: $\alpha=0.66$, $\alpha_1=1.24$ (region III), $\langle I\rangle=1.41$, $\lambda_{\text{eff}}=0.32$. Curve 3: $\alpha=0.32$, $\alpha_1=0.42$ (region II), $\langle I\rangle=1.38$, $\lambda_{\text{eff}}=0.07$. (c) $\alpha=0.21$, $\alpha_1=0.3$ (region III), $\langle I\rangle=0.67$, $\lambda_{\text{eff}}=0.066$.

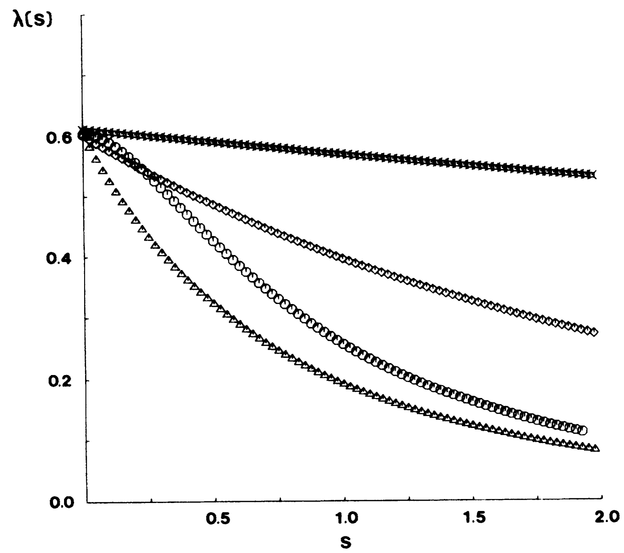

FIG. 6. Intensity correlation functions. Colored loss-noise model (○): $\alpha=1.13$, $\tau=0.3$, $\langle I\rangle=1.13$. Remaining lines are for white gain-noise model (□): $\alpha=0.32$, $\alpha_1=0.417$, $\langle I\rangle=1.38$; (◇): $\alpha=0.76$, $\alpha_1=1.63$, $\langle I\rangle=1.42$; (△): $\alpha=1.30$, $\alpha_1=8$, $\langle I\rangle=1.55$.

teau of the correlation function mentioned above. As a separate matter we also note that the value of $\lambda_{\rm eff}$ itself is a good quantity for discriminating between white gain and loss models. In this respect experimental work on gas lasers with fluctuating gain parameters similar to the recent work on fluctuating loss parameter[4] would certainly be interesting.

ACKNOWLEDGMENTS

Financial support from Dirección General de Investigación Científica y Técnica (Spain), Grant No. PB-86-0534, is acknowledged. Help with numerical routines to calculate $\lambda(0)$ from J. Pons and Ll. Serra is also acknowledged. We thank E. Gage for critical comments on the original manuscript.

APPENDIX

The stationary solution of the Fokker-Planck equation (2.16)–(2.18) is given by

$$P_{\rm st}(I)=N(I+\alpha_1)(I-x_+)^{E_+}(I-x_-)^{-E_-}\times\exp\left[-\frac{\alpha_2 Q}{\alpha_1 D}I\right], \tag{A1}$$

where N is a normalization constant and

$$x_\pm\equiv\frac{\alpha_1}{2D}[-2D-Q\pm(Q^2+4DQ)^{1/2}]<0\ , \tag{A2}$$

$$E_\pm\equiv Q\frac{\left[1+\dfrac{Q\alpha_2}{D\alpha_1}\right]x_\pm^2+\left[2\alpha_1+\dfrac{Q\alpha_2}{D}\right]x_\pm+\alpha_1(\alpha_1-1)}{(\alpha_1+x_\pm)(Q^2+4DQ)^{1/2}}\ . \tag{A3}$$

[1]R. Roy, A. W. Yu, and S. Zhu, in *Noise in Nonlinear Dynamical Systems,* edited by F. Moss and P. McClintock (Cambridge University Press, Cambridge, England, 1988).

[2]M. San Miguel, in *Instabilities and Chaos in Quantum Optics II,* edited by N. Abraham, F. T. Srecchi, and L. Lugiato (Plenum, New York, 1988).

[3](a) K. Kaminishi, R. Roy, S. Short, and L. Mandel, Phys. Rev. A **24**, 370 (1981); (b) R. Short, L. Mandel, and R. Roy, Phys. Rev. Lett. **49**, 647 (1982).

[4]M. R. Young and S. Singh, Opt. Lett. **13**, 21 (1988); Phy. Rev. A **38**, 238 (1988).

[5]P. Lett, E. C. Gage, and T. H. Chyba, Phys. Rev. A **35**, 746 (1987).

[6]M. Aguado and M. San Miguel, Phys. Rev. A **37**, 450 (1988).

[7]P. Lett, Ph.D. thesis, University of Rochester, 1986; P. Lett and L. Mandel, J. Opt. Soc. Am. B **32**, 1165 (1985).

[8]A. Hernández-Machado, M. San Miguel, and S. L. Katz, Phys. Rev. A **31**, 2362 (1985).

[9]M. San Miguel, L. Pesquera, M. A. Rodríguez, and A. Hernández-Machado, Phys. Rev. A **35**, 208 (1987).

[10]F. de Pasquale, J. M. Sancho, M. San Miguel, and P. Tartaglia, Phys. Rev. Lett. **56**, 2473 (1986).

[11]A. W. Yu, G. P. Agrawal, and R. Roy, Opt. Lett. **12**, 806 (1987).

[12]A. Schenzle, in *Optical Instabilities,* edited by R. W. Boyd, M. G. Raymer, and L. M. Narducci (Cambridge University Press, Cambridge, England, 1986), p. 376.

[13]Fluctuations in the saturation term may not be significant in the transient processes associated with the laser switch-on. Such fluctuations are largely determined by linear terms and spontaneous-emission noise (Ref. 10). See also, S. Zhu, A. W. Yu, and R. Roy, Phys. Rev. A **34**, 4333 (1986).

[14]H. Haken, *Laser Theory* (Springer-Verlag, New York, 1984).

[15]For a detailed analysis, see the appendix of Ref. 1, and references therein.

[16]A model in which the fluctuating saturation term is expanded and in which spontaneous-emission noise is neglected was first introduced in Ref. 12. An analysis of the positivity requirements of fluctuating gain and loss parameters in this context was given by J. M. Sancho, M. San Miguel, L. Pesquera, and M. Rodriguez, Physica **142A**, 532 (1987).

[17]H. Risken, *The Fokker-Planck Equation: Methods of Solution and Applications* (Springer-Verlag, New York, 1984).

[18]When taking into account spontaneous-emission noise, a more useful rescaling of variables than (2.13) is considered in Ref. 6.

[19]E. T. Whittaker and G. N. Watson, *A Course of Modern Analysis,* 4th ed. (Cambridge University Press, Cambridge, England, 1952).

[20]The present discussion of the stationary distribution neglects

spontaneous-emission noise. For $D\neq 0$, $P_{st}(I=0)$ is always finite. For sufficiently small D, essential changes in the diagram in Fig. 2 and in the values of the normalized intensity distributions are not expected. Changes in $P_{st}(I)$ are only significant for small $\langle I \rangle$ values.

[21]A. Hernández-Machado, M. San Miguel, and J. M. Sancho, Phys. Rev. A **29**, 3388 (1984).

[22]The numerical simulation has been carried out following the algorithm explained by J. M. Sancho, M. San Miguel, S. Katz, and J. D. Gunton, Phys. Rev. A **26**, 1589 (1982). Initial conditions for the process have been given by sampling the known stationary distribution so that we do not need to wait to reach steady-state behavior. The step of integration used is $\Delta=0.005$. Averages were taken over 1200 realizations and 15 000 points in each realization. The error found in $\lambda(0)$ compared with the results in Fig. 1 is less than 1%.