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Quantum propagation: Squeezing via modulational polarization instabilities in a birefringent nonlinear medium

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We calculate the spectrum of quadrature fluctuations of orthogonal polarizations of light for a beam propagating along a birefringence axis of a dispersive transparent Kerr medium. Reshaping and tuning of the spectrum is achievable by variation of the pump power or the linear birefringence: Wide-band squeezing is predicted, and its relationship to modulational polarization instabilities discussed.

Squeezed states of light have noise less than the fundamental quantum limit in one quadrature, at the expense of correspondingly larger fluctuations in the complementary quadrature.[1] There has been considerable success in the generation and detection of squeezed light by a variety of parametric interactions, once extraneous noise is eliminated to a large degree.[2] Recently, promising predictions were made in relation to large wide-band squeezing of *scalar* solitons, i.e., when only a single linear polarization is excited and maintained along an optical fiber.[3]

When an intense light beam propagates in a weakly nonlinear and anisotropic medium, one must include both field polarizations in the description.[4] The classical wave polarization dynamics in nonlinear birefringent dispersive media has recently been discussed, and new modulational polarization instabilities predicted.[5] Modulational instabilities lead to an exponential growth of sidebands, and have recently been observed experimentally in optical fibers operating in the anomalous dispersion regime.[6] Here we calculate squeezing spectra for the two propagating polarization states using a unified quantum theory of nonlinear birefringent dispersive media, characterized by the nonlinear electric displacement[4]

$$\mathbf{D}^{\mathrm{NL}}=\epsilon_0\chi[(1-B)(\mathbf{E}\cdot\mathbf{E}^*)\mathbf{E}+B(\mathbf{E}\cdot\mathbf{E})\mathbf{E}^*] , \qquad (1)$$

where $\chi\equiv3\chi_{xxxx}$, and the value of $B\equiv\chi_{xyyx}/\chi_{xxxx}$, which lies between 0 and 1, depends on the nonlinearity mechanism ($\frac{1}{3}$ for a glass fiber, $\frac{3}{4}$ for liquid CS_2). Equation (1) implies that the isotropy condition $\chi_{xxxx}=\chi_{xyyx}+2\chi_{xyxy}$ holds between the components of the nonlinear susceptibility tensor χ. The complex electric field $\mathbf{E}$ is written

$$\mathbf{E}=[\Phi_x e^{ik_xz}\mathbf{x}+\Phi_y e^{ik_yz}\mathbf{y}]e^{-i\omega_0t} , \qquad (2)$$

where $k_{x,y}$ are the wave vectors for the components polarized along the x and y axes, respectively, and ω_0 is the mean frequency.

After quantization of the classical field theory, including group-velocity dispersion, we use coherent-state phase-space methods[3,7] to derive stochastic equations for the scaled field envelopes $\phi_j(\tau,\zeta)$ ($j=x,y$ labels the polarization state). For ease of comparison, we adopt the same scaling and coordinates as Ref. 3, in which τ and ζ are dimensionless local time and propagation distance, respectively. We find

$$\frac{\partial\boldsymbol{\phi}(\tau,\zeta)}{\partial\zeta}=\mathcal{A}\left[\frac{\partial}{\partial\tau}\right]+\mathcal{B}\boldsymbol{\eta}(\tau,\zeta) , \qquad (3)$$

where $\boldsymbol{\phi}$, $\mathcal{A}$, and $\boldsymbol{\eta}$ are vectors of the form $\mathbf{v}\equiv(v_x, v_x^\dagger, v_y, v_y^\dagger)^T$, with

$$\begin{aligned}\mathcal{A}_x&=-\frac{i}{2}\left[1\pm\frac{\partial^2}{\partial\tau^2}-2\phi_x^\dagger\phi_x-2(1-B)\phi_y^\dagger\phi_y\right]\phi_x+iB\phi_y^2\phi_x^\dagger ,\\ \mathcal{A}_y&=-\frac{i}{2}\left[1\pm\frac{\partial^2}{\partial\tau^2}-4\kappa-2\phi_y^\dagger\phi_y-2(1-B)\phi_x^\dagger\phi_x\right]\phi_y+iB\phi_x^2\phi_y^\dagger ,\end{aligned} \qquad (4)$$

and

$$\mathcal{B}\mathcal{B}^T=\frac{i}{p_0}\begin{bmatrix}\phi_x^2+B\phi_y^2 & 0 & (1-B)\phi_x\phi_y & 0\\ 0 & -(\phi_x^{\dagger2}+B\phi_y^{\dagger2}) & 0 & -(1-B)\phi_x^\dagger\phi_y^\dagger\\ (1-B)\phi_x\phi_y & 0 & \phi_y^2+B\phi_x^2 & 0\\ 0 & -(1-B)\phi_x^\dagger\phi_y^\dagger & 0 & -(\phi_y^{\dagger2}+B\phi_x^{\dagger2})\end{bmatrix} , \qquad (5)$$

where $2\kappa \equiv (k_x - k_y)z_0$ is the birefringence parameter, and z_0 and p_0 are the characteristic distance scale (inversely proportional to input intensity) and photon number defined in Ref. 3. Upper and lower signs hold for normal and anomalous group-velocity dispersion $\partial^2 k_x/\partial\omega^2 \cong \partial^2 k_y/\partial\omega^2$, respectively. The $\mathcal{A}_j^\dagger$ are found from $\mathcal{A}_j$, by interchange of ϕ_j and $\phi_j^\dagger$, and i and $-i$. The zero-mean, real Gaussian noise vector $\eta(\tau,\zeta)$ has the correlation

$$\langle \eta(\tau,\zeta)\eta^T(\tau',\zeta')\rangle = \mathbf{I}\delta(\tau-\tau')\delta(\zeta-\zeta') , \tag{6}$$

where $\mathbf{I}$ is the identity matrix. Equation (6) indicates that the components of the noise vector are independent, and consequently ϕ_j and $\phi_j^\dagger$ are only complex conjugate in the mean. Ignoring the noise term in Eq. (3), gives the classical deterministic equations.[5] Note that, unlike the scalar case, modulational instability of sidebands with polarization orthogonal to the pump is possible also in the normal-dispersion regime. Inclusion of the noise term gives a complete quantum-mechanical description in terms of a stochastic process defined on a phase space twice as large as the classical phase space.[3,8] To study the quantum fluctuations we linearize Eq. (3) about the classical continuous wave solution $\boldsymbol{\phi}_0 = 1/\sqrt{2}(1,1,0,0)^T$, representing an input polarized along the x axis (the "slow" axis if $\kappa > 0$, or the "fast" axis if $\kappa < 0$). The linearized equations for the fluctuation vector $\delta\boldsymbol{\phi} = \boldsymbol{\phi} - \boldsymbol{\phi}_0$, correct to first order in Fourier space, are given by

$$\frac{\partial}{\partial\zeta}\delta\boldsymbol{\phi}_j(\omega,\zeta) = \mathbf{A}_j\delta\boldsymbol{\phi}_j(\omega,\zeta) + \sqrt{\mathbf{D}_j}\eta_j(\omega,\zeta), \; j = x,y , \tag{7}$$

where

$$\delta\boldsymbol{\phi}_j(\omega,\zeta) \equiv [\delta\phi_j(\omega,\zeta),\, \delta\phi_j^\dagger(\omega,\zeta)]^T ,$$

$$\mathbf{A}_x = \frac{i}{2}\begin{bmatrix} 1\pm\omega^2 & 1 \\ -1 & -(1\pm\omega^2) \end{bmatrix},$$

$$\mathbf{A}_y = \frac{i}{2}\begin{bmatrix} -B-4\kappa\pm\omega^2 & B \\ -B & B+4\kappa\mp\omega^2 \end{bmatrix} , \tag{8}$$

$$\mathbf{D}_x = \frac{i}{2p_0}\mathrm{diag}(1,-1), \; \mathbf{D}_y = \frac{iB}{2p_0}\mathrm{diag}(1,-1) ,$$

and as in Ref. 4, the Fourier transform is defined as

$$\delta\boldsymbol{\phi}(\omega,\zeta) \equiv \frac{1}{\sqrt{2\pi}}\int_{-T/2t_0}^{T/2t_0} d\tau e^{i\omega\tau}\delta\boldsymbol{\phi}(\tau,\zeta) , \tag{9}$$

with T the total observation time, and t_0 a characteristic time scale for a given input power. We note that for such inputs, the quantum fluctuations of the x and y polarizations are independent to first order.

The spectrum of quadrature fluctuations of polarization j, at detuning ω, position ζ, and angle θ (defined by the local oscillator phase) is defined[3]

$$S_j(\omega,\zeta,\theta) \equiv \frac{4\pi t_0 p_0}{T}\mathrm{Re}[e^{i2\theta}\langle\delta\phi_j(\omega,\zeta)\delta\phi_j(-\omega,\zeta)\rangle + \langle\delta\phi_j(\omega,\zeta)\delta\phi_j^\dagger(-\omega,\zeta)\rangle] . \tag{10}$$

Since the equations of motion for $\delta\boldsymbol{\phi}_x$ are precisely those derived from the scalar stochastic nonlinear Schrödinger equation,[3] the squeezing spectrum also coincides with that found previously.[3] The spectrum associated with the orthogonal polarization y at the phase angle for maximum reduction of the fluctuations is

$$S_{y,\max}(\omega,\zeta) = B\left[\frac{B}{\lambda^2}(\cosh\lambda\zeta - 1) - \left| i\frac{\sinh\lambda\zeta}{\lambda} + \frac{\mu}{\lambda^2}(1-\cosh\lambda\zeta)\right|\right] , \tag{11}$$

where

$$\lambda \equiv [(2B \mp \omega^2 + 4\kappa)(\pm\omega^2 - 4\kappa)]^{1/2} , \tag{12}$$

$$\mu \equiv \pm\omega^2 - 4\kappa - B . \tag{13}$$

In Figs. 1-3 we display squeezing spectra for the polarization orthogonal to the pump, in a fiber ($B = \frac{1}{3}$). In Fig. 1 ($\kappa = 0$, normal dispersion) exponential squeezing occurs at $\omega^2 = B$, the phase matching condition for peak gain of the instability (this may be deduced from the eigenvalues of A_y). Note that in the scalar case,[3] modulational instability and associated large squeezing occurs only in the anomalous dispersion regime. Birefringence alters the phase-matching condition for peak gain of the instability: this occurs for $\omega^2 = B + 4\kappa$ in Fig. 2. The exponential squeezing at the pump frequency $\omega = 0$ [Fig. 2(b)], is a particularly interesting special case. Moreover, nearly 100% squeezing is achievable well within a one beat length ($\equiv \pi/\kappa$) long sample. Figure 3 illustrates that birefringence is essential in order to induce complete squeezing orthogonal to the pump in the anomalous dispersion regime.

The conditions necessary to attain $|\kappa| = B/4$ are as follows. The liquid CS_2 ($n_2 = 3.0\times10^{-14}$ cm^2/W) with a

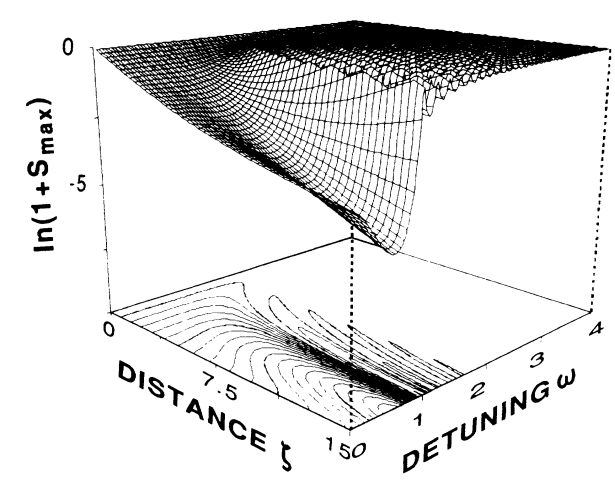

FIG. 1. Maximum squeezing vs normalized frequency offset and propagation distance for sideband fluctuations orthogonally polarized with respect to a linearly polarized carrier propagating in an isotropic (i.e., $\kappa = 0$) Kerr medium (with $B = \frac{1}{3}$) and in the normal-dispersion regime. Contour plot shown below.

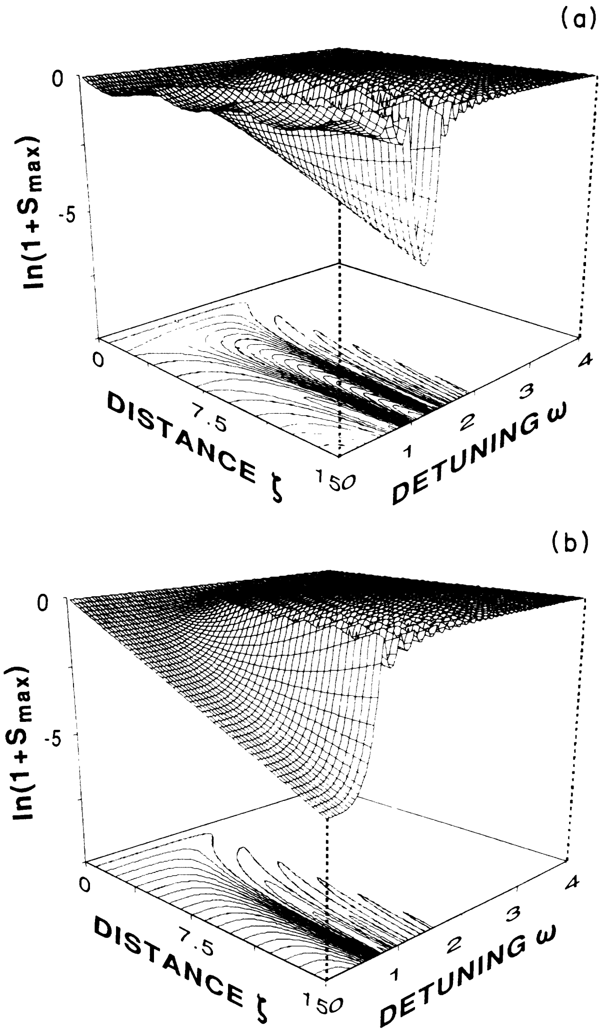

FIG. 2. Same as in Fig. 1, expect that the pump propagates (a) along the slow axis of a birefringent fiber ($\kappa=0.25$, $B=\frac{1}{3}$, normal dispersion) and (b) along the fast axis ($\kappa=-B/4=-\frac{1}{12}$, anomalous dispersion).

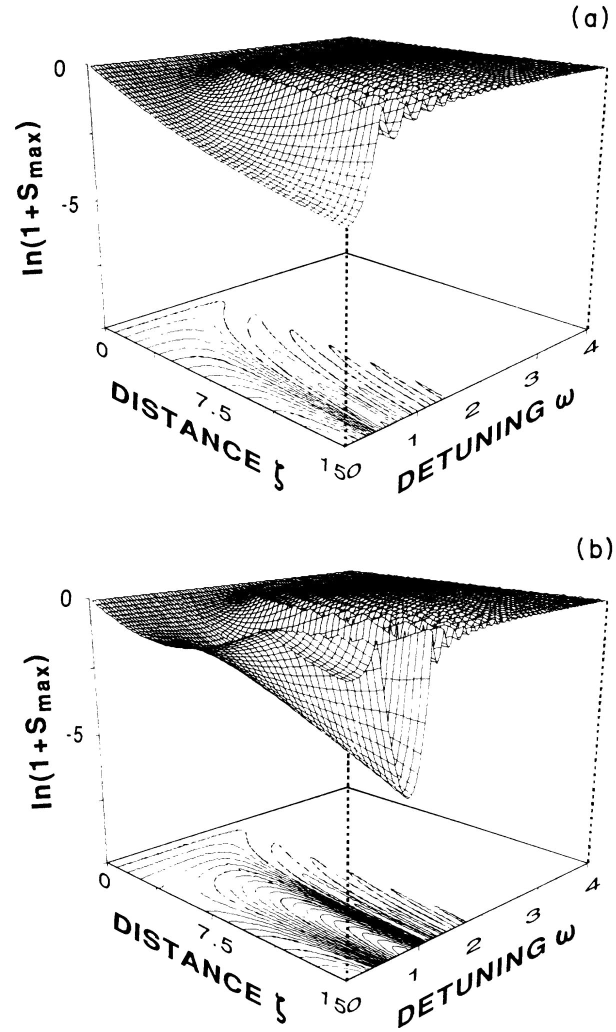

FIG. 3. Same as in Fig. 1, for a pump propagating in the anomalous dispersion regime with (a) $\kappa=0$, (b) $\kappa=-0.25$ (carrier along the fast axis).

10-cm beat length,[9] requires a pump intensity of 470 MW/cm^2 at 1.06 μm. A low birefringence optical fiber ($n_2=3.2\times10^{-16}$ cm^2/W), with a 1-m beat length and an effective area of 10^{-7} cm^2, requires a *peak* power of 496 W at $\lambda=0.53$ μm.[10] Here we have in mind quasi-cw experiments with long pulses (100 ps), which may reduce stimulated scattering losses. Power requirements for squeezing may be reduced by using longer samples of birefringent (polarization maintaining) fiber.

To summarize, we have extended the scalar theory of nonlinear quantum propagation including dispersion, to a vector theory, and investigated new possibilities for squeezing offered by modulational polarization instabilities.

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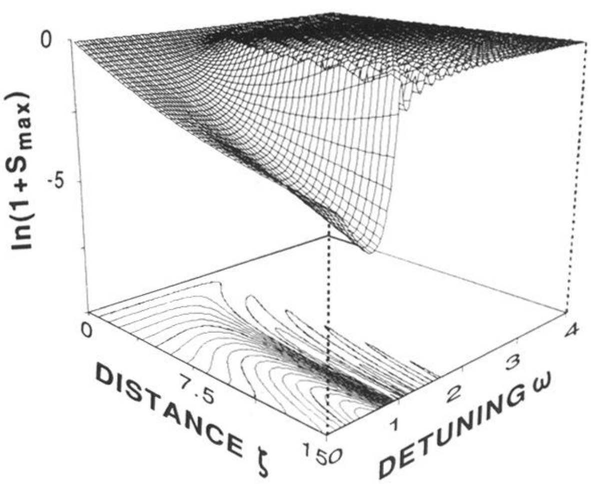

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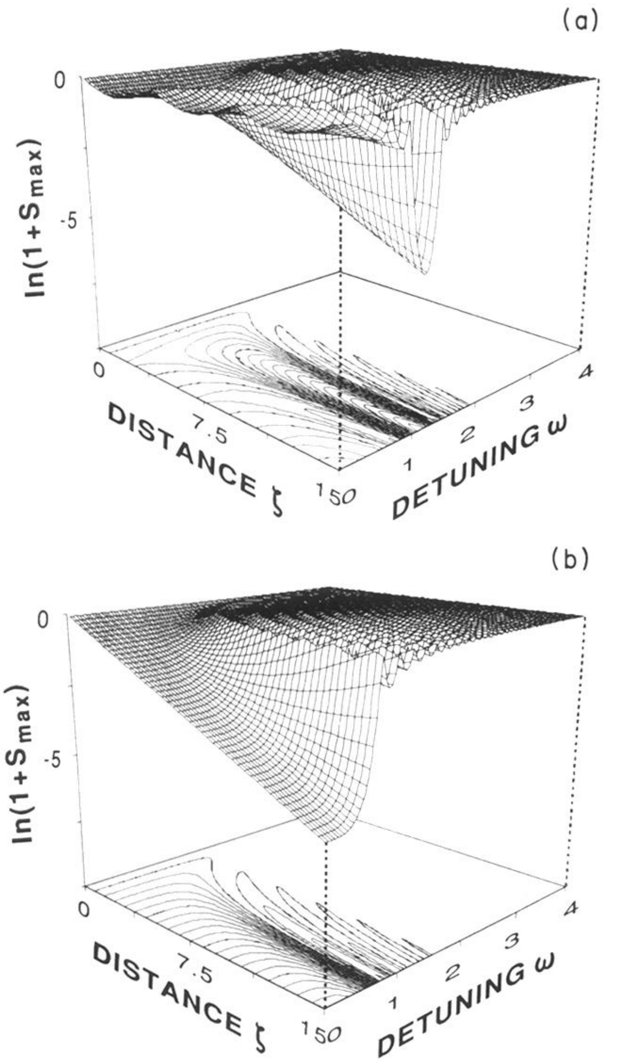

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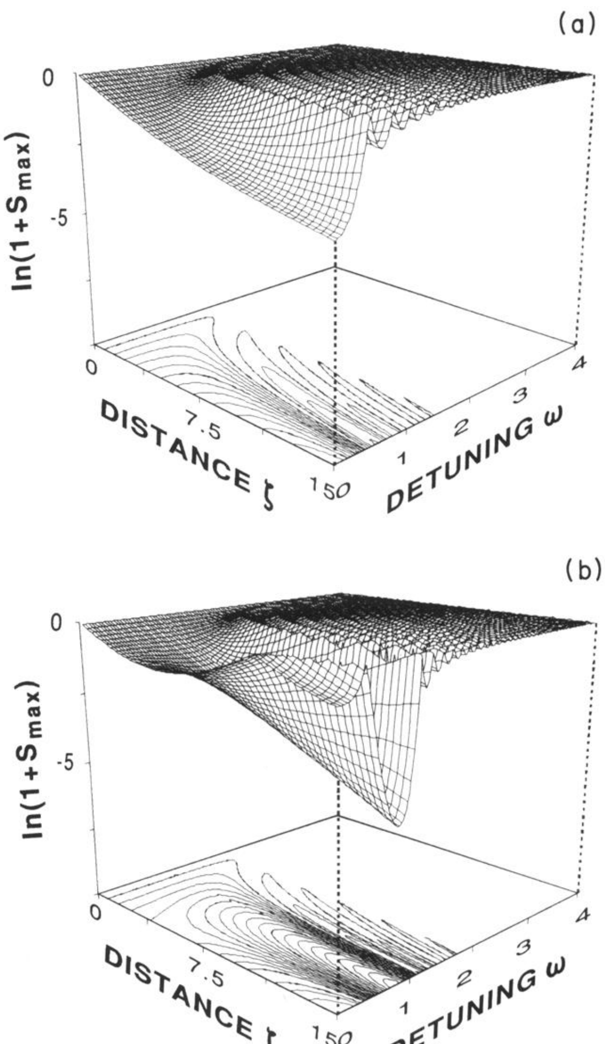

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