

## Low-energy electron drift and scattering in krypton and xenon

S. R. Hunter, J. G. Carter, and L. G. Christophorou\*

*Atomic, Molecular, and High Voltage Physics Group, Health and Safety Research Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831-6122*

(Received 9 May 1988)

The drift velocity of electrons in Ar, Kr, and Xe has been measured at a gas temperature  $T=301$  K using gas pressures of 300 and 600 kPa over the  $E/N$  range  $2 \times 10^{-20} \leq E/N \leq 3 \times 10^{-17}$  V cm<sup>2</sup>. The measurements have been used to derive the low-energy momentum-transfer cross section  $\sigma_m(\epsilon)$  in Kr and Xe at electron energies up to 8 eV using a Boltzmann-equation analysis along with a four-parameter modified-effective-range-theory procedure. Particular attention has been paid to the effects of impurities on the measured electron drift velocities and the uniqueness of the derived momentum-transfer cross sections. The present measurements and calculated cross sections are compared with published values from experiments and calculations.

### I. INTRODUCTION

Low-energy ( $\lesssim 10$  eV) electron scattering in Ar, Kr, and Xe has been the subject of considerable recent experimental and theoretical research. This interest has been generated in part by the need for accurate electron-scattering cross sections for these gases for various gas-discharge applications, including gaseous lasers, diffuse discharge switches, and radiation counters.<sup>1</sup> This information is also needed in attempts to relate the electron motion (and hence electron-scattering processes) in low-pressure gases to those found in high-pressure gases, and ultimately, in the liquid phase.<sup>2,3</sup>

The differential elastic electron-scattering cross sections of Ar, Kr, and Xe possess complicated dependences on the electron energy and scattering angle at low electron energies,  $\epsilon$ , and several recent theoretical<sup>4-11</sup> and experimental<sup>12-14</sup> attempts have been made to quantify these dependences. The total scattering cross section  $\sigma_t(\epsilon)$  and the momentum-transfer cross section  $\sigma_m(\epsilon)$  of these atoms have been known for many years to possess deep minima at low electron energies ( $< 1$  eV). These were discovered in the pioneering work of Ramsauer<sup>15</sup> using electron beam techniques and of Townsend and Bailey<sup>16</sup> using electron-swarm methods. Although these original measurements are now more than 50 years old, research into the low-energy electron-scattering cross sections in these gases is still active<sup>12-14,17-21</sup> due in large measure to the considerable uncertainties that exist in the magnitude and energy dependence of  $\sigma_t(\epsilon)$  and  $\sigma_m(\epsilon)$  near the Ramsauer-Townsend (R-T) minima.

Electron-swarm techniques can be used to determine accurately the low-energy  $\sigma_m(\epsilon)$  in atomic gases where only elastic electron scattering is involved, and hence questions of cross-section uniqueness do not arise.<sup>22</sup> The earliest estimates of  $\sigma_m(\epsilon)$  at low electron energies for these atoms using electron-swarm experiments are those of Frost and Phelps<sup>23</sup> using Boltzmann-equation analyses

of electron-drift-velocity ( $w$ ) measurements.<sup>24</sup> Studies during the past decade using both electron beam<sup>14,18-20</sup> and electron-swarm techniques<sup>25-29</sup> have greatly reduced the uncertainties in the experimentally measured low-energy  $\sigma_m(\epsilon)$  and  $\sigma_t(\epsilon)$  scattering cross sections in Ar. In contrast, recent theoretical<sup>4-11</sup> and experimental electron beam<sup>14,17,18,21</sup> and swarm<sup>30-34</sup> estimates of the  $\sigma_m(\epsilon)$  and  $\sigma_t(\epsilon)$  cross sections in Kr and Xe indicate that the position and magnitude of the R-T minimum in these two atoms is uncertain to within a factor of 2 or more.<sup>34</sup>

The recent evaluation of  $\sigma_m(\epsilon)$  from the measurements of the ratio of transverse diffusion coefficient to electron mobility,  $D_T/\mu$ , by Kiozumi *et al.*<sup>34</sup> in Kr and Xe have not clarified the situation. The scatter in their  $D_T/\mu$  measurements in these two gases was up to  $\pm 15\%$  and the  $\sigma_m(\epsilon)$  obtained from these measurements by a Boltzmann-equation analysis gives calculated  $D_T/\mu$  values which differ from the experimental values by  $\pm 15\%$  and predict electron drift velocities which are up to 20% different from those measured by Pack *et al.*<sup>24</sup> This comparison indicates that either the  $w$  or the  $D_T/\mu$  measurements in these two gases are in serious error.

In the present study, we have measured  $w$  in Ar, Kr, and Xe using a pulsed Townsend experimental technique<sup>35</sup> over a wide  $E/N$  (ratio of electric field to gas number density) range which enables us to cover the mean electron energy,  $\langle \epsilon \rangle$ , range from thermal energy ( $\langle \epsilon \rangle \approx 0.038$  eV) up to  $\langle \epsilon \rangle \approx 4$  eV. The  $w$  measurements have been used to calculate  $\sigma_m(\epsilon)$  for Kr and Xe over the electron energy range  $0.01 \leq \epsilon \leq 8$  eV using a conventional "two-term" Boltzmann-equation analysis.<sup>36</sup> The experimental technique is briefly described in Sec. II along with an analysis of the uncertainties in the measurements, and the  $w$  measurements are discussed in Sec. III in comparison with the previous literature values. The  $\sigma_m(\epsilon)$  cross sections are derived from the  $w$  measurements in Sec. IV and compared with recent theoretical and experimental data for Kr and Xe.

## II. EXPERIMENTAL METHOD

### A. Apparatus

The apparatus and technique used in the present  $w$  measurements has been described in detail previously.<sup>35</sup> A pulsed-Townsend technique is used with the detection circuit operating in the voltage integrating mode where the transit time of the electron swarm,  $t_e$ , is determined from the discontinuity in the voltage transient upon the arrival of the electron swarm at the anode. The experimental apparatus consists of an ultrahigh vacuum (UHV) pumped stainless-steel chamber with a base pressure  $\lesssim 2 \times 10^{-6}$  Pa, containing two contoured stainless-steel electrodes with central flat regions of 6 cm in diameter and a drift gap spacing of  $\sim 2$  cm.

Electron swarms are produced photoelectrically at the cathode by a pulsed uv excimer laser ( $\lambda_L = 193$  nm, pulse width  $\simeq 8$  ns), and the voltage induced in the anode circuit by the motion of the electrons in the drift gap is detected by a fast (rise time  $\simeq 1$  ns), high-impedance ( $Z_i \approx 10^{11} \Omega$ ) unitary-gain emitter-follower preamplifier. The voltage transient is digitized by a Biomation model 6500 waveform recorder (2 ns per channel, 6 bits vertical resolution), and the resultant waveforms are transferred to a PDP11 computer where multiple waveforms are averaged and analyzed to obtain the electron-swarm transit time.

### B. Error analysis

One of the major aims of the present study was to determine the momentum-transfer cross sections  $\sigma_m(\epsilon)$  for Kr and Xe as uniquely as possible in an attempt to decide on the relative merits of the various recent theoretical and experimental scattering cross sections. This requires that the electron drift velocities be accurately measured, which in turn requires that the uncertainty in the experimental parameters be minimized. The drift gap spacing was measured with precision depth gauges and has an estimated uncertainty of  $\pm 0.1\%$ . The gas temperature was determined from the average reading of four calibrated thermocouple gauges located at various positions around the chamber, each having a resolution of  $\pm 0.1$  K and an absolute uncertainty of  $\pm 0.1\%$ . The voltage was measured to an uncertainty of  $\pm 0.1\%$  using a calibrated digital multimeter and the time base calibration of the transient waveform recorder has also been found to be accurate to within  $\pm 0.1\%$ .

At the gas pressures used in the present measurements (300 and 600 kPa) these gases depart significantly from ideal-gas behavior and the measured gas pressure (obtained with MKS Series 315 capacitance manometers with an uncertainty of  $\pm 0.2\%$ ) must be corrected for compressibility effects. The values of the second virial coefficient used in the present study are those listed by Friedmann<sup>37</sup> (higher-order coefficients are negligible at the gas temperatures and pressures used in the present study). The compressibility corrections at  $P = 600$  kPa ( $T = 301$  K) are 3.7%, 1.2%, and 0.4% for Xe, Kr, and Ar, respectively.

TABLE I. Sources of uncertainty in the experimental electron drift velocity measurements.

Source	Error (%)
Drift distance	$\pm 0.1$
Chamber temperature	$\pm 0.1$
Electric field	$\pm 0.1$
Gas pressure	$\pm 0.2$
Waveform digitizer calibration	$\pm 0.1$
Statistical uncertainty in determining the electron-swarm transit time	$\pm 0.5-1.0$
Total uncertainty	$\pm 1.0-2.0$

The last source of error in the present measurements is the statistical uncertainty in estimating the electron-swarm transit time from the digitized voltage transient. The uncertainty results from two sources: the finite temporal resolution of the digitized signal (maximum error is  $\pm 0.2\%$ ) and the rounding of the discontinuity in the voltage transient as the electron swarm arrives at the anode. The rounding is due to electron diffusion, the finite laser pulse width, and electrical noise and leads to an increased uncertainty in determining  $t_e$  of  $\lesssim 1.0\%$ . These errors are listed in Table I and indicate that at the high  $E/N$  values, the total uncertainty in  $w$  is  $\pm 1\%$ , and increases to  $\pm 2\%$  when diffusion broadening of the swarm is pronounced.

### C. Gas Purity

Robertson<sup>25</sup> has shown that the use of high purity gases is critically important if accurate  $w$  measurements are to be obtained in Ar, where impurity levels of the order of 1 ppm (parts per million) can significantly affect the results. This problem is expected to be exacerbated in the heavier rare gases due to the smaller elastic energy losses, and careful attention has been paid in the present study of the purity of Kr and Xe. Mass-analyzed Xe and Kr with purity specifications of better than 99.999% supplied by Spectra Gases and Alphagaz, respectively, and research purity Ar from Matheson Gas Products with a purity of 99.999% were slowly passed through a titanium getter cell, heated to  $> 600^\circ\text{C}$  to remove molecular impurities (principally  $\text{N}_2$  with a stated concentration of  $\lesssim 3$  ppm in these gas samples) and stored at liquid-nitrogen temperatures. The gases were subjected to repeated gettering and freeze-pump-thaw cycles until the  $w$  measurements obtained after each cycle did not change on further attempts at purification. An example of such measurements is shown in Fig. 2(a) (Sec. III) for Kr where the initial  $w$  measurements for a gas sample straight from the cylinder without purification are considerably higher at high  $E/N$  ( $> 3 \times 10^{-19}$  V cm<sup>2</sup>) and lower at small  $E/N$  values in comparison with the final  $w$  values for the extensively purified gas sample.

## III. RESULTS

The electron-swarm drift velocity  $w$  is equivalent to the measured electron drift velocity  $w_m = d/t_e$  (where  $d$  is

the drift distance) in the absence of diffusion broadening and other nonequilibrium effects within the swarm and at the drift chamber boundaries.<sup>22</sup> In general, these processes are not negligible and in the absence of electron nonconservation processes,  $w$  has been shown to be related to  $w_m$  by the following:<sup>22</sup>

$$w_m = \frac{d}{t_e} = w \left[ 1 + C \left[ \frac{ND_L}{w} \right] \left[ \frac{1}{Nd} \right] \right] \\ = w \left[ 1 + \frac{C(D_L/\mu)}{(E/N)(Nd)} \right], \quad (1)$$

where  $D_L$  is the longitudinal electron diffusion coefficient and  $C$  is an empirically measured constant with a value near unity. The  $D_L/\mu$  values in the present gases are abnormally large at low  $E/N$  (Ref. 38) and to minimize the effects of diffusion broadening on  $w_m$ , the present measurements were performed at gas pressures of 300 and 600 kPa since the correction term in Eq. (1) is dependent on the gas pressure (or number density  $N$ ). No dependence of  $w_m$  on pressure was observed at these pressures in all three gases within the statistical uncertainty of the measurements, indicating that the correction term to  $w_m$  in Eq. (1) is negligibly small (i.e.,  $w \simeq w_m$ ). Several studies<sup>39-41</sup> have also shown that in Ar and Xe, pressure dependences in  $w_m$  due to dimer formation and/or multiple scattering effects become significant only at gas pressures 2-3 times larger than those used in the present study.

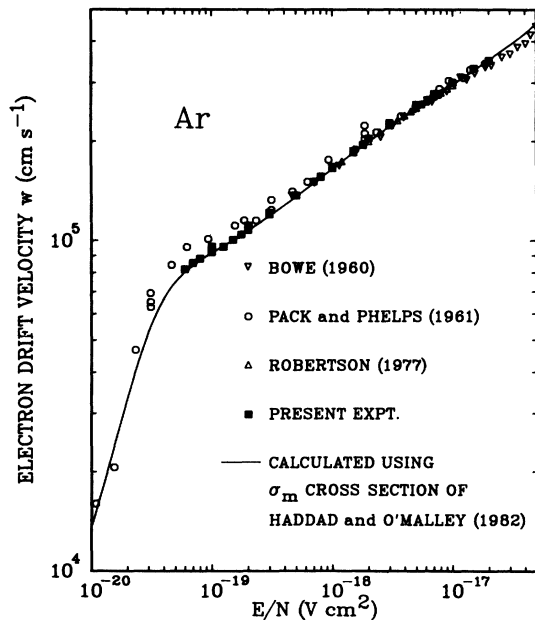


FIG. 1. Present electron drift velocity  $w$  measurements in Ar in comparison with the measurements of Bowe (Ref. 43), Pack and Phelps (Ref. 42), and Robertson (Ref. 25), and with the calculated  $w$  values using the  $\sigma_m(\epsilon)$  of Haddad and O'Malley (Ref. 28).

### A. Argon

The present  $w$  measurements in Ar were performed over the  $E/N$  range of  $6 \times 10^{-20} \leq E/N \leq 2 \times 10^{-17}$  V cm<sup>2</sup> and were made primarily to check the accuracy of the apparatus and gas purification procedures. These measurements are compared in Fig. 1 with the previous literature data,<sup>25,42,43</sup> and the values calculated using the  $\sigma_m(\epsilon)$  derived by Haddad and O'Malley.<sup>28</sup> The present  $w$  values are in excellent agreement with the high accuracy measurements of Robertson<sup>25</sup> to within  $\pm 1\%$ , indicating that the present technique is capable of measuring  $w$  in these gases to within the stated uncertainty limits. The

TABLE II. Electron drift velocities and density-normalized electron mobilities in Kr and Xe.

$E/N$ ( $10^{-18}$ V cm <sup>2</sup> )	Kr $w$ ( $10^5$ cm s <sup>-1</sup> )	Xe $w$ ( $10^5$ cm s <sup>-1</sup> )
0.020	0.0237	
0.025	0.0297	
0.030	0.0357	
0.035	0.0419	
0.04	0.0483	0.0152
0.05	0.0622	0.0192
0.06	0.0777	0.0230
0.07	0.0950	0.0268
0.08	0.1146	0.0311
0.10	0.1668	0.0387
0.12	0.238	0.0466
0.14	0.330	0.0544
0.17	0.482	0.0673
0.20	0.625	0.0820
0.25	0.827	0.1147
0.30	0.924	0.1654
0.35	0.963	0.247
0.4	0.992	0.336
0.5	1.039	0.526
0.6	1.082	0.662
0.7	1.115	0.743
0.8	1.148	0.805
1.0	1.199	0.858
1.2	1.234	0.900
1.4	1.274	0.924
1.7	1.324	0.956
2.0	1.367	0.981
2.5	1.424	1.020
3.0	1.476	1.052
3.5	1.531	
4.0	1.572	1.105
5.0	1.654	1.149
6.0	1.717	1.185
7.0	1.784	
8.0	1.840	1.249
10.0	1.935	1.311
12.0	2.02	1.367
14.0	2.10	1.414
17.0	2.20	1.486
20.0	2.30	1.554
25.0	2.46	1.654
30.0	2.63	1.787

measurements of Pack and Phelps<sup>42</sup> are generally in agreement with the present values for  $E/N > 10^{-18}$  V cm<sup>2</sup>, but are increasingly larger at lower  $E/N$  values, being  $> +10\%$  at  $3 \times 10^{-20} < E/N < 3 \times 10^{-19}$  V cm<sup>2</sup>.

### B. Krypton

The electron drift velocity measurements in Kr were performed at a gas temperature  $T = 301.2$  K over the  $E/N$  range  $2 \times 10^{-20} \leq E/N \leq 3 \times 10^{-17}$  V cm<sup>2</sup>. These are listed in Table II and shown in Fig. 2(a) in comparison with the unpurified gas sample measurements, and those of Bowe<sup>43</sup> and Pack *et al.*<sup>24</sup> The total uncertainty in the present measurements in Kr is estimated to be  $\pm 2\%$  at low  $E/N$  values ( $\leq 5 \times 10^{-19}$  V cm<sup>2</sup>) and decreases to  $\pm 1\%$  at the higher  $E/N$  values. The present results are in good agreement with those of Bowe<sup>43</sup> at the lower  $E/N$  range of his measurements ( $E/N \approx 3 \times 10^{-19}$  V cm<sup>2</sup>), but are increasingly higher at larger  $E/N$  values with a maximum difference at  $+8\%$  at  $E/N = 3 \times 10^{-17}$  V cm<sup>2</sup>.

In contrast, the present measurements are in better agreement with those of Pack *et al.*<sup>24</sup> at the high  $E/N$  values but deviate significantly at lower  $E/N$  values with the maximum deviation being  $\approx 20\%$  at  $E/N = 1.5 \times 10^{-19}$  V cm<sup>2</sup>. During the early stages in the purification of our gas sample, we were able to reproduce the results of Pack *et al.*, but with further purification cycles, the final drift velocity curves given in Fig. 2 were obtained. Since our unpurified gas sample contained approximately 3 ppm of N<sub>2</sub>, it appears that the Kr sample used by Pack *et al.* might have been contaminated with  $\sim 1$  ppm of N<sub>2</sub> or the other molecular impurity.

The density-normalized electron mobility  $\mu N [= w / (E/N)]$  measurements obtained from the  $w(E/N)$  data in Kr are plotted in Fig. 2(b) in comparison with the

data of Pack *et al.*<sup>24</sup> These data clearly indicate that the peak in the present mobility measurements occurs at a lower  $E/N$  value than that of Pack *et al.* and the present density-normalized thermal electron mobility  $(\mu N)_{th}$  is also significantly smaller. Electron thermal equilibrium with the surrounding gas occurs in Kr at  $E/N \leq 4 \times 10^{-20}$  V cm<sup>2</sup>, with  $(\mu N)_{th} = (1.17 \pm 0.03) \times 10^{23}$  V<sup>-1</sup> cm<sup>-1</sup> s<sup>-1</sup>. This compares with the value  $(\mu N)_{th} = 1.32 \times 10^{23}$  V<sup>-1</sup> cm<sup>-1</sup> s<sup>-1</sup> obtained by Pack *et al.* These values are listed in Table III in comparison with several calculated values obtained from Boltzmann analyses of previous low-energy electron-scattering cross-section measurements and swarm studies. None of the previously measured or calculated  $(\mu N)_{th}$  values are compatible with the present measured value to within the experimental uncertainty of the present measurement. The calculated  $(\mu N)_{th}$  value obtained from the swarm study of the electron drift velocity in Kr-H<sub>2</sub> gas mixtures by England and Elford<sup>44</sup> is in close agreement ( $\approx 5.0\%$  lower) with the present value but still outside the combined error limits of the measurements.

### C. Xenon

The electron drift velocity measurements in Xe were performed at a gas temperature of 301.2 K over the  $E/N$  range  $4 \times 10^{-20} \leq E/N \leq 3 \times 10^{-17}$  V cm<sup>2</sup>. They are listed in Table II and shown in Fig. 3(a) in comparison with several previous measurements.<sup>24,39,41,43,45</sup> The total uncertainty in the present measurements is  $\pm 2\%$  for  $E/N < 1 \times 10^{-18}$  V cm<sup>2</sup>, and  $\pm 1\%$  at higher  $E/N$  values. In contrast to the situation in Kr, the present results in Xe are in fair agreement with those of Pack *et al.*<sup>24</sup> to within the statistical scatter of their data (maximum differences are  $\pm 8\%$ ), but substantially higher than those of Bowe<sup>43</sup> at the lower end of his range of  $E/N$  values

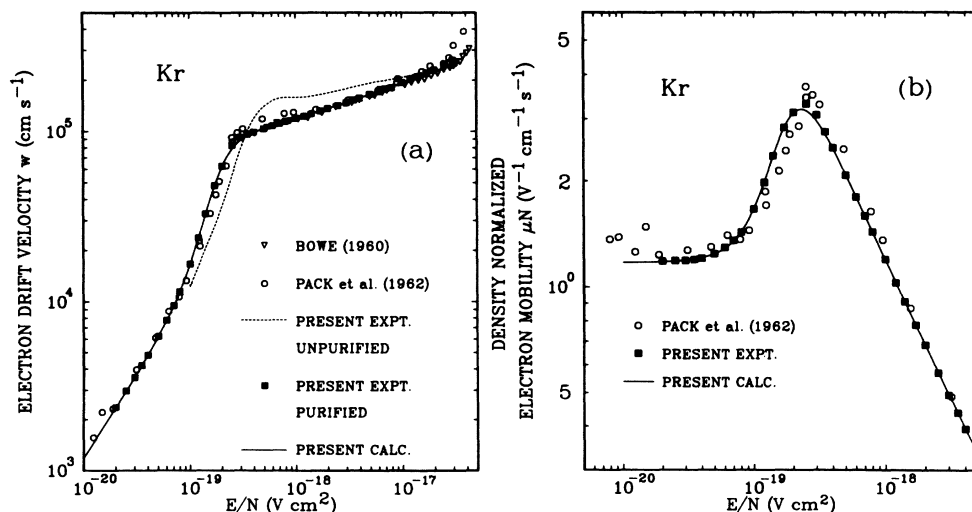


FIG. 2. Experimental and calculated (a) electron drift velocity and (b) density-normalized electron mobility measurements in Kr in comparison with the measurements of Bowe (Ref. 43) and Pack *et al.* (Ref. 24).

TABLE III. Density-normalized thermal electron mobilities  $(\mu N)_{th}$  for Kr and Xe.

	Kr ( $10^{23} \text{ V}^{-1} \text{ cm}^{-1} \text{ s}^{-1}$ )	Xe ( $10^{22} \text{ V}^{-1} \text{ cm}^{-1} \text{ s}^{-1}$ )
Experimental		
Present measurements	1.17±0.03	3.81±0.07
Pack, Voshall, and Phelps <sup>a</sup>	1.32	3.87
Huang and Freeman <sup>b</sup>		2.96
Dmitrenko <i>et al.</i> <sup>c</sup>		2.89
Calculated (Swarm)		
Hoffmann and Skarsgard <sup>d</sup>	1.64	4.79
Koizumi <i>et al.</i> <sup>e</sup>	1.27	3.76
England and Elford <sup>f</sup>	1.11	
Calculated (Electron beam)		
O'Malley <sup>g</sup>	1.07	4.24
Gus'kov <i>et al.</i> <sup>h</sup>	0.95	3.59
Jost <i>et al.</i> <sup>i</sup>	0.92	4.23
Buckman and Lohmann <sup>j</sup>	1.29	
Weyhreter <i>et al.</i> <sup>k</sup>	1.08	3.16

<sup>a</sup>Reference 24.<sup>b</sup>Reference 39.<sup>c</sup>Reference 41.<sup>d</sup>Reference 30.<sup>e</sup>Reference 34.<sup>f</sup>Reference 44.<sup>g</sup>Reference 51.<sup>h</sup>Reference 17.<sup>i</sup>Reference 18.<sup>j</sup>Reference 21.<sup>k</sup>Reference 14.

(maximum difference is +15%).

The measurements of Brooks *et al.*<sup>45</sup> were made in a chamber contaminated with halogen-containing compounds<sup>46</sup> and indicate the importance of using very high purity gases and good vacuum techniques when performing electron transport measurements in these gases. The source of the large difference between the present measurements and those of Huang and Freeman<sup>39</sup> and Dmi-

trenko *et al.*<sup>41</sup> at low  $E/N$  values is harder to identify. The differences between these sets of data are clearly evident in the  $\mu N(E/N)$  plots given in Fig. 3(b). The present  $(\mu N)_{th}$  value is in good agreement with that of Pack *et al.*<sup>24</sup> but is  $\approx 25\%$  larger than the low-pressure values obtained by Huang and Freeman<sup>39</sup> and Dmitrenko *et al.*<sup>41</sup> (see Table III). Previous analyses<sup>47,48</sup> have indicated that there may be substantial errors in the tech-

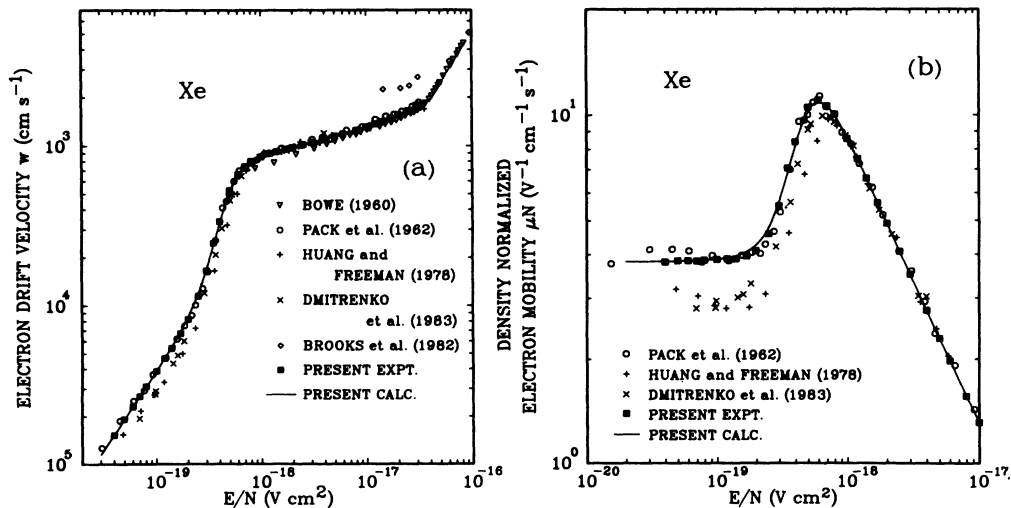


FIG. 3. Experimental and calculated (a) electron drift velocity and (b) density-normalized electron mobility measurements in Xe in comparison with the measurements of Bowe (Ref. 43), Pack *et al.* (Ref. 24), Huang and Freeman (Ref. 39), Dmitrenko *et al.* (Ref. 41), and Brooks *et al.* (Ref. 45).

nique used by Freeman and co-workers,<sup>39</sup> particularly due to their simplified treatment of diffusion corrections<sup>47,48</sup> and other possible sources of experimental error.<sup>49</sup> Dmitrenko *et al.*<sup>41</sup> used a standard pulsed Townsend technique to obtain their  $w$  measurements with a claimed uncertainty of  $\pm 2\%$ . The difference between the present measurements and their data is not understood. The other  $(\mu N)_{th}$  values for Xe shown in Table III obtained from Boltzmann-equation analyses of low-energy electron beam and other swarm studies are clearly not compatible with the present measurement.

#### IV. DATA ANALYSIS

##### A. Procedure

The motion of electrons in a gas under the influence of a uniform electric field is related to the electron scattering cross sections in the gas by the Boltzmann equation. In the absence of inelastic scattering processes, the Boltzmann equation can be expressed as<sup>22,23</sup>

$$\frac{e^2 E^2}{3N} \frac{d}{d\varepsilon} \left[ \frac{\varepsilon}{\sigma_m(\varepsilon)} \frac{df}{d\varepsilon} \right] + \frac{2mNkT}{M} \frac{d}{d\varepsilon} \left[ \varepsilon^2 \sigma_m(\varepsilon) \frac{df}{d\varepsilon} \right] + \frac{2mN}{M} \frac{d}{d\varepsilon} \left[ \varepsilon^2 \sigma_m(\varepsilon) f(\varepsilon) \right] = 0, \quad (2)$$

where  $m$  and  $M$  are the electron and atomic masses,  $e$  is the electron charge,  $k$  is the Boltzmann constant, and  $f(\varepsilon)$  is the electron energy distribution function and is normalized such that

$$\int_0^\infty \varepsilon^{1/2} f(\varepsilon) d\varepsilon = 1.$$

The electron transport coefficients  $w$  and  $D_T$  are related to  $f(\varepsilon)$  and  $\sigma_m(\varepsilon)$  by<sup>22</sup>

$$w = -\frac{eE}{3N} \left[ \frac{2}{M} \right]^{1/2} \int_0^\infty \frac{\varepsilon}{\sigma_m(\varepsilon)} \frac{df(\varepsilon)}{d\varepsilon} d\varepsilon, \quad (3)$$

$$ND_T = \frac{1}{3} \left[ \frac{2}{M} \right]^{1/2} \int_0^\infty \frac{\varepsilon f(\varepsilon)}{\sigma_m(\varepsilon)} d\varepsilon,$$

where  $D_T$  is the transverse electron diffusion coefficient. The experimentally measured parameter  $D_T/\mu = ND_T/[w/(E/N)]$  is then

$$D_T/\mu = -e^{-1} \int_0^\infty \frac{\varepsilon f(\varepsilon)}{\sigma_m(\varepsilon)} d\varepsilon / \int_0^\infty \frac{\varepsilon}{\sigma_m(\varepsilon)} \frac{df}{d\varepsilon} d\varepsilon. \quad (4)$$

The relations are obtained by assuming that  $f(\varepsilon)$  can be expanded in Legendre polynomials and truncated after the first two terms: the so-called two-term expansion. Several Monte Carlo and "multiterm" Boltzmann code studies<sup>50</sup> have been performed to investigate the validity of this approximation in gases such as Ar, which possess deep R-T minima in their  $\sigma_m(\varepsilon)$  cross section. All these studies have shown that negligible error occurs from the neglect of higher-order terms in the expansion of the electron energy distribution function. We have used a Boltzmann code developed by Gibson<sup>36</sup> to determine the low-energy ( $\varepsilon < 10$  eV)  $\sigma_m(\varepsilon)$  in Kr and Xe from the

present  $w(E/N)$  measurements. This involves assuming a trial  $\sigma_m(\varepsilon)$  and solving the Boltzmann equation [Eq. (2)] to find  $f(\varepsilon)$  as a function of  $E/N$ , which is then substituted into Eq. (3) to obtain  $w$ . The calculated values of  $w$  are compared with the experimental results and the  $\sigma_m(\varepsilon)$  is iteratively modified to improve the agreement between the experimental and calculated  $w$  values.

The uniqueness of the derived  $\sigma_m(\varepsilon)$  at low electron energies can be improved by using the modified-effective-range theory (MERT) developed by O'Malley<sup>51</sup> to extend  $\sigma_m(\varepsilon)$  to zero energy and thereby obtain an estimate for the scattering length  $A$  in each of these gases. The four-parameter MERT expansions for the scattering phase shifts are given by<sup>27</sup>

$$\tan \eta_0 = -Ak [1 + (4\alpha/3a_0)k^2 \ln(ka_0)] - (\pi\alpha/3a_0)k^2 + Dk^3 + Fk^4, \quad (5)$$

$$\tan \eta_1 = (\pi/15a_0)\alpha k^2 [1 - (\varepsilon/\varepsilon_1)^{1/2}], \quad (6)$$

$$\tan \eta_l = \pi\alpha k^2 / [(2l+3)(2l+1)(2l-1)a_0], \quad (7)$$

where  $\alpha$  is the polarizability of the atom [ $\alpha = 16.737a_0^3$  for Kr and  $\alpha = 27.292a_0^3$  for Xe (Ref. 52)],  $a_0$  is the Bohr radius,  $k$  is the electron wave number in a.u. [ $k$  is related to the electron energy  $\varepsilon$  (in eV) by  $\varepsilon = 13.605(ka_0)^2$ ],  $l$  is the angular-momentum quantum number, and  $A$ ,  $D$ ,  $F$ , and  $\varepsilon_1$ , are four adjustable parameters. The  $\eta_1$  ( $l=1$ ) phase shift may alternatively be expressed as<sup>27</sup>

$$\tan \eta_1 = (\pi/15a_0)\alpha k^2 - A_1 k^3, \quad (8)$$

where  $A_1 (= \pi\alpha\sqrt{13.605}/15\sqrt{\varepsilon_1})$  is now the adjustable parameter. The higher-order phase shifts  $\eta_l$  [Eq. (7)] are assumed to be given with a sufficient accuracy by the Born approximation for the polarization potential. Within this approximation,  $\sigma_m(\varepsilon)$  and  $\sigma_i(\varepsilon)$  are given by

$$\sigma_m = \frac{4\pi a_0^2}{k^2} \sum_{l=0}^{\infty} (l+1) \sin^2(\eta_l - \eta_{l+1}), \quad (9)$$

$$\sigma_i = \frac{4\pi a_0^2}{k^2} \sum_{l=0}^{\infty} (2l+1) \sin^2 \eta_l. \quad (10)$$

Previous studies have shown that the MERT expansion may be valid only over a limited range of electron energies (i.e.,  $\lesssim 0.5$  eV) in these rare gases.<sup>14,21,28</sup>

##### B. $\sigma_m(\varepsilon)$ cross sections

The present  $\sigma_m(\varepsilon)$  in Kr and Xe have been obtained at low  $\varepsilon$  (up to  $\approx 0.35$  eV in Kr and up to  $\approx 0.75$  eV in Xe) using the MERT analysis, while at higher  $\varepsilon$  the trial  $\sigma_m(\varepsilon)$  has been adjusted until the calculated  $w$  values generally agreed with the experimental measurements to within  $\pm 1.0\%$  for both gases. The influence of electronic excitation and ionization processes on the calculated electron drift velocities become significant in both gases at  $E/N > 1.5 \times 10^{-17}$  V cm<sup>2</sup>. The total electronic excitation cross sections given by Specht *et al.*<sup>53</sup> and the total ionization cross sections given by Rapp and Englander-Golden<sup>54</sup> for both Kr and Xe have been used in the

present calculations without modification. The inclusion of these processes changes the calculated  $w$  values by only a few percent at  $E/N=3.0\times 10^{-17}$  V cm<sup>2</sup>, the highest  $E/N$  value used in the present study.

The present  $\sigma_m(\epsilon)$  for both Kr and Xe are listed in Table IV and the  $\sigma_m(\epsilon)$  for Kr is shown in Fig. 4(a) in comparison with previous cross sections derived from swarm analyses,<sup>23,30,31,34,44</sup> in Fig. 4(b) in comparison with several electron-beam analyses,<sup>12,14,17,18,21</sup> and in Fig. 4(c) with the theoretically<sup>6,7,11</sup> derived cross sections. The  $\sigma_m(\epsilon)$  for Xe obtained from previous swarm analyses<sup>23,30,32,34</sup> are given in Fig. 5(a), the previous electron-beam<sup>13,14,17,18</sup> measurements are given in Fig. 5(b), and the theoretically derived cross sections<sup>6,7</sup> are given in Fig. 5(c) in comparison with the present cross section.

### C. Accuracy of the derived cross sections

A significant source of error in the Boltzmann analysis of the transport data in these gases occurs due to the rapidity with which  $\sigma_m$  varies with  $\epsilon$  at electron energies near the R-T minimum. Accurate transport data may be calculated using very small energy step sizes in the calculations (typically 3000 energy points were used which resulted in energy step sizes  $\approx 10^{-3}$  eV at the lower  $E/N$  values), a large input cross section data set ( $\approx 100$  points for electron energies up to 10 eV) and interpolating between these data points using a quadratic interpolation routine (as compared with the standard linear interpolation).<sup>44</sup> Under these conditions, the calculated transport data are stable to within  $\pm 0.03\%$ .

TABLE IV. Momentum-transfer cross sections for electron scattering from krypton and xenon.

Energy $\epsilon$ (eV)	Kr $\sigma_m$ ( $10^{-16}$ cm <sup>2</sup> )	Xe $\sigma_m$ ( $10^{-16}$ cm <sup>2</sup> )	Energy $\epsilon$ (eV)	Kr $\sigma_m$ ( $10^{-16}$ cm <sup>2</sup> )	Xe $\sigma_m$ ( $10^{-16}$ cm <sup>2</sup> )
0.00	39.73	130.5	0.70	0.268	0.8069
0.01	26.03	84.25	0.72	0.290	0.8437
0.02	21.14	67.14	0.74	0.312	0.8899
0.03	17.82	55.71	0.76	0.336	0.9456
0.04	15.32	47.24	0.78	0.361	1.005
0.06	11.68	35.33	0.80	0.386	1.055
0.08	9.132	27.29	0.84	0.442	1.175
0.10	7.237	21.53	0.88	0.501	1.30
0.12	5.780	17.24	0.92	0.563	1.44
0.14	4.633	13.96	0.96	0.632	1.57
0.16	3.716	11.40	1.00	0.705	1.72
0.18	2.975	9.392	1.05	0.800	1.91
0.20	2.374	7.787	1.1	0.893	2.12
0.22	1.883	6.497	1.2	1.11	2.55
0.24	1.482	5.453	1.3	1.32	3.03
0.26	1.156	4.603	1.4	1.57	3.53
0.28	0.8929	3.908	1.6	2.08	4.68
0.30	0.6823	3.337	1.8	2.63	5.98
0.32	0.5168	2.866	2.0	3.19	7.39
0.34	0.3902	2.477	2.2	3.78	8.95
0.36	0.2980	2.154	2.4	4.41	10.6
0.38	0.2370	1.885	2.6	5.05	12.4
0.40	0.2000	1.661	2.8	5.71	14.3
0.42	0.1820	1.474	3.0	6.35	16.1
0.44	0.1720	1.318	3.3	7.32	18.8
0.46	0.1680	1.189	3.6	8.28	21.4
0.48	0.1665	1.081	4.0	9.51	24.1
0.50	0.1660	0.9928	4.4	10.7	26.2
0.52	0.1675	0.9208	4.8	11.9	27.6
0.54	0.171	0.8634	5.2	13.15	28.7
0.56	0.175	0.8191	5.6	14.2	29.2
0.58	0.181	0.7866	6.0	15.15	29.5
0.60	0.191	0.7652	6.5	16.2	29.55
0.62	0.204	0.7541	7.0	17.2	29.2
0.64	0.219	0.7530	7.5	18.15	
0.66	0.234	0.7615	8.0	19.1	
0.68	0.250	0.7795			

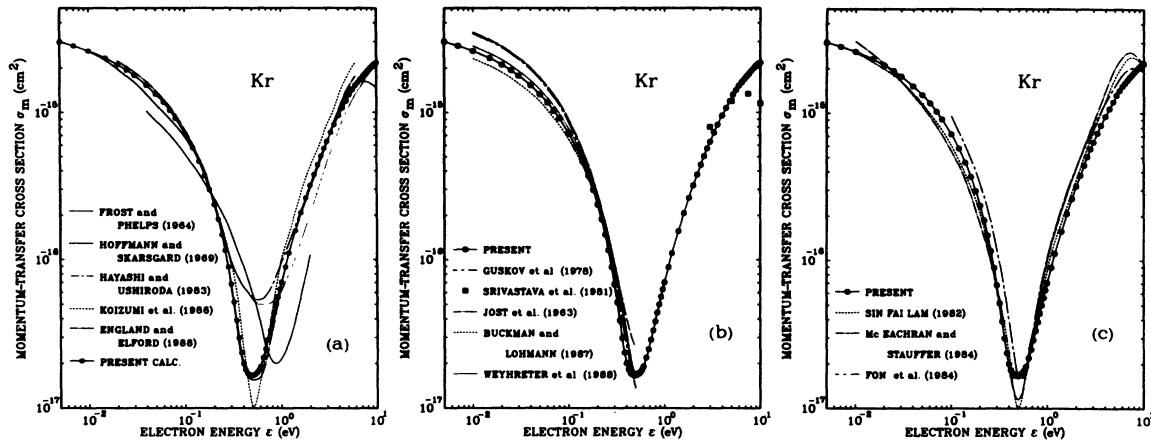


FIG. 4. Present calculated momentum-transfer cross section  $\sigma_m(\epsilon)$  for Kr in comparison with (a) the previous swarm-derived cross sections of Frost and Phelps (Ref. 23), Hoffmann and Skarsgard (Ref. 30), Hayashi and Ushiroda (Ref. 31), Koizumi *et al.* (Ref. 34), and England and Elford (Ref. 44); (b) electron-beam measurements of Guskov *et al.* (Ref. 17), Srivastava *et al.* (Ref. 12), Jost *et al.* (Ref. 18), Buckman and Lohmann (Ref. 21), and Weyhreter *et al.* (Ref. 14); and (c) theoretical cross sections of Sin Fai Lam (Ref. 7), McEachran and Stauffer (Ref. 6), and Fon *et al.* (Ref. 11).

The overall magnitude of the present  $\sigma_m(\epsilon)$  cross sections have an uncertainty which is not significantly greater than the experimental error in the electron drift velocity measurements (i.e.,  $\approx 2\%$ ), assuming that the “two-term” Boltzmann code approximation is valid for the electron-swarm analyses in these gases. However, the absolute accuracy of the derived cross sections at a given energy varies considerably with electron energy due to the depth of the R-T minimum in these gases. At electron energies considerably above and below the R-T minimum, the calculated electron drift velocities are sensitive to the magnitude of the trial  $\sigma_m(\epsilon)$  cross section, allowing  $\sigma_m(\epsilon)$  to be found with good accuracy (error  $\approx 5\%$ ). In contrast, the calculated  $w$  values are relatively insensitive to the depth of the R-T minimum, thereby increasing the uncertainty with which the  $\sigma_m(\epsilon)$  cross sec-

tion can be obtained in this energy region. Similar observations have previously been made by Milloy *et al.*<sup>27</sup> in their analysis of  $w$  and  $D_T/\mu$  data to obtain the  $\sigma_m(\epsilon)$  in Ar, and Koizumi *et al.*<sup>34</sup> in their analysis of  $D_T/\mu$  measurements in Kr and Xe.

The uncertainty in the depth of the R-T minimum in Xe and Kr has been examined by observing the variation in the calculated  $w$  values with changes in the depth of the minimum. These comparisons are shown in Fig. 6 for Xe where the  $\sigma_m(\epsilon)$  used are those having the upper and lower limits on the R-T minimum obtained from the previous swarm analyses given in Fig. 5(a). The cases B and C clearly do not give the best fit to the experimental measurements, but even with the best fit to the measurements (case A) the calculated  $w$  values differ from the experimental values by up to  $\pm 4\%$  over the  $E/N$  range

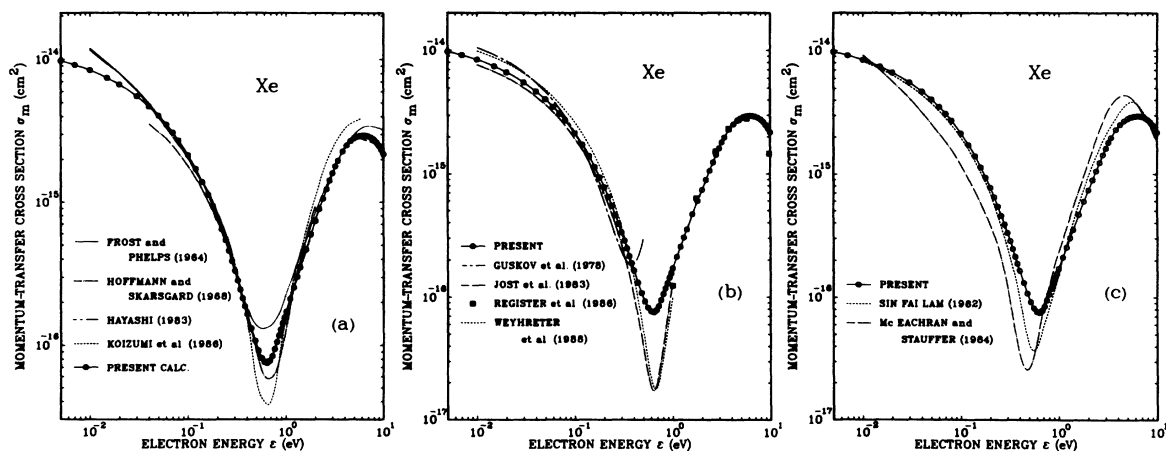


FIG. 5. Present calculated momentum-transfer cross section  $\sigma_m(\epsilon)$  for Xe in comparison with (a) the previous swarm-derived cross sections of Frost and Phelps (Ref. 23), Hoffmann and Skarsgard (Ref. 30), Hayashi (Ref. 32), and Koizumi *et al.* (Ref. 34); (b) electron-beam measurements of Guskov *et al.* (Ref. 17), Jost *et al.* (Ref. 18), Register *et al.* (Ref. 13), and Weyhreter *et al.* (Ref. 14); and (c) theoretical cross sections of Sin Fai Lam (Ref. 7) and McEachran and Stauffer (Ref. 6). Note that in Fig. 5(a) the  $\sigma_m(\epsilon)$  of Hayashi is superimposed on the present cross section and is not clearly visible in the figure.



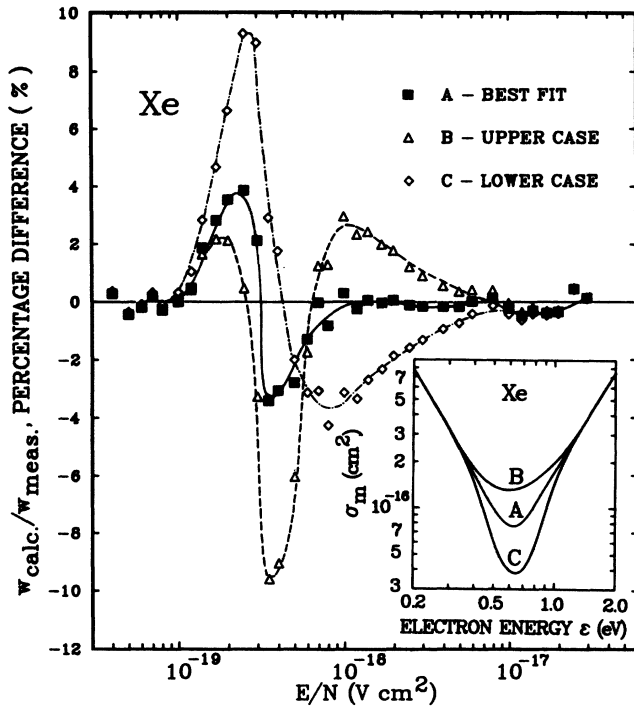


FIG. 6. Percentage difference between the present measured  $w$  values in Xe and the  $w$  values calculated using the  $\sigma_m(\epsilon)$  shown in the figure inset.

$0.015 < E/N < 0.05 \times 10^{-17}$  V cm<sup>2</sup>, which is considered to be outside the error limits of the experimental measurements. Similar, though smaller, differences occur between the experimental and the best-fit calculated  $w$  values in Kr over the  $E/N$  range  $0.02 < E/N < 0.04 \times 10^{-17}$  V cm<sup>2</sup> [Fig. 7(a)]. These differences are

not understood at the present time, but the following observations can be made. The differences occur at the  $E/N$  values which are most sensitive to the minimum in  $\sigma_m(\epsilon)$ , and where  $D_L/\mu$  reaches a local maximum as has been shown in the calculations of Lowke and Parker.<sup>38</sup> Experimentally, this can be seen in a diffusion broadening of the voltage transient due to the swarm motion in the gas, which leads to a larger uncertainty in estimating the transit time  $t_e$ . The diffusion corrections to the measured transit time are also larger in this situation [Eq. (1)], but we found no discernible pressure dependence in the  $w$  measurements at 300 and 600 kPa outside the experimental uncertainty ( $\pm 2\%$  over this  $E/N$  range). Similar problems occur when an attempt is made to derive  $\sigma_m(\epsilon)$  from the  $w$  measurements of Pack *et al.*<sup>24</sup> for, although the scatter in their data are larger, significant deviations between the experimental and calculated  $w$  values occur at these  $E/N$  values. Failure of the “two-term” expansion in this  $E/N$  region for Kr and Xe is unlikely.<sup>55</sup> The nonuniqueness of the calculated  $w$  values in this region leads to uncertainty limits of  $\pm 20\%$  on the depth of the R-T minimum in the  $\sigma_m(\epsilon)$  cross sections in these two gases, which reduces to  $\pm 5\%$  on the upper and lower energy wings of  $\sigma_m(\epsilon)$ .

#### D. Comparison with previous $\sigma_m(\epsilon)$ cross sections

##### 1. Kr

With the possible exception of the  $\sigma_m(\epsilon)$  given by England and Elford,<sup>44</sup> none of the previous swarm-derived cross sections given in Fig. 4(a) are compatible with the present  $w$  data. This can be seen in Fig. 7(a), where the percentage difference between the calculated  $w$  values using these  $\sigma_m(\epsilon)$  [Fig. 4(a)] and the present  $w$  measure-

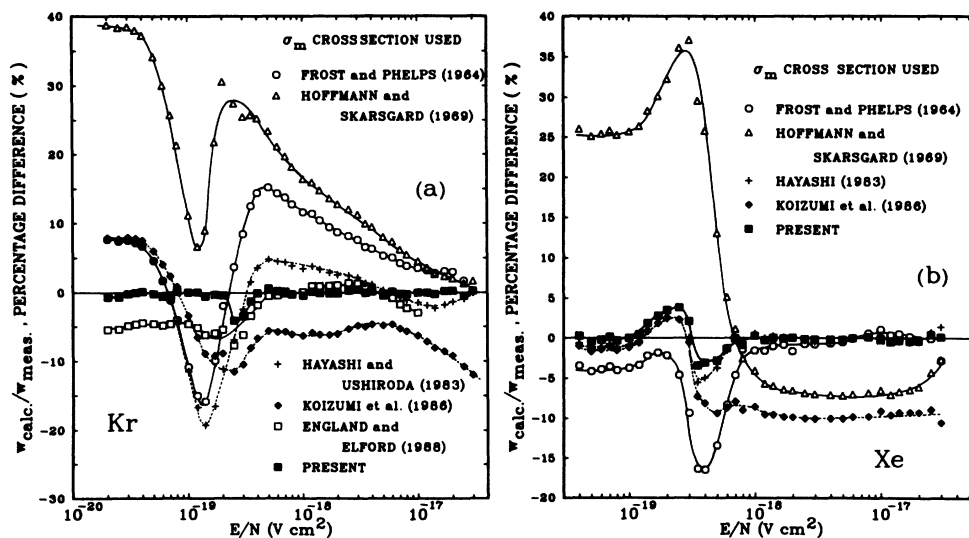


FIG. 7. Percentage difference between the present measured  $w$  values and the  $w$  values calculated using the  $\sigma_m(\epsilon)$  cross sections of (a) Frost and Phelps (Ref. 23), Hoffmann and Skarsgard (Ref. 30), Hayashi and Ushiroda (Ref. 31), Koizumi *et al.* (Ref. 34), and England Elford (Ref. 44) in Kr; and (b) Frost and Phelps (Ref. 23), Hoffmann and Skarsgard (Ref. 30), Hayashi (Ref. 32), and Koizumi *et al.* (Ref. 34) in Xe.

ments are plotted. The  $\sigma_m(\epsilon)$  reported by Frost and Phelps<sup>23</sup> and Hayashi and Ushiroda<sup>31</sup> were obtained from an analysis of the drift velocity data of Pack *et al.*<sup>24</sup> which have been shown in Sec. III to be in error due to the possible presence of impurities in their gas sample. The  $\sigma_m(\epsilon)$  obtained by Hoffman and Skarsgard<sup>30</sup> was determined from an analysis of the transient electric conductivity in a microwave experiment and may also have been affected by impurity and data analysis problems. The calculated  $w$  values using any of these cross sections lie well outside the estimated uncertainty in the present experimental data [Fig. 7(a)]. In particular, the recent  $\sigma_m(\epsilon)$  of Koizumi *et al.*<sup>34</sup> for Kr predicts  $w$  values that are up to 12% different from the present values. Impurities and perhaps other experimental problems in the  $D_T/\mu$  measurements of Koizumi *et al.*<sup>34</sup> are thought to be responsible for the large differences in the calculated  $\sigma_m(\epsilon)$ , particularly at electron energies above the R-T minimum.<sup>56</sup> In contrast to these earlier studies, the  $\sigma_m(\epsilon)$  cross section obtained in the concomitant study by England and Elford<sup>44</sup> is in better agreement with the present  $\sigma_m(\epsilon)$  cross section. The  $w$  measurements in their study were performed in Kr-H<sub>2</sub> gas mixtures to reduce the effect of diffusion broadening on  $w_m(E/N)$  as a function of gas pressure, and to increase the sensitivity of the calculated  $w$  values to the depth of the R-T minimum. A comparison of the percentage difference in the calculated to experimental drift velocity ratio using their  $\sigma_m(\epsilon)$  cross section (Fig. 7a) indicates that at  $E/N \gtrsim 4 \times 10^{-19}$  V cm<sup>2</sup> the present and calculated  $w$  values are in fair agreement, but at lower  $E/N$  values, the

calculated  $w$  values are up to 6% lower than the present measurements which is considered to be outside the experimental uncertainty of the present measurements.<sup>57</sup> This difference results primarily from the larger value of the scattering length  $A$  obtained in their MERT analysis of the electron drift velocity measurements in the Kr-H<sub>2</sub> gas mixtures (see Sec. IV E).

None of the electron beam or theoretically derived  $\sigma_m(\epsilon)$  cross sections shown in Figs. 4(b) and 4(c), respectively, are in good agreement with the present swarm-derived  $\sigma_m(\epsilon)$  cross section; the best agreement is with the semirelativistic calculation of Sin Fai Lam<sup>7</sup> at low electron energies. This can also be seen in a comparison of the calculated and experimental thermal electron mobility values  $(\mu N)_{th}$  given in Table III and the scattering length  $A$  calculations given in Table V (see Sec. IV E). The present  $\sigma_m(\epsilon)$  is in reasonable agreement with the experimental  $\sigma_m(\epsilon)$  of Srivastava *et al.*<sup>12</sup> which was obtained from the differential cross section measurements at electron energies  $\epsilon \geq 3$  eV and with an estimated error limit of  $\pm 30\%$ . There is also fair agreement with the  $\sigma_m(\epsilon)$  of Buckman and Lohmann<sup>21</sup> which was derived from a low-energy MERT analysis of their total scattering cross section  $\sigma_t(\epsilon)$  measurements to obtain the  $\sigma_m(\epsilon)$ . The  $\sigma_m(\epsilon)$  of Jost *et al.*<sup>18</sup> and Weyhreter *et al.*<sup>14</sup> given in Fig. 4(b) were also obtained by a similar procedure, and Buckman and Lohmann<sup>21</sup> have argued that the comparison between the  $\sigma_m(\epsilon)$  and  $\sigma_t(\epsilon)$  cross sections in Kr using the MERT analysis is valid for  $\epsilon < 0.3$  eV.

TABLE V. Comparison of the four parameters obtained with the present MERT analysis in krypton with those obtained in previous analyses.

Investigator	$A/a_0$	$D/a_0^3$	$F/a_0^4$	$A_1/a_0^3$	Energy range used (eV)
Present <sup>a</sup>	-3.36( $\pm 0.03$ )	178.8	-283.3	12.5	(0.01-0.35)
England and Elford <sup>b,a</sup>	-3.434	178.6	-291.2	12.47	(0.01-0.35)
O'Malley <sup>c</sup> from Pack <i>et al.</i> <sup>d,a</sup>	-3.2				
Buckman and Lohmann <sup>e,f</sup>	-3.19	184.75	-300.8	12.12	(0.175-0.5)
Weyhreter <i>et al.</i> <sup>g,f</sup>	-3.478	169.3	-198.0	12.71	(0.05-0.5)
Jost <i>et al.</i> <sup>h,f</sup>	-3.83	139.0		12.7	(0.3-0.5)
Guskov <i>et al.</i> <sup>i,f</sup>	-3.8	136.0		14.0	(0.025-2.0)
O'Malley <sup>c</sup> from Ramsauer and Kollath <sup>h,f</sup>	-3.7	132.0		12.8	
Fon <i>et al.</i> <sup>k,l</sup>	-3.79				
McEachran and Stauffer <sup>m,l</sup>	-3.103				
O'Connell and Lane <sup>n,l</sup>	-3.1				
Sin Fai Lam <sup>o,l</sup>	-3.34				
Yau <i>et al.</i> <sup>p,l</sup>	-4.201				

<sup>a</sup>Swarm.

<sup>b</sup>Reference 44.

<sup>c</sup>Reference 51.

<sup>d</sup>Reference 24.

<sup>e</sup>Reference 21.

<sup>f</sup>Beam.

<sup>g</sup>Reference 14.

<sup>h</sup>Reference 18.

<sup>i</sup>Reference 17.

<sup>j</sup>Reference 15.

<sup>k</sup>Reference 11.

<sup>l</sup>Theory.

<sup>m</sup>Reference 6.

<sup>n</sup>Reference 8.

<sup>o</sup>Reference 7.

<sup>p</sup>Reference 4.

## 2. Xe

The present  $\sigma_m(\epsilon)$  in Xe is in better agreement with the earlier swarm-derived cross sections shown in Fig. 5(a) than was the case in Kr, although significant differences exist between the present and all previous cross section estimates. The  $\sigma_m(\epsilon)$  above  $\epsilon \approx 0.04$  eV derived by Hayashi<sup>32</sup> from a reanalysis of the  $w$  measurements of Pack *et al.*<sup>24</sup> (which were shown in Sec. III to be in agreement with the present measurements to within the combined experimental uncertainty) is in very good agreement with the present cross section as is expected. A comparison of the percentage difference between the calculated  $w$  values using these  $\sigma_m(\epsilon)$  [Fig. 5(a)] and the present measurements in Xe is given in Fig. 7(b). These calculations also indicate that the best agreement with the present cross section occurs with the  $\sigma_m(\epsilon)$  cross section of Hayashi.<sup>32</sup> At low electron energies ( $\epsilon < 0.75$  eV) the MERT analysis has been used in the present study to derive  $\sigma_m(\epsilon)$  where the uniqueness of the calculated cross section is poor. The  $\sigma_m(\epsilon)$  obtained by Koizumi *et al.*<sup>34</sup> is in poor agreement with the present cross section for  $\epsilon > 0.4$  eV possessing a R-T minimum twice as deep as in the present calculations, while at higher electron energies ( $\epsilon > 1.2$  eV), their  $\sigma_m(\epsilon)$  is considerably larger than the present estimate. These differences are similar to those for the Kr cross sections.

The experimental  $\sigma_m(\epsilon)$  of Register *et al.*<sup>13</sup> (obtained from relative differential electron-scattering measurements normalized to independent total electron-scattering cross-section measurements) shown in Fig. 5(b) is in good agreement with the present  $\sigma_m(\epsilon)$  to within their experimental error ( $\pm 25\%$ ). However, the electron-beam and theoretically derived cross sections shown in Figs. 5(b) and 5(c), respectively, possess R-T

minima at electron energies below that of the present cross section, and are up to two times smaller than the present  $\sigma_m(\epsilon)$ .

## E. MERT parameters for Kr and Xe

The four parameters of the MERT analyses which were used to derive the present low energy  $\sigma_m(\epsilon)$  in Kr and Xe are listed in Tables V and VI, respectively, along with the parameters obtained in recent total electron-scattering cross-section experiments, electron-swarm studies, and theoretical calculations. The values for all four parameters obtained in the studies of Buckmann and Lohmann<sup>21</sup> and England and Elford<sup>44</sup> in Kr are in good agreement with the present values. The smaller value for the scattering length  $A$  obtained by Buckmann and Lohmann is thought to be due to the restricted energy range over which the MERT analysis was applied in their measurements (0.175–0.5 eV). One of the primary advantages of swarm analyses is the ability to accurately access the low-energy ( $0.01 \lesssim \epsilon \lesssim 0.1$  eV) electron-scattering processes. The present values of  $A$  in both Kr and Xe are in reasonable agreement with the previous values obtained by O'Malley<sup>51</sup> from an analysis of the  $w$  measurements of Pack *et al.*,<sup>24</sup> but are considerably different from those obtained in the more recent electron-beam studies.<sup>14,17,18</sup> The best agreement obtained between the present estimates for  $A$  in Kr and Xe and the several recent theoretical analyses of the low-energy electron scattering in these two gases are those obtained by Sin Fai Lam.<sup>7</sup> These values, and that of Fon *et al.*<sup>11</sup> in Kr, were obtained from a MERT fit to their tabulated low-energy  $\sigma_m(\epsilon)$  calculations.

TABLE VI. Comparison of the four parameters obtained with the present MERT analysis in xenon with those obtained in previous analyses.

Investigator	$A/a_0$	$D/a_0^3$	$F/a_0^4$	$A_1/a_0^3$	Energy range used (eV)
Present <sup>a</sup>	-6.09( $\pm 0.05$ )	490.2	-627.5	22.0	(0.01–0.75)
O'Malley <sup>b</sup> from Pack <i>et al.</i> <sup>c,a</sup>	-6.0				
Weyhreter <i>et al.</i> <sup>d,e</sup>	-6.527	517.0	-717.8	21.65	(0.05–0.5)
Jost <i>et al.</i> <sup>f,e</sup>	-5.83	490.0	-708.0	22.8	(0.1–0.5)
Guskov <i>et al.</i> <sup>g,e</sup>	-6.8	406.0		21.0	(0.025–2.0)
O'Malley <sup>b</sup> from Ramsauer and Kollath <sup>h,e</sup>	-6.5	388.0		23.2	
McEachran and Stauffer <sup>j,i</sup>	-5.232				
O'Connell and Lane <sup>k,i</sup>	-6.0				
Sin Fai Lam <sup>l,i</sup>	-6.04				
Yau <i>et al.</i> <sup>m,i</sup>	-7.816				

<sup>a</sup>Swarm.

<sup>b</sup>Reference 51.

<sup>c</sup>Reference 24.

<sup>d</sup>Reference 14.

<sup>e</sup>Beam.

<sup>f</sup>Reference 18.

<sup>g</sup>Reference 17.

<sup>h</sup>Reference 15.

<sup>i</sup>Theory.

<sup>j</sup>Reference 6.

<sup>k</sup>Reference 8.

<sup>l</sup>Reference 7.

<sup>m</sup>Reference 4.

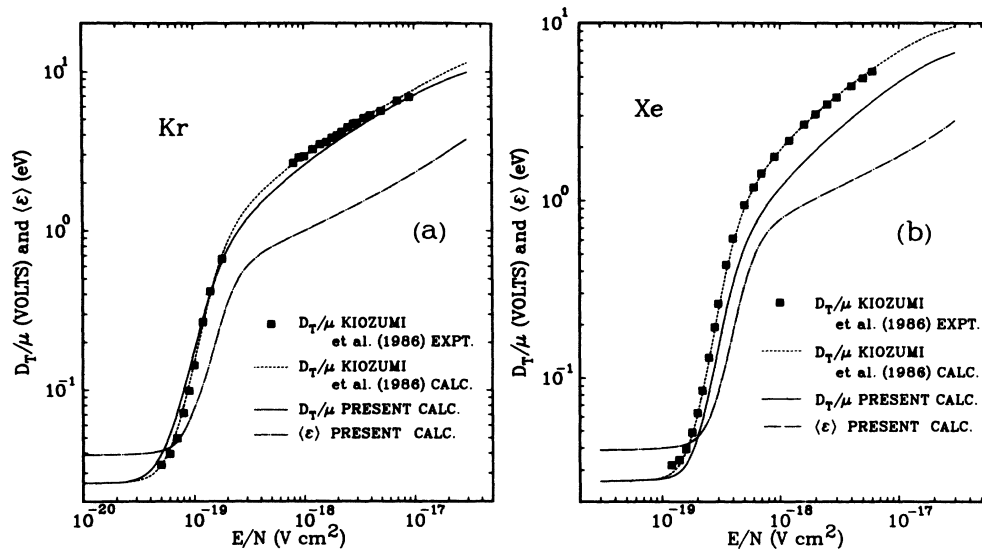


FIG. 8. Experimental  $D_T/\mu$  measurements in (a) Kr and (b) Xe of Koizumi *et al.* (Ref. 34) in comparison with the present calculated  $D_T/\mu$  and  $\langle \epsilon \rangle$  values, and the calculated  $D_T/\mu$  values using the  $\sigma_m(\epsilon)$  cross sections of Koizumi *et al.* (Ref. 34).

## V. CONCLUSIONS

From the present study of the low-energy electron scattering in Kr and Xe, the following conclusions can be made

(i) The present  $w$  measurements and calculated  $\sigma_m(\epsilon)$  for Xe are in good agreement to within the experimental accuracy with the earlier measurements of Pack *et al.*<sup>24</sup> and the Boltzmann analysis of Hayashi,<sup>32</sup> respectively. In contrast, the present  $w$  measurements are considerably higher ( $\approx 25\%$ ) at thermal electron energies than the more recent measurements of Huang and Freeman<sup>39</sup> and Dmitrenko *et al.*<sup>41</sup>

(ii) The  $w$  measurements of Pack *et al.*<sup>24</sup> in Kr may have been influenced by a low concentration molecular impurity, in a similar manner to that observed in Ar in the present and the previous study by Robertson.<sup>25</sup>

(iii) The  $\sigma_m(\epsilon)$  for Kr and Xe derived by a Boltzmann-equation analysis of the present  $w$  measurements are not compatible with those obtained from the  $D_T/\mu$  measurements of Koizumi *et al.*<sup>34</sup> This is evident in a comparison of the cross sections given in Figs. 4(a) and 5(a). It is also clearly seen in Figs. 8(a) and 8(b) for Kr and Xe, respectively, where the experimental  $D_T/\mu$  measurements of Koizumi *et al.* are plotted in comparison with the  $D_T/\mu$  values calculated using their published  $\sigma_m(\epsilon)$  and the present  $\sigma_m(\epsilon)$  cross sections. The scatter between the calculated and experimental  $D_T/\mu$  values of Koizumi *et al.* is  $>10\%$ , but in Xe the differences between the experimental  $D_T/\mu$  values and those calculated using the present  $\sigma_m(\epsilon)$  are  $>50\%$  at

the higher  $E/N$  values. The magnitude of these differences indicates that either impurities and/or experimental problems were present in their study.

(iv) The  $\sigma_m(\epsilon)$  for Kr derived by England and Elford<sup>44</sup> from their electron drift velocity measurements in Kr-H<sub>2</sub> gas mixtures is in reasonable agreement with the present  $\sigma_m(\epsilon)$  derived from  $w$  measurements in pure Kr to within the combined uncertainty in the calculations. The percentage difference in the calculated  $w$  values using England and Elford's  $\sigma_m(\epsilon)$  and the present experimental  $w$  measurements at low  $E/N$  values ( $<4 \times 10^{-19}$  V cm<sup>2</sup>), however, is outside the experimental uncertainty of the present measurements [1–2%; see Fig. 7(a)]. At higher  $E/N$  values the agreement between the calculated and present experimental  $w$  measurements is considerably better.

(v) We believe that the higher accuracy  $\sigma_m(\epsilon)$  cross sections derived from the present work can be used to improve our understanding of the scattering of low-energy electrons from Kr and Xe.

## ACKNOWLEDGMENTS

We wish to thank Dr. M. T. Elford, Mr. J. P. England, Dr. S. J. Buckman, Professor F. Linder, Dr. K. Jost, Mr. Y. Tomisawa, Mr. A. Naito, and Professor M. Hayashi for providing papers and tabulated data prior to publication. This work was sponsored by the Office of Health and Environmental Research, U. S. Department of Energy, under Contract No. DE-AC05-84OR21400 with Martin Marietta Energy Systems, Inc.

- \*Also at Department of Physics, The University of Tennessee, Knoxville, TN 37996.
- <sup>1</sup>L. G. Christophorou and S. R. Hunter, in *Electron-Molecule Interactions and Their Applications*, edited by L. G. Christophorou (Academic, Orlando, 1984), Vol. 2, Chap. 5.
  - <sup>2</sup>L. G. Christophorou and K. Siomos, in *Electron-Molecule Interactions and Their Applications*, edited by L. G. Christophorou (Academic, Orlando, 1984), Vol. 2 Chap. 4.
  - <sup>3</sup>L. G. Christophorou, S. R. Hunter, and J. G. Carter (unpublished).
  - <sup>4</sup>A. W. Yau, R. P. McEachran, and A. D. Stauffer, *J. Phys. B* **13**, 377 (1980).
  - <sup>5</sup>R. P. McEachran and A. D. Stauffer, *J. Phys. B* **16**, 4023 (1983).
  - <sup>6</sup>R. P. McEachran and A. D. Stauffer, *J. Phys. B* **17**, 2507 (1984).
  - <sup>7</sup>L. T. Sin Fai Lam, *J. Phys. B* **15**, 119 (1982).
  - <sup>8</sup>J. K. O'Connell and N. F. Lane, *Phys. Rev. A* **27**, 1893 (1983).
  - <sup>9</sup>K. L. Bell, N. S. Scott, and M. A. Lennon, *J. Phys. B* **17**, 4757 (1984).
  - <sup>10</sup>W. C. Fon, K. A. Berrington, P. G. Burke, and A. Hibbert, *J. Phys. B* **16**, 307 (1983).
  - <sup>11</sup>W. C. Fon, K. A. Berrington, and A. Hibbert, *J. Phys. B* **17**, 3279 (1984).
  - <sup>12</sup>S. K. Srivastava, H. Tanaka, A. Chutjian, and S. Trajmar, *Phys. Rev. A* **23**, 2156 (1981).
  - <sup>13</sup>D. F. Register, L. Vuskovic and S. Trajmar, *J. Phys. B* **19**, 1685 (1986); J. C. Nickel, K. Imre, D. F. Register, and S. Trajmar, *ibid.* **18**, 125 (1985).
  - <sup>14</sup>M. Weyhreter, B. Barzick and F. Linder, in *Abstracts of Contributed Papers, International Conference on the Physics of Electronic and Atomic Collisions, Berlin, 1983*, edited by J. Eichler, I. V. Hertel, and N. Stolterfoht (North-Holland, Amsterdam, 1984), p. 547; M. Weyhreter, B. Barzik, A. Mann, and F. Linder, *Z. Phys. D*, **7**, 333 (1988).
  - <sup>15</sup>C. Ramsauer, *Ann. Phys. (Leipzig)* **66**, 546 (1921); **72**, 345 (1923); C. Ramsauer and R. Kollath, *ibid.* **12**, 529; **12**, 837 (1932).
  - <sup>16</sup>J. S. E. Townsend and V. A. Bailey, *Philos. Mag.* **43**, 593 (1922).
  - <sup>17</sup>Y. K. Gus'kov, R. V. Savvov, and V. A. Slobodyanyuk, *Zh. Tekh. Fiz.* **48**, 277 (1978) [*Sov. Phys. Tech. Phys.* **23**, 167 (1978)].
  - <sup>18</sup>K. Jost, P. G. F. Bisling, F. Eschen, M. Felsmann, and L. Walther, in *Abstracts of the Proceedings of the Thirteenth International Conference on the Physics of Electronic and Atomic Collisions, Berlin, 1983*, edited by J. Eichler, W. Fritch, I. V. Hertel, N. Stolterfoht, and U. Wille (North-Holland, Amsterdam, 1984), p. 91; (private communication).
  - <sup>19</sup>J. Ferch, B. Granitza, C. Masche, and W. Raith, *J. Phys. B* **18**, 967 (1985).
  - <sup>20</sup>S. J. Buckman and B. Lohmann, *J. Phys. B* **19**, 2547 (1986).
  - <sup>21</sup>S. J. Buckman and B. Lohmann, *J. Phys. B* **20**, 5807 (1987).
  - <sup>22</sup>L. G. H. Huxley and R. W. Crompton, *The Diffusion and Drift of Electrons in Gases* (Wiley, New York, 1974).
  - <sup>23</sup>L. S. Frost and A. V. Phelps, *Phys. Rev.* **136**, A1538 (1964).
  - <sup>24</sup>J. L. Pack, R. E. Voshall, and A. V. Phelps, *Phys. Rev.* **127**, 2084 (1962).
  - <sup>25</sup>A. G. Robertson, *Aust. J. Phys.* **30**, 39 (1977).
  - <sup>26</sup>H. B. Milloy and R. W. Crompton, *Aust. J. Phys.* **30**, 51 (1977).
  - <sup>27</sup>H. B. Milloy, R. W. Crompton, J. A. Rees, and A. G. Robertson, *Aust. J. Phys.* **30**, 61 (1977).
  - <sup>28</sup>G. N. Haddad and T. F. O'Malley, *Aust. J. Phys.* **35**, 35 (1982).
  - <sup>29</sup>Z. Lj. Petrovic, Ph.D. thesis, Australian National University, Canberra, 1985.
  - <sup>30</sup>C. R. Hoffmann and H. M. Skarsgard, *Phys. Rev.* **178**, 168 (1969).
  - <sup>31</sup>M. Hayashi and S. Ushiroda, *J. Chem. Phys.* **78**, 2621 (1983).
  - <sup>32</sup>M. Hayashi, *J. Phys. D* **16**, 581 (1983).
  - <sup>33</sup>M. Hayashi (private communication).
  - <sup>34</sup>T. Koizumi, E. Shirakawa, and I. Ogawa, *J. Phys. B* **19**, 2331 (1986).
  - <sup>35</sup>S. R. Hunter, J. G. Carter, and L. G. Christophorou, *J. Appl. Phys.* **60**, 24 (1986).
  - <sup>36</sup>D. K. Gibson, *Aust. J. Phys.* **23**, 683 (1970).
  - <sup>37</sup>A. S. Friedman, in *American Institute of Physics Handbook*, edited by D. E. Gray (McGraw-Hill, New York, 1957), pp. 4-118.
  - <sup>38</sup>J. L. Lowke and J. H. Parker, *Phys. Rev.* **181**, 302 (1969).
  - <sup>39</sup>S. S.-S. Huang and G. R. Freeman, *J. Phys. Chem.* **68**, 1355 (1978).
  - <sup>40</sup>S. S.-S. Huang and G. R. Freeman, *Phys. Rev. A* **24**, 714 (1981).
  - <sup>41</sup>V. V. Dmitrenko, A. S. Romanyuk, S. I. Suchkov, and Z. M. Uteshev, *Zh. Tekh. Fiz.* **53**, 2343 (1983) [*Sov. Phys. Tech. Phys.* **28**, 1440 (1983)].
  - <sup>42</sup>J. L. Pack and A. V. Phelps, *Phys. Rev.* **121**, 798 (1961).
  - <sup>43</sup>J. C. Bowe, *Phys. Rev.* **117**, 1411 (1960).
  - <sup>44</sup>J. P. England and M. T. Elford, *Aust. J. Phys.* (to be published).
  - <sup>45</sup>H. L. Brooks, M. C. Cornell, J. Fletcher, I. M. Littlewood, and K. J. Nygaard, *J. Phys. D* **15**, L51 (1982).
  - <sup>46</sup>K. J. Nygaard, H. L. Brooks, and S. R. Hunter, *IEEE J. Quantum Electronics*, **QE-15**, 1216 (1979).
  - <sup>47</sup>R. A. Cassidy, *Aust. J. Phys.* **34**, 677 (1981).
  - <sup>48</sup>R. E. Robson, *Phys. Rev. A* **31**, 3492 (1985).
  - <sup>49</sup>R. W. Crompton and M. A. Morrison, *Phys. Rev. A* **26**, 3695 (1982).
  - <sup>50</sup>See discussion in S. R. Hunter and L. G. Christophorou, in *Electron-Molecule Interactions and Their Applications*, edited by L. G. Christophorou (Academic, Orlando, 1984), Vol. 2, Chap. 3.
  - <sup>51</sup>T. F. O'Malley, *Phys. Rev.* **130**, 1020 (1963).
  - <sup>52</sup>R. R. Teachout and R. T. Pack, *At. Data* **3**, 195 (1971).
  - <sup>53</sup>L. T. Specht, S. A. Lawton, and T. A. DeTemple, *J. Appl. Phys.* **51**, 166 (1980).
  - <sup>54</sup>D. Rapp and P. Englander-Golden, *J. Chem. Phys.* **43**, 1464 (1965).
  - <sup>55</sup>"Multiterm" Boltzmann code calculations by K. E. Ness and R. E. Robson have failed to find any deviation from the results obtained using a "two-term" Boltzmann code over this  $E/N$  range in these two gases except at unrealistically large anisotropies in the electron scattering (Ref. 56).
  - <sup>56</sup>J. P. England (private communication).
  - <sup>57</sup>It is possible to derive a  $\sigma_m(\epsilon)$  cross section in Kr which gives calculated  $w$  values in the Kr-H<sub>2</sub> gas mixtures to within  $\pm 0.7\%$  while maintaining the agreement in pure Kr to within  $\pm 1\%$  [Ref. 56 and the Electron Physics Group, Research School of Physical Sciences, Australian National University, Canberra, Quarterly Report No. 109 (unpublished)].