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Theoretical study of a two-photon double-beam laser

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A nondegenerate two-photon laser in a cascade three-level atomic system is studied. The difference between the laser and that derived from an effective Hamiltonian is discussed. Photon number distributions about field 1 are presented. The effects of field 2 on field 1 are studied in detail. Field 2 can play a role of gain enhancement or gain reduction for field 1. This is due to two actions of field 2: reducing the population of the intermediate level and changing the detuning really acting upon field 1, which is caused by the ac Stark shift.

The problem of interaction between light and three-level atoms has been extensively studied for two decades. In 1981, the first two-photon experiment was reported,[1] which was followed by Gao's in 1984.[2] Most of the theoretical studies made use of an effective interaction Hamiltonian of the type $a_1 a_2 \sigma^+ + \text{H.c.}$, where a_1 is the field annihilation operator and σ^+ is the two-level rising operator.[3] Here, we derive the master equation for a nondegenerate two-photon laser directly from the exact Hamiltonian by using the Scully-Lamb[4–7] method, and study the operation of some of the properties. The difference between our results and previous results is pointed out. A discussion is given about the influence of the ac Stark effect on the laser operation, which may lead to mutual attenuation rather than always support between each other.

I. HAMILTONIAN AND MASTER EQUATION

The Hamiltonian for the atom-field system is

$$H=\sum_{a,b,c}\omega_\alpha A_\alpha^\dagger A_\alpha+\sum_{j=1,2}\Omega_j(a_j^\dagger a_j+\tfrac{1}{2})+(g_1a_1A_a^\dagger A_b+g_2a_2A_b^\dagger A_c+\text{H.c.})\ . \tag{1}$$

For simplicity, the same decay constant γ for all three levels is assumed and only a pumping to the upper level $|a\rangle$ with pumping rate R is considered.

The initial state vector of the atom-field system is

$$|\psi_{Af}^I(t_0)\rangle=\sum_{n_1,n_2}F_{n_1,n_2}(t_0)\,|a,n_1,n_2\rangle\ . \tag{2}$$

At time t, it develops into[4,5]

$$|\psi_{Af}^I(t)\rangle=\sum_{n_1,n_2}[\,a_{n_1,n_2}(t)\,|a,n_1,n_2\rangle+b_{n_1+1,n_2}(t)\,|b,n_1+1,n_2\rangle+c_{n_1+1,n_2+1}(t)\,|c,n_1+1,n_2+1\rangle\,]\ , \tag{3}$$

$$\alpha_{n_1,n_2}(t)=\sum_{i=1}^{3}\alpha_{n_1,n_2}^{(i)}e^{-i\omega_i(t-t_0)}F_{n_1,n_2}(t_0)\quad(\alpha=a,b,c)\ , \tag{4}$$

$$a_{n_1,n_2}^{(i)}=\frac{(\mu_i+\delta_2)(\mu_{i+1}+\delta_1)(\mu_{i+2}+\delta_1)}{(\delta_2-\delta_1)(\mu_i-\mu_{i+1})(\mu_i-\mu_{i+2})}\ , \tag{5}$$

$$V_1 b_{n_1+1,n_2}^{(i)}=(\mu_i+\delta_1)a_{n_1,n_2}^{(i)}\ , \tag{6}$$

$$(\mu_i+\delta_2)c_{n_1+1,n_2+1}^{(i)}=V_2^* b_{n_1+1,n_2}^{(i)}\ , \tag{7}$$

where $\omega_i=\mu_i\gamma$, $\delta_i=\Delta_j/\gamma$, $V_j=g_j\sqrt{n_j+1}/\gamma$, and μ_i $(i=1,2,3)$ are the three roots of a cubic equation. Adopting the Scully-Lamb method,[6,7] the master equation controlling the laser operation can be obtained,

$$\begin{aligned}\dot p(n_1,n_2)=&-A_1(n_1+1)F_1(n_1,n_2)p(n_1,n_2)-A_1(n_1+1)F_2(n_1,n_2)p(n_1,n_2)\\&+A_1n_1F_1(n_1-1,n_2)p(n_1,n_2)+A_1n_1F_2(n_1-1,n_2-1)p(n_1-1,n_2-1)\\&+C_1(n_1+1)p(n_1+1,n_2)+C_2(n_2+1)p(n_1,n_2+1)-C_1n_1p(n_1,n_2)-C_2n_2p(n_1,n_2)\ ,\end{aligned} \tag{8}$$

$$\begin{aligned}F_1(n_1,n_2)=&\frac{(\mu_1+\delta_2)(\mu_2+\delta_2)}{(\mu_2-\mu_3)(\mu_1-\mu_3)[(\mu_1-\mu_2)^2+1]}+\frac{(\mu_1+\delta_2)(\mu_3+\delta_2)}{(\mu_1-\mu_2)(\mu_3-\mu_2)[(\mu_1-\mu_3)^2+1]}\\&+\frac{(\mu_2+\delta_2)(\mu_3+\delta_2)}{(\mu_2-\mu_1)(\mu_3-\mu_1)[(\mu_2-\mu_3)^2+1]}\ ,\end{aligned} \tag{9}$$

where C_1 (C_2) stands for the cavity loss for field 1 (2) and $A_1=2R\,|g_1/\gamma|^2$, and $F_2(n_1,n_2)$ is the same as $F_1(n_1,n_2)$ with the numerators replaced by $|V_2|^2$.

It is very clear from the master equation that there are two kinds of processes: the single-photon process, which is represented by the first and third terms in the right-hand side of Eq. (8), and the two-photon process, which is represented by the second and fourth terms. In the previous two-photon theory derived from the effective Hamiltonian,[3,8,7] the single-photon process was absent. For certain n_1 and n_2, which satisfy the following conditions, at two-photon resonance, $\delta=\delta_1=-\delta_2$, the first and third terms in Eq. (8) can be discarded compared with the second and fourth terms:

$$\delta^2 \gg |g_1/\gamma|^2(n_1+1) \approx |g_1/\gamma|^2(n_2+1) \gg 1 \ . \tag{10}$$

The master equation approximately becomes[8]

$$\begin{aligned}\dot{p}(n_1,n_2)=&-\frac{A'(n_1+1)(n_2+1)}{1+(B'/A')(n_1+1)(n_2+1)}p(n_1,n_2)+\frac{A'n_1n_2}{1+(B'/A')n_1n_2}p(n_1-1,n_2-1)\\&+C_1(n_1+1)p(n_1+1,n_2)+C_2(n_2+1)p(n_1,n_2+1)-C_1n_1p(n_1,n_2)-C_2n_2p(n_1,n_2) \ ,\end{aligned} \tag{11}$$

where $A'=2R|g_1g_2/\gamma^2\delta|^2$ and $B'/A'=4|g_1g_2/\gamma^2\delta|^2$. This equation is consistent with the previous result derived from the effective Hamiltonian,[8] but we must notice that it is valid only in the limited regions of n_1 and n_2 indicated by Eq. (10).

If we neglect correlations, the equations for the average photon numbers,

$$\langle n_i\rangle = \sum_{n_1,n_2} n_i p(n_1,n_2) \ ,$$

can be deduced,

$$\begin{aligned}\langle \dot{n}_1\rangle =& A_1(\langle n_1\rangle+1)F_1(\langle n_1\rangle,\langle n_2\rangle)\\&+A_1(\langle n_1\rangle+1)F_2(\langle n_1\rangle,\langle n_2\rangle)-C_1(\langle n_1\rangle) \ ,\end{aligned} \tag{12}$$

$$\langle \dot{n}_2\rangle = A_1(\langle n_1\rangle+1)F_2(\langle n_1\rangle,\langle n_2\rangle)-C_2(\langle n_2\rangle) \ . \tag{13}$$

If the intensities of the two fields are not very high so that the conditions $1+\delta_i^2 \gg |g_i/\gamma|^2\langle n_i\rangle$ are still valid, $F_1(\langle n_1\rangle,\langle n_2\rangle)$ can be expanded into the Taylor series and only the first and second nonzero terms are kept. Then we have

$$\langle \dot{n}_1\rangle = \left[\left[\frac{A_1}{1+\delta^2}-C_1\right]+\beta_1\langle n_1\rangle+\theta_{12}\langle n_2\rangle\right]\langle n_1\rangle, \tag{14}$$

$$\begin{aligned}\langle \dot{n}_2\rangle =& [(\beta_2\langle n_1\rangle - C_2)+\theta_{21}\langle n_1\rangle^2\\&+\theta_{22}\langle n_1\rangle\langle n_2\rangle]\langle n_2\rangle \ ,\end{aligned} \tag{15}$$

where the relations $F_1(-1,-1)=(1+\delta^2)^{-1}$, $F_2(-1,-1)=0$, and $\langle n_i\rangle \gg 1$ have been used.

The coupling constant θ_{12} in Eq. (14) may be positive or negative. Therefore the influence of field 2 on field 1 may be either to enhance or reduce field 1. It seems a little strange that field 2 may result in reduction on field 1. An increase of field 2 will lead to a more rapid decay of the intermediate level's population, which makes field 1 become stronger. On the other hand, the ac Stark effect of the intermediate level caused by the second field has to be considered. This effect makes the level shift up or down, and then the actual energy difference between the upper and intermediate levels seen by field 1 may become larger or smaller, and the detuning for field 1 may be widened or narrowed. Large detuning brings about small gain. If the detuning is widened, field 2 has two opposite actions: (1) depleting the population of the intermediate level (gain enhancement for field 1), (2) widening the detuning which field 1 really "sees" (gain reduction for field 1). The overall effect of field 2 on field 1 is the result of the competition between the two actions. For some situations the field 2 causes a gain reduction for field 1, where θ_{12} is negative. This phenomenon will be seen later in the photon statistics. This effect of gain reduction of field 1 caused by field 2 was not considered previously.

If Eq. (10) is met, we have $F_1 \ll F_2$. Consequently, Eq. (12) reduces to

$$\langle \dot{n}_1\rangle = A_1(\langle n_1\rangle+1)F_2(\langle n_1\rangle,\langle n_2\rangle)-C_1\langle n_1\rangle \ ,$$

which is the same as Eq. (13). Thus we have $C_1\langle n_1\rangle = C_2\langle n_2\rangle$ in the steady state, the conclusion from the effective Hamiltonian method. Here we must mention that in order to meet Eq. (10), not only is the proper pumping required (to meet $\delta^2 \gg |g_1/\gamma|^2\langle n_1\rangle$, $|g_2/\gamma|^2\langle n_2\rangle \gg 1$) but also proper cavity losses (to meet $|g_1/\gamma|^2\langle n_1\rangle \approx |g_2/\gamma|^2\langle n_2\rangle$). Therefore the effective Hamiltonian is not fully equivalent to the exact one even under the perturbation approximation.

II. PHOTON STATISTICS

Now we consider the photon number distribution in the steady state. From Eq. (8), we obtain under decorrelation approximation

$$p(n_1+1)=\frac{A_1}{C_1}[F_1(n_1,\langle n_2\rangle)+F_2(n_1,\langle n_2\rangle)]p(n_1) \ , \tag{16}$$

$$p(n_2+1)=\frac{A_1}{C_2}F_2(\langle n_1\rangle,n_2)p(n_2) \ . \tag{17}$$

Then the photon statistics for the two fields are ready to be obtained,

$$p(n_1)=\left[\frac{A_1}{C_1}\right]^{n_1}\prod_{n=0}^{n_1-1}[F_1(n,\langle n_2\rangle_e)+f_2(n,\langle n_2\rangle_e)] \ , \tag{18}$$

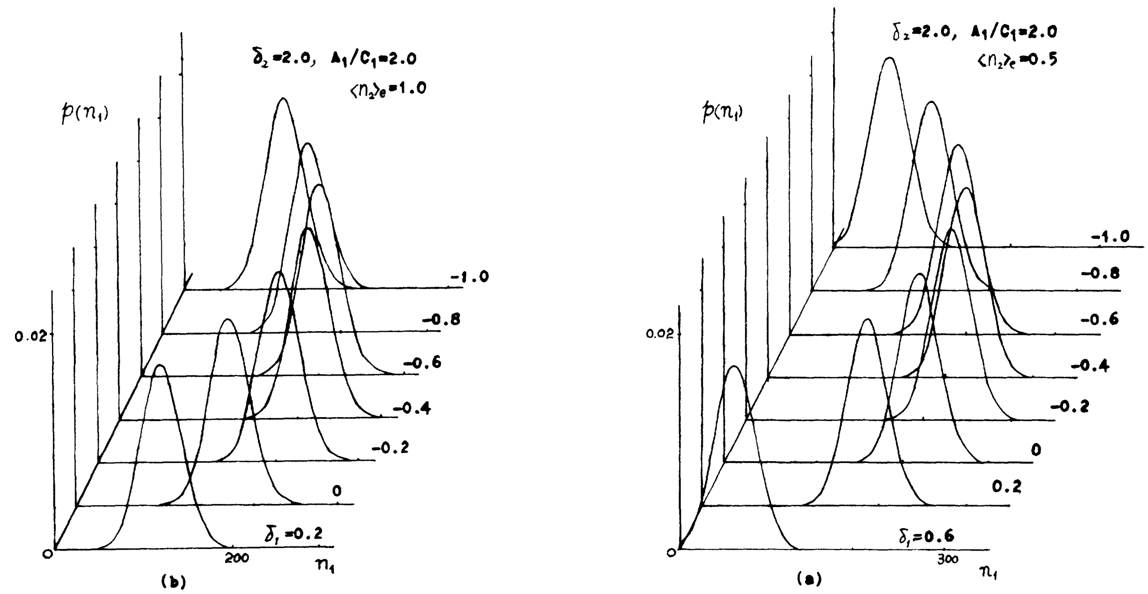

FIG. 1. Photon number distributions for field 1 with different δ_1; (a) $\langle n_2 \rangle_e = 0.5$, (b) $\langle n_2 \rangle_e = 1.0$.

$$p(n_2) = \left[\frac{A_1}{C_2} \right]^{n_2} \prod_{n=0}^{n_2-1} F_2(\langle n_1 \rangle_e, n) , \tag{19}$$

where $\langle n_i \rangle_e = B_i \langle n_i \rangle / 4A_i$.

The photon number distributions of field 1 for different δ_1 and $\langle n_2 \rangle_e$ are shown in Fig. 1. The peak position of the distribution reaches a maximum when δ_1 is about -0.2 for $\langle n_2 \rangle_e = 0.5$ [Fig. 1(a)] or -0.4 for $\langle n_2 \rangle_e = 1.0$ [Fig. 1(b)], but not equal to zero. This is due to the upwards ac Stark shift of the intermediate level caused by field 2. Because of field 2, the middle level splits into two sublevels. If $\delta_2 = 0$, the two sublevels have the same weight and the same distance from the original position. For $\delta_2 \neq 0$, the weights of the two sublevels are different, and if δ_2 is large enough the weight of one sublevel is considerably larger than the other, so that approximately only the former needs to be considered. When $\delta_2 > 0$ (or < 0), the intermediate level (the main sublevel) shifts up (or down) by an amount of

$$-(\delta_2/2) + [(\delta_2/2)^2 + \langle n_2 \rangle_e]^{1/2}$$

or

$$(\delta_2/2)[(\delta_2/2)^2 + \langle n_2 \rangle_e]^{1/2} .$$

Hence the actual energy difference seen by field 1 is less (or bigger) than ω_{ab}. When the actual energy difference is equal to Ω_1, the peak position of the photon number distribution is the maximum. For $\langle n_2 \rangle_e = 0.5$ and 1.0, the upwards shift of the intermediate level is 0.22 and 0.41, respectively, which is consistent with Fig. 1.

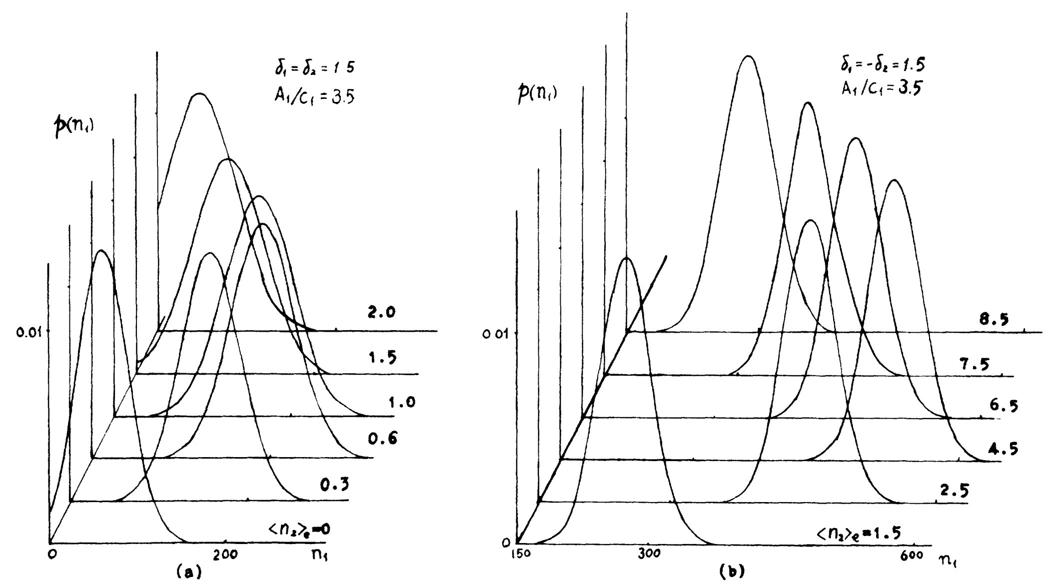

FIG. 2. Photon number distribution for field 1 with different $\langle n_2 \rangle_e$; (a) $\delta_1 = \delta_2 = 1.5$, (b) $\delta_1 = -\delta_2 = 1.5$.

As mentioned in Sec. I, field 2 has two actions on field 1: (1) reducing the population in the middle level, which leads to an increase of field 1, and (2) changing the detuning that field 1 really sees due to the ac Stark effect, which may lead to a decrease of field 1 if the detuning is widened. If δ_1 and δ_2 have the opposite signs, the actual detuning for field 1 is widened and the two actions have opposite influence on field 1. On the other hand, if they have the same sign, the actual detuning for field 1 is narrowed and the two actions have the same influence on field 1. The second action was not considered before.[9] Because of the second action, the effect of field 2 on field 1 is gain reduction rather than gain increase. The two actions of field 2 can obviously be seen from the curves of the photon number distribution of field 1 in Fig. 2. Figure 2(a) is for the case of $\delta_1=\delta_2$, where the two actions have the opposite influence, while Figs. 2(b) is for the case of $\delta_1=-\delta_2$, where the two actions have the same influence for small intensity of field 2. In Fig. 2(a), as $\langle n_2\rangle_e$ increases, the peak position of the distribution first increases a little because of the first action, and quickly reaches the maximum at $\langle n_2\rangle_e=0.6$, and then decreases because of the second action. Without the second action, the peak position would monotonically increase as $\langle n_2\rangle_e$ increases. In fact, in some cases, for example, $\delta_1=\delta_2=0$, the peak position monotonically goes to the origin when $\langle n_2\rangle_e$ increases. It has been noticed that in Fig. 2(a) the distribution at $\langle n_2\rangle_e=2.0$ is similar to that at $\langle n_2\rangle_e=0$, which indicates that the two actions of field 2 on field 1 cancel each other. In Fig. 2(b), the peak position increases until $\langle n_2\rangle_e=4.5$ because of the same influence of the two actions. At $\langle n_2\rangle_e=4.5$, the upwards shift of the middle level is

$$-(\delta_2/2)+[(\delta_2/2)^2+\langle n_2\rangle_e]^{1/2}=1.5\ ,$$

which just makes the actual detuning of field 1 be zero, so that the peak position reaches the maximum. For $\langle n_2\rangle>4.5$, the actual detuning for field 1 increases, so that the peak position goes towards the origin, as shown in Fig. 2(b).

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