Broadband squeezing via degenerate parametric amplification

David D. Crouch
Theoretical Astrophysics 130-33, California Institute of Technology, Pasadena, California 91125
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We show that parametric amplification is capable of generating squeezed light over a wide band if materials with large $\chi^{(2)}$ nonlinearities can be found, and that the squeezing bandwidth can be enhanced considerably by phase matching away from degeneracy. We compare our results with similar results recently found for four-wave mixing in an optical fiber.

Squeezed light has been generated recently using degenerate parametric amplification in both oscillator[1] and traveling-wave[2] configurations. In the experiment of Slusher *et al.*,[2] pulsed squeezed light was generated in a traveling-wave degenerate parametric amplifier (DPA), using a pulsed pump to increase the effective nonlinearity. If materials with larger $\chi^{(2)}$ nonlinearities can be found, one could generate squeezed light over a wide band using a continuous pump. Here we present a first-order analysis of a DPA with a cw (monochromatic) pump; we ignore losses and pump quantum fluctuations, which have been studied previously,[3] but we include dispersion.

The spatial differential equation describing the DPA is given by[3]

$$\frac{da_s(\Omega+\epsilon,z)}{dz}=-g_0\,e^{i[\Delta k(\epsilon)z+2\phi]}a_s^\dagger(\Omega-\epsilon,z)\ , \tag{1}$$

where the parametric gain g_0 is given by

$$g_0=\frac{2\pi\chi^{(2)}\Omega}{n_0^{3/2}}\left[\frac{8\pi P_p}{c^3\sigma}\right]^{1/2}, \tag{2}$$

and the phase mismatch $\Delta k(\epsilon)$ by

$$\Delta k(\epsilon)\equiv K_p(2\Omega)-k(\Omega+\epsilon)-k(\Omega-\epsilon)$$
$$=\frac{1}{c}[2\Omega n(2\Omega)-(\Omega+\epsilon)n(\Omega+\epsilon)-(\Omega-\epsilon)n(\Omega-\epsilon)]. \tag{3}$$

Here $\phi_p=2\phi$ is the pump phase, P_p the pump power, $\Omega_p=2\Omega$ the pump frequency, $n(\omega)$ the index of refraction, $\chi^{(2)}$ the nonlinear susceptibility (assumed nondispersive over the frequencies of interest), and σ an effective cross-sectional area used to account crudely for the transverse structure of the waves. The operators $a_s(\omega,z)$ are Fourier components of the magnetic field operator

$$B_s^{(+)}(z,t)=\int_{\beta_s}\frac{d\omega}{2\pi}\left[\frac{c}{n(\omega)v_g(\omega)}\right]^{1/2}\left[\frac{2\pi n(\omega)\hbar\omega}{c\sigma}\right]^{1/2}\times a_s(\omega,z)\,e^{-i\omega[t-n(\omega)z/c]}\ . \tag{4}$$

Here β_s is the signal bandwidth, and $v_g(\omega)$ the group velocity.

We introduce a set of quadrature-phase amplitudes,[3–5] defined by

$$\bar{a}_1(\epsilon,z)=\tfrac{1}{2}[e^{-i[\Delta k(\epsilon)z/2+\phi]}a_s(\Omega+\epsilon,z)+e^{i[\Delta k(\epsilon)z/2+\phi]}a_s^\dagger(\Omega-\epsilon,z)]\ , \tag{5a}$$

$$\bar{a}_2(\epsilon,z)=-\frac{i}{2}[e^{-i[\Delta k(\epsilon)z/2+\phi]}a_s(\Omega+\epsilon,z)-e^{i[\Delta k(\epsilon)z/2+\phi]}a_s^\dagger(\Omega-\epsilon,z)]\ . \tag{5b}$$

The quadrature-phase amplitudes $\bar{a}_1$ and $\bar{a}_2$ contain the spectral information about the squeezing produced by the DPA. If $z=L$, $\bar{a}_1(\epsilon,L)$ and $\bar{a}_2(\epsilon,L)$ can be detected by a balanced homodyne detector by changing the phase of the local oscillator as a function of rf frequency ϵ.

By combining Eqs. (1), (5a), and (5b) we can write the equation of motion for the DPA in terms of the barred quadrature-phase amplitudes,

$$\frac{d\bar{a}_1(\epsilon,z)}{dz}=-g_0\,\bar{a}_1(\epsilon,z)+\tfrac{1}{2}\Delta k(\epsilon)\,\bar{a}_2(\epsilon,z)\ , \tag{6a}$$

$$\frac{d\bar{a}_2(\epsilon,z)}{dz}=g_0\,\bar{a}_2(\epsilon,z)-\tfrac{1}{2}\Delta k(\epsilon)\bar{a}_1(\epsilon,z)\ . \tag{6b}$$

At phase-matched frequencies, where $\Delta k(\epsilon)=0$, the $\bar{a}_1$ quadrature is deamplified (squeezed) and the $\bar{a}_2$ quadrature is amplified. At other frequencies, where $\Delta k(\epsilon)\neq 0$, the phase mismatch degrades the squeezing by mixing part of the amplified quadrature with the squeezed quadrature. The solutions to Eqs. (6a) and (6b) are given by

$$\bar{a}_1(\epsilon,z)=\mathrm{Re}\,[\mu(\epsilon,z)+\nu(\epsilon,z)]\bar{a}_1(\epsilon,0)-\mathrm{Im}\,[\mu(\epsilon,z)-\nu(\epsilon,z)]\bar{a}_2(\epsilon,0)\ , \tag{7a}$$

$$\bar{a}_2(\epsilon,z)=\mathrm{Im}\,[\mu(\epsilon,z)+\nu(\epsilon,z)]\bar{a}_1(\epsilon,0)+\mathrm{Re}\,[\mu(\epsilon,z)-\nu(\epsilon,z)]\bar{a}_2(\epsilon,0)\ , \tag{7b}$$

where

$$\mu(\epsilon,z)=\cosh g(\epsilon)z-i\frac{\Delta k(\epsilon)}{2g(\epsilon)}\sinh g(\epsilon)z\ , \tag{8a}$$

$$\nu(\epsilon,z)=-\frac{g_0}{g(\epsilon)}\sinh g(\epsilon)z \tag{8b}$$

when $g_0^2>[\Delta k(\epsilon)/2]^2$, and

$$\mu(\epsilon,z)=\cos K(\epsilon)z-i\frac{\Delta k(\epsilon)}{2K(\epsilon)}\sin K(\epsilon)z\ , \tag{9a}$$

$$\nu(\epsilon,z)=-\frac{g_0}{K(\epsilon)}\sin K(\epsilon)z \tag{9b}$$

when $g_0^2<[\Delta k(\epsilon)/2]^2$. Here

$$g(\epsilon)=iK(\epsilon)=\{g_0^2-[\Delta k(\epsilon)/2]^2\}^{1/2}\ . \tag{10}$$

Spectral information about the squeezing produced by the DPA is contained in the spectral-density matrix[3–5]

$\bar{S}_{mn}(\epsilon)$ of the output quadrature-phase amplitudes $\bar{a}_1(\epsilon,L)$ and $\bar{a}_2(\epsilon,L)$;

$$\langle \Delta\bar{a}_m^\dagger(\epsilon',L)\Delta\bar{a}_n(\epsilon,L)\rangle_{\rm sym}=\pi\bar{S}_{mn}(\epsilon)\delta(\epsilon-\epsilon') , \quad m,n=1,2 . \tag{11}$$

Here, for any operator θ, $\Delta\theta=\theta-\langle\theta\rangle$, and sym denotes a symmetrized product. Using the continuum commutation relation

$$[a(\omega,z),a^\dagger(\omega',z)]=2\pi\delta(\omega-\omega') , \tag{12}$$

one can show that for a vacuum input

$$\langle \Delta\bar{a}_m^\dagger(\epsilon',0)\Delta\bar{a}_n(\epsilon,0)\rangle_{\rm sym}=\frac{\pi}{2}\delta_{mn}\delta(\epsilon-\epsilon') . \tag{13}$$

Although the barred spectral-density matrix $\bar{S}_{mn}(\epsilon)$ contains all the spectral information about squeezing, it does not give directly the maximum and minimum spectra at each ϵ. This is obtained by diagonalizing the spectral-density matrix by applying a suitable frequency- and position-dependent rotation $\theta(\epsilon,L)$ to the output quadrature-phase amplitudes $\bar{a}_1(\epsilon,L)$ and $\bar{a}_2(\epsilon,L)$. We define $\tilde{a}_1(\epsilon,L)$ and $\tilde{a}_2(\epsilon,L)$ by

$$\tilde{a}_1(\epsilon,L)=\bar{a}_1(\epsilon,L)\cos\theta+\bar{a}_2(\epsilon,L)\sin\theta , \tag{14a}$$

$$\tilde{a}_2(\epsilon,L)=-\bar{a}_1(\epsilon,L)\sin\theta+\bar{a}_2(\epsilon,L)\cos\theta , \tag{14b}$$

and the new spectral-density matrix $\tilde{S}_{mn}(\epsilon)$ by

$$\langle \Delta\tilde{a}_m^\dagger(\epsilon',L)\Delta\tilde{a}_n(\epsilon,L)\rangle_{\rm sym}=\pi\tilde{S}_{mn}(\epsilon)\delta(\epsilon-\epsilon') . \tag{15}$$

Using Eqs. (7a) and (7b), we find that the elements of the rotated spectral-density matrix $\tilde{S}_{mn}(\epsilon)$ are given by

$$\tilde{S}_{11}(\epsilon)=\tfrac{1}{2}\{\,|\mu(\epsilon,L)|^2+2\,{\rm Re}[\mu(\epsilon,L)\nu(\epsilon,L)e^{-2i\theta}] + |\nu(\epsilon,L)|^2\} , \tag{16a}$$

$$\tilde{S}_{12}(\epsilon)=\tilde{S}_{21}(\epsilon)={\rm Im}[\mu(\epsilon,L)\nu(\epsilon,L)e^{-2i\theta}] , \tag{16b}$$

$$\tilde{S}_{22}(\epsilon)=\tfrac{1}{2}\{\,|\mu(\epsilon,L)|^2-2\,{\rm Re}[\mu(\epsilon,L)\nu(\epsilon,L)e^{-2i\theta}] + |\nu(\epsilon,L)|^2\} . \tag{16c}$$

We diagonalize the spectral-density matrix by choosing θ to satisfy

$$\mu(\epsilon,L)\nu(\epsilon,L)e^{-2i\theta}=-|\mu(\epsilon,L)|\,|\nu(\epsilon,L)| . \tag{17}$$

The matrix element $\tilde{S}_{11}(\epsilon)$ gives the spectrum of the differenced photocurrent from a balanced homodyne detector when the phase of the local oscillator is chosen to yield the maximum squeezing at rf frequency ϵ. The elements of the spectral-density matrix resulting from this choice are

$$\tilde{S}_{11}(\epsilon)=\tfrac{1}{2}[\,|\mu(\epsilon,L)|-|\nu(\epsilon,L)|\,]^2 , \tag{18a}$$

$$\tilde{S}_{22}(\epsilon)=\tfrac{1}{2}[\,|\mu(\epsilon,L)|+|\nu(\epsilon,L)|\,]^2 . \tag{18b}$$

Equations (17), (18a), and (18b) are formally equivalent to results obtained recently by Potasek and Yurke for four-wave mixing in an optical fiber.[6]

The frequency dependence of the phase mismatch $\Delta k(\epsilon)$—and thus the index of refraction—must be specified before we can study the squeezing spectrum. Since the index of refraction varies only a small amount over the phase-matched bandwidth, we can expand it in a Taylor series about Ω,

$$n(\Omega\pm\epsilon)=n(\Omega)\pm n^{(1)}(\Omega)\epsilon+\tfrac{1}{2}n^{(2)}(\Omega)\epsilon^2+\cdots , \quad \epsilon\ll\Omega \tag{19}$$

where $n^{(j)}(\Omega)$ denotes the jth derivative of n evaluated at Ω. One normally assumes phase matching at degeneracy, i.e., $n(2\Omega)=n(\Omega)$. Here we will not make such an assumption, but will assume $n(2\Omega)=n(\Omega+\epsilon_0)$, where $\epsilon_0\ll\Omega$. Equation (3) then becomes

$$\Delta k(\epsilon)=\frac{1}{c}[2\Omega n(\Omega+\epsilon_0)-(\Omega+\epsilon)n(\Omega+\epsilon) -(\Omega-\epsilon)n(\Omega-\epsilon)] . \tag{20}$$

Substituting Eq. (19) into Eq. (20), we find to fourth order in ϵ/Ω,

$$\Delta k(\epsilon)=\frac{\Omega}{c}[\Delta-p(\epsilon/\Omega)^2-q(\epsilon/\Omega)^4] , \tag{21a}$$

where the dimensionless parameters Δ, p, and q are given by

$$\Delta=2[n(\Omega+\epsilon_0)-n(\Omega)] , \tag{21b}$$

$$p=2\Omega n^{(1)}(\Omega)+\Omega^2n^{(2)}(\Omega) , \tag{21c}$$

$$q=\tfrac{1}{12}[4\Omega^3n^{(3)}(\Omega)+\Omega^4n^{(4)}(\Omega)] . \tag{21d}$$

Phase matching occurs at frequency $\Omega+\epsilon_m$ where $\Delta k(\epsilon_m)=0$; setting Eq. (21a) equal to zero and solving for ϵ_m, we find

$$\epsilon_m=2\pi f_m=\Omega\left\{-\frac{p}{2q}\left[1\pm\left[1+\frac{4\Delta q}{p^2}\right]^{1/2}\right]\right\}^{1/2} . \tag{22}$$

We wish to investigate the case where $\Delta=0$ and $p=0$ simultaneously, so we must find a frequency Ω_0 such that $n(2\Omega_0)=n(\Omega_0)$ and $p(\Omega_0)=0$; then $\Delta k(\epsilon)\propto(\epsilon/\Omega)^4$ varies from zero only slowly as long as $\epsilon\ll\Omega$. For example, using a modified Sellmeier equation for the ordinary refractive index in lithium niobate[7] and assuming that phase matching is possible at any frequency, we find that $p=0$ at $\lambda_0=2\pi c/\Omega_0\simeq1.9025\,\mu$m; we also find that $p>0$ for $\lambda<\lambda_0$, $p<0$ for $\lambda>\lambda_0$, and $q<0$ for all wavelengths in the neighborhood of λ_0. In the following we will assume $g_0=1.0\,{\rm m}^{-1}$, and $L=1.0$ m. Figure 1 is a plot of the squeezed spectral density S (where $S=2\tilde{S}_{11}$, so that $S=1$ is the vacuum level) as a function of $f=\epsilon/2\pi$ for relatively large values of p. The solid line is for $\lambda=1.935\,\mu$m, where $p=-3.208\times10^{-3}$ and $q=-4.883\times10^{-2}$. The short-dashed line is for $\lambda=1.875\,\mu$m, where $p=2.76\times10^{-3}$ and $q=-4.545\times10^{-2}$. The bandwidth over which squeezing occurs at these frequencies can be improved by taking Δ to be nonzero; the result is a nonzero phase-matching frequency, as is seen from Eq. (22). The medium-dashed line in Fig. 1 shows the broadened squeezing band for $\lambda=1.935\,\mu$m obtained by taking $f_0=\epsilon_0/2\pi=-1$ GHz, resulting in $\Delta=-7.86\times10^{-7}$ and, from Eq. (22), a new phase-matching frequency $f_m=2.42$ THz. The long-dashed line in Fig. 1 shows the broadened squeezing band for $\lambda=1.875\,\mu$m obtained by taking $f_0=1$ GHz, resulting in $\Delta=7.38\times10^{-7}$ and a new phase-matching frequency $f_m=2.62$ THz.

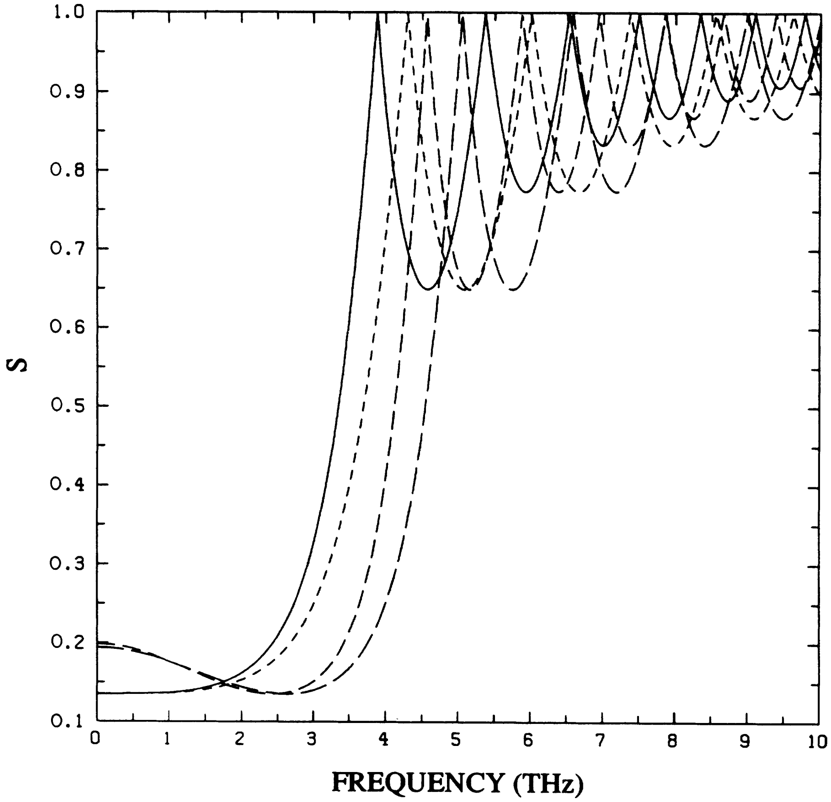

FIG. 1. Spectral-density S as a function of detuning f for degenerate parametric amplification in lithium niobate; $S=1$ is the vacuum level. Solid line: $\lambda=1.935\,\mu$m, $f_0=0$. Short-dashed line: $\lambda=1.875\,\mu$m, $f_0=0$. Medium-dashed line: $\lambda=1.935\,\mu$m, $f_0=-1$ GHz. Long-dashed line: $\lambda=1.875\,\mu$m, $f_0=1$ GHz.

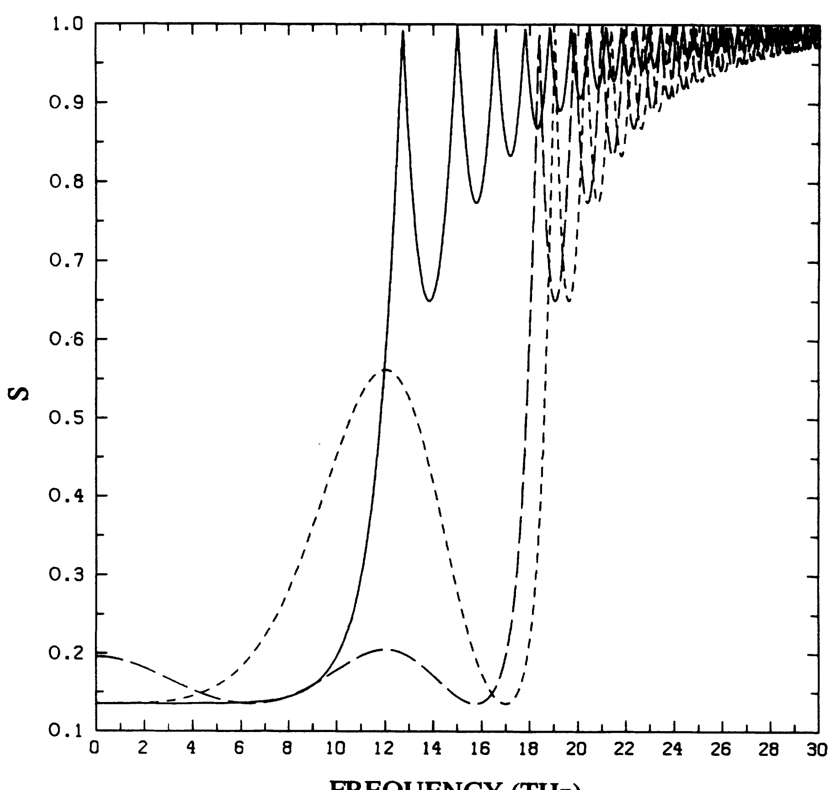

FIG. 2. Broadened squeezing spectra near $\lambda_0=1.9025\,\mu$m, where the dispersion parameter $p=0$. Solid line: $\lambda=\lambda_0$, $f_0=0$. Short-dashed line: $\lambda=1.8971\,\mu$m, $f_0=0$. Long-dashed line: $\lambda=1.8971\,\mu$m, $f_0=1$ GHz.

Squeezing over much greater bandwidths is obtained near λ_0, where p is small. The solid line in Fig. 2 shows S as a function of f for $\lambda=\lambda_0$, where $p=0$, $q=-4.7\times10^{-2}$, and $\Delta=0$. In practice, p can be small but not identically zero. The short-dashed line in Fig. 2 is for $\lambda=1.8971\,\mu$m, where $p=5.39\times10^{-4}$, $q=-4.67\times10^{-2}$, and $\Delta=0$. Because p and q are of opposite sign, Eq. (22) yields *two* phase-matching frequencies: one at $f_m=0$ and another at $f_m=16.99$ THz. The long-dashed line in Fig. 2 is for $\lambda=1.8971\,\mu$m and $f_0=1$ GHz, where $\Delta=7.55\times10^{-7}$. Again we have phase matching at two frequencies, $f_m=6.39$ THz and $f_m=15.75$ THz, resulting in squeezing of roughly 80% or better over a bandwidth of 17 THz.

Similar results are obtained for four-wave mixing in an optical fiber.[6,8] The equation of motion (found using the method of Ref. 3) is

$$\frac{db_s(\Omega+\epsilon,z)}{dz}=2i\gamma b_s(\Omega+\epsilon,z)+i\gamma e^{2i(\gamma z+\theta)}e^{i\Delta k_s(\epsilon)z}b_s^\dagger(\Omega-\epsilon,z)\;, \tag{23}$$

where

$$\gamma=\frac{48\pi^2\Omega\chi^{(3)}}{n^2(\Omega)c^2\sigma}P_p\;, \tag{24}$$

$$\Delta k_s(\epsilon)\equiv 2K_p(\Omega)-k(\Omega+\epsilon)-k(\Omega-\epsilon)=\frac{1}{c}[2\Omega n(\Omega)-(\Omega+\epsilon)n(\Omega+\epsilon)-(\Omega-\epsilon)n(\Omega-\epsilon)]\;. \tag{25}$$

Here $\theta_p=\theta$ is the pump phase, P_p the pump power, and $\Omega_p=\Omega$ the pump frequency. Introducing $b_s(\Omega+\epsilon,z)=c_s(\Omega+\epsilon,z)\exp(2i\gamma z)$, we find

$$\frac{dc_s(\Omega+\epsilon,z)}{dz}=-\gamma e^{i[\Delta k_{\text{eff}}(\epsilon)z+2\phi_s]}c_s^\dagger(\Omega-\epsilon,z)\;, \tag{26}$$

where $\phi_s=\theta-\pi/4$ and

$$\Delta k_{\text{eff}}(\epsilon)=\Delta k_s(\epsilon)-2\gamma\;. \tag{27}$$

Equation (26) has the same form as the equation for the DPA, Eq. (1); the solution of Eq. (26) follows from Eqs. (7), (8), and (9) if $\Delta k_{\text{eff}}(\epsilon)$ is substituted for $\Delta k(\epsilon)$, γ for g_0, and ϕ_s for ϕ. The solution so obtained is different from that of the nonlinear Schrödinger equation only in the absence of odd-order dispersion terms that have been shown by Potasek and Yurke[6] to have no effect on the squeezing. Using Eq. (20), we expand $\Delta k_{\text{eff}}(\epsilon)$ in a Taylor series:

$$\Delta k_{\text{eff}}(\epsilon)=\frac{\Omega}{c}[\Delta_s-p_s(\epsilon/\Omega)^2-q_s(\epsilon/\Omega)^4]\;, \tag{28a}$$

where

$$\Delta_s=-\frac{2\gamma c}{\Omega}\;. \tag{28b}$$

Here p_s and q_s are given by Eqs. (21c) and (21d), respectively. Equations (26) and (28a) are the four-wave-mixing analogs of Eqs. (1) and (21a) for parametric amplification; in this sense, four-wave mixing and parametric amplification are equivalent processes for generating squeezed light.

Maximum squeezing occurs not when $\Delta k_s(\epsilon)=0$, but when $\Delta k_{\text{eff}}(\epsilon)=0$; we find the "phase-matching" frequencies f_s by substituting p_s for p, q_s for q, and Δ_s for Δ in Eq. (22). If q_s and Δ_s are both negative—as they tend to be for optical fibers—then only one real solution f_s will exist regardless of the sign of p_s. If Δ_s could be made positive—as is possible with the DPA—then for $p_s>0$ one would obtain two real solutions f_{s1} and f_{s2}. For $p_s<0$ there are no real solutions when $\Delta_s>0$. It is the possibility of obtaining two phase-matching frequencies that distinguishes parametric amplification from four-wave mixing in an optical fiber. As we saw in Fig. 2, two phase-matching frequencies result in squeezing over a very wide band. This is not possible with four-wave mixing.

In our example, we have assumed that one could phase match lithium niobate at or near $\lambda_0=1.9025\,\mu$m, a wavelength somewhat into the infrared. We are not proposing lithium niobate as a candidate material for the generation of broadband squeezed light, but use it merely as an illustrative example. Whether or not suitable materials can be found is a problem we have not addressed; the point we wish to make is that *if* one can find a nonlinear material in which it is possible to phase match at a frequency Ω at which $p(\Omega)\sim0$, one can then obtain squeezing over a large bandwidth.

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