Plasma heating by two laser fields

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The heating of a plasma by electrons in the simultaneous presence of weak and strong laser fields by the inverse bremsstrahlung process is considered from a quantum-mechanical viewpoint. A kinetic equation is derived and the change in kinetic energy of the plasma electrons is calculated. Results show that the joint action of the two laser beams gives a nonvanishing but large heating rate in the regime of high intensity of the strong field. A physical explanation for this behavior is provided.

I. INTRODUCTION

The development of powerful and tunable lasers has caused a rapid increase of multiphoton experiments in atoms,[1–5] solids,[6–9] and plasmas.[10–12] In particular, when plasma is mentioned, nuclear fusion immediately comes to mind. To induce nuclear fusion reactions effectively by overcoming the repulsive forces between atomic nuclei, those nuclei have to collide vigorously with each other, which only happens at very high temperatures. The heating of a plasma is thus an essential problem in realization of the controlled release of nuclear fusion energy which can be attained, for instance, by a strong laser field. In the process of interaction of laser field with a plasma it may happen that the plasma gains energy from the external field and may be heated to the desired thermonuclear temperature. One of the chief mechanisms for rapid absorption of the laser radiation by plasma is the multiphoton inverse-bremsstrahlung (MIB) mechanism.[11] In this process an electron absorbs energy from the field during a collision with the nucleus. The heating of a plasma by inverse bremsstrahlung in the presence of an intense laser field has been discussed from the quantum-mechanical viewpoint by Seely and Harris,[11] who have calculated an effective collision frequency for electrons and nuclei in the intense field regime of the electromagnetic wave. It was shown[11] that the rate of energy absorption by the MIB is proportional to $I^{-1/2}$ (I is the intensity of the laser beam), clearly showing that it vanishes[13] in the limit of ultrahigh intensities ($I \rightarrow \infty$). Since in order to reach the required nuclear fusion temperature the plasma electrons should be given a finite and large rate of energy absorption by the external field, the preceding heating mechanism[11] is not effective in the regime of high intensities.

Here in this paper we propose the heating of a plasma via multiphoton inverse bremsstrahlung,[11] taking into account the additional presence of a weak laser field. Results show that the joint action of the two laser beams gives a nonvanishing and rather large heating rate in the regime of high intensity of the strong laser, showing that the plasma heating by two laser fields due to the inverse bremsstrahlung process may be one of the most efficient mechanisms for the heating of a plasma by radiation.

The approach we deal with in this paper follows closely that of Ref. 11. The plasma is assumed to be infinite and homogeneous. The laser beams are treated as classical plane electromagnetic waves in the dipole approximation (spatial dependence of the wave neglected) and the electrons treated quantum mechanically. The former is valid provided there is a large number of photons in the same state. The inverse bremsstrahlung process is treated using first-order perturbation theory. The unperturbed electron states are taken to be the solutions of the Schrödinger equation for a plasma electron in the fields of the two electromagnetic waves. The field of the nucleus is treated as the perturbation. Transition probabilities are then calculated between the unperturbed electron states and a kinetic equation is derived for plasma electrons from which the rate of charge of kinetic energy of the electrons is calculated.

II. TRANSITION PROBABILITIES

We begin with a brief derivation of the transition probabilities. We assume two linearly polarized eletromagnetic waves propagating along the z direction. The total vector potential is (dipole approximation)

$$\mathbf{A}_T(t) = A_1 \mathbf{e}_x \cos(\omega_1 t) + A_2 \mathbf{e}_x \cos(\omega_2 t) . \tag{1}$$

The time-dependent Schrödinger equation

$$-\frac{\hbar}{i}\frac{\partial \psi(\mathbf{x},t)}{\partial t} = \frac{1}{2m}\left[\frac{\hbar}{i}\nabla - \frac{e}{c}\mathbf{A}_T(t)\right]^2 \tag{2}$$

has the solution (normalized in a box of unit volume)

$$\psi(\mathbf{x},t) = \exp\left[\frac{i}{\hbar}\mathbf{p}\cdot\mathbf{x} - \frac{i}{2m\hbar}\int_0^t \left|\mathbf{p} - \frac{e}{c}\mathbf{A}_T(t')\right|^2 dt'\right] . \tag{3}$$

Using first-order perturbation theory and Eq. (1), the transition-probability amplitude for a transition from a state 1 with momentum $\mathbf{p}_1$ to a state 2 with momentum $\mathbf{p}_2$ is found to be

$$a(1\to 2) = -\frac{i}{\hbar}\int\int_{-\tau/2}^{\tau/2} \psi_2^*(\mathbf{x},t)V(\mathbf{x})\psi_1(\mathbf{x},t)dt\, d^3x$$

$$= -\frac{i}{\hbar}V(\mathbf{q})\int_{-\tau/2}^{\tau/2} dt \exp\left[\frac{i}{\hbar}\Omega t\right]$$

$$\times \exp\left[-i\frac{\lambda_1}{\omega_1}\sin(\omega_1 t) - i\frac{\lambda_2}{\omega_2}\sin(\omega_2 t)\right],$$

where $V(\mathbf{x})$ is the potential of the perturbing nucleus, $V(\mathbf{q}\equiv(\mathbf{p}_2-\mathbf{p}_1)/\hbar)$ is the Fourier transform of $V(\mathbf{x})$, and

$$\Omega = (p_2^2 - p_1^2)/2m ,$$

$$\lambda_i = \frac{eE_i(\mathbf{p}_2-\mathbf{p}_1)\cdot\mathbf{e}_x}{m\omega_i}, \quad i=1,2 .$$

Here $(\mathbf{p}_2-\mathbf{p}_1)$ is the momentum transfer during the collision. We take $V(x)$ to be the Coulomb potential. Then

$$\left|V\left[\frac{\mathbf{p}_2-\mathbf{p}_1}{\hbar}\right]\right|^2 = \left[\frac{4\pi Ze^2\hbar^2}{|\mathbf{p}_2-\mathbf{p}_1|^2}\right]^2 . \tag{4}$$

In writing Eq. (4) we have as in Ref. 11 not included screening effects to take account of the background plasma.[14–16] Although the screening effect actually lowers the Coulomb interaction, one might accomplish a reduction of this screening effect, for instance, by illuminating the plasma with the two electromagnetic waves having a difference of frequency nearly equal to the plasma frequency.[17]

Proceeding further, the transition probability per unit time is

$$\frac{a(1\to 2)^2}{\tau} = \frac{2}{\hbar}|V(\mathbf{q})|^2$$

$$\times \sum_{\substack{n=-\infty \\ (n\neq 0)}}^{+\infty} \sum_{\substack{m=-\infty \\ (m\neq 0)}}^{+\infty} J_n^2\left[\frac{\lambda_1}{\hbar\omega_1}\right] J_m^2\left[\frac{\lambda_2}{\hbar\omega_2}\right]$$

$$\times \delta(\Omega - n\hbar\omega_1 - m\hbar\omega_2) , \tag{5}$$

where J_i is the Bessel function of order i ($i=n,m$). Positive values of $i=n,m$ correspond to the absorption of i photons and negative values to the emission of $|i|$ photons from the laser fields.

III. KINETIC EQUATION

From Eq. (4), we see that the transition probability per unit time for the transition from state 1 to state 2 with the absorption ($n,m>0$) or emission ($n,m<0$) of $|n|,|m|$ photons is

$$T(n,m,\mathbf{p}_1-\mathbf{p}_2) = \left[\frac{2\pi}{\hbar}\right]|V(\mathbf{q})|^2 J_n^2\left[\frac{\lambda_1}{\hbar\omega_1}\right] J_m^2\left[\frac{\lambda_2}{\hbar\omega_2}\right]$$

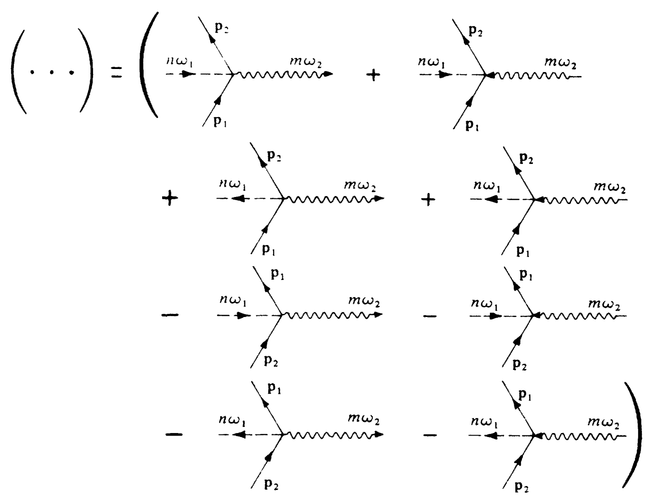
$$\times \delta(\Omega - n\hbar\omega_1 - m\hbar\omega_2) . \tag{6}$$

The δ function implies that the energy of the electron-photon system is conserved. The momentum of the electron-photon system is not conserved since the nucleus may carry off some momentum and a sum over momentum will appear in the kinetic equation.

The change in $N_e(\mathbf{p}_2)$, the number of electrons with momentum $\mathbf{p}_2$, may be written schematically as[15]

$$\frac{\partial N_e(\mathbf{p}_2)}{\partial t} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{\mathbf{p}_1}(\cdots) ,$$

(7)



In (7) the processes in which an electron with momentum $\mathbf{p}_2$ is destroyed are subtracted from the process in which an electron with momentum $\mathbf{p}_2$ is created. This difference gives the increase in $N_e(\mathbf{p}_2)$. The schematical equation may be converted to a mathematical equation by replacing the diagrams in Eq. (7) by the transition probability per unit time for the process given by Eq. (6). This mathematical equation is

$$\frac{\partial N_e(\mathbf{p}_2)}{\partial t} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{p_1}\{T(n\omega_1,-m\omega_2,\mathbf{p}_1{}_-\mathbf{p}_2)N_e(\mathbf{p}_1)[1-N_e(\mathbf{p}_2)]$$

$$+T(n\omega_1,m\omega_2,\mathbf{p}_1-\mathbf{p}_2)N_e(\mathbf{p}_1)[1-N_e(\mathbf{p}_2)]+T(-n\omega_1,-m\omega_2,\mathbf{p}_1-\mathbf{p}_2)N_e(\mathbf{p}_1)[1-N_e(\mathbf{p}_2)]$$

$$+T(-n\omega_1,m\omega_2,\mathbf{p}_1-\mathbf{p}_2)N_e(\mathbf{p}_1)[1-N_e(\mathbf{p}_2)]-T(n\omega_1,-m\omega_2,\mathbf{p}_2-\mathbf{p}_1)N_e(\mathbf{p}_2)[1-N_e(\mathbf{p}_1)]$$

$$-T(n\omega_1,m\omega_2,\mathbf{p}_2-\mathbf{p}_1)N_e(\mathbf{p}_2)[1-N_e(\mathbf{p}_1)]-T(-n\omega_1,m\omega_2,\mathbf{p}_2-\mathbf{p}_1)N_e(\mathbf{p}_2)[1-N_e(\mathbf{p}_1)]$$

$$-T(-n\omega_1,m\omega_2,\mathbf{p}_2-\mathbf{p}_1)N_e(\mathbf{p}_2)[1-(\mathbf{p}_1)]\} . \tag{8}$$

In Eq. (8), $[1-N_e(\mathbf{p})]$ is the square of the matrix element of the fermion creation operator and $N_e(\mathbf{p})$ is the square of the matrix element of the fermion destruction operator. These factors appear in the transition probabilities when the electrons are treated using the second quantized theory rather than the first quantized theory used in Sec. II. It is clear from (8) that

$$T(\mp n\omega_1, \mp m\omega_2, \mathbf{p}_1-\mathbf{p}_2)=T(\pm n\omega_1, \pm m\omega_2, \mathbf{p}_2-\mathbf{p}_1) \ .$$

We assume now that the electrons are far from degeneracy so that $N_e(\mathbf{p}) \ll 1$. Equation (8) then becomes

$$\frac{\partial N_e}{\partial t}(\mathbf{p}_2)=\sum_{\substack{n=-\infty \\ (n\neq 0)}}^{\infty}\sum_{\substack{m=-\infty \\ (m\neq 0)}}^{\infty}\sum_{\mathbf{p}_1} T(n,m,\mathbf{p}_1-\mathbf{p}_2) \times[N_e(\mathbf{p}_1)-N_e(\mathbf{p}_2)] \ . \quad (9)$$

We let the volume of the box in which the system is normalized become infinite so that

$$\sum_{\mathbf{p}_1} \to \int \frac{d\mathbf{p}_1}{(2\pi\hbar)^3} \ .$$

A Maxwellian distribution is assumed for the electrons. Using Eqs. (4) and (6), Eq. (9) becomes

$$\frac{\partial f_e(\mathbf{v}_2)}{\partial t}=\frac{4Z^2e^4N}{m}\left[\frac{m}{2\pi k_B T}\right]^{3/2}\int \frac{d^3v_1[\exp(-mv_1^2/2k_BT)-\exp(-mv_2^2/2k_BT)]}{|\mathbf{v}_2-\mathbf{v}_1|^4} \times \sum_{\substack{n=-\infty \\ (n\neq 0)}}^{\infty}\sum_{\substack{m=-\infty \\ (m\neq 0)}}^{\infty} J_n^2\left[\frac{\lambda_1}{\hbar\omega_1}\right] J_m^2\left[\frac{\lambda_2}{\hbar\omega_2}\right]\delta(\Omega-n\hbar\omega_1-m\hbar\omega_2) \ , \quad (10)$$

where $f_e(\mathbf{v})$ is the electron distribution function and N is the electron density. Equation (10) is the kinetic equation for electrons.

IV. RATE OF ENERGY ABSORPTION

The rate of change of the average kinetic energy of the plasma electrons should now be evaluated in the strong-field regime of laser 2 (intense) and compared with that found by Seely and Harris.[11] This is done using Eq. (10) for the kinetic equation of the electrons. The result for the rate of change $d\langle \epsilon \rangle /dt$ is then

$$\begin{aligned}\frac{d\langle \epsilon \rangle}{dt}&=\int d^3v_2\frac{mv_2^2}{2}\cdot\frac{\partial f(\mathbf{v}_2)}{\partial t}\\ &=\frac{4Z^2e^4N}{m}\left[\frac{m}{2\pi k_BT}\right]^{3/2}\int d^3v_2\frac{mv_2^2}{2}\int d^3v_1\frac{[\exp(-mv_1^2/2k_BT)-\exp(-mv_2^2/2k_BT)]}{|\mathbf{v}_2-\mathbf{v}_1|^4}\\ &\quad\times\sum_{\substack{n=-\infty \\ (n\neq 0)}}^{+\infty}\sum_{\substack{m=-\infty \\ (m\neq 0)}}^{+\infty} J_n^2\left[\frac{\lambda}{\hbar\omega_1}\right]J_m^2\left[\frac{\lambda}{\hbar\omega_2}\right]\delta(\Omega-n\hbar\omega_1-m\hbar\omega_2) \ . \end{aligned} \quad (11)$$

In the following we assume that laser 2 is the intense fields and laser 1 is the weak one. In the regime of intense field, $\lambda_2 \gg \hbar\omega_2$ and the argument of the Bessel function of order m of Eq. (11) is large. According to Seely and Harris,[11] for $\lambda_2 \gg \hbar\omega_2$ the sum over m in Eq. (11) may be written approximately as

$$\sum_{\substack{m=-\infty \\ (m\neq 0)}}^{+\infty} J_m^2(\lambda_2/\hbar\omega_2)\delta(\tilde{\Omega}-m\hbar\omega_2)\approx\tfrac{1}{2}[\delta(\tilde{\Omega}-\lambda_2)+\delta(\tilde{\Omega}+\lambda_2)] \ , \quad (12)$$

where $\tilde{\Omega}\equiv\Omega-n\hbar\omega_1$. In the weak-field case, laser 1, $\lambda_1 \ll \hbar\omega_1$. In this case, the Bessel function of order n can be approximated by

$$J_n^2(\lambda_1/\hbar\omega_1)\simeq[1/(n!)^2](\lambda_1/2\hbar\omega_1)^{2|n|} \ , \quad (13)$$

and consequently only the $n=1$ term should be retained; i.e., in the weak-field limit only single-photon processes are significant. Using Eqs. (12) and (13) in (11) and retaining only the $n=1$ term, we obtain

$$\begin{aligned}\frac{d\langle \epsilon \rangle}{dt}&=Z^2e^4N\left[\frac{m}{2\pi k_BT}\right]^{3/2}\int d^3v_2\, v_2^2\left[\frac{\lambda_1}{2\hbar\omega_1}\right]^2\exp(-mv_2^2/2k_BT)\\ &\quad\times\int\frac{d^3v_1}{|\mathbf{v}_2-\mathbf{v}_1|^4}\{(\exp[(\lambda_2+\hbar\omega_1)/k_BT]-1)\delta(\Omega-\hbar\omega_1-\lambda_2)\\ &\quad+(\exp[(-\lambda_2+\hbar\omega_1)/k_BT]-1)\delta(\Omega-\hbar\omega_1+\lambda_2)\} \ . \end{aligned} \quad (14)$$

The first term in the brackets of (14) corresponds to the absorption of m photons and the second to the emission of m photons from the strong laser field.

We now assume the amplitude of laser 2 is such that $\lambda_2 \gg k_B T$. Alternatively, assume the temperature to be low so that $\lambda_2 \gg k_B T$. Then Eq. (14) becomes

$$\frac{d\langle\epsilon\rangle}{dt} = Z^2 e^4 N \left[\frac{m}{2\pi k_B T}\right]^{3/2} \times \int d^3 v_2\, v_2^2 \left[\frac{eE_1(\mathbf{v}_2-\mathbf{v}_1)\cdot\mathbf{e}_x}{2\hbar\omega_1^2}\right]^2 \times \int \frac{d^3 v_1 \exp(-mv_1^2/2k_B T)}{|\mathbf{v}_2-\mathbf{v}_1|^4} \times \delta(\Omega - \hbar\omega_1 - \lambda_2) \; . \tag{15}$$

For low temperature, the contribution to the rate of energy absorption of processes in which the laser-2 photons are emitted is negligible compared to the contribution of processes in which the laser-2 photons are absorbed. From the δ function of Eq. (15), we have $v_2 \gg v_1$ for the intense field case. Then, $|\mathbf{v}_2-\mathbf{v}_1| \simeq v_2$ and $(\mathbf{v}_2-\mathbf{v}_1)\cdot\mathbf{e}_x \simeq \mathbf{v}_2\cdot\mathbf{e}_x$. Assuming for the sake of simplicity that the electrons are moving parallel to the x direction, which is actually the direction of maximum absorption of the m photons ($\mathbf{v}_2\cdot\mathbf{e}_x$ is maximum). Equation (15) finally becomes (apart from an angular integral which is convergent)

$$\frac{\partial\langle\epsilon\rangle}{\partial t} = \frac{Z^2\alpha^2 N}{2m}\left[\frac{eE_1 c}{\omega_1^2}\right]^2 \left[\frac{eE_2}{m\omega_2}\right] , \tag{16}$$

where α is the fine-structure constant and c is the speed of light.

V. CONCLUSION

Equation (16) is the expression for the absorption rate of energy from the two laser fields by the plasma electrons we now want to discuss. We notice that $d\langle\epsilon\rangle/dt$ is proportional to the intensity of the weak laser I_1 and proportional to $I_2^{1/2}$, I_2 being the intensity of the strong laser. This is a surprising feature since it shows firstly the direct influence of I_1 on the change of energy as expected, and secondly that the rate of energy absorption is finite and can be made rather larger in the regime of ultrahigh intensity for laser 2 (strong), contrary to the result of Ref. 11 in which $d\langle\epsilon\rangle/dt$ vanishes in the limit of I_2 very large. The presence of the weak field is noticed through the Bessel function which in the weak-field regime provides a factor which is proportional to the momentum transfer of the electrons squared. Physically, the effect of the weak field is to promote initially the electrons in the plasma to a situation of preheating since in this case the electrons speed is not altered significantly. This may be better understood as follows. In the presence of the intense field only,[11] it may happen that in the regime of ultrahigh intensities the electron has such a very large kinetic energy that it is no longer affected by the nucleus, i.e., the electron-nucleus interaction becomes "frozen." In other words, since $d\langle\epsilon\rangle/dt$ depends on the electron-nucleus interaction it must be vanishing small when this interaction is frozen out as it is the case in Ref. 11. The introduction of the extra laser (weak), promoting the electrons to a state of preheating, ensures that the electron-nucleus interaction will then regain strength, and therefore an enhancement of the plasma heating collisional absorption should be expected.

In closing, it has been proposed in this paper that plasma be heated to thermonuclear temperature by the rapid absorption of electromagnetic energy from two laser fields. We have shown that the joint action of the two laser beams results in a nonvanishing, but rather large heating rate in the regime of high intensity of the strong laser in contrast to the case where only one laser (strong) is present.[11] This shows that the plasma heating by two laser fields (weak and strong) due to the inverse bremsstrahlung process may be one of the most efficient mechanisms for the heating of a plasma by radiation.

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