Noise-reduced and anisotropy-enhanced Eden and screened-growth models

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(Received 8 June 1987)

The effects of noise reduction and anisotropy enhancement on the Eden and screened-growth models have been explored using computer simulations. In the case of the Eden model a transition from an almost circular asymptotic shape to a diamondlike shape is observed with increasing noise reduction or anisotropy enhancement. This shape transition is continuous and the asymptotic cluster shape seems to be reached at quite small cluster sizes. For the screened-growth model a similar transition from a more or less circular shape to a crosslike shape in the cluster envelope is observed. The simulations are consistent with but do not provide strong support for the idea that noise reduction accelerates the approach to asymptotic behavior without compromising the asymptotic geometric scaling properties of the clusters. As was found earlier for the diffusion-limited aggregation model, noise reduction causes the effective exponents describing the cluster geometry to oscillate with increasing cluster mass and this makes an unambiguous interpretation of the results impossible without a great deal of additional work. This oscillatory behavior is a consequence of the formation of an almost discrete hierarchy of sidebranches with large amounts of noise reduction.

INTRODUCTION

Simple computer models for nonequilibrium growth and aggregation processes have been of inteest for many years. Recently, this interest has been heightened by the realization that some of these models lead to the formation of fractal[1] structures and by the introduction of the Witten-Sander[2] model for diffusion-limited aggregation (DLA) which provides a basic description for a variety of physical phenomena.[3–7]

One of the main objectives of this work on growth models has been to determine the geometric scaling relationships (fractal dimensionalities) which characterize these structures in the asymptotic (large-size) limit. Another quantity which has been of considerable interest is the overall shape of the clusters, or aggregates generated by these models.

Initial prejudices based on the concept of universality (and supported to some extent by computer-simulation results) led to the idea that the geometric scaling relationships and the overall culster shapes should be insensitive to lattice details and other model details in lattice-based models. However, as early as 1973, Richardson[8] had found a transition from a more or less circular shape to a diamond shape in the square-lattice Eden[9] model as the probability p that each of the unoccupied surface sites will be filled at each step in the growth process is increased. More recent, much larger scale, simulations using Eden models with $p \rightarrow 0$ (i.e., only one site is filled at each step in the growth process) have shown that the overall shape of square-lattice Eden clusters deviates slightly from that of a circle in the asymptotic ($M \rightarrow \infty$, where M is the cluster mass) limit.[10–13] The formation of diamondlike shapes has also been found in other models that are more or less closely related to the Eden model.[14,15] Recently, Savit and Ziff[16] have shown that the shapes of the clusters grown using the algorithms of Alexandrowicz[17] and Leath[18] change from a random fractal (percolation cluster) for a growth probability p equal to the percolation threshold value p_c to a compact diamond shape for $p = 1.0$. For $0.705 \lesssim p \lesssim 1.0$, the clusters have smooth faces with rounded corners and, for $p \lesssim 0.705$, the cluster has a smooth curved shape but still displays the effects of lattice anisotropy. Savit and Ziff have also shown how the qualitative features of these shape changes can be understood in terms of a simple random-walk model. Early small-scale simulations using the DLA model[2,19] led to the belief that the overall shape and fractal dimensionality of clusters generated using this model were insensitive to the lattice structure. However, as clusters of larger[20–22] and larger[23,24] sizes were grown, it became apparent that both the overall shape and asymptotic scaling properties were sensitive to the lattice structure. This work stimulated and was in turn stimulated by the theoretical ideas of Turkevich and Scher[25] and Ball *et al.*[26] which indicated that the structures generated by the DLA process should be sensitive to lattice and other types of anisotropy.

It is now apparent that the overall shape of square-lattice DLA clusters evolves from a more or less circular shape via a diamond shape to a crosslike shape with increasing cluster size M. However, a precise quantitative description of the asymptotic structure still eludes us. A variety of methods has now been found to generate crosslike shapes in relatively small clusters using DLA-like models by either reducing the noise[27–31] or enhancing the anisotropy[26,32–34] in the growth process. The effects of lattice anisotropy can also be seen in deterministic models that are more or less closely related to DLA.[35–37]

The fact that noise reduction and/or anisotropy enhancement leads to clusters with an overall shape very similar to that exhibited by very much larger clusters grown without noise reduction suggests that we might also be able to learn about the asymptotic geometric scaling properties associated with models such as DLA by using relatively small clusters with noise reduction or anistropy enhancement. However, at this time, the relationship between these models and DLA is not well understood, and the DLA process itself is not at all well understood.

In an attempt to obtain insight into the effects of noise reduction and anisotropy on nonequilibrium growth models, simulations have been carried out using both the Eden[9] and the screened-growth models.[15,38] While the Eden and screened-growth models are not yet fully understood, there is a satisfactory theoretical basis for understanding their properties,[8] and their asymptotic fractal dimensionalities seem to be well determined. In this paper the results obtained from the Eden and screened-growth models are described.

COMPUTER SIMULATIONS

Simulations were carried out using a simplified version of the original Eden[9] model in which surface sites (unoccupied sites with one or more occupied nearest neighbors) are selected randomly and filled with equal probability. After each site has been filled, new surface sites are identified and added to the list of sites from which the random selection is made. Noise reduction is carried out using the procedure introduced by Tang[27] and by Kertesz and Vicsek.[28] Surface sites are selected at random, as in the ordinary Eden model, but are not actually occupied until they have been selected m times. The case $m=1$ corresponds to the ordinary Eden model. Anisotropy enhancement is carried out using the method developed by Matsushita and Kondo[33] for DLA. A closely related model has been developed by Chen and Wilkinson.[32] In this model, growth is carried out on a square lattice, but only those sites with coordinates (i,j) satisfying $i=lk$ and $j=ln$ can be filled where l, k, and n are integers. The integer l characterizes the degree of anisotropy and the case $l=1$ corresponds to the ordinary Eden model.

In the case of the screened-growth model, the surface sites are selected at random with relative probabilities given by

$$P_i \propto \prod_{j=1}^{M} e^{-(A/r_{ij}^{\epsilon})} , \qquad (1)$$

where P_i is the growth probability associated with the ith surface site and r_{ij} is the distance between the ith surface site and the jth occupied site in the growing cluster (M is the total number of occupied sites). The parameter A can be given any value, but in our simulation a value of 1.0 was used. The parameter ϵ determines the fractal dimensionality ($D=\epsilon$).[15,39,40] In the noise-reduced model, a record is kept of how many times each of the surface sites has been selected and these sites are filled after they have been selected m times. Anisotropy enhancement is carried out in the same manner as that for the Eden model described above.

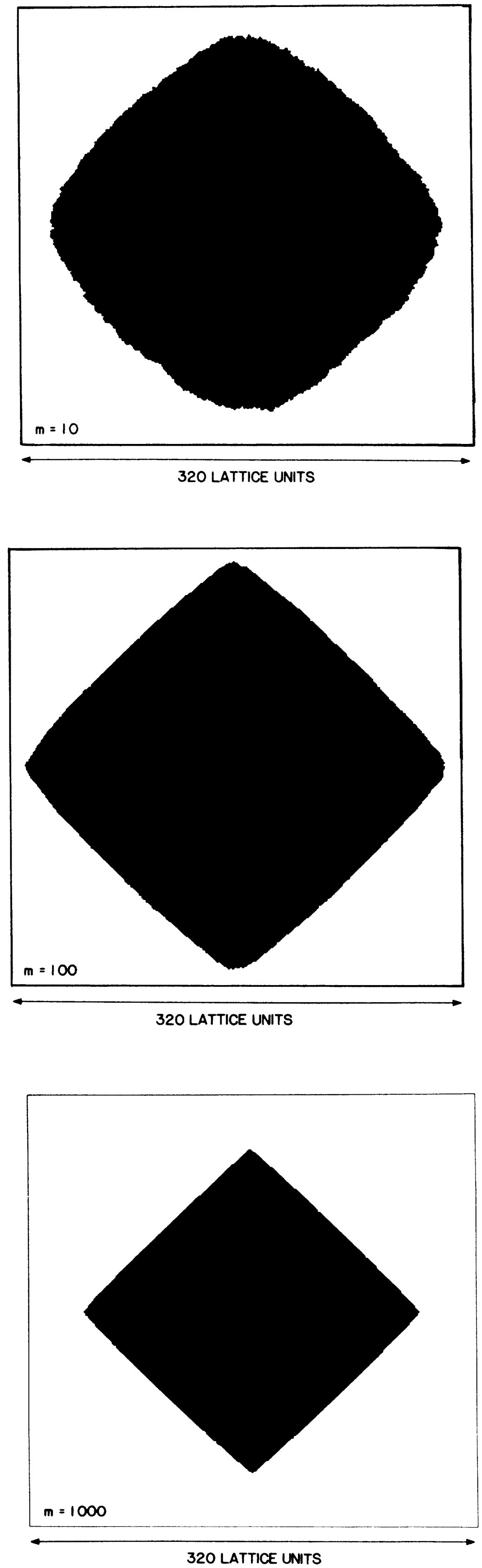

FIG. 1. Square-lattice Eden clusters grown with noise-reduction parameters m of 10, 100, and 1000. Each cluster contains 50 000 occupied sites.

RESULTS

Figure 1 shows three clusters generated using the Eden model with noise-reduction parameters m of 10, 100, and 1000. Each cluster contains 50 000 sites and ordinary Eden model clusters of this size appear to be round, although a very small distortion towards a diamondlike shape probably exists even at this relatively small size. Figure 1 shows a transition from a more or less circular shape at small m to a sharp diamondlike shape at large values of m. Although Eden clusters do show a small deviation from a circular shape[10–13] and theoretical con-

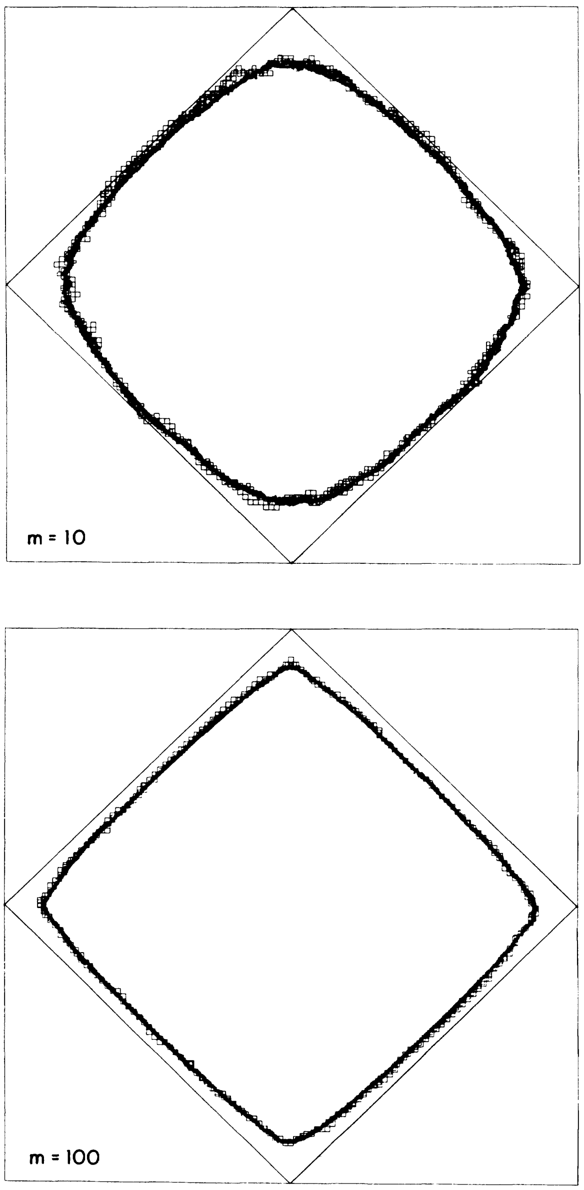

FIG. 2. Superposition of the unoccupied interface sites for Eden clusters containing 5000, 10 000, 20 000, 40 000, 80 000, 160 000, 320 000, and 500 000 occupied sites. The results obtained with a noise-reduction parameter m of 10 and 100 are shown.

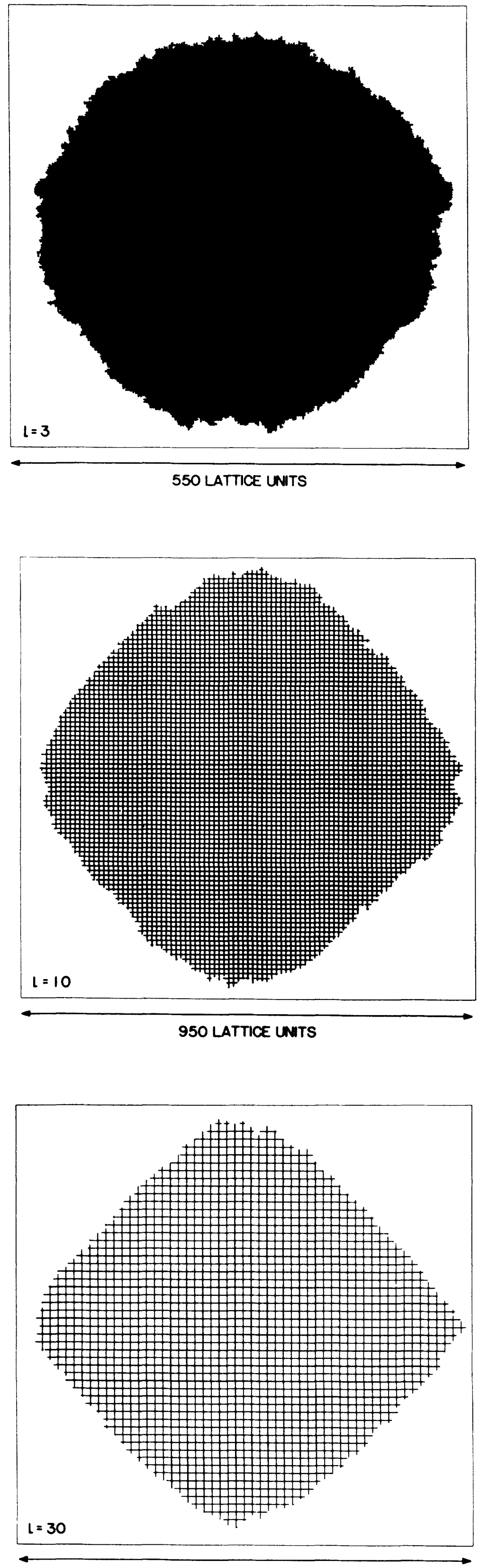

FIG. 3. Eden growth on a square lattice with anisotropy enhancement. Clusters grown with the parameter l (described in the text) set to values of 3, 10, and 30 are shown.

siderations predict a deviation from a hyperspherical shape on high-dimensional hypercubic lattices,[41] there is no indication that the asymptotic shape for square-lattice Eden clusters is a diamond.

In Fig. 2 the unoccupied perimeter sites are shown for clusters containing $M=5000$, 10 000, 20 000, 40 000, 80 000, 160 000, 320 000, and 500 000 occupied sites. The length scale used to display the perimeters is proportional to $M^{1/2}$ and the perimeters for clusters of different sizes are superimposed. Figure 2 shows the results obtained using a noise-reduction perimeter of 10 and the results obtained with $m=100$. This figure demonstrates that the shape of the cluster does not change significantly when the cluster mass is changed by a factor of 100 (i.e., the length scale is changed by a factor of 10). It seems that the asymptotic shape is reached quite quickly in this model. It should be noted that, unlike the clusters generated using the model of Savit and Ziff,[16] the shapes of these clusters cannot be described in terms of flat faces with rounded corners. Instead, the faces of the cluster are slightly curved even for very large values of m (Figs. 1, $m=100,1000$ and Fig. 2, $m=100$).

Figure 3 shows some typical results obtained from the modified Eden model in which growth is allowed only in sites for which the coordinates (i,j) satisfy $i=lk$ and $j=ln$ for $l=3$, 10, and 30. Again, a transition from a more or less circular shape to a diamond shape can be seen.

Clusters grown using the screened-growth model with values for the screening exponent ϵ [Eq. (1)] of 1.25, 1.5,

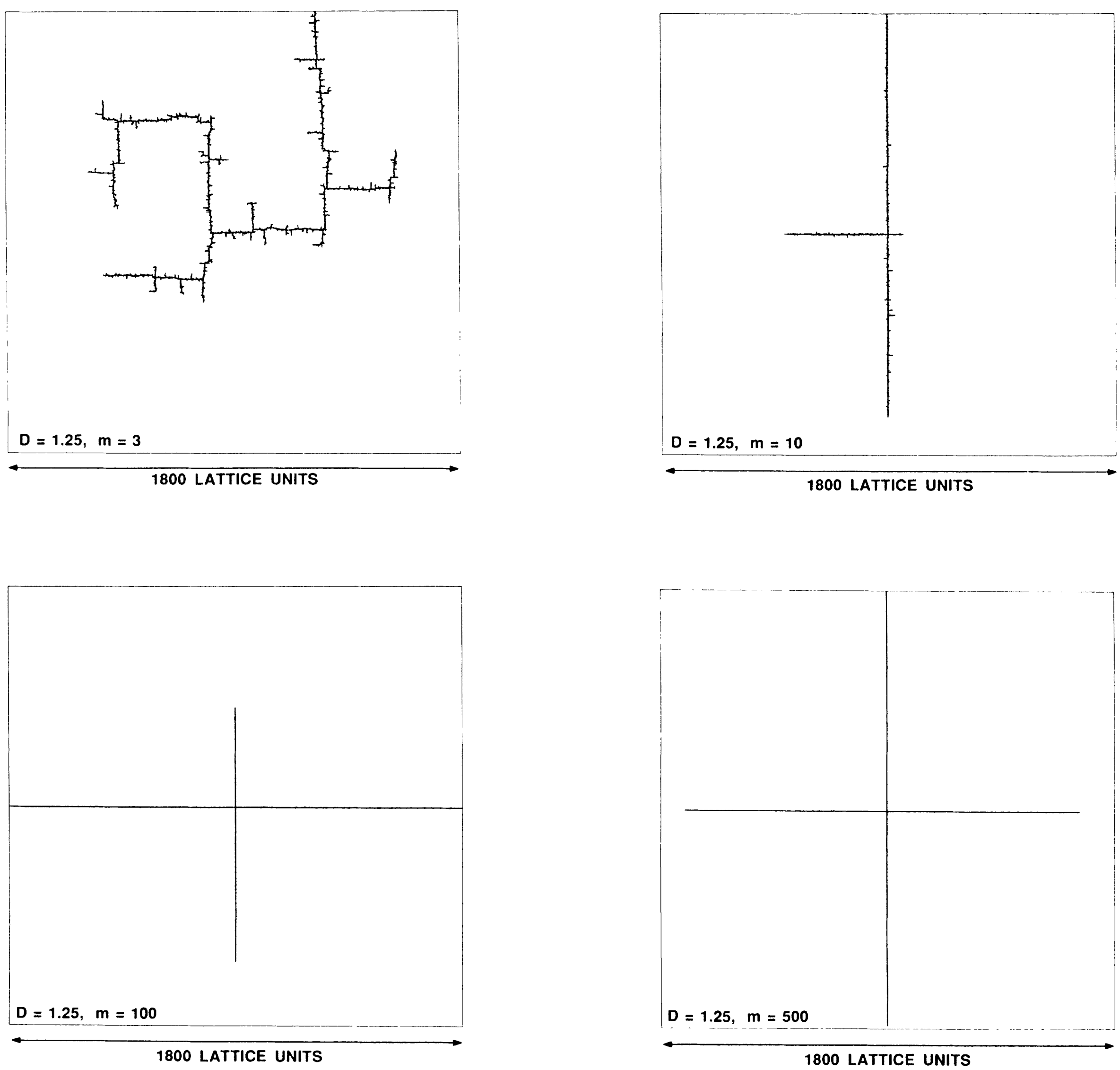

FIG. 4. Clusters grown using the noise-reduced screened-growth model with noise-reduction parameters of 3, 10, 100, and 500. In all cases the screening-function exponent ϵ has a value of 1.25 and A [Eq. (1)] is 1.0.

and 1.75 for several values of the noise-reduction parameter m are shown in Figs. 4–6, respectively. These clusters were grown from the center of 1800×1800 square lattices and, in most cases, the growth process was continued until the clusters either grew to the edge of the lattice or reached a size of 20 000 occupied lattice sites. For small values of ϵ ($\epsilon=1.25$, Fig. 4), the effects of the lattice anisotropy become very strong for quite small values of m and, for large values of m (Fig. 4, $m\geq10$), the arms of the cluster reach the edge of the lattice with little or no branching corresponding to an effective fractal dimensionality of 1. However, it is likely that sidebranching would occur for sufficiently large clusters and that the asymptotic fractal dimensionality (for finite m) might be larger than 1.0.

Figure 5 shows the results obtained with ϵ set to a value of 1.5. The clusters seem to exhibit a quite linear structure on short-length scales but are branched on longer-length scales. This might lead us to expect a crossover from an effective fractal dimensionality of $\simeq1$ on short-length scales to a limiting asymptotic ($M\rightarrow\infty$) value which might be equal to the screening exponent ϵ on long-length scales.

Figue 7 shows the two-point density-density correlation functions that were obtained from clusters grown with a screening exponent ϵ of 1.5 and noise-reduction parameters of $m=3$, 10, 30, and 100, respectively. The coordinates from five clusters were used to obtain $C(r)$ for each value of m, using all of the distances from 2000 randomly selected occupied lattice sites to calculate $C(r)$ for each cluster. The correlation functions shown in Fig. 7 are reasonably linear over almost two decades in r (distance) for all four values of m. There is no indication of the anticipated crossover and the results shown in Fig. 7

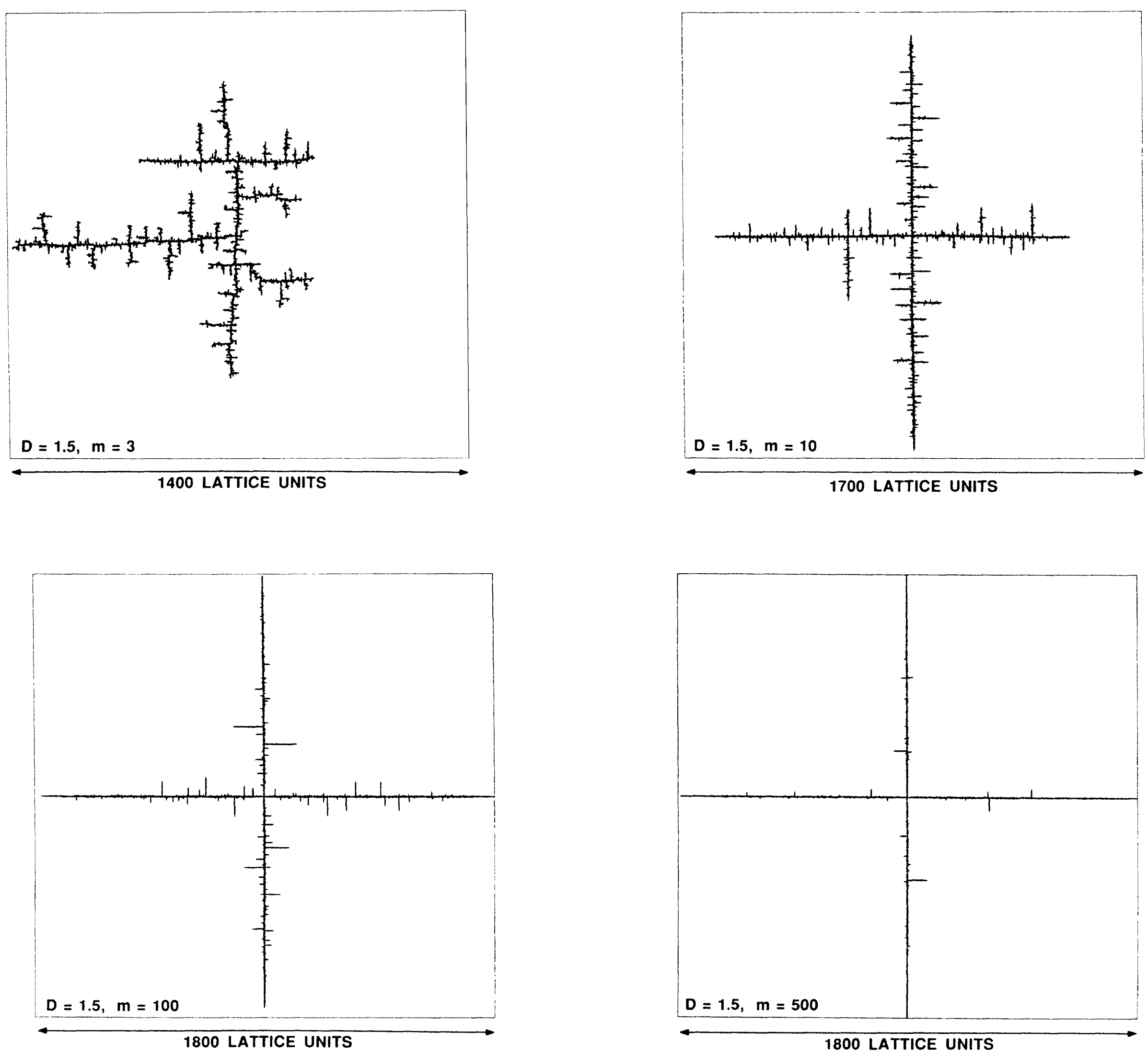

FIG. 5. Clusters grown using the noise-reduced screened-growth model with $\epsilon=1.5$ and $m=3$, 10, 100, and 500.

suggest that the fractal dimensionality might vary continuously from 1.5 for $m=1$ to 1.0 for $m \to \infty$.

Figure 8 shows the dependence of $\ln(M^{1/\epsilon}/R_g)$ on $\ln(M)$, where R_g is the radius of gyration for the same set of clusters ($\epsilon=1.5$, $m=3$, 10, 30, and 100). For all four values of m, $M^{1/\epsilon}R_g$ seems to be approaching a constant value (different for each value of m). This suggests that D_β, the effective fractal dimensionality that is obtained from the scaling relationship

$$R_g \sim M^{\beta} \quad (D_\beta = 1/\beta) , \tag{2}$$

has a limiting ($M \to \infty$) value equal to that of the screening-function exponent ε. However, plots similar to that shown in Fig. 7 for noise-reduced square-lattice DLA show pronounced oscillations[30] in the effective exponent β as the cluster mass changes, and it is quite possible that similar oscillations occur in the screened-growth model. Consequently, the results shown in Fig. 7 could be misleading.

As the clusters grow, the distance ω, at which the particles are deposited from the closest lattice axis passing through the origin (growth site), can be measured and averaged over small increments in the cluster mass and over a number of clusters. Figure 9 shows the dependence of $\bar{\omega}$ on M can be represented reasonably well by

$$\bar{\omega} \sim M^{\nu_\perp} , \tag{3}$$

where the exponent $\nu_\perp$ has a value of about 0.62. For the cases $m=30$ and 100, the dependence of $\nu_\perp$ on M can be represented reasonably well by Eq. (3) for the largest clus-

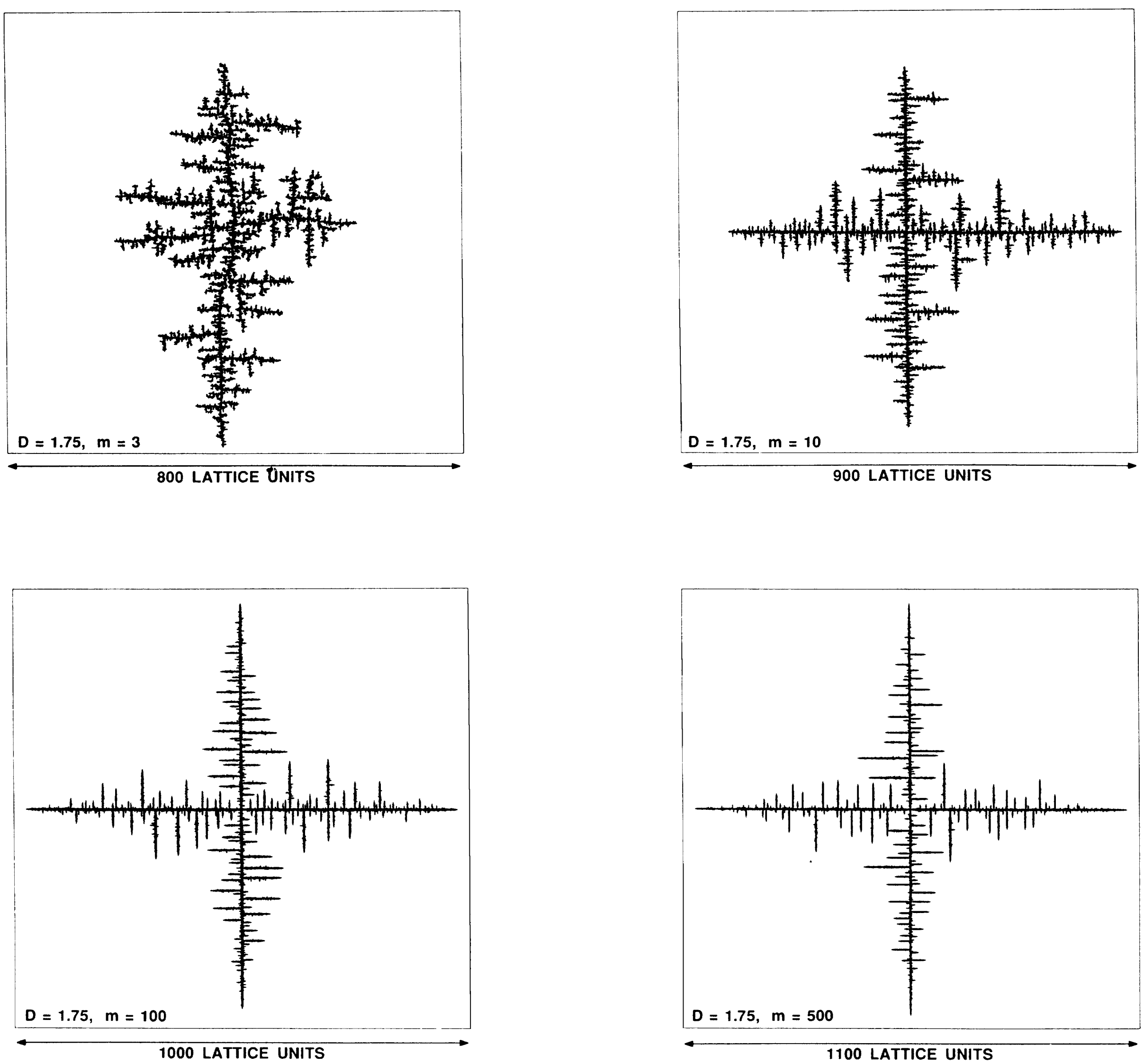

FIG. 6. Clusters grown with the noise-reduced screened-growth model using a screening-function exponent ϵ of 1.75 and noise-reduction parameters m of 3, 10, 100, and 500.

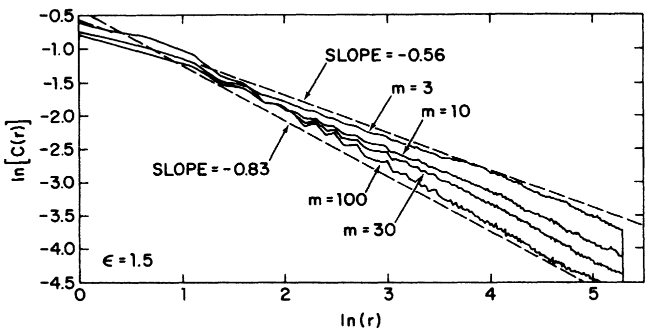

FIG. 7. The two-point density-density correlation functions $C(r)$ obtained from the screened-growth model using the parameters $\epsilon=1.5$, $A=1.0$, and noise-reduction parameters m of 3, 10, 30, and 100.

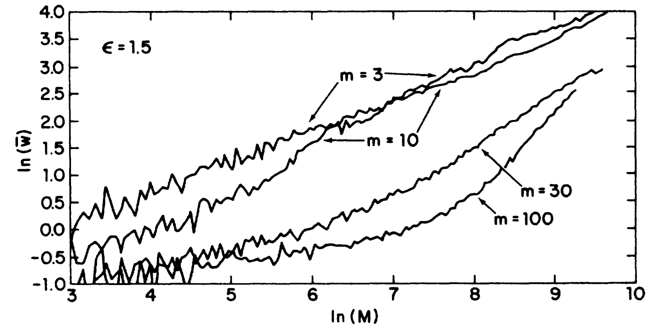

FIG. 9. Dependence of $\ln(\omega)$ on $\ln(M)$ for $\epsilon=1.5$ screened-growth clusters with noise-reduction parameters m of 3, 10, 30, and 100. ω is the mean deposition distance from the closest cluster axis averaged over 5% increments in M and over several clusters (five for $m=3$ and 100, ten for $m=10$ and 30).

ter sizes. However, this cannot be the true asymptotic behavior, since $\nu_{\perp}\simeq 1$ for $m=30$ and $\nu_{\perp}\simeq 1.5$ for $m=100$. A related quantity is the angle $\delta\theta$ between the vector from the origin to an occupied site and the vector along the nearest lattice axis. The angle $\delta\theta$ has been averaged over 5% increments in the cluster mass and over several clusters to obtain the quantity $\overline{\delta\theta}$. Figure 10 shows the dependence of $\ln(\overline{\delta\theta})$ on $\ln(M)$ obtained from the same set of clusters which were used to generate Fig. 9. In this case oscillations in $\overline{\delta\theta}$ with increasing cluster mass can be clearly seen for $m=10,30$ and, particularly, $m=100$.

Figure 11 shows clusters grown using the screened-growth model ($\epsilon=1.5$) in which growth is restricted to lattice sites with coordinates (i,j) satisfying $i=lk$, $j=ln$, where l, k, and n are integers. In this case the effects of lattice anisotropy are relatively weak.

DISCUSSION

The asymptotic shape of Eden clusters is determined by the structure of the growing surface. A variety of numerical studies and theoretical considerations[42–44] has shown that the width of the surface or active zone[42] ξ is related to the height h and width L for growth on a strip with periodic boundary conditions by

$$\xi \sim L^{\beta} f(h/L^{\gamma}) \;, \tag{4}$$

where the exponent β has a value of about $\frac{1}{2}$. For small values of x, the scaling function $f(x)$ has the form $f(x)\sim x^{\nu}$ ($\nu\simeq\frac{1}{3}$), so that $\xi\sim h^{\nu}$ and $\gamma=\beta/\nu\simeq 1.5$. In the limit $x\gg 1$ ($h\gg L^{\gamma}$), the number of surface sites is directly proportional to the strip width L,

$$N_s = aL \;, \tag{5}$$

but the coefficient a in Eq. (5) is slightly dependent on the direction of growth. The dependence of a on growth direction has been measured directly for Eden model C (Ref. 13) (in which occupied surface sites are randomly selected and growth occurs in a randomly selected unoccupied nearest neighor). It is the variation of a with growth direction which determines the shape of Eden

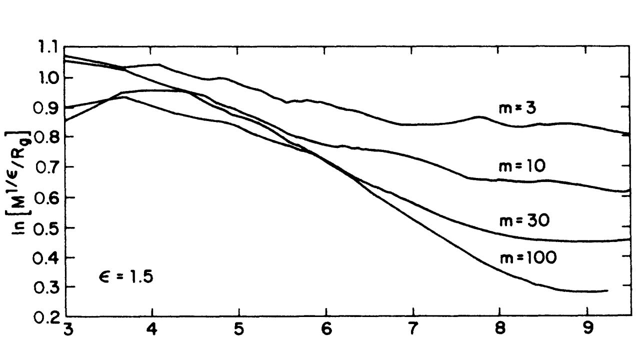

FIG. 8. Dependence of $\ln(M^{1/\epsilon}/R_g)$ on $\ln(M)$ for screened-growth clusters growth generated using $\epsilon=1.5$ and $m=3$, 10, 30, and 100. The same set of clusters was used to generate Figs. 7 and 8.

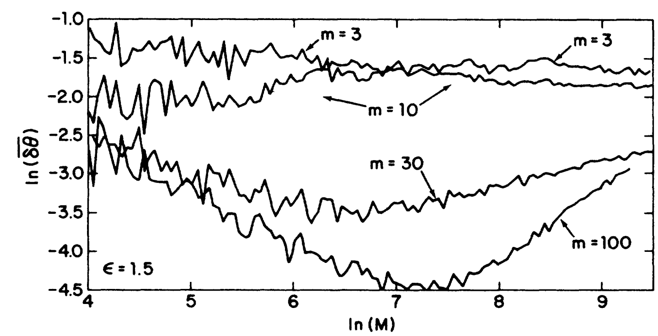

FIG. 10. Dependence of $\ln(\overline{\delta\theta})$ on $\ln(M)$ obtained from the same clusters which were used to generate Fig. 9. Here $\delta\theta$ is the angle between the vector from the origin to the last occupied lattice site and the vector along the nearest lattice axis. The quantity $\delta\theta$ is the average over 5% mass increments and over all clusters with the same value of m (3, 10, 30, or 100).

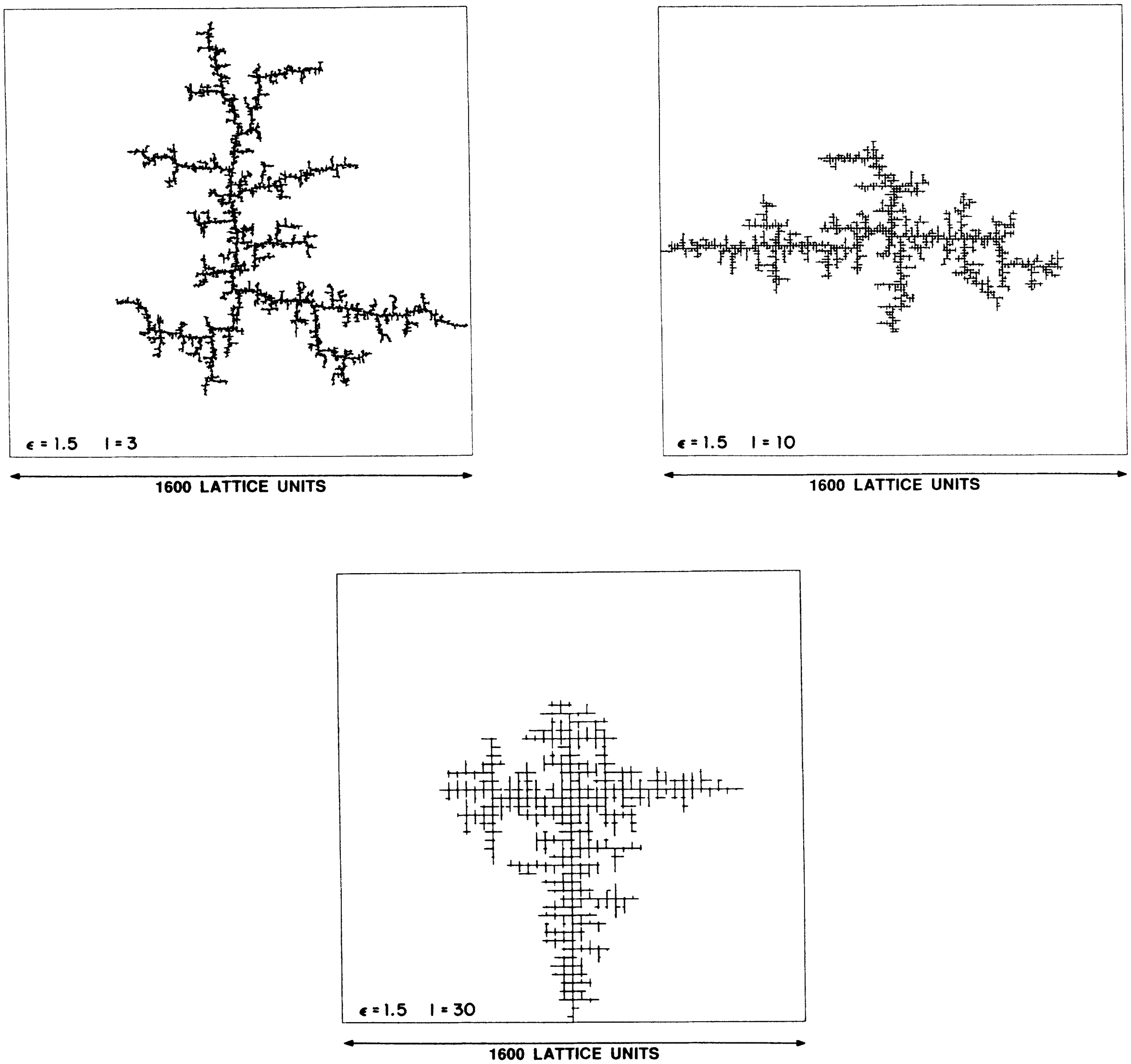

FIG. 11. Clusters grown using the screened-growth model with coordinates (i,j) satisfying $i=lk$ and $j=ln$ (with l, k, and n being integers). Clusters generated using a screening exponent ϵ of 1.5 and $l=3$, 10, and 30, respectively, are shown.

clusters.[45] For the simplified Eden model A (Ref. 46) used in this work, the anisotropy in a has been inferred from the shape of large clusters.[12]

Very recently,[47] simulations have been carried out for Eden growth on strips of width L using the noise-reduction method discussed above. A value of 0.326 ± 0.015 was found for the exponent ν, indicating that the exponents in the scaling form given in Eq. (4) are not changed by noise reduction.

Wolf[45] has shown how the shape of Eden clusters can be obtained from the direction dependence of the growth velocity (direction dependence of a) using a Wulff construction[48,49] approach. However, no theory has yet been developed for the direction dependence of the growth velocity for any version of the Eden model. The formation of a diamond shape can be associated with a smooth surface. As the growth process becomes more random, the cluster surface becomes rougher and the shape deviates from a diamond towards a circle. For $m=1$, the surface is quite rough and the deviation from a circular shape is so small that it has only recently been detected.[10–13] Noise-reduction parameters m less than 1 have no meaning in the context of the models described above. However, the number of surface sites associated with an Eden-model cluster of a given size can be increased by associating a growth probability selected at random from a distribution of probabilities as each of the surface sites is formed. In this way, it might be possible to approach more closely a circular shape in the limit $M\rightarrow\infty$.

The anisotropy-enhanced and noise-reduced models are very closely related. The anisotropy-enhanced models can be considered in terms of growth on a square network of bonds. Only after l growth events is a bond filled and growth onto adjacent bond is possible. In view of this, it is not surprising that noise reduction and anisotropy enhancement lead to similar results.

In the case of the screened-growth model, oscillatory behavior similar to that found earlier for noise-reduced diffusion-limited aggregation[30] makes unambiguous interpretation of the simulation results difficult. However, the results seem to be consistent with the idea that noise reduction does not change the asymptotic scaling behavior associated with the cluster geometry. However, there is a distinct change in the cluster shape for even quite small values of the noise-reduction parameter m, and the results presented here do not provide very strong support for a universal fractal dimensionality ($D=\epsilon$) independent of the value of m.

The oscillatory behavior found in noise-reduced DLA and screened-growth models appears to be related to the development of an almost discrete hierarchy of sidebranches. This feature can be seen clearly in Figs. 5 and 6. For example, in Fig. 5, when $m=10$, the cluster exhibits three generations in the growth hierarchy, while when $m=100$ and 500 (with larger noise-reduction parameters), only two generations can be seen.

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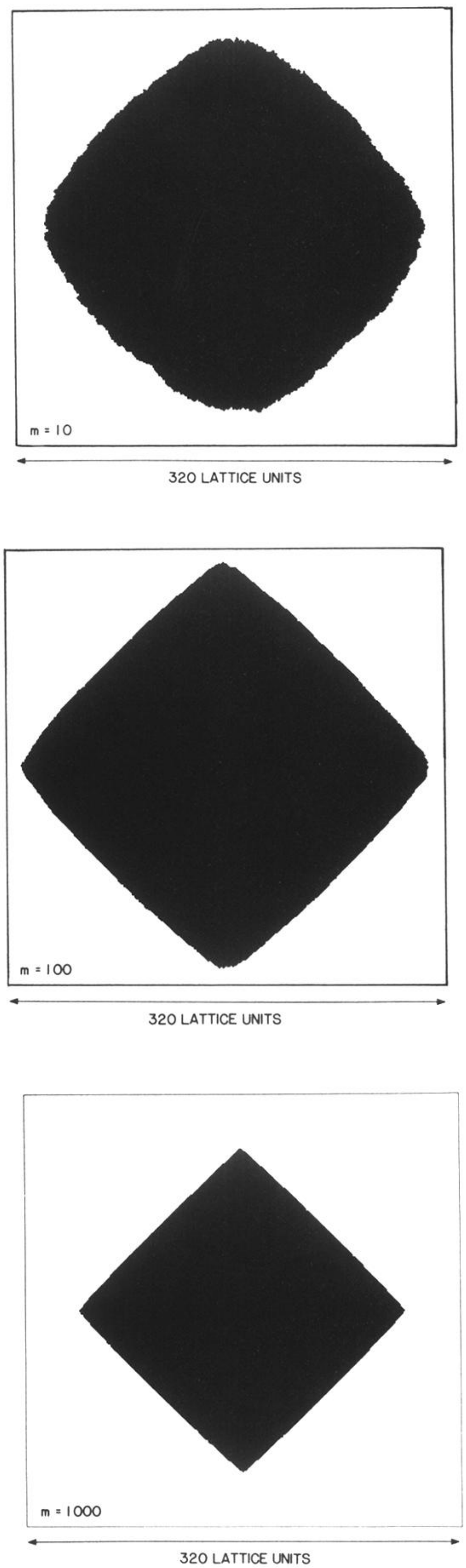

FIG. 1. Square-lattice Eden clusters grown with noise-reduction parameters m of 10, 100, and 1000. Each cluster contains 50 000 occupied sites.

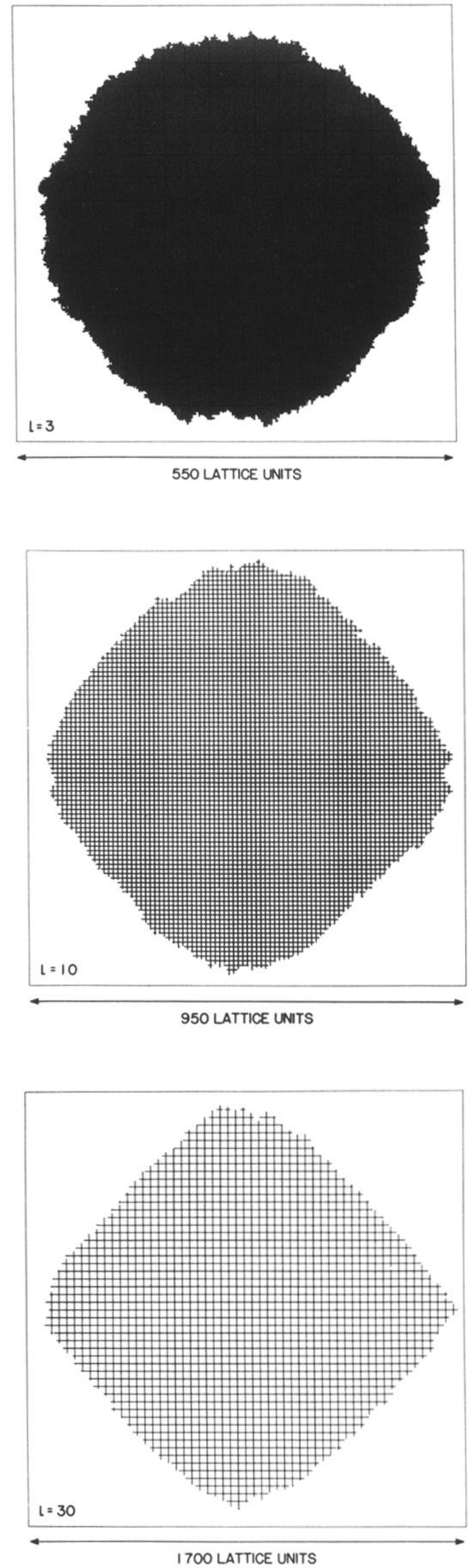

FIG. 3. Eden growth on a square lattice with anisotropy enhancement. Clusters grown with the parameter l (described in the text) set to values of 3, 10, and 30 are shown.