Transport of low- and medium-energy electron and ion beams in seawater and its vapors

Daniel A. Erwin and Joseph A. Kunc
Department of Aerospace Engineering and Department of Physics, University of Southern California, Los Angeles, California 90089-1191
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A general theory of stopping power for electrons and ions in a target medium (gas, liquid, or thin solid foil) containing neutral as well as positive and negative ions is developed. The approach is quite accurate in the low- and medium-energy range (<1 MeV), where the Bethe stopping-power cross section is inaccurate. The energy transfer during individual collisions is treated by the binary-encounter approximation. The theory is applied to determine the stopping power of seawater, whose major components are H_2O molecules and Na^+ and Cl^- ions. The stopping-power cross sections for low- and medium-energy protons in water show excellent agreement with existing measurements. The theory is extended to relativistic energies and to cases where the excitation of the incident and target particles is allowed. The approach is quite general and can be applied to other ionic, atomic, and molecular systems.

INTRODUCTION

We present in this paper an analysis of the electron- and ion-impact stopping power of seawater and its three major components, H_2O, Na^+, and Cl^-, in the low- and medium-energy range (<1 MeV) where traditional approaches such as the Bethe stopping-power theory[1] are inaccurate. The stopping power of seawater is important in many applications associated with energy storage, insulators, and deep-water naval salvage operations. However, available work on the ion-impact stopping-power cross sections for H_2O, Na^+, and Cl^- species is limited to pure water and to heavy water and deals mainly with proton and neutral transport.[2-5] But even in these cases, the Bethe theory (valid in situations when the incident particles are completely stripped high-energy ions) gives inaccurate results at low and medium impact energy. If the incident particles are ions other than bare nuclei, i.e., ions having electronic shells, then the Bethe theory can be inaccurate also at high energies (above 1 MeV). This is because the theory does not take into account such processes as Z^3 corrections, polarization effects, excitation of the incident ion, and so on. The recent analysis of Kim and Cheng[6] has included several processes, important at high impact energy, to the Bethe stopping-power theory. Their approach has led to a new Bethe-like formula that shows substantial superiority, when compared with experimental data, over the original Bethe formula at the high energies of the incident ions. However, at low and medium impact energy, existing ion-impact stopping-power theories are inaccurate.

Possibilities of an accurate determination of the electron-impact stopping-power cross section for water are also very limited, as the Bethe theory does not apply to cases when the incident particles are electrons. Theoretical predictions of Berger and Seltzer[7] exist only for interaction of electrons with pure water; they seem to underestimate somewhat the low- and medium-energy cross section because the mean excitation energy used for the H_2O molecule appears to be too high. Therefore, we propose here a consistent approach, which is accurate for slowing down of low- and medium-energy ions and electrons. This approach is quite general and can be applied in a straightforward way to other ionic, atomic, and molecular systems.

Since the stopping-power cross sections for Na^+ and Cl^- are not known, it is impossible *a priori* to predict the contribution of these ions to the stopping power of seawater compared to that of the H_2O molecules, even though the density of the ions in seawater is about two orders of magnitude lower than the density of H_2O molecules. This difficulty results from the fact that the ionization energies of Cl^- ions are smaller by an order of magnitude than the corresponding energies in the H_2O molecule. As a result, the contribution of the Cl^- ions to the inelastic energy loss of the incident particles may be higher than the contribution of the other particles. Therefore, accurate stopping-power cross sections for H_2O, Na^+, and Cl^-, in a broad energy range, are needed for determination of the contribution of each component to the total stopping power of seawater.

The energy losses of a charged particle (projectile) moving in the target medium are caused by (1) inelastic collisions with the target particles, (2) interaction with Coulomb field of the target nuclei, (3) interaction by nuclear forces, and (4) radiation losses. The interactions of the first category, inelastic collisions, are the most important in the low- and medium-energy range and cause the projectile to lose energy through one of the following processes: (a) excitation or ionization of the target particle while the projectile remains in the ground state, (b) excitation or ionization of the projectile while the target particle remains in the ground state, and (c) excitation of both the projectile and the target particle during the collision.

Calculations of the cross sections for the inelastic energy loss of incident electrons and ions with H_2O, Na^+, and Cl^- species are difficult, requiring complex and time-

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consuming quantum-mechanical calculations, usually based on the Born approximation or one of its modifications.[8] However, the Born approximation traditionally overestimates the inelastic energy-loss cross sections at their maxima, which are located in the low- and medium-energy range. Therefore, we use here the binary-encounter approximation[9] (BEA) to determine the energy exchanged during slowing-down inelastic collisions (and consequently the stopping-power cross sections) of the incident particles with the target particles; the BEA has been quite successful in describing inelastic processes involving molecules, atoms, and ions.[9-11] Numerical calculations are made for the stopping-power cross sections for electrons and protons colliding with H_2O, Na^+, and Cl^-; the results are compared with available theories and measurements. Using the cross sections we calculate the stopping power of seawater, the "penetration time" (the time after which the incident beam loses its collimation) and the corresponding penetration range of the incident electrons and protons in seawater. Finally, we extend the validity of our approach to relativistic energies, including the possibility of excitation of the incident and target particles [process (c)] during the collisions. It should be added that we consider here neither vibrational excitation of H_2O molecules nor elastic collisions; these collisions are far less important than the inelastic collisions considered here, in the energy range considered, for slowing down the energetic incident particles.

The efficiency of slowing down of a particle beam along its way throughout a target medium consisting of several components is characterized by the total stopping power of the medium:

$$\frac{d\varepsilon_h}{dh}=\sum_j \left[\frac{d\varepsilon_h}{dh}\right]^{(j)}, \tag{1}$$

where the stopping power of the jth component of the medium is

$$\left[\frac{d\varepsilon_h}{dh}\right]^{(j)}=-n^{(j)}S^{(j)}(\varepsilon_h), \tag{2}$$

and where ε_h and h are the energy and the depth of penetration of the target medium by the incident particles, respectively, $n^{(j)}$ is the particle density of the jth component and $S^{(j)}$ is the average stopping-power cross section for collision of the incident particle with a target particle of the jth component.

STOPPING-POWER CROSS SECTIONS

The most common stopping-power cross section for interaction of a neutral, ground-state target particle with a completely stripped incident ion of charge Z_he (e is the electronic charge) with velocity v_h, is that given by the Bethe formula[1,6] which in the nonrelativistic case has the form

$$S_b(v_h)=16N_tZ_h^2\frac{\pi a_0^2\mathcal{R}^2}{m_lv_h^2}\ln\left[\frac{2m_lv_h^2}{\langle E_l\rangle}\right], \tag{3}$$

where a_0 is the Bohr radius, $\mathcal{R}$ is the Rydberg energy, N_t is the number of electrons in the target particle, m_l is the electron mass, and $\langle E_l\rangle$ is the mean excitation energy; this can be taken as equal to half of the first ionization potential of the target particle. [The Bethe formula in the form (3) takes into account the fact that glancing collisions and knock-on collisions contribute equally to the stopping power.] The formula sometimes gives results of acceptable accuracy at very high energies (greater than a few MeV/amu). However, it loses accuracy at low and medium energies of the incident ions. In addition, as stated above, the Bethe formula cannot be used in situations when the incident particles are electrons.

A collision between a charged projectile (an electron or a completely stripped ion) and a target particle is treated in the binary encounter approximation as an encounter of two electric charges interacting through Coulomb forces, the incident charge and one of the electrons belonging to an orbital of the target particle. The energy exchanged in such a binary collision can be accurately described by both classical and quantum mechanics. Consequently, the cross section for scattering of the incident charge from the electron of the target particle can be averaged over the velocity distribution of the electron. Summing the effects of all the binary interactions of the incident charge with all the electrons of the target particle gives the average stopping-power cross section for a bare nucleus interacting with an atomic, ionic, or molecular particle of the target medium.

The stopping-power cross section for the binary collision of two free charges can be written as[9]

$$S_f(\mathbf{v}_h,\mathbf{v}_l)=-2\pi\frac{V}{v_h}\int_0^{D_{\max}}\Delta\varepsilon(\mathbf{v}_h,\mathbf{v}_l,D)D\,dD, \tag{4}$$

where $\mathbf{v}_h$ and $\mathbf{v}_l$, respectively, are the velocities of the incident charge and the target charge, and $V=|\mathbf{v}_h-\mathbf{v}_l|$ is the magnitude of their relative velocity. $\Delta\varepsilon$ is the energy exchanged during the collision; $D_{\max}$ is the maximum value of the impact parameter D.

Using the BEA relationship for $\Delta\varepsilon$ one obtains the cross section S_f for the scattering of two electric charges,

$$S_f(\mathbf{v}_h,\mathbf{v}_l)=2\pi\frac{(Z_hZ_le^2)^2}{\mu v_hV}\left[\frac{m_h-m_l}{m_h+m_l}+\frac{v_h^2-v_l^2}{V^2}\right]\times\ln\left[1+\left[\frac{\mu V^2}{Z_hZ_le^2}D_{\max}\right]^2\right], \tag{5}$$

where $\mu=m_hm_l/(m_h+m_l)$ is the reduced mass of the colliding system, with m_h and m_l the masses of the incident charge and the target charge, respectively. (In this paper the target charge will always be one of the electrons belonging to an orbital of the target particle (H_2O, Na^+, or Cl^-). Z_he and Z_le are the electric charges of the incident ion and the orbital electron, respectively. Averaging the cross section (5) over the orbital electron velocity distribution, $f_i(\mathbf{v}_l)$, for each orbital of the target particle, and summing over all the orbitals, one obtains the average stopping-power cross section for collision of the incident particle with the entire target particle:

$$S(v_h)=\sum_i \int_0^\infty N_i S_{f,i}(\mathbf{v}_h,\mathbf{v}_l) f_i(\mathbf{v}_l) d\mathbf{v}_l \ , \tag{6}$$

where N_i is the number of the electrons occupying the ith orbital and the sum is over all the orbitals of the target particle.

The spatial distribution $p_i(\theta_l,\vartheta_l)$ of the velocity vectors $\mathbf{v}_l$ in the target medium (in the spherical coordinates r_l, θ_l, and ϑ_l, where the angle θ_l is measured from the vector $\mathbf{v}_h$) can be assumed to be isotropic in the absence of strong external electric or magnetic fields. Then the distributions $p_i(\theta_l,\vartheta_l)$ and $f_i(v_l,\theta_l,\vartheta_l)$ are

$$p_i(\theta_l,\vartheta_l)d\theta_l d\vartheta_l=\frac{1}{4\pi}\sin\theta_l d\theta_l d\vartheta_l \ , \tag{7}$$

and

$$\begin{aligned} f_i(\mathbf{v}_l)d\mathbf{v}_l &= f_i(v_l,\theta_l,\vartheta_l)dv_l d\theta_l d\vartheta_l \\ &= p_i(\theta_l,\vartheta_l)g_i(v_l)dv_l d\theta_l d\vartheta_l \ , \end{aligned} \tag{8}$$

where $0\le\theta_l\le\pi$ and $0\le\vartheta_l\le 2\pi$ and where $g_i(v_l)$ is the absolute velocity distribution of the electrons occupying the ith orbital. Consequently, one has from Eqs. (5)–(8),

$$S(v_h)=\sum_i \int_0^\infty N_i S_{v,i}(v_l) g_i(v_l) dv_l \ , \tag{9}$$

where the stopping-power cross section $S_{v,i}$ is

$$\begin{aligned} S_{v,i}(v_l) &= \int_0^{2\pi}\int_0^{\pi} S_{f,i}(v_l,\theta_l) p_i(\theta_l,\vartheta_l) d\theta_l d\vartheta_l \\ &= \frac{\pi Z_h^2 Z_l^2 e^4}{2\varepsilon_l d}\left[1+\frac{m_l}{m_h}\right]\left[\frac{v_l}{v_h}\right]^2 \\ &\quad\times\sum_{k=1}^{4}[\beta_k(s^2)-\beta_k(u^2)] \ , \end{aligned} \tag{10}$$

with

$$\varepsilon_l=m_l v_l^2/2 \ , \tag{11}$$

$$d=\frac{v_l}{e}\left[\frac{\mu D_{\max,i}}{Z_h Z_l}\right]^{1/2} \ , \tag{12}$$

$$s=d\left[1+\frac{v_h}{v_l}\right] \ , \tag{13}$$

$$u=d\left[1-\frac{v_h}{v_l}\right] \ , \tag{14}$$

$$\beta_1(x)=\frac{t-us}{\sqrt{2}}\begin{cases}\tan^{-1}\left[\dfrac{\sqrt{2x}}{1-x}\right] & \text{if } x\le 1\\ \pi-\tan^{-1}\left[\dfrac{\sqrt{2x}}{x-1}\right] & \text{otherwise,}\end{cases} \tag{15}$$

$$\beta_2(x)=\frac{t+us}{\sqrt{2}}\ln\left[\frac{x+\sqrt{2x}+1}{\sqrt{1+x^2}}\right] \ , \tag{16}$$

$$\beta_3(x)=\frac{1}{2}\left[\frac{us}{\sqrt{x}}+t\sqrt{x}\right]\ln(1+x^2) \ , \tag{17}$$

$$\beta_4(x)=-2t\sqrt{x} \ , \tag{18}$$

and

$$t=\frac{m_h-m_l}{m_h+m_l} \ . \tag{19}$$

The maximum impact parameter $D_{\max,i}$ can be estimated in the way proposed by Bethe:

$$D_{\max,i}=\frac{Z_h e^2}{\Delta E_{\min,i}} \ , \tag{20}$$

where $\Delta E_{\min,i}$ is the mean ionization energy for the ith orbital. In case of scattering from atoms and ions this energy can be taken as $U_i/2$ (outer-shell electrons) or U_i (inner-shell electrons), where U_i is the ionization potential for the ith orbital. Since it is rather difficult to distinguish the outer shell in the H_2O molecule we assume in what follows that $\Delta E_{\min,i}=U_i$ for all the molecular orbitals.

Since the Koopman theorem is well fulfilled in Na^+ and Cl^- ions and in the H_2O molecule,[12] it is justified to assume that the absolute value of the binding energy of the electron in the ith orbital is equal to the orbital ionization potential U_i. Therefore, we use here the δ function for the velocity distributions of the orbital electrons, which leads to the following electron (ion) impact average stopping-power cross section:

$$\begin{aligned} S(v_h) &= \sum_i \int_0^\infty N_i S_{v,i}(v_l)\delta_i(v_l-v_i)dv_l \\ &= \sum_i N_i S_{v,i}(v_i) \ , \end{aligned} \tag{21}$$

where

$$v_i=(2U_i/m_l)^{1/2} \ . \tag{22}$$

The choice of the δ function as the velocity distribution of the orbital electrons has been dictated by the fact that the cross section for the exciting and ionizing collisions is much less sensitive to the shape of the distribution than to its average energy; this differs, for example, from the situation in charge transfer collisions where the cross section is sensitive to the distribution at high velocities of relative motion. Therefore, the representation of the average energy of the orbital electron by relationship (22) is quite a reasonable approximation. This conclusion is also supported by the work of Bates and Kingston[13]

TABLE I. Ionization potentials U_i (in eV) for molecular orbitals (MO) of $H_2O(X\,^1A_1)$. Theoretical values were taken from Refs. 14 and 15 while the measured values are those of Ref. 12. The values denoted by the asterisks are assumed in the present calculations.

MO	Symmetry orbitals	U_i Theory	U_i Expt.
$1a_1$	O $1s$	559.48*	
$2a_1$	O $2s$, $H_1\,1s+H_2\,1s$	34.83*	
$1b_2$	O $2p_y$, $H_1\,1s-H_2\,1s$	16.87	18.02*
$3a_1$	O $2p_z$, O $2s$, $H_1\,1s+H_2\,1s$	12.79	14.23*
$1b_1$	O $2p_x$	10.88	12.61*

TABLE II. Ionization potentials U_i (in eV) for ionic orbitals (AO) in $Na^+(1s^22s^22p^6\,{}^1S)$ and $Cl^-(1s^22s^22p^63s^23p^6\,{}^1S)$ ions. All values are taken from Ref. 16.

Ion/AO	1s	2s	2p	3s	3p
Na^+	1109.07	83.63	48.90		
Cl^-	2843.58	278.36	209.38	19.94	4.08

analyzing a large class of inelastic collisions with different velocity distributions of target electrons.

The electronic configuration of the ground-state H_2O molecule (three orbitals of a_1 symmetry, one of b_1, and one of b_2 symmetry) is given in Table I. The ionization potentials for molecular orbitals (MO's) of the H_2O molecule (Table I) were taken from measurements[12] (by either photoelectron spectroscopy or threshold methods) and from self-consistent-field linear combination of atomic orbitals (SCF LCAO) calculations.[14,15] The ionization potentials for the ionic orbitals of Na^+ and Cl^- were taken from the high-accuracy calculations of Clementi and Roetti[16] and they are summarized in Table II.

One may add that the BEA approach should not be used for calculating the stopping-power cross sections when the incident particles are electrons with energies below about 10 keV, since then the stopping-power cross sections based on the binary-encounter approximation become inaccurate. This loss of accuracy results from the BEA requirements limiting the amount of energy and momentum exchanged during a binary collision;[9] this is a strong function of the charge-mass ratios of the two colliding particles.

The results of numerical calculations of the stopping-power cross sections (21) for the electron- and proton-impact on H_2O, Na^+, and Cl^- are given in Figs. 1 and 2. As can be seen from Fig. 1, the electron-H_2O stopping-power cross section of the present work is in good agreement with the theoretical predictions of Berger and Seltzer.[7] Since our results are somewhat larger than the results of Berger and Seltzer, it seems (see discussion in the Introduction) that our results are closer to reality than those.

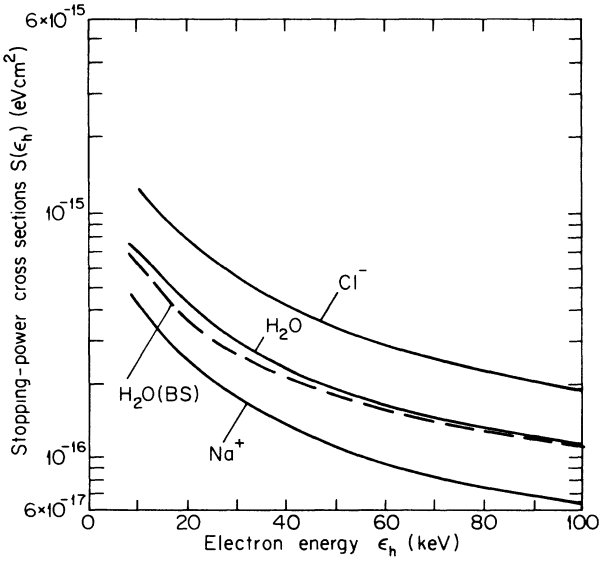

FIG. 1. Stopping-power cross sections for interaction of electrons with H_2O molecules and Na^+ and Cl^- ions. The solid lines represent the results of the present work. The dashed line represents the theoretical calculations of Berger and Seltzer (Ref. 7) (BS) for the electron-H_2O interaction.

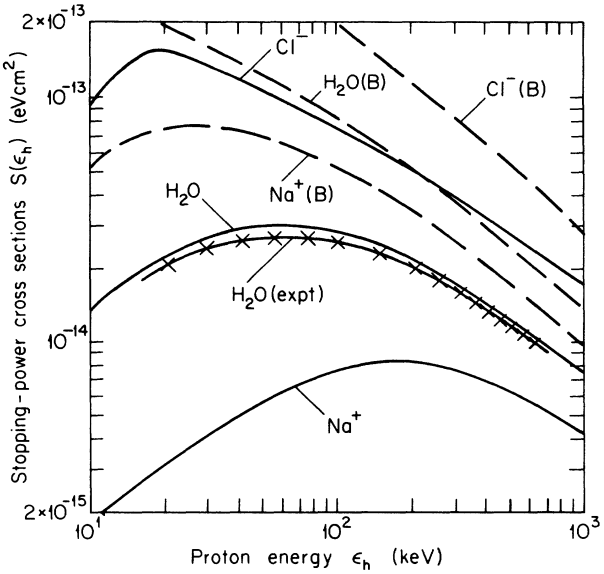

FIG. 2. Stopping-power cross sections for interaction of protons with H_2O molecules and Na^+ and Cl^- ions. The solid lines represent the results of the present work. The solid-crossed line represents the measurements (expt) of Reynolds *et al.* (Ref. 3) for the proton-H_2O interaction, while the dashed lines represent the results obtained from the Bethe (*B*) formula [Eq. (3)].

The stopping-power cross sections of the present work for proton-H_2O interaction [Eq. (21)] are in excellent agreement (Fig. 2) with the measurements of Reynolds *et al.*[3] in the entire experimental energy range, whereas the corresponding Bethe cross sections are inaccurate for all components of seawater in the entire energy range considered.

STOPPING POWER OF SEAWATER

The total stopping power of seawater can be calculated from relationships (1) and (2) if the dependence of the seawater temperature T_{sw} and particle densities on the depth z_{sw} (measured from the surface) are known. The temperature and density of seawater are very weak functions of z_{sw}.[17] The temperature T_{sw} ($\approx$280 K) decreases by a few percent when z_{sw} increases up to 8 km, while the density of H_2O molecules increases by a few percent. In addition, the fractions $n^{(H_2O)}/n$, $n^{(Na^+)}/n$, and $n^{(Cl^-)}/n$ (where n is the total seawater particle density) are also very weakly dependent on z_{sw} and can be taken as

$$n^{(H_2O)}/n = 0.982, \quad n^{(Na^+)}/n = 0.0082$$

and (23)

$$n^{(Cl^-)}/n = 0.0096 .$$

Taking this and the results discussed in the previous section into account, one may say that although the electron- and proton-impact stopping-power cross sections for Cl^- are much higher (Figs. 1 and 2) than the corresponding cross sections for H_2O, the difference is not large enough to compensate the fact that the density of Cl^- ions is 2 orders of magnitude lower than the density of H_2O molecules. (It is also clear from the above that the Na^+ ions do not play an important role in slowing beams in seawater.) Thus, the H_2O molecules are clearly the major contributors to the total stopping power of sea water, both for electron and ion beams. Since the particle densities n and $n^{(H_2O)}$ change very little with z_{sw}, the total stopping power of seawater for incident particles of energy ε_h can be given [Eqs. (1) and (2)] for practical purposes as

$$\frac{d\varepsilon_h}{dh} = -n_0^{(H_2O)} S^{(H_2O)}(\varepsilon_h) , \tag{24}$$

regardless of the depth below the surface. $n_0^{(H_2O)} = 3.35\times10^{22}$ cm^{-3} is the mean value of the density of the H_2O molecules in seawater.

PENETRATION RANGE AND PENETRATION TIMES

A beam of monoenergetic particles (electrons or ions) can penetrate the seawater to a distance l (called hereafter the "penetration range") during a time τ (called hereafter the "penetration time") after which the beam loses its collimation and formula (2) ceases to be valid. Taking into account the remarks of the previous section, the penetration range in seawater is independent of the depth z_{sw} and can be estimated as

$$l(\varepsilon_h^0) = -\int_{\varepsilon_h^0}^{\varepsilon_h^f} \left[\sum_j n^{(j)}(z_{sw}) S^{(j)}(\varepsilon_h)\right]^{-1} d\varepsilon_h$$
$$= (n_0^{(H_2O)})^{-1} I_l(\varepsilon_h^0) , \tag{25}$$

where ε_h^0 is the initial energy of the particles of the incident beam, ε_h^f is the final energy (see below), and where

$$I_l(\varepsilon_h^0) = \int_{\varepsilon_h^f}^{\varepsilon_h^0} [S^{(H_2O)}(\varepsilon_h)]^{-1} d\varepsilon_h . \tag{26}$$

The penetration time for the incident particles of initial energy ε_h^0 can be given (regardless of depth) as

$$\tau(\varepsilon_h^0) = -\left[\frac{m_h}{2}\right]^{1/2} \times \int_{\varepsilon_h^0}^{\varepsilon_h^f} \left[\varepsilon_h^{1/2} \sum_j n^{(j)}(z_{sw}) S^{(j)}(\varepsilon_h)\right]^{-1} d\varepsilon_h$$
$$= A_\tau I_\tau , \tag{27}$$

where

$$A_\tau = \left[\frac{m_h}{2}\right]^{1/2} (n_0^{(H_2O)})^{-1} \tag{28}$$

and

$$I_\tau(\varepsilon_h^0) = \int_{\varepsilon_h^f}^{\varepsilon_h^0} [\varepsilon_h^{1/2} S^{(H_2O)}(\varepsilon_h)]^{-1} d\varepsilon_h . \tag{29}$$

The final energy of the beam, ε_h^f, is difficult to determine. The particles of the incident beam are scattered mainly by interactions with H_2O electrons. When the incident particles are protons, the efficiency of redistribution of the proton energy is comparable with the efficiency of redistribution of its momentum. Therefore, the proton beam can still be considered a "beam" [in the sense that Eqs. (2), (25), and (27) are valid] if the proton energy is greater than the average velocity of the molecular electrons. Thus, in the case of a proton beam, $\varepsilon_h^f = m_p v_i^2/2$. When the incident particles are electrons, the redistribution of electron momentum is much faster than that of energy. Therefore, the electron beam loses its identity at energies much higher than $m_e v_i^2/2$. To estimate the electron final energy we follow the procedure of Nelms[18] who estimated the energy value as 5 keV. (We use in this work a conservative estimate of 10 keV.) Summarizing the above, we can say that the penetration range and time defined here are those at which the energy of the incident proton and electron beams drops to 10 keV.

The functions $I_l(\varepsilon_h^0)$ and $I_\tau(\varepsilon_h^0)$ for slowing down of the electron and proton beams in seawater are shown in Figs. 3 and 4. [The values of A_τ are 6.37×10^{-37} g$^{1/2}$cm^3 (electron beam) and 2.73×10^{-35} g$^{1/2}$cm^3 (proton beam).] It can be seen from there and from Eqs. (25) and (27) that the penetration range of seawater by low- and medium-energy electron beams is between 2×10^{-3} cm (at $\varepsilon_h^0 = 20$ keV) and 2×10^{-2} cm (at $\varepsilon_h^0 = 100$ keV) while the proton penetration range changes from 3×10^{-5} cm (at 20 keV) to 2×10^{-3} cm (at 1 MeV).

It is clear from the above discussion that seawater is highly nonpenetrable for nonsustained beams of low- and medium-energy electrons and ions. However, its vapor can be penetrable to distances of the order of centimeters and more if the density of the vapor is a fraction of atmospheric.

It should be remembered that the cross sections discussed above are evaluated for two-particle inelastic collisions (ion-ion and ion-molecule) and are atomic quantities, i.e., they do not depend on the seawater state. Using these cross sections, the total stopping power of seawater is calculated by statistical superposition of all the two-particle collisions. Such an approach, used commonly in gases and vapors, may be inappropriate in the liquid phase where multiparticle scatterings can dominate inelastic interactions (the mean distance between the particles of seawater in liquid phase is about 3 Å). The validity of the superposition of two-particle interactions in the liquid phase can be verified experimentally by measuring the penetration range of seawater and comparing the results with the theoretical results of the present work. Since our theoretical stopping-power cross sections are in

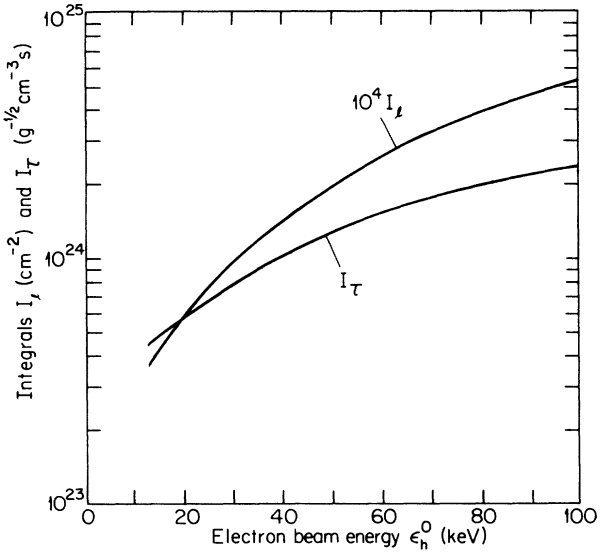

FIG. 3. The integrals I_l [Eq. (26)] and I_τ [Eq. (29)] characterizing the penetration range l and penetration time τ for monoenergetic electron beams in seawater. ε_h^0 is the initial energy of the incident electrons.

excellent agreement with the measured cross sections (Fig. 2) the discrepancy between the theoretical and measured penetration ranges of the beams in seawater would be a reliable measure of the validity of the statistical superposition (as a function of density of H_2O molecules) of the two-particle collisions for description of the slowing down and other inelastic collisions in the liquid phase of seawater. It may be added that a similar superposition is

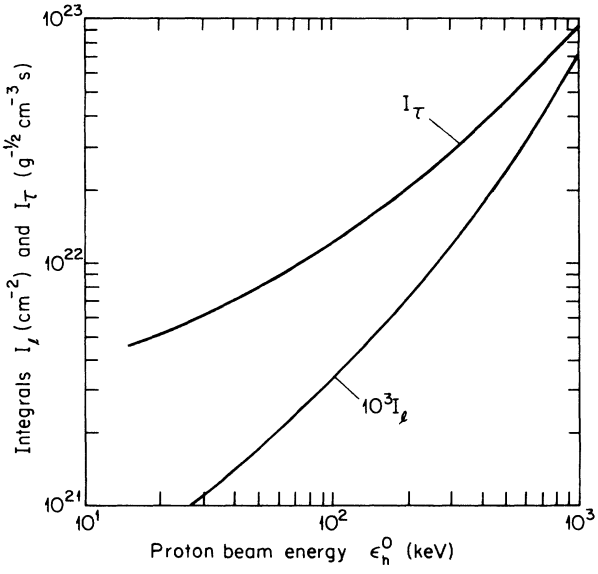

FIG. 4. The integrals I_l [Eq. (26)] and I_τ [Eq. (29)] characterizing the penetration range l and penetration time τ for monoenergetic proton beams in seawater. ε_h^0 is the initial energy of the incident protons.

often successful in describing highly-ionized plasmas where long-range, multiparticle Coulomb interactions dominate collisional processes, for it has been shown by several authors[19,20] that the assumption of superposition of two-particle collisions is acceptable in highly-ionized plasmas if the density of the plasma and the strength of the applied magnetic field are not very high. Two-particle superposition seems to be a quite valid procedure for determination of the stopping power of seawater vapor even at relatively high pressures. This is because the procedure leads to values of the penetration range which are in good agreement with experiment in other, fairly dense (3×10^{19} cm^{-3}) molecular gases.[21–23]

INCLUSION OF PROCESS (c)

As discussed in the Introduction, an additional energy-loss mechanism [process (c)] may occur if the incident ion has electronic shells; this was recently investigated in detail by Kim and Cheng.[6] Adopting their conclusions, and assuming that the stopping-power cross section is poorly sensitive to changes in the mean excitation energy of the target particles, one can estimate the stopping-power cross section, when process (c) is included, as

$$S_c(v_h)=\lambda_c S(v_h)\ , \tag{30}$$

where the cross section $S(v_h)$ is given by Eq. (21) and, in the case when the target particle is an ion,

$$\lambda_c=\frac{N_t(Z_p^2+\delta_p^2)+N_p(Z_t^2+\delta_t^2)+2N_pN_t}{2N_tZ_p^2}\ . \tag{31}$$

In the above, Z, N, and $\delta=Z-N$ denote, respectively, the charge numbers, the total numbers of electrons, and the net charges. The subscripts p and t stand for the incident and target particle, respectively. The relationship (31) was proposed by Kim and Cheng[6] for ionic and atomic targets; in the latter case, $\delta_t=0$. Therefore, it should be used with caution when adopted for molecular targets such as H_2O.

EXTENSION TO RELATIVISTIC ENERGIES

The validity of the cross sections (21) can be extended to relativistic energies in a way similar to that proposed by Kim and Cheng.[6] The contribution to the cross sections resulting from these energies is represented in that approach by a factor $\gamma_{\rm rel}$ such that the relativistic stopping-power cross section can be given as

$$S_{\rm rel}(v_h)=\gamma_{\rm rel}S_c(v_h)\ , \tag{32}$$

with

$$\gamma_{\rm rel}=\frac{m_l v_h^2}{2\mathcal{R}}\left[\frac{\alpha}{\beta}\right]^2\frac{\ln\left[\dfrac{4\mathcal{R}\beta^2}{(1-\beta^2)\alpha^2\langle E_l\rangle}\right]-\beta^2}{\ln\left[\dfrac{2m_l v_h^2}{\langle E_l\rangle}\right]}\ , \tag{33}$$

where $\mathcal{R}$ is the Rydberg constant, α is the fine-structure constant, and $\beta=v_h/c$, with c being the speed of light.

SUMMARY

In this paper we have developed an approach to determination of the stopping-power cross sections for low- and medium-energy (10 keV $< \varepsilon <$ 1 MeV) electrons and ions in a medium that may contain atoms and molecules as well as positive and negative ions. The approach is applied to calculate the stopping power, the penetration range, and the penetration times of electron and proton beams in seawater, a good example of a target medium containing positive (Na^+) and negative (Cl^-) ions and molecules (H_2O). We generalized our approach to high (also relativistic) energies and included the possibility of excitation (ionization) of both the incident and target particles during the collisions. The results obtained are directly applicable to analysis of ionic and molecular processes in other gases and liquids.

The stopping-power cross sections of this work seems to be quite accurate in the range of energy considered. It has been found that at these energies the main contribution to the stopping power of seawater comes from excitation and ionization of H_2O molecules by particles of the incident beam. Also, the total stopping power of seawater is proportional to the stopping-power cross section for the H_2O molecules since the molecular density and temperature change only a few percent with depth from the surface down to 8 km.

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