Quantum correlation and state reduction of photon twins produced by a parametric amplifier

K. Watanabe and Y. Yamamoto

Nippon Telephone and Telegraph Corporation Basic Research Laboratories, Musashino-shi, Tokyo 180, Japan

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Signal and idler waves produced by a nondegenerate parametric amplifier feature strong positive correlation between their photon numbers, or in-phase amplitudes. They feature equally strong negative correlation between their phases, or quadrature-phase amplitudes. These photon twins can produce arbitrarily squeezed states via state reduction by appropriate measurements of an idler wave. When the quadrature amplitude of an idler wave is measured, the signal wave is collapsed to a quadrature-amplitude squeezed state. When the two-quadrature amplitudes of an idler wave are measured simultaneously, the signal wave becomes a coherent state.

I. INTRODUCTION

Nonclassical light is usually generated by one of two schemes. The first is a phase-sensitive amplification-deamplification in a four-wave mixer[1] or a degenerate parametric amplifier.[2] The second is amplitude saturation and phase diffusion in a pump-noise-suppressed laser.[3] This paper discusses a completely different approach using nonunitarity state reduction by quantum measurement.

Arthurs and Kelly demonstrated that the system wave function can be reduced to a new state after the simultaneous measurement of two conjugate observables. This reduction realizes the arbitrary distribution of quantum noise determined by the measurement resolution of the two observables.[4] However, these authors have not shown any physically realizable Hamiltonian for such measurement. At optical frequencies, the photon number $\hat{n}$, the quadrature amplitude $\hat{a}_1$, and the two-quadrature amplitudes $\hat{a}_1$ and $\hat{a}_2$ are physically measured by a photon counter, homodyne detector, and heterodyne detector, respectively.[5] Von Neumann's projection postulate has been generalized to characterize these measurements quantum mechanically.[6,7] The generalized projectors, or operation-valued measures for these measurements, are given by $|n\rangle\langle n|$, $|\alpha_1\rangle\langle\alpha_1|$, and $|\alpha\rangle\langle\alpha|$. However, the measurements themselves are not considered as processes for generating number states $|n\rangle$, quadrature-amplitude eigenstates $|\alpha_1\rangle$, and coherent states $|\alpha\rangle$, because the electromagnetic fields are completely absorbed after these measurements are made.

To produce quantum light via state reduction by quantum measurement, quantum correlation between a signal and probe waves must first be established. Destructive measurement can be then performed on the probe wave. An example is the quantum nondemolition measurement of photon number, in which the signal waves is collapsed into a number-phase squeezed state after the signal photon number is nondestructively measured via destructive measurement of the probe phase.[8]

When a signal and idler waves are amplified in a nondegenerate parametric amplifier, the outputs are correlated both in photon number and phase.[9–11] Measurement of the idler output wave provides information on the signal output. This suggests the generation of quantum light is possible via nonlinear parametric amplification of idler and signal waves, and subsequent measurement of the idler output.

This paper is organized as follows. In Sec. II we briefly review the evolution of a combined signal-and-idler density operator in a nondegenerate parametric amplifier. The reduced density operator of a signal wave is discussed, which corresponds to the signal quantum state without measurement. State reduction resulting from measurement of idler quadrature amplitude is treated in Sec. III. State reduction resulting from simultaneous measurement of the two idler quadrature amplitudes is studied in Sec. IV. A projection operator that maps a coherent state onto a number-phase squeezed state, quadrature-amplitude squeezed state, and coherent state is derived in Sec. V using the results of Secs. III and IV. In Sec. VI feedfoward manipulation of a signal wave according to the measurement results to continuously generate nonclassical light with a fixed eigenvalue is determined. Finally, in Sec. VII, quantum correlations of signal and idler waves are studied using a Heisenberg picture. We present a physical interpretation of the results in Secs. III and IV.

II. STATE EVOLUTION IN A NONDEGENERATE PARAMETRIC AMPLIFIER

In this section we briefly review the evolution operator $\hat{U}$ and the density operator $\hat{\rho}_{si}$ for signal and idler waves for the nondegenerate parametric amplifier.[12] The interaction Hamiltonian is

$$\hat{H}_{\text{int}}=\hbar[\kappa\hat{a}_s^\dagger\hat{a}_i^\dagger+\kappa^*\hat{a}_s\hat{a}_i] , \tag{1}$$

and thus the differential equation for $\hat{U}$ is

$$\frac{d\hat{U}}{dt}=-i\kappa_0[e^{i\theta}\hat{a}_s^\dagger\hat{a}_i^\dagger+e^{-i\theta}\hat{a}_s\hat{a}_i]\hat{U} , \tag{2}$$

where $\kappa=\kappa_0 e^{i\theta}$, κ_0 is a real constant, κ is a parametric interaction coefficient, and $\hat{a}_s$ and $\hat{a}_i$ are signal and idler annihilation operators.

From Refs. 12 and 13, we obtain the evolution operator, the output density operator with number state $|n_0\rangle$ as the input-signal state and the output density operator with coherent state $|\alpha_0\rangle$ as the input-signal state:

$$\hat{U}=\exp[-\ln\cosh(\kappa_0 t)]\exp[-ie^{i\theta}\tanh(\kappa_0 t)\hat{a}_i^\dagger\hat{a}_s^\dagger]\exp\{-\ln[\cosh(\kappa_0 t)\hat{a}_i^\dagger\hat{a}_i]\}$$
$$\times\exp\{-\ln[\cosh(\kappa_0 t)\hat{a}_s^\dagger\hat{a}_s]\}\exp[-ie^{-i\theta}\tanh(\kappa_0 t)\hat{a}_i\hat{a}_s] \ , \quad (3)$$

$$\hat{\rho}_{si,n_0}=N^2\cosh^{-2(n_0+1)}(\kappa_0 t)\sum_{l,h}(-ie^{i\theta}\tanh\kappa_0 t)^l$$
$$\times(ie^{-i\theta}\tanh\kappa_0 t)^h\left[\begin{bmatrix} n_0+l\\ n_0\end{bmatrix}\begin{bmatrix} n_0+h\\ n_0\end{bmatrix}\right]^{1/2}|n_0+l\rangle_s|l\rangle_{i\,i}\langle h|_s\langle n_0+h| \ , \quad (4)$$

$$\hat{\rho}_{si}=N'^2e^{-|\alpha_0|^2}\sum_{k,l,h,n}\left[\begin{bmatrix} k+l\\ k\end{bmatrix}\begin{bmatrix} h+n\\ h\end{bmatrix}\right]^{1/2}\frac{(-ie^{i\theta}\tanh\kappa_0 t)^l(ie^{-i\theta}\tanh\kappa_0 t)^n}{\cosh^2\kappa_0 t}$$
$$\times\left[\frac{\alpha_0}{\cosh\kappa_0 t}\right]^k\left[\frac{\alpha_0^*}{\cosh\kappa_0 t}\right]^h\frac{1}{\sqrt{k!}}\frac{1}{\sqrt{h!}}|k+l\rangle_s|l\rangle_{i\,i}\langle n|_s\langle h+n| \ , \quad (5)$$

where N and N' are normalization constants and

$$\begin{bmatrix} k\\ m\end{bmatrix}=\frac{k!}{(k-m)!m!} \ .$$

When we simply want to know the signal state after parametric amplification,the reduced density operator can be calculated. No measurement process is involved. The reduced density operator for a number-state input signal is calculated by taking the trace of (4) with respect to idler variables,[12]

$$\hat{\rho}_{s,n_0}^{(\mathrm{red})}\equiv\mathrm{Tr}_i\hat{\rho}_{si,n_0}=N^2\left[\frac{1}{\cosh\kappa_0 t}\right]^{2(n+1)}\sum_l\tanh^{2l}\kappa_0 t\begin{bmatrix} n_0+l\\ n_0\end{bmatrix}|n_0+l\rangle_{s\,s}\langle n_0+l| \ . \quad (6)$$

The quasiprobability density ${}_s\langle\alpha|\hat{\rho}_{s,n_0}^{(\mathrm{red})}|\alpha\rangle_s$ is schematically shown in Fig. 1 for $n_0=0$ and $n_0\neq 0$. The reduced density operator for a coherent-state input signal is calculated by taking the trace of (5),[12]

$$\hat{\rho}_s^{(\mathrm{red})}=N'^2e^{-|\alpha_0|^2}\sum_{k,l,h}\left[\begin{bmatrix} k+l\\ k\end{bmatrix}\begin{bmatrix} h+l\\ h\end{bmatrix}\right]^{1/2}\frac{\tanh^{2l}\kappa_0 t}{\cosh^2\kappa_0 t}\left[\frac{\alpha_0}{\cosh\kappa_0 t}\right]^k\left[\frac{\alpha_0^*}{\cosh\kappa_0 t}\right]^h\frac{1}{\sqrt{k!}}\frac{1}{\sqrt{h!}}|k+l\rangle_{s\,s}\langle h+l| \ . \quad (7)$$

The quasiprobability density ${}_s\langle\alpha|\hat{\rho}_s^{(\mathrm{red})}|\alpha\rangle_s$ is schematically shown in Fig. 2.

III. STATE REDUCTION BY MEASUREMENT OF IDLER QUADRATURE AMPLITUDE

In this section and Sec. IV and Appendix A, we discuss the effect of idler output measurement on signal output. The conditional density operator is calculated by the well-defined generalized projection[6,7] (operator-valued measure).

A measurement of idler quadrature amplitude with readout α_1' is described by

$$|\alpha_1'\rangle_{i\,i}\langle\alpha_1'| \ . \quad (8)$$

Since α_1 is defined over all real values of α_1, $|\alpha_1\rangle_{i\,i}\langle\alpha_1|$ means continuous projection. The operation-valued measure is introduced and removes the difficulty in the continuous spectrum.[14] The density operator of a signal wave after readout α_1' is

$$\hat{\rho}_s^{(\mathrm{meas},\alpha_1')}\equiv\mathrm{Tr}_i\hat{\rho}_i^{(\mathrm{read},\alpha_1')}\hat{\rho}_{si}$$
$$=N'^2\sum_{k,l,h,m}\frac{1}{\sqrt{m!l!}}\left[\frac{2}{\pi}\right]^{1/2}\exp(-2\alpha_1'^2)\frac{H_m(\sqrt{2}\alpha_1')}{2^{m/2}}\frac{H_l(\sqrt{2}\alpha_1')}{2^{l/2}}\exp(-|\alpha|^2)\left[\begin{bmatrix} h+m\\ h\end{bmatrix}\begin{bmatrix} k+l\\ k\end{bmatrix}\right]^{1/2}$$
$$\times\frac{[-i\tanh(\kappa_0 t)e^{+i\theta}]^l[+i\tanh(\kappa_0 t)e^{-i\theta}]^m}{\cosh^2\kappa_0 t}\frac{\alpha_A^k}{\sqrt{k!}}\frac{\alpha_A^{*h}}{\sqrt{h!}}|k+l\rangle_{s\,s}\langle h+m| \ , \quad (9)$$

where a coherent-state input signal is assumed and

$$\hat{\rho}_i^{(\mathrm{read},\alpha_1')} = 1_s \otimes |\alpha_1'\rangle_{i\,i}\langle\alpha_1'| \ . \tag{10}$$

In deriving (9), we used the following equation:[5]

$$\langle m \mid \alpha_1'\rangle = \frac{1}{\sqrt{m!}}\left[\frac{2}{\pi}\right]^{1/4}\exp(-\alpha_1'^2)\frac{H_m(\sqrt{2}\alpha_1')}{2^{m/2}} \ . \tag{11}$$

Next the quasiprobability density of the conditional density operator is calculated. Without loss of generality, we assume $\theta=\pi/2$. Then the quasiprobability density is

$$\begin{aligned} Q^{(\mathrm{meas},\alpha_1')}(\alpha) &\equiv {}_s\langle\alpha|\hat{\rho}_s^{(\mathrm{meas},\alpha_1')}|\alpha\rangle_s \\ &= N_Q \exp[-(1+\tanh^2\kappa_0 t)(\alpha_1-\langle\alpha_1\rangle)^2]\exp\left[-\frac{1}{\cosh^2\kappa_0 t}(\alpha_2-\langle\alpha_2\rangle)^2\right] , \end{aligned} \tag{12}$$

where

$$N_Q = \left[\frac{1+\tanh^2(\kappa_0 t)}{\pi}\right]^{1/2}\left[\frac{1}{\pi\cosh^2(\kappa_0 t)}\right]^{1/2} ,$$

$$\langle\alpha_1\rangle = \frac{2\tanh(\kappa_0 t)\alpha_1' + \dfrac{\alpha_{0,1}}{\cosh(\kappa_0 t)}}{1+\tanh^2(\kappa_0 t)} ,$$

$$\langle\alpha_2\rangle = \alpha_{0,2}\cosh(\kappa_0 t) \ .$$

The quasiprobability density is Gaussian, centered at $\langle\alpha_1\rangle$ and $\langle\alpha_2\rangle$. The dispersion of the quadrature amplitudes can be calculated by using (12) as

$$\langle\Delta\alpha_1^2\rangle = \langle\Delta\alpha_1^2\rangle_Q - \tfrac{1}{4} = \frac{1}{4\cosh(2\kappa_0 t)} , \tag{13}$$

$$\langle\Delta\alpha_2^2\rangle = \langle\Delta\alpha_2^2\rangle_Q - \tfrac{1}{4} = \frac{\cosh(2\kappa_0 t)}{4} \ . \tag{14}$$

Here $\langle\Delta\alpha_1^2\rangle_Q$ and $\langle\Delta\alpha_2^2\rangle_Q$ are the variances of the quasiprobability density (12) that are larger by $\frac{1}{4}$ than the intrinsic variances or $\hat{\rho}_s^{(\mathrm{meas},\alpha_1')}$. The output-signal wave is reduced to a quadrature-amplitude squeezed state satisfying the minimum uncertainty product

$$\langle\Delta\alpha_1^2\rangle\langle\Delta\alpha_2^2\rangle = \tfrac{1}{16} \ . \tag{15}$$

The quasiprobability density ${}_s\langle\alpha|\hat{\rho}_s^{(\mathrm{meas},\alpha_1')}|\alpha\rangle_s$ is compared with that of the reduced density operator in Fig. 3.

IV. STATE REDUCTION BY A MEASUREMENT OF TWO IDLER QUADRATURE AMPLITUDES

A simultaneous measurement of two-quadrature amplitudes is described by the operation-valued measure

$$|\alpha'\rangle_{i\,i}\langle\alpha'| \ . \tag{16}$$

This corresponds to approximate simultaneous measurement.[4] Following the idea of operation-valued measure,[6] measurement of α_i by heterodynamic involves the product spaces $|\alpha_i\rangle_1\,|0\rangle_2$, where 1 stands for the idler band and 2 stands for the image band that is in a vacuum state. Measurement of $|\alpha_i\rangle$ couples unavoidingly to the zero-point fluctuations of the image band. Thus a measurement of $|\alpha_i\rangle$ must be interpreted as taking a trace of the product density matrix

$$\hat{\rho}_1 \otimes |0\rangle_{2\,2}\langle 0| ,$$

i.e., forming the expression

$$\mathrm{Tr}_1(\hat{\rho}_1 \otimes |0\rangle_{2\,2}\langle 0|\alpha\rangle_{1\,1}\langle\alpha|) \ . \tag{17}$$

The conditional density operator after the readout α' is

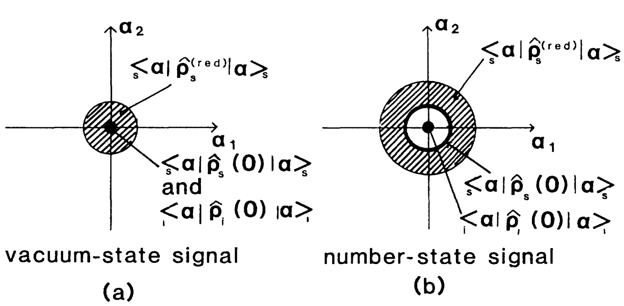

FIG. 1. (a) Quasiprobability densities of the initial density operators $\hat{\rho}_s(0)=|0\rangle_{s\,s}\langle 0|$ and $\hat{\rho}_i(0)=|0\rangle_{i\,i}\langle 0|$ and the reduced density operator $\hat{\rho}_s^{(\mathrm{red})}$. (b) Quasiprobability densities of the initial density operators $\hat{\rho}_s(0)=|n_0\rangle_{s\,s}\langle n_0|, \hat{\rho}_i(0)=|0\rangle_{i\,i}\langle 0|$ and the reduced density operator $\hat{\rho}_s^{(\mathrm{red})}$.

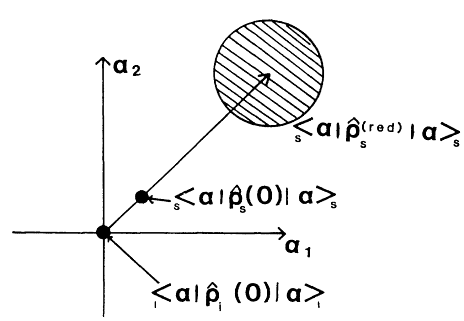

FIG. 2. Quasiprobability densities of the initial density operators $\hat{\rho}_s(0)=|\alpha_0\rangle_{s\,s}\langle\alpha_0|$ and $\hat{\rho}_i(0)=|0\rangle_{i\,i}\langle 0|$ and the reduced density operator $\hat{\rho}_s^{(\mathrm{red})}$.

$$\hat{\rho}_s^{(\text{meas},\alpha')} \equiv \text{Tr}_i \hat{\rho}_i^{(\text{read},\alpha')} \hat{\rho}_{si}$$
$$= N'^2 \sum_{k,h,l,n} \exp(-|\alpha'|^2) \exp(-|\alpha|^2) \frac{\alpha'^n}{\sqrt{n!}} \frac{\alpha'^l}{\sqrt{l!}} \left[\begin{pmatrix} h+n \\ h \end{pmatrix} \begin{pmatrix} k+l \\ k \end{pmatrix} \right]^{1/2} \frac{[-i\tanh(\kappa_0 t)e^{i\theta}]^l [i\tanh(\kappa_0 t)e^{-i\theta}]^n}{\cosh^2(\kappa_0 t)}$$
$$\times \frac{\alpha_A^k}{\sqrt{k!}} \frac{\alpha_A^{*h}}{\sqrt{h!}} |k+l\rangle_{s\,s}\langle h+n| \ . \tag{18}$$

Here

$$\hat{\rho}_i^{(\text{read},\alpha')} = \mathbf{1}_s \otimes |\alpha'\rangle_{i\,i}\langle \alpha'| \ . \tag{19}$$

The quasiprobability density is

$$Q^{(\text{meas},\alpha')}(\alpha) \equiv {}_s\langle \alpha | \hat{\rho}_s^{(\text{meas},\alpha')} | \alpha \rangle_s$$
$$= \frac{1}{\pi} \exp - |\alpha - [i\tanh(\kappa_0 t)\alpha'^* e^{-i\theta} - \alpha_A]|^2 \ . \tag{20}$$

If $\theta = \pi/2$, the quasiprobability density is circular centered at $\langle \alpha_1 \rangle = \tanh(\kappa_0 t)\alpha_1' + \alpha_{0,1}/\cosh(\kappa_0 t)$, $\langle \alpha_2 \rangle = -\tanh(\kappa_0 t)\alpha_2' + \alpha_{0,2}/\cosh(\kappa_0 t)$. If the difference between the real dispersion and the dispersion of the quasiprobability density is taken into account, the output signal is reduced to a coherent state,

$$\langle \Delta\alpha_1^2 \rangle = \tfrac{1}{4} \ ,$$

and

$$\langle \Delta\alpha_2^2 \rangle = \tfrac{1}{4} \ . \tag{21}$$

The quasiprobability density ${}_s\langle \alpha | \hat{\rho}_s^{(\text{meas},\alpha')} | \alpha \rangle_s$ is compared with that of the reduced density operator in Fig. 4.

V. PROJECTION OPERATORS GENERATING A NUMBER-PHASE SQUEEZED STATE, QUADRATURE-AMPLITUDE SQUEEZED STATE, AND COHERENT STATE

In Secs. III and IV we have shown that a nondegenerate parametric amplification process followed by a measurement produces various quantum light. Such a quantum-light-generation process based on state reduction resulting from measurement is characterized by a projection operator[15]

$$\hat{P} = {}_p\langle \psi | \hat{U} | \phi \rangle_p \ . \tag{22}$$

Here $|\phi\rangle_p$ and $|\psi\rangle_p$ are the initial (prepared) state and final (measured) state of the probe system and $\hat{U}$ is the evolution operator.

The projection operator $\hat{P}^{(m)}$ that characterizes an idler photon-number measurement scheme is written as

$$\hat{P}^{(m)} \equiv {}_i\langle m | \hat{U} | 0 \rangle_i$$
$$= \frac{1}{\cosh(\kappa_0 t)} \frac{[-ie^{i\theta}\tanh(\kappa_0 t)]^m}{\sqrt{m!}} \hat{a}_s^{\dagger m}$$
$$\times \exp\{-\ln[\cosh(\kappa_0 t)\hat{a}_s^\dagger \hat{a}_s]\} \ . \tag{23}$$

This projection operator generates a number state $|n_0 + m\rangle_s$ from a number state $|n_0\rangle_s$. It also generates a number-phase squeezed state (41) in Appendix A from a coherent state $|\alpha_0\rangle_s$ (Ref. 11) (see also Appendixes A, B, and C).

The projection operator $\hat{P}^{(\alpha_1')}$ that characterizes an idler quadrature-amplitude measurement scheme is given by

$$\hat{P}^{(\alpha_1')} \equiv {}_i\langle \alpha_1' | \hat{U} | 0 \rangle_i$$
$$= N'' e^{-\alpha_1'^2} \exp\left[-2i\alpha_1' \tanh(\kappa_0 t) e^{i\theta} \hat{a}_s^\dagger + \frac{\tanh^2(\kappa_0 t) e^{2i\theta}}{2} \hat{a}_s^{\dagger 2} \right]$$
$$\times \exp[-\ln(\cosh\kappa_0 t)\hat{a}_s^\dagger \hat{a}_s] \ . \tag{24}$$

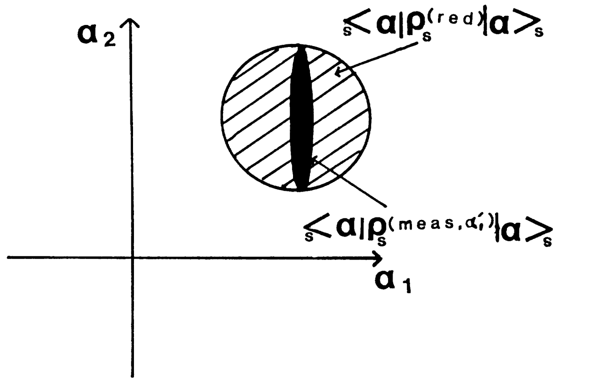

FIG. 3. Quasiprobability densities of the reduced density operator and conditional density operator for and idler wave quadrature-amplitude measurement. $\hat{\rho}_s(0) = |\alpha_0\rangle_{s\,s}\langle \alpha_0|$.

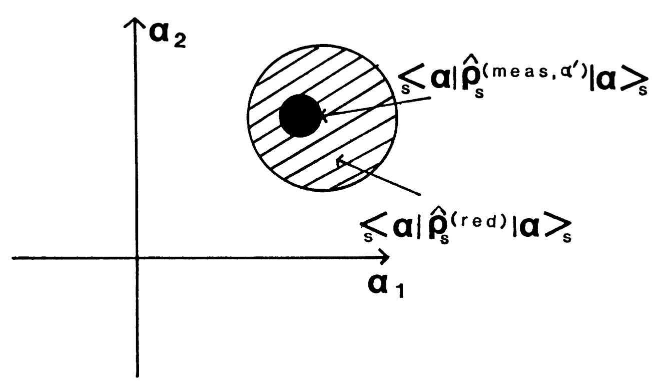

FIG. 4. Quasiprobability densities of the reduced density operator and conditional density operator for a measurement of the two idler wave quadrature amplitudes. $\hat{\rho}_s(0) = |\alpha_0\rangle_{s\,s}\langle \alpha_0|$.

Here $N''=[1/\cosh(\kappa_0 t)](2/\pi)^{1/4}$. This projection operator generates a quadrature-amplitude squeezed state (9) from a coherent state $|\alpha_0\rangle_s$.

The projection operator $\hat{P}^{(\alpha')}$ that characterizes a simultaneous-measurement scheme of the two idler quadrature amplitude is written as

$$\hat{P}^{(\alpha')} \equiv {}_i\langle \alpha' | \hat{U} | 0 \rangle_i$$
$$= \frac{e^{-|\alpha|^2/2}}{\cosh(\kappa_0 t)} \exp[-\alpha'^{*}\tanh(\kappa_0 t) e^{i\theta} \hat{a}_s^{\dagger}]$$
$$\times \exp\{-\ln[\cosh(\kappa_0 t)\hat{a}_s^{\dagger}\hat{a}_s]\} . \quad (25)$$

This projection operator generates a coherent state (18) from a coherent state $|\alpha_0\rangle_s$.

VI. FEEDFORWARD

Even though an output signal wave is reduced to a number-phase squeezed state or quadrature-amplitude squeezed state for a specific readout m or α_1' as demonstrated above, it is not considered to be a practical quantum-light-generation scheme. This is because each measurement produces a different readout and, if we want to know the quantum-statistical properties of all samples over all possible readouts, it is nothing but a signal-reduced density operator, as shown in Fig. 5.

In this section, we propose a method to overcome the preceding difficulty and produce a squeezed state continuously. Suppose a feedforward process operates on the output signal such that all the conditional density operators are translated to the same mean values $\langle \alpha_1 \rangle$ and $\langle \alpha_2 \rangle$ by using the measurement result as shown in Fig. 5. If the readout α_1'' is different from the most probable value α_1', then a translation operator

$$\hat{D}(\alpha_1'') = \exp[(\alpha_1' - \alpha_1'')(\hat{a}_s^{\dagger} + \hat{a}_s)] \quad (26)$$

acts on the conditional density operator $\hat{\rho}_s^{(\text{meas},\alpha'')}$. Then the result is

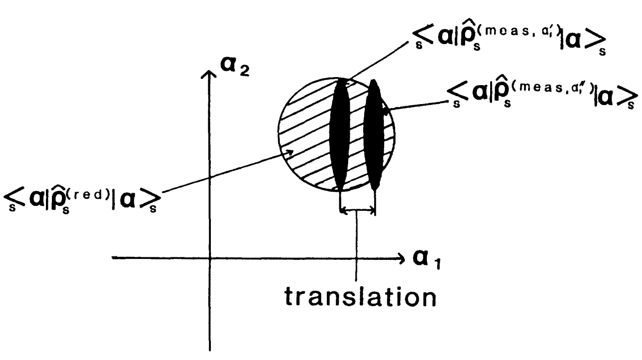

FIG. 5. Quasiprobability densities of the conditional density operator for different readouts α' and α''. Unitary translation operator $D(\alpha'')$ maps $\hat{\rho}_s^{(\text{meas},\alpha_1'')}$ onto $\hat{\rho}_s^{(\text{meas},\alpha_1')}$.

$${}_s\langle \alpha | \hat{D}(\alpha_1'') \hat{\rho}_s^{(\text{meas},\alpha_1'')} \hat{D}^{\dagger}(\alpha_1'') | \alpha \rangle_s$$
$$= \left[\frac{1+\tanh^2(\kappa_0 t)}{\pi}\right]^{1/2} \left[\frac{1}{\pi\cosh^2(\kappa_0 t)}\right]^{1/2}$$
$$\times \exp\{-[1+\tanh^2(\kappa_0 t)](\alpha_1 - \langle \alpha_1 \rangle^2\}$$
$$\times \exp\left[-\frac{1}{\cosh^2(\kappa_0 t)}(\alpha_2 - \langle \alpha_2 \rangle)^2\right] . \quad (27)$$

It is seen from (27) that a signal wave is always reduced to the same quadrature-amplitude squeezed state irrespective of the readout α_1''. The translation operator $D(\alpha_1'')$ is practically realized by a phase modulator.[10]

VII. QUANTUM CORRELATION OF PHOTON TWINS EXPRESSED BY A HEISENBERG PICTURE

In this section we study the quantum correlation of photon twins in a Heisenberg picture and will come to the same conclusions we have previously obtained using a Schrödinger picture. If phase angle is appropriately selected, the input-output operators in a Heisenberg picture are described by

$$\hat{b}_s = \cosh(\kappa_0 t)\hat{a}_s + \sinh(\kappa_0 t)\hat{a}_i^{\dagger} , \quad (28)$$

and

$$\hat{b}_i^{\dagger} = \sinh(\kappa_0 t)\hat{a}_s + \cosh(\kappa_0 t)\hat{a}_i^{\dagger} . \quad (29)$$

From (28) and (29), and taking account of the commutation relation $[\hat{a}_s, \hat{a}_i^{\dagger}]=0$, the difference between the signal photon number and probe photon number satisfies the following Manley-Rowe operator relation:

$$\hat{m}_s - \hat{m}_p = \hat{n}_s - \hat{n}_p . \quad (30)$$

Here $\hat{m}_s = \hat{b}_s^{\dagger}\hat{b}_s$, $\hat{m}_i = \hat{b}_i^{\dagger}\hat{b}_i$, $\hat{n}_s = \hat{a}_s^{\dagger}\hat{a}_s$, and $\hat{n}_i = \hat{a}_i^{\dagger}\hat{a}_i$. The dispersion difference between the signal and probe photon number is

$$\langle \Delta(\hat{m}_s - \hat{m}_i)^2 \rangle = \langle \Delta(\hat{n}_s - \hat{n}_i)^2 \rangle$$
$$= \begin{cases} 0 & \text{(number-state input)} \\ |\alpha_0|^2 & \text{(coherent-state input)} . \end{cases} \quad (31)$$

Here we use the fact that the number dispersion of the input idler wave in a vacuum state is zero.

The complete photon-number correlation for an input signal in a number state corresponds to the result of (40). When an input signal is in a coherent state, on the other hand, the photon-number correlation is partly degraded by the photon-number variance $\langle \Delta\hat{n}_s^2 \rangle = |\alpha_0|^2$ of an input signal. At first sight this result seems to contradict the conclusion of (47) in Appendix A. But notice that (47) represents a dispersion in photon number for a readout m close to the most provable value $|\alpha_0|^2\cosh^2\kappa_0 t$ and that (31) represents the ensemble average of the photon-number correlation for all possible m_i values. In fact, the dispersion in photon number for a readout m much smaller than $|\alpha_0|^2\cosh\kappa_0 t$ is reduced to zero, and that for a readout m much greater than $|\alpha_0|^2\cosh\kappa_0 t$ is greater than $|\alpha_0|^2$. When these different dispersions are integrated over all possible readout values with a proper probability density, we find a dispersion of not $|\alpha_0|^2/2$, but $|\alpha_0|^2$ (see Appendix D).

From (28) and (29), the output in-phase and phase-quadrature operators are related to the input operators by

$$\hat{b}_{s,1}=\cosh(\kappa_0 t)\hat{a}_{s,1}+\sinh(\kappa_0 t)\hat{a}_{i,1} , \tag{32}$$

$$\hat{b}_{i,1}=\sinh(\kappa_0 t)\hat{a}_{s,1}+\cosh(\kappa_0 t)\hat{a}_{i,1} . \tag{33}$$

The difference of output in-phase operators is

$$\hat{b}_{s,1}-\hat{b}_{i,1}=\exp(-\kappa_0 t)(\hat{a}_{s,1}-\hat{a}_{i,1}) , \tag{34}$$

therefore its dispersion is

$$\langle \Delta(\hat{b}_{s,1}-\hat{b}_{i,1})^2=\tfrac{1}{2}\exp(-2\kappa_0 t) . \tag{35}$$

This result corresponds to the squeezed quadrature-amplitude noise (13). On the other hand, the quadrature component dispersion itself is

$$\langle \Delta\hat{b}_{s,2}\rangle^2=\tfrac{1}{8}[\exp(-2\kappa_0 t)+\exp(2\kappa_0 t)] . \tag{36}$$

This corresponds to the enhanced quadrature-amplitude noise (14).

The dispersion of the sum of quadrature phase operators is similarly calculated as

$$\langle \Delta(\hat{b}_{s,2}+\hat{b}_{i,2})^2\rangle=\tfrac{1}{2}\exp(-2\kappa_0 t) . \tag{37}$$

They feature equally strong negative correlation.

VIII. CONCLUSION

Photon twins (signal and idler waves) produced by a high-gain parametric amplifies feature strong positive correlation for both their photon numbers and in-phase amplitudes, and equally strong negative correlation for their phases and quadrature-phase amplitudes. If the idler photon number, the single-quadrature amplitude, and the two-quadrature amplitudes are measured by a photon counter, homodyne detector, and heterodyne detector, the signal wave is reduced to a number-phase squeezed state, quadrature-amplitude squeezed state, and coherent state, respectively. The projection operators characterizing these nonunitary processes are given. With a feedforward technique, these can be used to generate various quantum lights.

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APPENDIX A

We discuss the effect of a photon-number measurement of idler output on signal output. A measurement of idler photon number with count m is described by the projection operator

$$|m\rangle_{i\,i}\langle m| . \tag{38}$$

The density operator of a signal wave after the readout is calculated by

$$\hat{\rho}_s^{(\mathrm{meas},m)}\equiv \mathrm{Tr}_i \hat{\rho}_i^{(\mathrm{read},m)}\hat{\rho}_{si} , \tag{39}$$

where $\hat{\rho}_i^{(\mathrm{read},m)}=1_s\otimes|m\rangle_{i\,i}\langle m|$ and 1_s is an identity operator for a single wave.

For a number state input case, we obtain

$$\hat{\rho}_s^{(\mathrm{meas},m)}=|n_0+m\rangle_{s\,s}\langle n_0+m| . \tag{40}$$

From (4) the output signal wave is also in a number state translated from the initial value n_0 by the exact number of readout m.

For coherent-state input, we get the following from (5):[12]

$$\hat{\rho}_s^{(\mathrm{meas},m)}=N'^2\sum_{k,h}\exp(-|\alpha_0|^2)\left[\begin{bmatrix}h+m\\h\end{bmatrix}\begin{bmatrix}k+m\\k\end{bmatrix}\right]^{1/2}\frac{\tanh^{2m}(\kappa_0 t)}{\cosh^2(\kappa_0 t)} \times\left[\frac{\alpha_0}{\cosh(\kappa_0 t)}\right]^k\left[\frac{\alpha_0^*}{\cosh(\kappa_0 t)}\right]^h\left[\frac{1}{h!k!}\right]^{1/2}|k+m\rangle_{s\,s}\langle h+m| . \tag{41}$$

The photon-number probability density function is calculated as

$$P(n)\equiv{}_s\langle n|\hat{\rho}_s^{(\mathrm{meas},m)}|n\rangle_s =\frac{1}{F(1+m,1;N_A)}\begin{bmatrix}n\\n-m\end{bmatrix}N_A^{n-m}\frac{1}{(n-m)!} , \tag{42}$$

where $N_A=|\alpha_0|^2/\cosh^2(\kappa_0 t)$ and $F(a,b;c)$ is Kummer function given by

$$F(a,b;c)=\frac{\Gamma(1+a)}{\Gamma(1+b)}\sum_j\frac{\Gamma(1+a+j)c^j}{\Gamma(1+b+j)j!} . \tag{43}$$

The average photon number and its dispersion is obtained using the probability $P(n)$ as

$$\langle n\rangle\equiv\sum_{j>m}jP(j)=(m+1)\frac{L_{m+1}(-N_A)}{L_m(-N_A)}-1 , \tag{44}$$

$$\langle\Delta n^2\rangle\equiv\sum_{j>m}(j-\langle j\rangle)^2P(j) =(m+2)(m+1)\frac{L_{m+2}(-N_A)}{L_m(-N_A)} -(m+1)^2\left[\frac{L_{m+1}(-N_A)}{L_m(-N_A)}\right]^2 -(m+1)\frac{L_{m+1}(-N_A)}{L_m(-N_A)} . \tag{45}$$

Derivation of (45) used the relation

$$F(1+m,\ 1;N_A)=L_m(-N_A)\exp(N_A)\ ,$$

where $L_m(x)$ is a Laggure polynomial. When the parametric-amplifier gain is high enough, $\kappa_0 t \gg 1$, and the readout m is in the vicinity of the most probable value, $m \sim |\alpha_0|^2\cosh^2\kappa_0 t$, the average photon number and its dispersion derived in Appendix B are approximately given by

$$\langle n\rangle \sim m+|\alpha_0|^2\ , \tag{46}$$

and

$$\langle \Delta n^2\rangle \sim \frac{|\alpha_0|^2}{2}\ . \tag{47}$$

Note that the input signal has the photon-number dispersion $\langle \Delta n(0)^2\rangle = |\alpha_0|^2$. It is reduced by a factor of 2 after the parametric amplification and photon-counting measurement. This result was first discovered by Yuen[16] and was later confirmed numerically by Kitagawa and Yamamoto.[12] Since m is much greater than $|\alpha_0|^2$ in a high-gain parametric amplifier, the signal wave features a strong sub-Poissonian distribution.

Next the phase dispersion is calculated. The sine and cosine probability density functions are calculated in a manner similar to the photon-number probability density function,

$$P(\cos\psi)={}_s\langle \cos\psi\,|\,\hat\rho_s^{(\mathrm{meas},m)}\,|\,\cos\psi\rangle_s$$
$$=\sum_{k,h}\frac{2}{\pi}\sin(k+m+1)\psi\sin(h+m+1)\psi\left[\binom{h+n}{h}\binom{k+m}{k}\right]^{1/2}\frac{\alpha_A^k}{\sqrt{k!}}\frac{\alpha_A^{*h}}{\sqrt{h!}}\ , \tag{48}$$

$$P(\sin\psi)\equiv{}_s\langle \sin\psi\,|\,\hat\rho_s^{(\mathrm{meas},m)}\,|\,\sin\psi\rangle_s$$
$$=\sum_{k,h}\frac{1}{2\pi}[e^{i(k+m+1)\psi}-e^{-i(k+m+1)(\psi-\pi)}][e^{i(h+m+1)\psi}-e^{-i(h+m+1)(\psi-\pi)}]$$
$$\times\left[\binom{h+m}{h}\binom{k+m}{k}\right]^{1/2}\frac{\alpha_A^k}{\sqrt{k!}}\frac{\alpha_A^{*h}}{\sqrt{h!}}\ , \tag{49}$$

where $\alpha_A=\alpha_0/\cos(\kappa_0 t)$. Straightforward calculation leads to

$$\langle\cos\psi\rangle\equiv\int_0^\pi \cos\psi\,P(\cos\psi)d\psi$$
$$=N_A^{1/2}[F(1+m,\ 1;N_A)]^{-1}\Psi_{1,m}(N_A)\cos\phi_{\alpha_0}\ , \tag{50}$$

$$\langle\sin\psi\rangle\equiv\int_{-\pi/2}^{\pi/2} \sin\psi\,P(\sin\psi)d\psi$$
$$=N_A^{1/2}[F(1+m,\ 1;N_A)]^{-1}\Psi_{1,m}(N_A)\sin\phi_{\alpha_0}\ , \tag{51}$$

$$\langle\cos^2\psi\rangle\equiv\int_0^\pi \cos^2\psi\,P(\cos\psi)d\psi$$
$$=\tfrac{1}{2}-\frac{\delta_{m,0}}{4}[F(1+m,\ 1;N_A)]^{-1}+\frac{N_A}{2}[F(1+m,\ 1;N_A)]^{-1}\Psi_{2,m}(N_A)\times\cos(2\phi_{\alpha_0})\ , \tag{52}$$

$$\langle\sin^2\psi\rangle\equiv\int_{-\pi/2}^{\pi/2} \sin^2\psi\,P(\sin\psi)d\psi$$
$$=\tfrac{1}{2}-\frac{\delta_{m,0}}{4}[F(1+m,\ 1;N_A)]^{-1}-\frac{N_A}{2}[F(1+m,\ 1;N_A)]^{-1}\Psi_{2,m}(N_A)\cos(2\phi_{\alpha_0})\ , \tag{53}$$

where $\phi_{\alpha_0}=\arctan\alpha_0$ and $\cos\phi_{\alpha_0}$ and $\sin\phi_{\alpha_0}$ are the average sine and cosine operator values of an input signal. $\delta_{i,j}$ is Kronecker's δ,

$$\Psi_{1,m}(N_A)=\sum_k\frac{(k+m)!}{k!m!}\left[\frac{k+m+1}{k+1}\right]^{1/2}\frac{N_A^k}{k!\sqrt{k+1}} \tag{54}$$

and

$$\Psi_{2,m}(N_A)=\sum_k\frac{(k+m)!}{k!m!}\left[\frac{(k+m+2)(k+m+1)}{(k+2)(k+1)}\right]^{1/2}\times\frac{N_A^k}{k!\sqrt{(k+2)(k+1)}}\ . \tag{55}$$

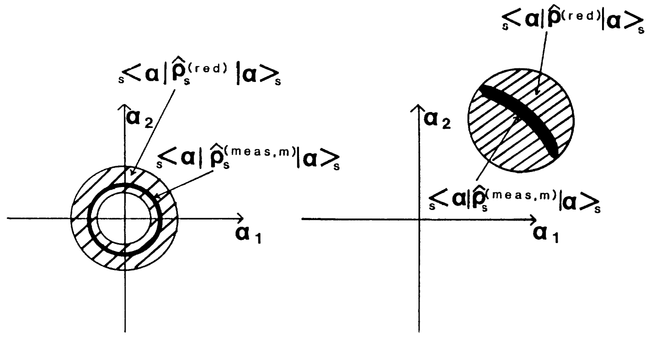
Note that $\Psi_{1,0}(N_A)=\Psi_1(N_A)$ and $\Psi_{2,0}(N_A)=\Psi_2(N_A)$, where $\Psi_1(N_A)$ and $\Psi_2(N_A)$ are shown in Ref. 17.

When the parametric-amplifier gain is high enough, $\kappa_0 t\gg 1$, and the readout m is in the vicinity of the most provable value,

$$m\sim|\alpha_0|^2\cosh^2(\kappa_0 t)\ ,$$

the average sine and cosine operator values and the normalized dispersion are approximately calculated (see Appendix C) as

$$\langle\cos\psi\rangle\sim\left[1-\frac{1}{4|\alpha_0|^2}\right]\cos\phi_{\alpha_0}\ , \tag{56}$$



number-state input signal (a)

coherent-state input signal (b)

FIG. 6. Quasiprobability densities of the reduced density operator and conditional density operator for an idler photon-number measurement. (a) $\hat{\rho}_s(0)=|n_0\rangle_{s\,s}\langle n_0|$ and (b) $\hat{\rho}_s(0)=|\alpha_0\rangle_{s\,s}\langle\alpha_0|$.

$$\langle \sin\psi\rangle \sim \left[1-\frac{1}{4|\alpha_0|^2}\right]\sin\phi_{\alpha_0} , \tag{57}$$

and

$$\frac{\langle \Delta\sin^2\psi\rangle}{\langle\cos\psi\rangle}\sim\frac{1}{2|\alpha_0|^2} . \tag{58}$$

Note that the normalized sine operator dispersion of an input-signal wave is $1/4|\alpha_0|^2$. It is enhanced by a factor of 2 after parametric amplification. This doubling of quantum noise is a manifestation of a general quantum limit of linear amplifiers and simultaneous measurement of the two conjugate observables.[18]

From (47) and (58) we see that the signal wave after measurement of idler photon number is reduced to a number-phase squeezed state, satisfying the minimum uncertainty product of photon number and sine operators,

$$\frac{\langle\Delta n^2\rangle\langle\Delta\sin^2\psi\rangle}{\langle\cos\psi\rangle^2}\sim\tfrac{1}{4} . \tag{59}$$

The quasiprobability density $_s\langle\alpha|\hat{\rho}_s^{(\mathrm{meas},m)}|\alpha\rangle_s$ of thus generated number states and number-phase squeezed states are compared with those for the reduced density operator in Fig. 6.

APPENDIX B

We derive the approximate value of photon-number dispersion (47). Suppose the readout m of the output idler photon number is in the vicinity of the most provable, $|\alpha_0|^2\cosh^2\kappa_0 t$. The relations

$$L_m(-z)=e^{-z}F(1+m,1;z) \tag{60}$$

and

$$F(a+1,b;x)=F(a,b;x)+\frac{x}{b}F(a+1,b+1;x) \tag{61}$$

from (45) give us

$$\begin{aligned}\langle\Delta n^2\rangle=&2(m+2)(m+1)z\frac{F(m+2,2,z)}{F(m+1,1,z)}\\&+(m+2)(m+1)\frac{z^2}{2}\frac{F(m+3,3,z)}{F(m+1,1,z)}\\&-(m+1)^2z^2\left[\frac{F(m+2,2,z)}{F(m+1,1,z)}\right]^2\\&-2(m+1)^2z\frac{F(m+2,2,z)}{F(m+1,1,z)}\\&-(m+1)z\frac{F(m+2,2,z)}{F(m+1,1,z)} ,\end{aligned} \tag{62}$$

where $z=|\alpha_0|^2/\cosh(\kappa_0 t)$. Further applying the relations,

$$F\left[a,c;\frac{x}{a}\right]\to\Gamma(c)x^{(1-c)/2}I_{c-1}(2\sqrt{x}) \quad \text{as } a\to\infty \tag{63}$$

and

$$I_{c+1}(x)=I_{c-1}(x)-\frac{2c}{x}I_c(x) , \tag{64}$$

and taking the limit as $z\to 0$ and $zm\to z_0$, we obtain

$$\langle\Delta n^2\rangle\to z_0\left[1-\left[\frac{I_1(2\sqrt{z})}{I_0(2\sqrt{z})}\right]^2\right] , \tag{65}$$

where $\Gamma(x)$ is the γ function and $I_k(x)$ is the modified Bessel function.

If the input average photon number is much greater than one, $z_0 \gg 1$,

$$I_0(2\sqrt{z_0})=e^{2\sqrt{z_0}}F(\tfrac{1}{2},1,-4\sqrt{z_0}) . \tag{66}$$

By using (66) and the expansion

$$F(a,b,-x)\sim x^{-a}\frac{\Gamma(b)}{\Gamma(b-a)}[1+a(1+a-b)]x^{-1} \tag{67}$$

in (65), we obtain

$$\frac{I_1(2\sqrt{z})}{I_0(2\sqrt{z})}\sim 1-\tfrac{1}{4}z_0^{-1/2} \tag{68}$$

and

$$\langle\Delta n^2\rangle\sim\tfrac{1}{2}z_0^{1/2}\sim\tfrac{1}{2}|\alpha_0|^2 . \tag{69}$$

APPENDIX C

We derive the normalized sine dispersion (58). Equation (54) can be rewritten as

$$\Psi_{1,m}(z)=\frac{1}{m^{1/2}}\sum_k\frac{(k+m)!}{(k+1)!(m-n)!}\left[1+\frac{k+1}{m}\right]^{1/2}\frac{z^k}{k!} . \tag{70}$$

Since m is sufficiently large, there is always an N_0 such that $k+1/m$ for $k<N_0$. Since the terms for $k>N$ do not contribute to the sum, we can set

$$\begin{aligned}\Psi_{1,m}(z)&\sim\frac{1}{m^{1/2}}\sum_k\frac{(k+m)!}{(k+1)!(m-1)!}\frac{z^k}{k!}\\&=m^{1/2}F(1+m,2;z) .\end{aligned} \tag{71}$$

From (50), (51), and (63), we obtain

$$\langle \cos\psi \rangle \sim \frac{I_1(2\sqrt{z_0})}{I_0(2\sqrt{z_0})}\cos\phi_{\alpha_0} \sim \left[1-\frac{1}{4|\alpha_0|^2}\right]\cos\phi_{\alpha_0} ,$$
$$\langle \sin\psi \rangle \sim \frac{I_1(2\sqrt{z_0})}{I_0(2\sqrt{z_0})}\sin\phi_{\alpha_0} \sim \left[1-\frac{1}{4|\alpha_0|^2}\right]\sin\phi_{\alpha_0} . \tag{72}$$

By a similar approximation, we get

$$\Psi_{2,m}(z) \sim \sum_k \frac{(k+m)!}{(k+2)!(m-1)!}\frac{z^k}{k!}$$
$$= \frac{m}{2}F(1+m, 3; z)$$
$$\sim m z_0^{-1} I_2(2\sqrt{z_0}) . \tag{73}$$

Here (63) was used. By (53), (63), and (73), we obtain

$$\langle \sin^2\psi \rangle \sim \frac{1}{2}\left[1-\frac{I_2(2\sqrt{z_0})}{I_0(2\sqrt{z_0})}\cos2\phi_{\alpha_0}\right] . \tag{74}$$

From (64) and (68), we get

$$\frac{\langle \sin^2\psi \rangle}{\langle \cos\psi \rangle^2} \sim \tan^2\phi_{\alpha_0} + \tfrac{1}{2} z_0^{-1/2} . \tag{75}$$

Since $|\langle \sin\psi\rangle / \langle \cos\psi \rangle|^2 \sim \tan^2\phi_{\alpha_0}$, the normalized sine uncertainty is given by

$$\frac{\langle \Delta\sin^2\psi \rangle}{\langle \cos\psi \rangle^2} \sim \tfrac{1}{2} z_0^{-1/2} \sim \frac{1}{2|\alpha_0|^2} . \tag{76}$$

APPENDIX D

We calculate $\langle \Delta(m_s - m_i)^2 \rangle$ using the Schrödinger picture. First we calculate $\langle m_s - m_i \rangle$. From (2), we obtain

$$\langle m_s - m_i \rangle = \mathrm{Tr}_{s,i}\hat{\rho}_{si}(\hat{n}_s \otimes \mathbf{1}_i - \mathbf{1}_s \otimes \hat{n}_i) = e^{-|\alpha_0|^2} \sum_i \frac{[\tanh^2(\kappa_0 t)]^i}{\cosh(\kappa_0 t)} \sum_j (j-1) \binom{j}{j-i} \frac{z^{j-1}}{(j-i)!}$$
$$= e^{-|\alpha_0|^2} \sum_i \frac{[\tanh^2(\kappa_0 t)]^i}{\cosh(\kappa_0 t)} z \frac{dF(1+i, 1; z)}{dz} . \tag{77}$$

z is defined below (62). By using

$$\frac{dF(a,b;x)}{dx} = \frac{a}{b}F(a+1, b+1; x), \quad F(a,b;z) = e^z F(b-a\ b, -z) ,$$

from (77) and

$$F(-a, 1+b, x) = \frac{\Gamma(b+1)\Gamma(a+1)}{\Gamma(b+a+1)} L_a^b(x) , \tag{78}$$

we obtain

$$\langle m_s - m_i \rangle = \frac{z e^{-|\alpha_0|^2}}{\cosh^2\kappa_0 t} \sum_i \tanh^{2i}\kappa_0 t L_i^1(-z) = |\alpha_0|^2 . \tag{79}$$

In deriving last the equality of (79), we use

$$\sum_k L_k^a(z) x^k = (1-x)^{-a-1} e^{xz/(x-1)} . \tag{80}$$

Next we derive $\langle (m_s - m_i)^2 \rangle$ with a similar procedure,

$$\langle (m_s - m_i)^2 \rangle = \mathrm{Tr}_{s,i}\hat{\rho}_{si}(\hat{n}_s \otimes \mathbf{1}_i - \mathbf{1}_s \otimes \hat{n}_i)^2$$
$$= e^{-|\alpha_0|^2} \sum_i \frac{[\tanh^2(\kappa_0 t)]^i}{\cosh(\kappa_0 t)} \left[z^2 \frac{d^2F(1+i, 1; z)}{dz^2} + z \frac{dF(1+i, 1; z)}{dz} \right]$$
$$= |\alpha_0|^4 + |\alpha_0|^2 . \tag{81}$$

In deriving of last equality of (81), we use (78) and (79). Therefore we obtain

$$\langle \Delta(m_s - m_i)^2 \rangle = |\alpha_0|^2 . \tag{82}$$

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