PHYSICAL REVIEW A VOLUME 38, NUMBER 7 OCTOBER 1, 1988

Excited states in stochastic electrodynamics

H. M. França* and T. W. Marshall†
Departamento de Física Moderna, Universidad de Cantabria Santander, 39005 Spain

(Received 4 December 1987)

We show that the set of Wigner functions associated with the excited states of the harmonic oscillator constitutes a complete set of functions over the phase space. An arbitrary probability distribution can be expanded in terms of these Wigner functions. By studying the time evolution, according to stochastic electrodynamics, of the expansion coefficients, we are able to separate explicitly the contributions of the radiative reaction and the vacuum field to the Einstein A coefficients for this system. We also supply a simple semiclassical explanation of the Weisskopf-Heitler phenomenon in resonance fluorescence.

I. INTRODUCTION

Stochastic electrodynamics[1,2] (SED) is a revival of Planck's second theory. Since it treats the electromagnetic field as a c number, it may be considered as the oldest semiclassical radiation theory. Reviews[3,4] of such theories have generally concentrated on the version of Jaynes, in which there is no vacuum field. The conclusion has been reached that semiclassical theories are inadequate, because of their inability to explain various phenomena, including the coincidence counting of the "photons" in an atomic cascade,[5] the "anticorrelation" in two channels of a beam splitter,[6] and the "antibunching" observed in resonance fluorescence.[7] Recently it has been demonstrated, however, that an optical theory based on SED is capable of explaining all of these phenomena,[8] and hence this conclusion is incorrect.

Although the modern revival of SED preceded Jaynes's theory, it has received little attention in the literature. This is probably because, in its original conception, it was overambitious. It treated the electron as a classical point particle, and set out to explain the whole of nonrelativistic quantum mechanics (QM) through the interaction of such a particle with the zero-point electromagnetic field. The program was remarkably successful for just one quantum system—the harmonic oscillator—but 30 years of study of nonlinear systems has given only one useful result—a correct estimate of the size of the hydrogen atom.[9]

At this point we may note that a certain convergence has occurred between quantum electrodynamics in the optical region and SED. Thirty years ago there was little recognition in quantum electrodynamics that the zero-point field was anything other than a source of "virtual photons." Now, with the emergence of quantum optics, especially the study of the atomic equations of motion in their Heisenberg form,[3] the vacuum field has a much more "real" status. The idea that the ground state of an atom represents some kind of balance between radiative reaction and vacuum-field effects certainly originated in SED, but it is now being proposed as a respectable quantum-mechanical description. Parallel with this is the idea that "spontaneous" emission, which Einstein, its discoverer, always considered so unsatisfactory, is actually stimulated by the zero-point field. We shall show, in this article, that by returning to the harmonic oscillator we can make more precise the contributions, respectively, of the radiative reaction and vacuum field to Einstein's A coefficients. This may also cast some light on the general problem of how frictional forces may be included in the quantum formalism.

Our treatment will also show more clearly the status of "negative probability" in physics. We shall argue that non-positive-definite distributions can enter into the description of physical systems only as a calculating device. They cannot be properly interpreted as distributions of probability and only those linear combinations which have the property of positive definiteness may be said to describe real physical states. This has a radical consequence which has been recognized in SED long ago:[10] no excited state of the harmonic oscillator is a real physical state, and hence a statement that a given atom "is" in a given excited state may not be correct.

One argument[11] in favor of "negative probabilities" involves the classical diffusion of a particle between two reflecting walls; the eigenfunctions of the diffusion operator are not positive definite. Since nobody questions that, in this classical process, the particle is always in some definite place, we infer that these eigenfunctions cannot, on their own, represent physical states of the diffusing particle. We shall show that the Wigner functions of the excited states of the harmonic oscillator play a similar role with respect to the Fokker-Planck operator, which gives the time evolution of an arbitrary distribution in phase space. None of these functions, except that representing the ground state, is, however, positive definite. We infer that only certain linear combinations of these excited-state distributions, that is, certain mixtures of the first kind, represent real physical states.

II. COMPLETENESS OF THE EXCITED STATES IN PHASE SPACE

A Hamiltonian H, for instance, a one-dimensional Hermitian operator, may, in certain cases, possess a complete set of excited states $\phi_n(x)$, such that

$$H\phi_n(x)=E_n\phi_n(x)\ , \tag{2.1}$$

where E_n are the corresponding energy levels ($n=0,1,2,\ldots$) and the eigenfunctions $\phi_n(x)$ satisfy the completeness relation

$$\sum_{n=0}^{\infty}\phi_n^*(x)\phi_n(y)=\delta(x-y)\ . \tag{2.2}$$

The time evolution, in quantum mechanics, of the Hamiltonian system specified by H is then given by

$$\psi(x,t)=\sum_{n=0}^{\infty}a_n\phi_n(x)\exp(-iE_nt/\hbar)\ , \tag{2.3}$$

where

$$a_n=\int_{-\infty}^{\infty}dx\ \phi_n^*(x)\psi(x,0)\ . \tag{2.4}$$

Consider now the Wigner function $W(x,p,t)$ associated with $\psi(x,t)$,

$$W(x,p,t)\equiv\int_{-\infty}^{\infty}\frac{dy}{\pi\hbar}\psi^*(x+y,\ t)\psi(x-y,t)\times\exp(2ipy/\hbar)\ . \tag{2.5}$$

The time evolution of $W(x,p,t)$ is

$$W(x,p,t)=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}a_m^*a_n\,W_{mn}(x,p)\exp[i(E_m-E_n)t/\hbar]\ , \tag{2.6}$$

where $a_m^*a_n$ are constant and

$$W_{mn}(x,p)=\int_{-\infty}^{\infty}\frac{dy}{\pi\hbar}\phi_m^*(x+y)\phi_n(x-y)\exp(2ipy/\hbar)\ . \tag{2.7}$$

Real physical systems do not evolve in time according to (2.6); all states, except the ground state ($n=0$), undergo decays by spontaneous emission of radiation. We propose to study such processes also through the evolution of $W(x,p,t)$, substituting for (2.6) the more general relation

$$W(x,p,t)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}C_{mn}(t)W_{mn}(x,p)\ . \tag{2.8}$$

The explicit expressions for $C_{mn}(t)$ will be obtained below, for the case of a free and forced harmonic oscillator, within the realm of SED.

In order to be sure that such an expansion is possible, we need a completeness property for the $\{W_{mn}\}$. This is guaranteed through the following theorem.

Theorem 1. The set of functions $W_{mn}(x,p)$ is complete and orthogonal over the space (x,p).

Proof. According to the definition (2.5) we can write

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}W_{mn}^*(x,p)W_{mn}(y,q)$$
$$=(\pi\hbar)^{-2}\int_{-\infty}^{\infty}d\xi\int_{-\infty}^{\infty}d\eta\exp\left[\frac{2i}{\hbar}(q\eta-p\xi)\right]\sum_{m=0}^{\infty}\phi_m(x+\xi)\phi_m^*(y+m)\sum_{n=0}^{\infty}\phi_n(y-\eta)\phi_n^*(x-\xi)=\frac{\delta(x-y)\delta(q-p)}{2\pi\hbar}\ , \tag{2.9}$$

where we have used (2.2) and the Fourier integral representation of the Dirac δ function. This establishes the property of completeness. The property of orthogonality is obtained in a similar manner and we give only the result

$$\int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dp\ W_{mn}^*(x,p)W_{ns}(x,p)=\frac{\delta_{mr}\delta_{ns}}{2\pi\hbar}\ . \tag{2.10}$$

In Secs. III and IV we shall consider the special case of the harmonic oscillator, for which the Hamiltonian is

$$H=-\frac{\hbar^2}{2m_0}\frac{\partial^2}{\partial x^2}+\tfrac{1}{2}m_0\omega_0x^2\ , \tag{2.11}$$

where m_0 is the mass of the particle and ω_0 the frequency. Then the excited-state wave functions can be expressed as

$$\phi_n(x)=(2^nn!)^{-1/2}\left[\frac{m_0\omega_0}{\pi\hbar}\right]^{1/4}H_n\left[x\left[\frac{m_0\omega_0}{\hbar}\right]^{1/2}\right]\times\exp\left[-\frac{m_0\omega_0x^2}{2\hbar}\right]\ , \tag{2.12}$$

where H_n are Hermite polynomials.

In this case each function of the complete set of Wigner functions $W_{mn}(x,p)$, defined in (2.7), can be calculated explicitly giving[12]

$$W_{mn}(x,p)=\frac{(-1)^m2^{(n-m)/2}}{\pi\hbar}\left[\frac{m!}{n!}\right]^{1/2}\times\exp\left[-\frac{m_0\omega_0x^2}{\hbar}-\frac{p^2}{m_0\omega_0\hbar}\right]\times\left[x\left[\frac{m_0\omega_0}{\hbar}\right]^{1/2}-\frac{ip}{\sqrt{\hbar m_0\omega_0}}\right]^{n-m}\times L_m^{n-m}\left[\frac{2m_0\omega_0x^2}{\hbar}+\frac{2p^2}{m_0\omega_0\hbar}\right]\ , \tag{2.13}$$

if $m\le n$. Here L_n^α are associated Laguerre polynomials. In the case $m>n$ we get a similar result, that is, we must interchange m with n everywhere and also replace i by $-i$.

We shall see that for studying transitions induced by thermal radiation, the diagonal subset of $\{W_{mn}\}$ is sufficient. The functions of this subset, which may be termed "Wigner functions of the excited states," are given by

$$W_{nn}(x,p)=\frac{(-1)^n}{\pi\hbar}L_n(2z)\exp(-z)\equiv U_n(z)\ , \qquad (2.14)$$

where $z=m_0\omega_0x^2/\hbar+p^2/m_0\omega_0\hbar$ and $L_n=L_n^0$ is a Laguerre polynomial. In this case it is possible to prove another completeness result.

Theorem 2. The set of functions $U_n(z)$ is complete and orthogonal in the sense that

$$\int_0^\infty dz\ U_m(z)U_n(z)=\frac{\delta_{mn}}{2\pi^2\hbar^2} \qquad (2.15)$$

and

$$\sum_{n=0}^{\infty} U_n(z)U_n(z')=\frac{\delta(z-z')}{2\pi^2\hbar^2}\ . \qquad (2.16)$$

These results follow directly from the standard orthogonality and completeness properties of the Laguerre polynomials.

III. EINSTEIN A AND B COEFFICIENTS IN SED

There has been a great deal of controversy about the respective roles of the vacuum field and radiative reaction in the spontaneous emission of radiation. Milonni[3] has shown, by making a suitable ordering of the operators in the second-quantized formalism, that the contribution of either of these fields may be reduced formally to zero. Such a treatment suggests that there is no real difference between the vacuum and radiative reaction fields.

In SED the electromagnetic field, instead of being an operator, is a c number. A product of fields is independent of its ordering, and "field commutators" are just a coded way of specifying the various autocorrelations of the zero-point field. We can therefore make an unambiguous distinction between the vacuum fields and the radiation reaction.

The motion of a harmonic oscillator in SED is given by a Langevin-type equation,

$$m_0\ddot{x}=-m_0\omega_0^2x-m_02\gamma\dot{x}+eE_x(t),\quad \gamma\equiv\frac{e^2\omega_0^2}{3m_0c^3}\ , \qquad (3.1)$$

where the term $-2\gamma m_0\dot{x}$ in the equation of motion has its origin in the radiation reaction force $2e^2\dddot{x}/3c^3$. The random force $eE_x(t)$ is generated by the x component of the fluctuating electromagnetic field, that is, $E_x(t)$ is a Gaussian stochastic process with a spectral density given by

$$I_0(\omega)=\int_{-\infty}^{\infty}d\tau\langle E_x(t)E_x(t+\tau)\rangle\exp(i\omega\tau)\ . \qquad (3.2)$$

Such a system differs from a Brownian harmonic oscillator in only one respect: the noise $E_x(t)$ is not white, so that I_0 is not a constant function of ω. If we choose for $I_0(\omega)$ the spectral density of the zero-point field

$$I_0(\omega)=\hbar\omega^3/6\pi^2c^3\ , \qquad (3.3)$$

then, because of the linearity of (3.1), $x(t)$ is also a Gaussian process. In particular, the stationary phase-space distribution is the ground-state distribution given by Eq. (2.14),

$$W(x,p)=U_0(z)=\frac{\exp(-z)}{\pi\hbar}\ . \qquad (3.4)$$

In order to understand the role of excited states in SED, we now consider the time-dependent case. It is convenient to treat "spontaneous" emissions along with those stimulated by incident radiation. This means considering a general spectral density $I(\omega)$ instead of just the zero-point field $I_0(\omega)$. However, because the zero-point field is not directly observable we write

$$I(\omega)=I_0(\omega)+I_{\rm obs}(\omega)\ , \qquad (3.5)$$

where $I_{\rm obs}(\omega)$ is the observable part of the incident spectrum. The phase-space distribution $W(x,p,t)$ satisfies the Fokker-Planck equation[10]

$$\frac{\partial W}{\partial t}=\gamma\frac{\partial}{\partial\xi}(\xi W)+\gamma\frac{\partial}{\partial\eta}(\eta W)+D\left[\frac{\partial^2W}{\partial\xi^2}+\frac{\partial^2W}{\partial\eta^2}\right]\ , \qquad (3.6)$$

where ξ and η are constants of the deterministic motion, given by

$$\xi=x\left[\frac{m_0\omega_0}{\hbar}\right]^{1/2}\cos(\omega_0t)-\frac{p}{\sqrt{m_0\omega_0\hbar}}\sin(\omega_0t)\ ,$$

$$\eta=x\left[\frac{m_0\omega_0}{\hbar}\right]^{1/2}\sin(\omega_0t)+\frac{p}{\sqrt{m_0\omega_0\hbar}}\cos(\omega_0t)\ ,$$

and

$$D=\frac{(\pi e)^2}{m_0\omega_0\hbar}I(\omega_0) \qquad (3.7)$$

is the diffusion coefficient. Because $\xi^2+\eta^2=z$ it is easy to show that the stationary solution of (3.6) is $W=U_0(z)=\exp(z)/(\pi\hbar)$ in the particular case in which

$$D=D_0\equiv\frac{(\pi e)^2I_0(\omega_0)}{m_0\omega_0\hbar}=\gamma/2\ .$$

A further transformation is convenient,

$$\xi=\sqrt{z}\cos\varphi,\quad \eta=\sqrt{z}\sin\varphi\ , \qquad (3.8)$$

where z is the same as before [see (2.14)]. Then (3.6) becomes

$$\frac{\partial W}{\partial t}=2\gamma\left[z\frac{\partial W}{\partial z}+W\right]+D\left[4z\frac{\partial^2W}{\partial z^2}+4\frac{\partial W}{\partial z}+\frac{1}{z}\frac{\partial^2W}{\partial\varphi^2}\right]\ . \qquad (3.9)$$

We observe that a system which, at $t=0$, is in equilibrium with the zero-point spectrum $[W=U_0(z)]$, and which, for $t>0$, interacts with any field of thermal (that is incoherent) radiation, is always homogeneous with respect to the variable φ. We may therefore, for such a system, discard the derivative with respect to φ in (3.9), and consider W as a function of z and t only. Then using theorem 2 of Sec. II, we can put

$$W(z,t)=\sum_{n=0}^{\infty} C_n(t)U_n(z) \ , \tag{3.10}$$

where $U_n(z)$ was defined before in (2.14) and

$$C_n(t)=2\pi^2\hbar^2\int_0^{\infty} dz\, U_n(z)W(z,t), \quad \sum_{n=0}^{\infty} C_n(t)=1 \ . \tag{3.11}$$

The time evolution of the coefficients $C_n(t)$ can be obtained by substituting (3.10) into (3.9). Using the recurrence relations between the Laguerre polynomials $L_n(2z)$ we get

$$\left[2\gamma\left[z\frac{\partial}{\partial z}+1\right]+4D\left[z\frac{\partial^2}{\partial z^2}+\frac{\partial}{\partial z}\right]\right]U_n(z)=\gamma[-(n+1)U_{n+1}(z)+U_n(z)+nU_{n-1}(z)]$$
$$+2D[(n+1)U_{n+1}(z)-(2n+1)U_n(z)+nU_{n-1}(z)] \tag{3.12}$$

for each term of the series on the right-hand side of (3.9).

Since the functions $U_n(z)$ are orthogonal we must have

$$\begin{aligned}\dot{C}_n(t)&=(n+1)(2D+\gamma)C_{n+1}+[\gamma-2D(2n+1)]C_n+(2D-\gamma)nC_{n-1}\\&\equiv P_{n+1,\,n}C_{n+1}+P_{n-1,\,n}C_{n-1}+(P_{n,n}-P_{n,\,n+1}-P_{n,\,n-1})C_n \ .\end{aligned} \tag{3.13}$$

With the above definition of $P_{n,\,n+1}$, $P_{n,n}$, and $P_{n,\,n-1}$ we can interpret these as "transition probabilities," per unit time, from the mathematical *(not physical)* states n to $n+1$, n to n, and n to $n-1$, respectively. The explicit expressions for these transition probabilities are

$$\begin{aligned}&P_{n,\,n+1}=(n+1)(2D-\gamma) \ ,\\&P_{n,n}=0 \ ,\\&P_{n,\,n-1}=n(2D+\gamma) \ .\end{aligned} \tag{3.14}$$

Here, for instance, in the expressions for $P_{n,\,n+1}$ and $P_{n,\,n-1}$, we can easily identify the role of radiation reaction due to the presence of γ. In a "downward transition," such as $P_{n,\,n-1}$, we have a positive contribution of radiation reaction as is expected on physical grounds. However, in an "upwards transition," such as $P_{n,\,n+1}$, we obtain a negative contribution of radiation reaction.

To obtain the Einstein A and B coefficients we substitute (3.5) in the expression for D given in (3.7). Then

$$\begin{aligned}&P_{n,\,n+1}=B_{n,\,n+1}I_{\text{obs}}(\omega_0)+A_{n,\,n+1} \ ,\\&P_{n,\,n+1}=B_{n,\,n-1}I_{\text{obs}}(\omega_0)+A_{n,\,n-1} \ ,\end{aligned} \tag{3.15}$$

where

$$B_{n,\,n+1}=\frac{2(n+1)\pi^2e^2}{\hbar m_0\omega_0}, \quad A_{n,\,n+1}=0 \tag{3.16}$$

and

$$B_{n,\,n-1}=\frac{2n\pi^2e^2}{\hbar m_0\omega_0}, \quad A_{n,\,n-1}=\frac{2ne^2\omega_c^2}{3m_0c^3} \ . \tag{3.17}$$

These results are in full agreement with the formal calculations of quantum electrodynamics. In particular, the coefficient $A_{n,\,n+1}$ of "spontaneous absorption" is zero and we now see that this is because of a balance between the effects of radiation reaction and vacuum fields ($\gamma=2D_0$, as was seen before). Also, the coefficient $A_{n,\,n-1}$ of "spontaneous emission" has equal contributions coming from these two sources (γ and $2D_0$). The equality between $B_{n,\,n+1}$ and $B_{n+1,\,n}$ also follows from (3.16) and (3.17).

We note that these conclusions agree with those reached by Milonni,[13] though his arguments had a more quantum-mechanical flavor. We also note that one of these results, Eq. (3.17) for $A_{n,\,n-1}$, is a rather old result in SED.[14]

We stress again, however, that the "states" given by the "distributions" $U_n(z)$ are, in SED, nothing more than a convenient complete set of functions. Their role with respect to the Fokker-Planck equation (3.9) is very similar to that of the functions

$$V_n(x)=\cos(nx), \quad n=0,1,2,\dots \ . \tag{3.18}$$

With respect to the diffusion problem,

$$\frac{\partial V}{\partial t}=\frac{\partial^2 V}{\partial x^2} \quad \left[\frac{\partial V}{\partial x}=0 \ \text{ at } x=0,\pi\right] \ . \tag{3.19}$$

None of the solutions (3.18) derived from (3.19) represents *by itself* a physical solution, with the sole exception of the "ground state," $n=0$. We could describe the solution

$$V(x,t)=1+\alpha\cos(x)\exp(-t) \tag{3.20}$$

as a decay of the "mixture" of states $n=0$ and $n=1$ into the state $n=0$ as t tends to infinity. But we would not, in this context, allow such language to obscure the fact that such a mixture may be a physical state only for those values of α, namely, $-1<\alpha<1$, for which V is positive definite. The same considerations apply to the excited states of the harmonic oscillator, and also, possibly, to the excited states of atoms. The state $n=1$ is represented by the Wigner function [see (2.14)]

$$W_{11}(x,p)=\frac{(2z-1)}{\pi\hbar}\exp(-z) \ , \tag{3.21}$$

which is negative for $z<\frac{1}{2}$. If we consider mixtures of this with the ground state W_{00}

$$W(x,p)=\frac{W_{00}(z)+\alpha W_{11}(z)}{1+\alpha} , \tag{3.22}$$

then such states correspond to physical states only if $0<\alpha<1$.

The language we are warning against is, of course, strongly entrenched in the literature. It is now becoming fashionable to describe resonance fluorescence as the radiation emitted by an atom which is oscillating coherently between two states. We shall show in Sec. IV that such language is no more justified than in the case we have just been studying.

IV. RESONANCE FLUORESCENCE IN SED

Resonance fluorescence occurs when an atom is driven by a coherent electromagnetic field whose frequency is close to one of its transition frequencies. It was predicted, long before sufficiently coherent sources became available, that the radiation emitted should have the linewidth of the driving field, rather than the natural linewidth of the transition.[15–17] Although the prediction was based on quantum electrodynamics, it has always been considered somewhat paradoxical, though it is now well verified experimentally.

In SED this Weisskopf-Heitler phenomenon receives a very natural explanation. A driven harmonic oscillator has a Langevin-type equation

$$\ddot{x}+2\gamma\dot{x}+\omega_0^2x=\frac{e}{m_0}[E_x(t)-F\sin(\omega t)] . \tag{4.1}$$

In the absence of the zero-point field $E_x(t)$ we have the deterministic solution

$$x_d=\mathrm{Re}\left[\frac{ieF\exp(i\omega t)}{m_0(\omega_0^2-\omega^2+2i\gamma\omega)}\right] . \tag{4.2}$$

In the absence of F we have the zero-point stationary solution x_z which is well known[1,2] in SED. Then the steady-state solution of (4.1) is

$$x=x_d+x_z . \tag{4.3}$$

Now the radiation emitted by this system at a given frequency is obtained from the autocorrelation of $\ddot{x}$, that is,

$$K(\tau)=\langle\ddot{x}(t)\ddot{x}(t+\tau)\rangle . \tag{4.4}$$

Because x_d and x_z are independent we have

$$K(\tau)=\langle\ddot{x}_d(t)\ddot{x}_d(t+\tau)\rangle+\langle\ddot{x}_z(t)\ddot{x}_z(t+\tau)\rangle . \tag{4.5}$$

The first term on the right-hand side represents radiation at the single frequency ω (which is the frequency of the deterministic motion x_d), while the second term represents a line of width γ centered on the frequency ω_0, as is well known in SED.[1,2] But we saw in Sec. III that the ground state of the oscillator, which is represented by the stochastic process x_z, is an equilibrium state in which the radiation emitted at a given frequency is equal, on the average, to the radiation absorbed at that frequency. Hence only the sharp line, at frequency ω of the deterministic force, is observed.

The phase-space distribution in the steady state, characterized by (4.3), is easily obtained. Since x_z is independent of x_d, the joint distribution of x_z and $p_z=m_0\dot{x}_z$ is simply the ground state, given by (3.4),

$$W(x_z,p_z)=\frac{1}{\pi\hbar}\exp\left[-\frac{m_0\omega_0}{\hbar}x_z^2-\frac{p_z^2}{m_0\omega_0\hbar}\right] . \tag{4.6}$$

Hence the distribution of $(x,p)=(x_d+x_z,p_d+p_z)$ is[18]

$$W(x,p)=\frac{1}{\pi\hbar}\exp\left[-\frac{m_0\omega_0}{\hbar}(x-x_d)^2-\frac{(p-p_d)^2}{m_0\omega_0\hbar}\right] . \tag{4.7}$$

According to theorem 1 of Sec. II we may write this as

$$W(x,p,t)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}C_{mn}(t)W_{mn}(x,p) , \tag{4.8}$$

where

$$C_{mn}(t)=\int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dp\,W_{mn}^*(x,p)W(x,p,t) . \tag{4.9}$$

These expansion coefficients may be obtained most easily by using the expression (2.7) for $W_{mn}(x,p)$, together with the generating functions for the Hermite polynomials. The result is

$$C_{mn}(t)=\left[\frac{\sigma(t)^{m+n}}{m!n!}\right]^{1/2}\times\exp\{-\sigma(t)+i(m-n)[\omega t+\delta(t)]\} , \tag{4.10}$$

where

$$\sqrt{\sigma(t)}\exp[i\delta(t)]=\left[\left[\frac{m_0\omega_0}{2\hbar}\right]^{1/2}x_d(t)+i\left[\frac{m_0}{2\omega_0\hbar}\right]^{1/2}\dot{x}_d(t)\right]\times\exp(-i\omega t) , \tag{4.11}$$

and $x_d(t)$ is given by (4.2). In general σ and δ both have a small periodic part, but for the case of resonance they take the constant values

$$\sigma=\frac{e^2F^2}{8m_0\gamma^2\omega_0\hbar},\quad \delta=0\quad(\omega=\omega_0) . \tag{4.12}$$

For this resonance case, (4.8) with (4.10) is readily recognized as the quantum-mechanical coherent state;[19] it corresponds to the state (2.6) with the coefficients

$$a_n=\left[\frac{\sigma^n}{n!}\right]^{1/2}\exp(-\sigma/2) . \tag{4.13}$$

This is remarkable because such a state has no formal radiative corrections, whereas the SED description corresponds to that of quantum electrodynamics rather than to QM. Just as the ground state of QM represents a balance between radiative reaction and zero-point field, so does the coherent state represent a balance between these two fields and the driving field. Because of the time variation in σ and δ, such an identification is not so simple for the general, off-resonance case, so that (4.7), or, equivalently, (4.10) and (4.11), give a SED generalization of the coherent state, by the inclusion of the radiative correction.

V. CONCLUSION

By a study of the harmonic oscillator in SED, we have shown that, for this system at least, the only role of the excited states is to provide us with a suitable complete set of Wigner functions in terms of which an arbitrary phase-space probability distribution may be expanded. Consideration of both spontaneous emission and resonance fluorescence leads us to the conclusion that these states have no independent physical existence. There are, of course, many elementary facts of atomic physics which can not be described by a harmonic oscillator model; for example, the Ritz combination principle for line spectra, which gave birth to the excited-state concept, remains unexplained in the completely classical version of SED.

Nevertheless, we think our results show that it is necessary to treat the excited states with some care. In quantum theory, and in SED, a harmonic oscillator driven by a coherent field at resonance does not alternate between its various excited states. The coefficients $a_n \exp(-iE_n t/\hbar)$ of (2.3) or $C_{mn}(t)$ given by (4.10) have a time-independent modulus; it is their varying phases which give rise to the emission of radiation at the frequency of the driving field. To be sure, quantum theory asserts that a measurement of the energy causes a collapse of ψ, leading to a discontinuous change in a_n. Furthermore, current descriptions of, for example, the "shelved-state" experiment^{20} interpret as a "measurement" the intervention caused by a second laser operating at a different resonant frequency. However, in view of many acknowledged difficulties of quantum theory of measurement, especially the nonlocal features associated with the collapse of the wave function, we think it is premature to conclude that this is the *only possible* explanation of such experiments. Another possible explanation may come from stochastic optics (a branch of SED), which has already given a completely local description for many phenomena for which it was previously thought displayed "quantum nonlocality."

ACKNOWLEDGMENTS

We want to thank Professor Emilio Santos for many valuable comments. One of us (T.W.M.) acknowledges financial support from the European Science Exchange under the joint agreement between the Royal Society and the Consejo Superior de Investigación Científica. Work of one of us (H.M.F.) was supported in part by Fundacão de Amparo a Pesquisa do Estado de São Paulo (FAPESP) and Conselho Nacional de Desenvolvimento Cientifico e Tecnologico (CNPq).

---

[*] On study leave from Instituto de Física, Universidade de São Paulo, São Paulo, Brazil.

[†] On study leave from Department of Mathematics, University of Manchester, Manchester M13 9PL, England.

[1] T. H. Boyer, Phys. Rev. D **11**, 809 (1975). See also T. H. Boyer, in *Foundations of Radiation Theory and Quantum Electrodynamics,* edited by A. O. Barut (Plenum, New York, 1980).

[2] L. de la Peña, in *Stochastic Processes Applied to Physics and Other Related Fields,* edited by B. Gomez, S. M. Moore, A. M. Rodriguez-Vargas, and A. Rueda (World Scientific, Singapore, 1982), p. 428.

[3] P. W. Milonni, Phys. Lett. C **25**, 1 (1976).

[4] L. Mandel, Prog. Opt. **13**, 27 (1976).

[5] A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. **49**, 91 (1982).

[6] P. Grangier, G. Roger, and A. Aspect, Europhys. Lett. **1**, 173 (1986).

[7] M. Dagenais and L. Mandel, Phys. Rev. A **18**, 2217 (1978).

[8] T. W. Marshall and E. Santos, Europhys. Lett. **3**, 293 (1987); Ann. N.Y. Acad. Sci. **480**, 400 (1986); in *Proceedings of the Second International Symposium on Foundations of Quantum Mechanics,* edited by M. Namiki, Y. Ohmuki, Y. Murayama, and S. Nomura, (Physical Society of Japan, Tokyo, 1986), p. 66; Found. Phys. **18**, 185 (1988).

[9] T. H. Boyer, Phys. Rev. D **11**, 790 (1975).

[10] T. W. Marshall; Proc. R. Soc. London, Ser. A **276**, 475 (1963).

[11] R. P. Feynman, *Quantum Implications Essays in Honour of David Bohm,* (Routledge and Kegan Paul, London, 1987) p. 235.

[12] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals Series and Products,* Academic, New York, 1965), p. 838, formula 7.377.

[13] P. W. Milonni, Am. J. Phys. **52**, 340 (1984).

[14] T. W. Marshall, Izv. Vuz. Fiz. **11**, 34 (1968).

[15] P. L. Knight and P. W. Milonni; Phys. Lett. C **66**, 21 (1980).

[16] V. Weisskopf, Ann. Phys. (N.Y.) **9**, 23 (1931).

[17] W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, New York, 1954), pp. 348–353.

[18] H. M. França and M. T. Thomaz, Phys. Rev. D **31**, 1337 (1985). See also the interesting series of papers by G. H. Goedecke, Found. Phys. **14**, 41 (1984).

[19] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill, New York, 1965), p. 74.

[20] R. G. Brewer and A. Schenzle, in *Proceedings of the Second International Symposium on Foundations of Quantum Mechanics,* edited by M. Namiki; Y. Ohmuki, Y. Murayama, and S. Nomura (Physical Society of Japan, Tokyo, 1986), p. 257.