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Passage time statistics in semiconductor laser turn on

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The statistical properties of the time delay between the switch on and the attainment of a fixed value of the output power in single-mode semiconductor lasers are analyzed and discussed. In particular, we show that the only parameter needed to describe the process, when the laser is initially operated quite below threshold, is the value of the stationary output power of the final state, a result supported by recent experimental measurements.

The study of the statistical properties of the electromagnetic radiation emitted by coherent sources during transients has fascinated a lot of researchers since the advent of the laser. First experimental works showed how, in Q-switched gas lasers, the contribution of spontaneous emission processes just after the cavity switch on gives rise to an anomalous broadening in the probability distribution of the number of photons inside the cavity, that is, quantum noise generates macroscopic effects.[1]

The above phenomenon is characterized on a quantitative basis by introducing the so-called "first passage time,"[2] the time elapsing between the instant of the switch on and that at which the photon number reaches a prefixed value, and by studying its statistical distribution.

A theoretical explanation of the phenomenon has been developed by Haake, Haus, and Glauber, who demonstrate the equivalence between the problem of the transient in Q-switched laser and the decay from an unstable equilibrium state.[3] Furthermore, the same phenomenology has been found to characterize several different systems so that the results obtained can describe a lot of phenomena also not directly related to physics, such as the distribution of time delays in superfluorescence,[4] the statistics of explosive chemical reactions,[5] the Malthus-Verhulst model for the population dynamics,[6] and, more recently, the Q-switching transient in dye lasers.[7] A similar phenomenon is also present in semiconductor lasers when they are suddenly switched on from below to above threshold. This fact must be accounted for, for example, when the above sources are employed in optical communication systems, because an indetermination in the "turn on instant," the time needed to reach a prefixed output power, can seriously affect the performances of the system by lowering its maximum attainable transmission rate. Apart from these considerations, whose importance is of applicative nature, semiconductor lasers present some peculiar differences with respect to other kinds of lasers which make it worthwhile to investigate their behavior during the transient. In the case of Q-switched lasers, in fact, one starts immediately from a condition of above threshold, so that stimulated emission is driven by the very photons that are spontaneously emitted just after the switching of the cavity, while, in semiconductor lasers, where the system gradually evolves from a condition of below to above threshold, during the transient subsequent to the turn on operation, only those photons which are spontaneously emitted when the laser is near threshold will cause stimulated emission to start and, hence, determine the successive evolution of the system. All the photons emitted before the transparency condition is achieved,[8] in fact, will be absorbed and the system will maintain no memory of them. It must also underline the relevant difference of the characteristic times of transients, which are of the order of 10^{-6} s in gas lasers and 10^{-10} s in semiconductor lasers. This very short time, which is taken advantage of in a high-speed optical communication system by directly modulating the semiconductor lasers used as transmitters, causes the measurement of the first passage time in such sources to be quite difficult.

In this paper, we present a theory which, according to the basic guidelines of the work of Haake *et al.*,[3] modified in such a way to account for the mentioned peculiar features of single-mode semiconductor lasers, allows us to express the variance of the first passage time distribution as a function of the stationary output power.

Comparison between the theoretical results and experimental measurements recently performed on distributed feedback (DFB) laser shows a satisfactory agreement.[9]

We start from the rate equations for $E(t)$, the slowly varying complex amplitude of the optical field oscillating at frequency ω_0, and $N(t)$, the minority carrier number, which in a single-mode semiconductor laser read[10]

$$\frac{dE}{dt}=\left[i\Delta\omega(N)+\frac{g(N)}{2}\right]E+\sqrt{R(N)}F(t)\,, \tag{1a}$$

$$\frac{dN}{dt}=-\frac{N}{\tau}-G(N)|E|^2+C\,, \tag{1b}$$

where $g(N)=-\Gamma_0+G(N)$, $G(N)$ being the gain and Γ_0 the total cavity-loss rate, $\Delta\omega(N)=\omega(N)-\omega_0$, $\omega(N)$ being the resonant frequency of the cavity mode, $R(N)$ is the spontaneous emission rate, τ is the spontaneous lifetime of the excited carriers, and C the current in electrons/s. $E(t)$ is normalized so that the intensity $I(t)=|E(t)|^2$ represents the number of photons into the cavity which belong to the lasing mode. $F(t)$, a Gaussian white noise accounting for the spontaneous-emission processes, is such that

$$\langle F(t)\rangle=0,\ \langle F(t_1)F(t_2)\rangle=0,\ \langle F(t_1)F^*(t_2)\rangle=\delta(t_1-t_2)\,, \tag{2}$$

where the brackets indicate ensemble averages. The system of stochastic differential Eqs. (1) has to be considered in the Ito sense. It this case, however, since the diffusion coefficient R is a function only of N and not of E, the correction term needed to obtain the equivalent Stratonovich equations vanishes, so that we could use, in the following, the rules of the ordinary calculus.

Our aim is the study of the transient which is generated when the laser is switched from below to above threshold by changing, through a step function, the biasing current from C_1 to $C_2 > C_1$. After the switching, there is a delay in the optical response as the injected carrier number rises to a level just beyond the threshold level. Up to this level, there is a negligible lasing emission,[8] i.e., the term $G(N)|E(t)|^2$ in Eq. (1b) can be neglected, so that solving Eq. (1a) for $E(t)$ and squaring one obtains

$$I(t)=|E(t)|^2=|\eta_1+\eta_2(t)|^2\exp\left(\int_{t_{\rm th}}^{t} g(N(t_1))dt_1\right), \tag{3}$$

where

$$\eta_1=E_0\exp\left[\int_0^{t_{\rm th}}\left(i\Delta\omega(N(t_1))+\frac{g(N(t_1))}{2}dt_1\right)\right], \tag{4a}$$

$$\eta_2(t)=\int_0^t dt_1 F(t_1)\sqrt{R(N(t_1))}\exp\left[-\int_{t_{\rm th}}^{t_1}\left(i\Delta\omega(N(t_2))+\frac{g(N(t_2))}{2}dt_2\right)\right]. \tag{4b}$$

Above, $E_0=E(t=0)$, and $t_{\rm th}$ is the time at which the number of injected carriers reaches its threshold value $N(t_{\rm th})$ so that $g(N(t_{\rm th}))=0$. η_1 and $\eta_2(t)$ (considered at any fixed time t), and so their sum $\eta(t)=\eta_1+\eta_2(t)$, are zero-mean, complex, independent, and circularly distributed Gaussian variables. This stems from the incoherent nature of the field E_0, from the properties of $F(t)$, and from the independence between E_0 and $F(t)$ at any $t>0$. Moreover, $\eta_2(t)$ and, hence, $\eta(t)$, can also be shown to be processes saturating to $\eta_2(\infty)$ and $\eta(\infty)$. To demonstrate this, we must evaluate the integral in (4b) which, in turn, due to the term $g(N(t))$ in the argument of the exponential is practically different from zero only in a narrow interval around $t_{\rm th}$ where $g(N(t))$ changes sign. Considering that near threshold one has

$$g(N(t))=G_N C_f(t-t_{\rm th}), \tag{5}$$

$G_N=(\partial G/\partial N)_{N=N_{\rm th}}$ and, from Eq. (1b),

$$C_f=\left(\frac{\partial N}{\partial t}\right)_{t=t_{\rm th}}=-\frac{N_{\rm th}}{\tau}+C_2=C_2-C_{\rm th}.$$

After some algebra one obtains

$$\langle\eta_2(t)\eta_2^*(t)\rangle=R(N_{\rm th})\left(\frac{\pi}{2G_N C_f}\right)^{1/2}\left\{\mathrm{erf}\left[\left(\frac{G_N C_f}{2}\right)^{1/2}t_{\rm th}\right]+\mathrm{erf}\left[\left(\frac{G_N C_f}{2}\right)^{1/2}(t-t_{\rm th})\right]\right\}. \tag{6}$$

We are interested in the evaluation of the statistical properties of the time $\bar{t}$ the laser spends to reach a prefixed value of the intensity $\bar{I}$ which is comparable with the final stationary value I_f. In the case in which $\bar{t}>t^*=t_{\rm th}+1.8[2/(G_N C_f)]^{1/2}$, a condition which, as shown below, is almost always satisfied, $\mathrm{erf}[(G_N C_f)^{1/2}(t-t_{\rm th})]=1$ within 1% so that one can reasonably set

$$\langle\eta_2(t)\eta_2^*(t)\rangle=\langle\eta_2(\infty)\eta_2^*(\infty)\rangle.$$

Under the above hypothesis, from Eqs. (3) and (5) one obtains

$$\bar{t}=t_{\rm th}+\left[\frac{2}{G_N C_f}\ln\left(\frac{\bar{I}}{|\eta(\infty)|^2}\right)\right]^{1/2}, \tag{7}$$

with $|\eta(\infty)|^2=|\eta_1+\eta_2(\infty)|^2$ a random variable with negative exponential distribution function, so that, after suitable averaging, the first two moments of $\bar{t}$ read

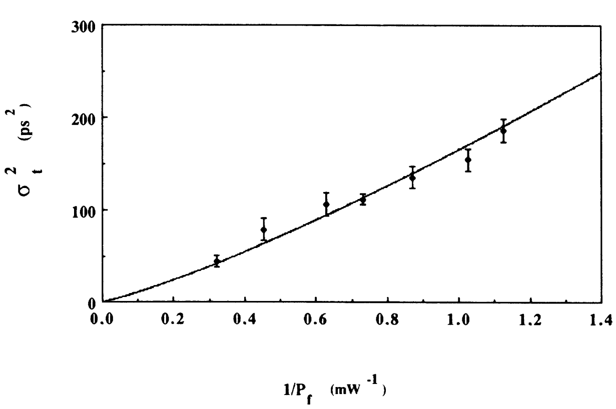

FIG. 1. Experimental and theoretical behavior of the variance of the first passage time vs the inverse of the final stationary output optical power for a distributed-feedback semiconductor laser.

$$\langle \bar{t} \rangle = t_{\text{th}} + \left[\frac{2}{G_N C_f} \right]^{1/2} \left[\sqrt{\ln\mu} + O\left[\frac{1}{\sqrt{\ln\mu}} \right] \right], \tag{8a}$$

$$\sigma_{\bar{t}}^2 = \langle (\bar{t} - \langle \bar{t} \rangle)^2 \rangle = \frac{2}{G_N C_f} \left[\frac{\psi'(1)}{4\ln\mu} + \frac{2\psi(1)\psi'(1) + \psi''(1)}{8(\ln\mu)^2} + O\left[\frac{1}{(\ln\mu)^3} \right] \right], \tag{8b}$$

where $\psi(x)$, $\psi'(x)$, and $\psi''(x)$ are the digamma function and its first and second derivatives, respectively,[11] and

$$\mu = \frac{\bar{I}}{\langle |\eta(\infty)|^2 \rangle} = \bar{I} \left\{ R(N_{\text{th}}) \left[\frac{\pi}{2G_N C_f} \right]^{1/2} \left\{ \text{erf}\left[\left[\frac{G_N C_f}{2} \right]^{1/2} t_{\text{th}} \right] + 1 \right\} + \langle |E_0|^2 \rangle \exp\left[\int_0^{t_{\text{th}}} g(N(t)) dt \right] \right\}^{-1}. \tag{9}$$

At this point, some considerations which, besides justifying the assumption made above, will give rise to further simplification in the obtained relations, are in order. Let us consider the situation, corresponding to the conditions of the experiment of Ref. 9, in which the initial biasing current is quite below the threshold value ($C_1/C_{\text{th}} < \frac{1}{3}$), and let us see if any cumulative effects of quantum noise during the transient can be evidenced. In this case, the term $\langle |E_0|^2 \rangle \exp[\int_0^{t_{\text{th}}} g(N(t)) dt]$ is practically zero. This can be immediately verified, once the physical parameters are known, considering that in strictly single-mode semiconductor lasers, the exponential term $g(N(t))$ is well approximated by a linear expansion[8] so that the argument of the exponent becomes $[-(G_N C_f) t_{\text{th}}^2/2]$. In our case, where the tested laser is a DFB with typical parameters, we have $G_N = 2\times 10^4\ \text{s}^{-1}$, t_{th} and C_f vary from 2×10^{-9} to 5×10^{-10} s and from 4×10^{17} to $20\times 10^{17}\ \text{s}^{-1}$, respectively, when C_2 is varied in such a way to obtain a final value of the output power ranging between 1 mW and 4 mW, and $\langle |E_0|^2 \rangle \approx 10^3$–$10^4$ photons for values of the initial biasing current C_1 which give output powers of the order of some tens of μW. Moreover, with such values for the parameters, $\text{erf}[(G_N C_f/2)^{1/2} t_{\text{th}}] = 1$ so that μ, and, hence, the variance of $\bar{t}$ becomes completely independent of the initial conditions: The system loses memory of its initial state and only spontaneous emission events which do take place around t_{th} determine its evolution. In Fig. 1, we report the set of measurements of $\sigma_{\bar{t}}^2$ vs $1/P_f$, P_f being the final stationary value of the output power, performed in Ref. 9. Here, $\bar{t}$ is the time at which the output power reaches the value $P_f/2$. This value has been chosen to minimize the experimental errors affecting the measurement. However, up to this level, the effect of stimulated emission is small enough to disregard saturation effects on the gain value.[8] The continuous curve is a plot of the theoretical results obtained remembering that $C_f = P_f/\eta\hbar\omega_0$ and $\bar{I} = 1/\Gamma_0\eta\hbar\omega_0(P_f/2)$, η being the incremental efficiency per facet (measured to be 0.212) $R(N_{\text{th}}) = 10^{12}\ \text{s}^{-1}$ and $\Gamma_0 = 6\times 10^{11}\ \text{s}^{-1}$. As far as the assumed condition $\bar{t} > t^* + 1.8[2/(G_N C_f)]^{1/2}$ is concerned, a simple check shows that it is practically always verified because $\langle \bar{t} \rangle > t^* + 2\sigma_{\bar{t}}$.

In conclusion, we have found simple analytical expressions for the first two moments of the first passage time in single-mode semiconductor lasers. The theory is also shown to be in agreement with the results of an experiment performed on a DFB laser.

During the completion of the present paper, the authors got acquainted with a theoretical work which, taking advantage of a similar technique, faces the problem of transient in gas lasers.[12]

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