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Probing the phase coherence of parametrically generated photon pairs: A new test of Bell's inequalities

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We show that the two-photon phase coherence of parametrically generated photon pairs, which is at the origin of squeezed-light generation, can be directly probed using an intensity-correlation measurement. The resulting intensity correlation leads to a new violation of Bell's inequalities, which could be experimentally tested.

Parametric processes are well known to produce quantum states of light such as squeezed states[1,2] and single-photon states,[3–5] which cannot be described by any classical wave model of light. These states have been demonstrated experimentally using, respectively, homodyne[1] and photon counting[3–5] techniques. On the other hand, some quantum states for light[6] or particles[7] have been demonstrated, which are in conflict with classical notions of causality and/or locality, and therefore cannot be described by any classical model: A criterion for this is the violation of the so-called "Bell's inequalities."[6–11] These inequalities restrict the possible amount of correlation between spatially separated events for a whole class of "local realistic theories"[6] which typify "classical" (i.e., nonquantum) theories according to ideas developed by Einstein, Podolsky, and Rosen.[12] An example of violation of Bell's inequalities is provided by polarization correlations between two photons emitted in pairs by atomic cascades[13–17] or direct two-photon decay processes.[18,19] Though the possibility of violation of Bell's inequalities in optics experiment without using polarization correlation has been considered,[20,21] the schemes proposed so far differ significantly from the one described below.

In this paper, we show that the two-photon phase coherence of photon pairs generated in parametric processes, probed using coherent "local oscillator" light beams, give rise to a violation of Bell's inequalities, which could be experimentally tested. The detection scheme to be used is in some sense (see below) a mixture of homodyne and intensity correlations (i.e., photon counting) techniques.[22]

A convenient way of describing parametric processes for our purposes is using the unitary transformation[23–25] which relates the input mode's destruction operators $\{a(\omega)\}$ to the output mode's destruction operators $\{b(\omega)\}$ for a parametric amplifier

$$b(\omega)=G(\omega)a(\omega)+M(\omega)a^{\dagger}(2\omega_0-\omega)\,, \quad (1)$$

where ω and ω_0 are, respectively, the signal and pump optical frequencies, and where $G(\omega)$ and $M(\omega)$ obey the unitarity relations

$$|G(\omega)|^2-|M(\omega)|^2=1\,, \quad (2a)$$

$$|G(\omega)|^2=|G(2\omega_0-\omega)|^2\,, \quad (2b)$$

$$|M(\omega)|^2=|M(2\omega_0-\omega)|^2\,, \quad (2c)$$

$$G(\omega)M(2\omega_0-\omega)=M(\omega)G(2\omega_0-\omega)\,. \quad (2d)$$

These equations give a correct account of a wide range of parametric processes involving second- or third-order nonlinear effects, as it has been established by squeezing experiments.[2] It can be said that Eq. (1) describes the creation of photon pairs at both sidebands ω and $(2\omega_0-\omega)$ with respect to the pump frequency (these are usually called signal and idler sidebands). Here, we want to separate spatially the signal and idler sidebands, which can be done using a dispersive element as shown in Fig. 1. One thus assumes that one selects two narrow bands of frequency B_1 and B_2:

$$\omega_1-\frac{\Delta\omega}{2}\leq\omega\leq\omega_1+\frac{\Delta\omega}{2}\,, \quad (3a)$$

$$\omega_2-\frac{\Delta\omega}{2}\leq\omega\leq\omega_2+\frac{\Delta\omega}{2}\,, \quad (3b)$$

which are directed to different detectors D1 and D2. We will assume in the following that $\omega_1+\omega_2=2\omega_0$. Using 50% beam splitters BS1 and BS2, the light beams from the dispersive element are mixed before detection with two weak coherent beams at frequencies ω_1 and ω_2, respectively. The destruction operators for modes a, c_1, and c_2 (see Fig. 1) are therefore given by

$$a(\omega)\,|\psi\rangle=0\,, \quad (4a)$$

$$c_1(\omega)\,|\psi\rangle=\beta_1\delta(\omega-\omega_1)\,|\psi\rangle\,, \quad (4b)$$

$$c_2(\omega)\,|\psi\rangle=\beta_2\delta(\omega-\omega_2)\,|\psi\rangle\,, \quad (4c)$$

where $|\psi\rangle$ is the quantum state we are considering and

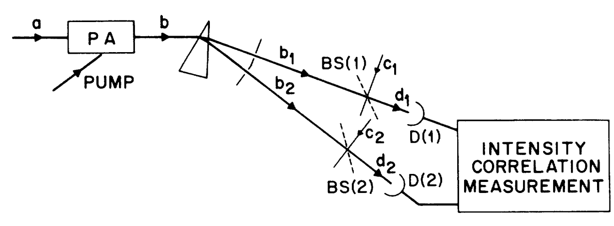

FIG. 1. Experimental scheme. The parametric amplifier PA transforms the (vacuum) input modes a to output modes b. The signal and idler parts of modes b are separated using a dispersive element (prism and slits), then mixed with modes c_1 and c_2 using beam splitters BS1 and BS2. One measures the intensity correlation between modes d_1 and d_2.

β_1,β_2 are coherent state amplitudes. The operators d_1 and d_2 for the detector modes are obtained from c_1 and c_2, and from the output modes of the dispersive element b_1 and b_2:

$$d_1(\omega)=\frac{1}{\sqrt{2}}[b_1(\omega)+c_1(\omega)]\,, \tag{5a}$$

$$d_2(\omega)=\frac{1}{\sqrt{2}}[b_2(\omega)+c_2(\omega)]\,. \tag{5b}$$

The detectors D1 and D2 are then set to measure the intensity correlation function between modes d_1 and d_2, which is given for a delay time τ by the relations

$$C_{12}(\tau)=\frac{1}{I_1I_2}\int_{B_1}d\omega\int_{B_1}d\omega'\int_{B_2}d\omega''\int_{B_2}d\omega'''e^{i(\omega-\omega')t}e^{i(\omega''-\omega''')(t+\tau)}\langle d_1^\dagger(\omega)d_1(\omega')d_2^\dagger(\omega'')d_2(\omega''')\rangle\,, \tag{6a}$$

where

$$I_1=\int_{B_1}d\omega\int_{B_1}d\omega'\langle d_1^\dagger(\omega)d_1(\omega')\rangle\,, \tag{6b}$$

$$I_2=\int_{B_2}d\omega''\int_{B_2}d\omega'''\langle d_2^\dagger(\omega'')d_2(\omega''')\rangle\,. \tag{6c}$$

As expected, $C_{12}(\tau)$ does not depend on the time variable t, which is eliminated when integrating over the frequencies. In order to proceed further, we will assume that the parametric gain is uniform over the frequency bandwidths B_1 and B_2, and therefore omit the ω dependencies. We define

$$|G(\omega_1)|=|G(\omega_2)|=|G|\,, \tag{7a}$$

$$|M(\omega_1)|=|M(\omega_2)|=|M|\,, \tag{7b}$$

$$\mathrm{Arg}[G(\omega_1)M(\omega_2)]=2\phi_0\,. \tag{7c}$$

It is also useful to introduce

$$\phi_1=\mathrm{Arg}(\beta_1),\;\;\phi_2=\mathrm{Arg}(\beta_2)\,. \tag{8}$$

The calculation is then straightforward, and we get

$$C_{12}(x)=1+\frac{N_{12}(x)}{I_1I_2}\,, \tag{9}$$

where we have introduced the normalized delay time

$$x=\Delta\omega\tau/2\,. \tag{10}$$

$N_{12}(x)$ and I_1I_2 are given by

$$N_{12}(x)=|G|^2|M|^2\left[\frac{\sin x}{x}\right]^2(\Delta\omega)^2 +2\cos(\phi_1+\phi_2-2\phi_0)\,|\beta_1\beta_2G^*M^*|\left[\frac{\sin x}{x}\right]\Delta\omega\,, \tag{11a}$$

$$I_1I_2=[\,|M|^2\Delta\omega+|\beta_1|^2][\,|M|^2\Delta\omega+|\beta_2|^2]\,. \tag{11b}$$

To get some insight in these results, let us consider first the case where $\beta_1=\beta_2=0$, i.e., when there is no coherent "local oscillator" field. We obtain

$$C_{12}(x)=1+\left|\frac{G}{M}\right|^2\left[\frac{\sin x}{x}\right]^2. \tag{12}$$

This is the well-known "pair production" correlation peak, which was observed for the first time by Burnham and Weinberg in 1970.[26] It is most apparent when $|M|\ll|G|$, i.e., when the parametric gain is very small.[27] On the other hand, when $|M|=0$ and $\beta_1,\beta_2\neq0$ we obtain

$$C_{12}(x)=1\,. \tag{13}$$

This is just the "flat" random coincidence spectrum for laser light.[28] Let us now assume that $|M|\ll|G|$, and adjust the coherent field intensities to get

$$|\beta_1|^2=|\beta_2|^2=|MG|\Delta\omega\,. \tag{14}$$

This means that the coincidence rate from the laser fields is equal to the pairs of photons counting rate for $\tau=0$. Then we obtain

$$C_{12}(x)=1+\left[\frac{\sin x}{x}\right]^2+2\left[\frac{\sin x}{x}\right]\cos(\phi_1+\phi_2-2\phi_0)\,. \tag{15}$$

Here the first term is due to laser-laser coincidences, the second one to pairs coincidences, and the third one is an interference term depending on the phases ϕ_1 and ϕ_2. $C_{12}(x)$ is plotted in Fig. 2 for various values of $\phi=(\phi_1+\phi_2-2\phi_0)$. The most dramatic effect is obtained

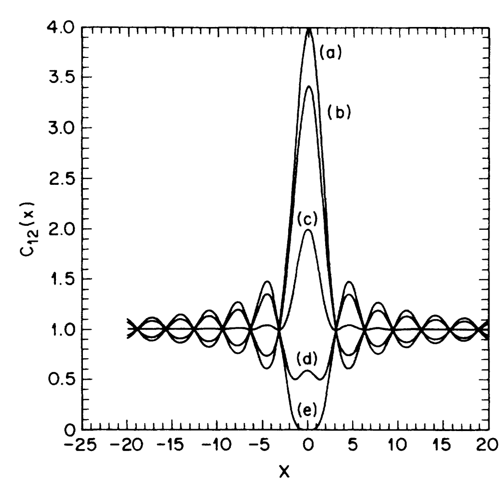

FIG. 2. Intensity correlation as a function of the normalized delay x for different values of the phase $\phi=\phi_1+\phi_2-2\phi_0$. Curves a, b, c, d, e correspond respectively to $\phi=0$, $\pi/4$, $\pi/2$, $3\pi/4$, π.

for $x=\tau=0$, where we get

$$C_{12}(0)=2[1+\cos(\phi_1+\phi_2-2\phi_0)]\,. \quad (16)$$

This correlation function is mathematically the same as the polarization correlation function[6–9] which is well known to yield a violation of Bell's inequalities in atomic cascade experiments.[13–17] Therefore, the experimental setup of Fig. 1 does yield a new test of Bell's inequalities. It has the advantage that the quantum correlated pairs of photons are emitted in well-defined directions, contrary to the atomic cascade experiments. Therefore, using this scheme, together with high-quantum efficiency solid-state photomultipliers recently developed,[29] would improve greatly over the best detection efficiency value achieved so far, which is about 10^{-3}, and includes both solid-angle collection efficiency and photomultiplier quantum efficiency.[16] Such an improvement is related to the so-called "supplementary assumption,"[7,30] which has always been made in order to derive usable Bell's inequalities[7] [see Eq. (A2) and discussion in Appendix].

As a conclusion, we have shown that parametrically generated pairs of photons could be used in a new experimental test of Bell's inequalities. Such an experiment would also be a direct evidence of two-photon phase coherence for the parametric pair. Indeed one can say that a photon from the parametric pair will be either transmitted or reflected by the beam splitters BS1 or BS2, depending on its "phase" relative to the coherent beam entering the other port of the beam splitter. Such a phase is undefined for a single photon, but the pair does have a two-photon coherence, which allows transmission or reflection by BS1 and BS2 to be strongly correlated. The quantum mechanical properties of this correlation are evidenced by Eq. (16), which leads to a violation of Bell's inequalities. Finally, let us note that the correlation could be probed "at the last instant,"[17,31] by allowing both the parametric pairs and the coherent beams to propagate separately a long way, and choosing their relative phase just before mixing and detection, using a fast electro-optical device.

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APPENDIX: BELL'S INEQUALITIES WITH LOCAL OSCILLATORS

This proof is closely related to the one given by Bell in Ref. 11, including "detector hidden variables." We will give here the essential features of the calculation, specifying clearly the hypothesis relevant to the present scheme.

A source emits pairs of correlated particles (here photons), counterpropagating to distant detectors. Each "detector" consists of two photomultipliers, in the two output ports of a beam splitter, and a "local oscillator" (LO) which enters the other input port. Consequently, the case is being considered where four photodetectors are employed, one in each output port of the beam splitter instead of just the two shown in Fig. 1. This allows one to follow closely the discussions of the more usual Bell's inequality experiments in which each polarization detector employs two photodetectors, one in each output port of a given polarization beam splitter. Both LO are supposed to be "locally" generated, i.e., are completely separated from the emission by the source. This is obviously possible in principle, by knowing well enough the frequency filters at the source (such filters are also used in atomic cascade experiments[13–17]). On each detector, one can adjust the phases ϕ_1 and ϕ_2 of the LO's. The result of a measurement will be $+1$ for a detection in the transmitted channel, -1 for a detection in the reflected channel.

Both the source and the detectors can be described within Bell's formalism, using local supplementary parameters as introduced in Ref. 11. We denote by λ these parameters for the source, and λ_1, λ_2 for the LO's. An important feature of the present scheme is that the detectors will register counts even without the source, due to the LO light. However, we are going to demonstrate that Bell's inequalities can be obtained for *any* pair of detections which occurs within a coincidence time W, which will be chosen to be of the order of the correlation time of the pairs emitted by the source.

Let us use $p_{12}^{ij}(\lambda,\lambda_1,\lambda_2,\phi_1,\phi_2)$ to denote the probabilities for joint detections within W, and $p_1^i(\lambda,\lambda_1,\phi_1)$, $p_2^j(\lambda,\lambda_2,\phi_2)$ to denote the probabilities for singles ($i,j=\pm 1$). The dependency of p_1^i (p_2^j) on λ and λ_1 (λ_2) allows a "redistribution" of the emitted pairs between both channels of the detectors, as a function of LO's parameters, which might be due to some kind of *local* interference effect at the detector. It also includes the fact that the photomultiplier may register a count, even with no emitted pair from the source. We remark however that in a naive "particle" point of view, the probability to get a LO photon *and* an emitted pair within the same W is very small.

In order to derive Bell's inequalities, we need two hypotheses:

$$p_{12}^{ij}(\lambda,\lambda_1,\lambda_2,\phi_1,\phi_2)=p_1^i(\lambda,\lambda_1,\phi_1)p_2^j(\lambda,\lambda_2,\phi_2)\,, \quad (A1)$$

$$p_n^+(\lambda,\lambda_n,\phi_n)+p_n^-(\lambda,\lambda_n,\phi_n)=p_n(\lambda,\lambda_n)\,, \quad (A2)$$

where $n=1,2$.

Equation (A1) is the well-known locality hypothesis, which is fundamental to obtain Bell's inequalities. It is written here in a form which was introduced first by Clauser and Horne[30] as a very general statement about "objective local theories."

Equation (A2) expresses that a change in ϕ_n *for a given set of parameters* can only redistribute the probabilities of detection between the transmitted and reflected channel, but does not change the *total* detection probability.

One then defines the correlation functions

$$E(\phi_1,\phi_2)=\frac{\left[\sum_{i,j} ij\,p_{12}^{ij}(\lambda,\lambda_1,\lambda_2,\phi_1,\phi_2)\right]_{\text{av}}}{\left[\sum_{i,j} p_{12}^{ij}(\lambda,\lambda_1,\lambda_2,\phi_1,\phi_2)\right]_{\text{av}}}\,, \quad (A3)$$

where av denotes an average over λ, λ_1 and λ_2. $E(\phi_1,\phi_2)$ is measured experimentally using the four twofold coincidence rates $N_{ij}(\phi_1,\phi_2)$:

$$E(\phi_1,\phi_2)_{\exp}=\frac{\sum_{i,j} ijN_{ij}(\phi_1,\phi_2)}{\sum_{i,j} N_{ij}(\phi_1,\phi_2)} . \quad \text{(A4)}$$

Using (A1) and (A2), the denominator in (A3) can be rewritten as $[p_1(\lambda,\lambda_1)p_2(\lambda,\lambda_2)]_{\text{av}}$, and corresponds experimentally to a coincidence in any of the four channels, i.e., to the denominator of (A4).

Using well-known calculations,[11] one then obtains

$$-2\leq S\leq 2 , \quad \text{(A5)}$$

where S is given as the result of four measurements:

$$S=E(\phi_1,\phi_2)-E(\phi_1,\phi_2')+E(\phi_1',\phi_2)+E(\phi_1',\phi_2') . \quad \text{(A6)}$$

On the other hand, let us consider quantum mechanical predictions for the scheme discussed in this paper, in the case where W is shorter than the correlation time $1/\Delta\omega$, such that the zero-delay result [Eq. (16)] can be used. Remembering that the relative phase between the local oscillator and the signal differs by π depending on whether the signal is transmitted through or reflected off of a beam splitter, Eq. (16) generalizes to

$$C_{12}^{ij}(0)=2[1+ij\cos(\phi_1+\phi_2-2\phi_0)] . \quad \text{(A7)}$$

One then gets

$$E(\phi_1,\phi_2)_{\text{QM}}=\cos(\phi_1+\phi_2-2\phi_0) . \quad \text{(A8)}$$

It is then easy to show that the choice

$$\phi_1+\phi_2-2\phi_0=\pi/4 , \quad \text{(A9a)}$$

$$\phi_1'=\phi_1-\pi/2 , \quad \text{(A9b)}$$

$$\phi_2'=\phi_2+\pi/2 , \quad \text{(A9c)}$$

yields $S_{\text{QM}}=2\sqrt{2}$, which strongly violates the inequality (A5). This violation is quantitatively the same as in atomic cascades experiments.[16] We emphasize that in this appendix hypothesis (A2) is a necessary assumption, which is related to the "no-enhancement hypothesis" of Ref. 6 or to the "nonbiased-sample hypothesis" of Ref. 16. A more detailed discussion of this assumption, in particular in the case of nearly perfect detection efficiency, will be presented in a forthcoming publication.

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