

Some parametric instabilities of an ordinary electromagnetic wave in magnetized plasmas

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Some of the parametric instabilities of an ordinary electromagnetic wave decaying into (i) a high-frequency upper hybrid wave and a low-frequency mixed-mode kinetic Alfvén wave and (ii) an obliquely propagating ordinary scattered electromagnetic wave and a low-frequency electrostatic-ion Bernstein wave (stimulated Brillouin scattering) have been investigated. In channel (i), the partially electrostatic nature of the kinetic Alfvén wave and the component of the low-frequency ponderomotive force along the direction of the external magnetic field lead to the dominant coupling. Explicit expressions for the growth rate and threshold power are given. It is found that, in the case of channel (ii), the homogeneous threshold power is lowest at the first harmonic of the low-frequency electrostatic-ion Bernstein wave. The relevance of the present investigation to fusion, ionospheric modification experiments, and space plasmas has been pointed out. For example, for fusion plasmas the convective threshold power for channel (i) is found to be approximately 20 kW/cm^2 .

I. INTRODUCTION

Heating of plasma by using ordinary electromagnetic (em) waves has been studied extensively both theoretically and experimentally.¹ These linear theories predict nearly complete absorption, in good agreement with experimental results. The linear absorption profile may change when the gyrotron source power exceeds a certain threshold and the pump is driven to the nonlinear regime, resulting in anomalous absorption through the excitation of parametric instabilities. The parametric instability involving a high-frequency wave, for example an upper hybrid wave, indirectly heats the ions. Most of the transferred energy of the pump is initially confined to the electrons in the tail. These electrons by electron-electron collisions transfer the energy to bulk electrons. The electrons ultimately transfer their energy to ions by electron-ion collisions. In this way both electrons and ions can be heated. Ions can also be heated directly when one of the parametric-decay waves is a low-frequency wave.

Shukla and Tagare,² and recently Heikkinen and Karttunen,³ have studied the parametric decay of an ordinary em wave into an ion cyclotron wave and an ordinary em wave. In this parametric scattering process the low-frequency response of the electrons has been taken to be adiabatic, and that of the ions to be nonadiabatic. In many physical situations, the phase velocity of the low-frequency wave along the field lines is greater than the electron thermal speed. In these situations, the low-frequency response of electrons cannot be assumed to be adiabatic. There is a possibility of excitation of some other low-frequency electrostatic waves for which the electron and ion inertia terms are to be retained. In those situations it is more appropriate to consider the low-frequency wave to be an ion Bernstein wave.

In the present paper we have shown that the ordinary em wave can be useful for anomalous heating of plasma by parametric decay of an ordinary em wave into a low-frequency ion Bernstein wave (IBW) and an ordinary em

wave (a stimulated Brillouin scattering process). Besides this, we have also considered another alternative decay channel in which the low-frequency wave can have its frequency less than the ion cyclotron frequency viz. decay of an ordinary em wave into a mixed-mode kinetic Alfvén wave (KAW) and an upper hybrid wave (UHW).

In Sec. II we set up the equations for the dynamics of the decay waves, i.e., the high-frequency upper hybrid wave, the low-frequency mixed-mode KAW, the high-frequency ordinary wave, and the low-frequency electrostatic IBW. Explicit expressions for the growth rate and threshold power have also been given in Sec. II. Section III presents the physical mechanism of coupling and relevance of this investigation not only to fusion plasmas but also to magnetospheric plasmas and ionospheric modification experiments.

II. BASIC EQUATIONS

Consider the propagation of an ordinary em pump wave along the x axis in a homogeneous magnetoplasma with the static magnetic field $\mathbf{B}_0 = B_0 \hat{z}$. The electric field of the pump wave is given by

$$\mathbf{E}_0 = \hat{z} E_{0z} \exp[-i(\omega_0 t - \mathbf{k}_0 \cdot \hat{\mathbf{x}})] + \text{c.c.}, \quad (1a)$$

where

$$k_0^2 = \frac{\omega_0^2 - \omega_{pe}^2}{c^2}. \quad (1b)$$

Here, ω_{pe} is the electron plasma frequency, c is the speed of light in vacuum, and $\mathbf{k}_0 = \hat{\mathbf{x}} k_{0x}$ is the propagation vector of the pump wave along the x direction.

One considers the decay instability of this ordinary em wave (ω_0, \mathbf{k}_0) into a mixed-mode KAW and an UHW. Both the decay waves are assumed to be propagating in the x - z plane. Energy and momentum conservation demand that

$$\omega_- = \omega - \omega_0 \quad \text{and} \quad \mathbf{k}_- = \mathbf{k} - \mathbf{k}_0.$$

The subscripts 0 and $-$ denote the pump and the scattered wave quantities, respectively, and the unsubscripted symbols denote the low-frequency quantities.

The equations governing the high-frequency wave

$$\epsilon_{-}(\omega_{-}, \mathbf{k}_{-})\phi_{-} = -\frac{\omega_{pe}^2}{e\omega_{-}^2 k_{-}^2 (\omega_{-}^2 - \omega_{ce}^2)} \left[i\omega_{-} k_{-x} F_{e-x}(\omega_{-}, \mathbf{k}_{-}) + \omega_{ce} k_{-x} F_{e-y}(\omega_{-}, \mathbf{k}_{-}) + \frac{i(\omega_{-}^2 - \omega_{ce}^2)}{\omega_{-}} k_{-z} F_{e-z}(\omega_{-}, \mathbf{k}_{-}) \right], \tag{2}$$

where

$$\epsilon_{-}(\omega_{-}, \mathbf{k}_{-}) = 1 - \frac{k_{-z}^2 v_{te}^2}{\omega_{-}^2} - \frac{k_{-x}^2 v_{te}^2}{(\omega_{-}^2 - \omega_{ce}^2)} - \frac{\omega_{pe}^2 \sin^2 \theta_{-}}{(\omega_{-}^2 - \omega_{ce}^2)} - \frac{\omega_{pe}^2}{\omega_{-}^2} \cos^2 \theta_{-}. \tag{3}$$

Here, θ_{-} is the angle between \mathbf{k}_{-} and \mathbf{B}_0 , ω_{ce} is the electron cyclotron frequency, and $v_{te}^2 (= T_e/m_e)$ is the electron thermal speed squared.

First, we set up the equations governing the dynamics of the low-frequency mixed-mode KAW. For low- β ($= 8\pi N_0 T_e / B_0^2$) plasmas, the perpendicular components of the wave electric field can be represented by⁴ $E_x = -\partial\phi/\partial x$ and $E_z = -\partial\Psi/\partial z$. Combining the two Maxwell's equations and the electron continuity equation, one obtains

$$\frac{e}{T_e} \rho_s^2 v_A^2 \frac{\partial^4}{\partial x^2 \partial z^2} (\phi - \psi) = \frac{\partial^2}{\partial t^2} \left[\frac{n_e}{N_0} \right], \tag{4}$$

where $\rho_s^2 = T_e / m_i \omega_{ci}^2$ and $v_A^2 = B_0^2 / 4\pi N_0 m_i$. Using quasineutrality condition and the ion continuity equation one obtains

$$v_A^2 \frac{\partial^4 (\phi - \psi)}{\partial x^2 \partial z^2} = \frac{\partial^4 \phi}{\partial t^2 \partial x^2} + \frac{T_e}{e \rho_s^2} \frac{\partial^2 v_{ix}^{nl}}{\partial t \partial x} \frac{\partial^2 v_{iz}^{nl}}{\partial t \partial x}. \tag{5}$$

When $\beta > m_e / m_i$, the adiabatic electrons attain equilibrium by streaming along the external magnetic field. Thus

$$\frac{n}{N_0} = \frac{e\psi}{T_e} - \frac{ie^2 E_{0z} \phi - k_{-z} \lambda_{e-}}{2T_e m_e \omega_0 \omega_{-}}, \tag{6}$$

$$\mu = -\frac{ie\lambda_{e-} v_A^2 k_z^3}{2m_e \omega_0 \omega_{-}^2} \left[\frac{m_e^2}{m_i^2} \frac{v_{te}^2}{\lambda_{e-} \omega_0^2 (\omega^2 - \omega_{ci}^2)} \left(k_x \frac{\omega \omega_{-}}{\omega_{ci}^2} [\omega_{ci}^2 k_{-x} + (\omega/\omega_{-})(k_x \omega_{-}^2 - k_{0x} \omega_{ci}^2)] - k_z^2 (\omega_0^2/\omega^2)(\omega^2 - \omega_{ci}^2) \right) + 1 \right] \tag{10}$$

is the coupling coefficient.

On the other hand, the nonlinear dispersion relation for high-frequency wave is obtained by simplifying Eq. (2) as

$$\epsilon_{-}(\omega_{-}, \mathbf{k}_{-})\phi_{-} = \mu E_{0z}^* \phi, \tag{11}$$

where $\epsilon_{-}(\omega_{-}, \mathbf{k}_{-})$ is given by Eq. (3) and the coupling coefficient μ_{-} is

$(\omega_{-}, \mathbf{k}_{-})$ can be obtained by using the Poisson's, continuity and momentum-balance equations. Combining these equations and assuming that the ions form the static background, we get

where

$$\lambda_{e-} = \left[1 - \frac{k_{-z}^2 v_{te}^2}{\omega_{-}^2} - \frac{k_{-x}^2 v_{te}^2}{\omega_{-}^2 - \omega_{ce}^2} \right]^{-1}.$$

Using this expression for n in Eq. (4) and using Eq. (5) one gets

$$\rho_s^2 \frac{\partial^4 \phi}{\partial t^2 \partial x^2} + \frac{T_e}{e} \left[\frac{\partial^2 v_{ix}^{nl}}{\partial t \partial x} + \frac{\partial^2 v_{iz}^{nl}}{\partial t \partial z} \right] = \frac{\partial^2}{\partial t^2} \left[\psi - \frac{ieE_{0z} \phi - k_{-z} \lambda_{e-}}{2m_e \omega_0 \omega_{-}} \right]. \tag{7}$$

Eliminating ψ with the help of Eq. (5) and Fourier transforming, one can obtain the nonlinear dispersion relation with the following result:

$$\epsilon(\omega, \mathbf{k})\phi = \mu\phi - E_{0z}, \tag{8}$$

where ϵ is the dielectric function given by

$$\epsilon(\omega, \mathbf{k}) = 1 - \frac{v_A^2 k_z^2}{\omega^2} - \frac{\rho_s^2 k_x^2 v_A^2 k_z^2}{\omega^2}, \tag{9}$$

and

$$\mu_{-} = \frac{ie\omega_{pe}^2 k_x k_{-x} k_z}{2m_e \omega_0 (\omega_{-}^2 - \omega_{ce}^2) v_A^2 k_z^2 k_{-}^2}. \tag{12}$$

Combining Eqs. (8) and (11) one can obtain the following nonlinear dispersion relation:

$$\epsilon(\omega, \mathbf{k})\epsilon_{-}(\omega_{-}, \mathbf{k}_{-}) = \mu\mu_{-} |E_{0z}|^2 \tag{13}$$

for this decay channel.

The nonlinear dispersion relation (13) can be solved for the homogeneous growth rate,⁵ which comes out to be

$$\gamma_0^2 = (\gamma_0 + \Gamma)(\gamma_0 + \Gamma_-) = - \frac{\mu\mu_- |E_{0z}|^2}{\frac{\partial\epsilon}{\partial\omega} \frac{\partial\epsilon_-}{\partial\omega_-}} \quad (14)$$

Now, one obtains the nonlinear dispersion relation for the decay into ordinary wave and IBW. The wave equation for the scattered ordinary wave after Fourier-analyzing can be written as

$$\vec{D} \cdot \mathbf{E}_- = - \left[\frac{4\pi i \omega_-}{c^2} \right] J_-^{nl} \quad (15)$$

where \vec{D} is the dispersion tensor

$$\vec{D} = -k_-^2 \vec{I} + \mathbf{k}_- \mathbf{k}_- + \frac{\omega_-^2}{c^2} \vec{\epsilon}_- \quad (16)$$

with $\vec{\epsilon}_-$ the linear dielectric tensor at (ω_-, \mathbf{k}_-) as given by Stix⁶ and J_-^{nl} is the nonlinear part of the current density at (ω_-, \mathbf{k}_-) due to the coupling of pump wave and low-frequency IBW.

The equation for the electrostatic IBW is obtained by using Poisson's equation. The total perturbed density n_α is given by

$$n_\alpha = \frac{\delta_\alpha q_\alpha N_0 \phi}{m_\alpha (\omega^2 - \omega_{c\alpha}^2)} \left[k_x^2 + k_z^2 \frac{(\omega^2 - \omega_{c\alpha}^2)}{\omega^2} \right] + \frac{\delta_\alpha}{\omega} \left[- \frac{N_0}{m_\alpha (\omega^2 - \omega_{c\alpha}^2)} \left(\omega_{c\alpha} \mathbf{k} \cdot (\mathbf{F} \times \hat{\mathbf{z}}) + i \frac{\omega_{c\alpha}^2}{\omega} F_{az} (\mathbf{k} \cdot \hat{\mathbf{z}}) - i \omega (\mathbf{k} \cdot \mathbf{F}_\alpha) \right) \right] \quad (17)$$

$$\delta_\alpha = \left[1 - \left(\frac{k_z^2}{\omega^2} + \frac{k_x^2}{(\omega^2 - \omega_{c\alpha}^2)} \right) v_{i\alpha}^2 \right]^{-1} \quad (18)$$

where the subscript α denotes the species level [electron (e) or ion (i)]; for electrons $q_\alpha = -e$ and for ions $q_\alpha = +e$, e is the electronic charge. The substitution of n_α from Eq. (17) into Poisson's equation gives

$$\epsilon k \phi = \beta \cdot \mathbf{E}_- \quad (19)$$

Here, ϵ is the linear dielectric function of the low-frequency wave given by

$$\epsilon = 1 - \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2} - \frac{\omega_{pe}^2}{\omega^2} \frac{k_z^2}{k^2} - \frac{k_x^2 v_{ti}^2}{(\omega^2 - \omega_{ci}^2)} \left[1 - \frac{\omega_{pe}^2}{\omega^2} \frac{k_z^2}{k^2} \right] \quad (20)$$

It is to be noted that the kinetic effects can be taken into account phenomenologically by replacing ϵ in Eq. (20) (Refs. 7 and 8) by

$$\epsilon = \frac{\omega_{pi}^2 \exp(-\lambda_i)}{\lambda_i} \sum_{n=m}^{\infty} \frac{I_n(\lambda_i) 2n^2}{(n^2 \omega_{ci}^2 - \omega^2)} - \frac{\omega_{pi}^2 \exp(-\lambda_i)}{\lambda_i} \sum_{n=1}^{m-1} \frac{I_n(\lambda_i) 2n^2}{(\omega^2 - n^2 \omega_{ci}^2)} + 1 - \frac{\omega_{pe}^2}{\omega^2} \frac{k_z^2}{k_x^2} = 0 \quad (21)$$

where $m\omega_{ci} \gtrsim \omega > (m-1)\omega_{ci}$ and we have used single ion species, cold electron plasma ($\omega \gg k_z v_{te}$), $\lambda_i = k_x^2 \rho_i^2$, $\rho_i^2 = T_i / m_i \omega_{ci}^2$, and I_n is the modified Bessel function of n th order. The nonvanishing relevant components of β in Eq. (19) are

$$\beta_x = \frac{e E_{0z} \omega_{pe}^2}{2m_e k \delta_i \omega^2 \omega_0 \omega_- (\omega^2 - \omega_{ce}^2) (\omega_-^2 - \omega_{ce}^2)} \left[\omega \omega_{ce}^2 (\omega + \omega_-) k_x k_{-z} - \frac{m_e^2}{m_i^2} \frac{\delta_i}{\delta_e} \frac{(\omega^2 - \omega_{ce}^2) (\omega_-^2 - \omega_{ce}^2)}{(\omega^2 - \omega_{ci}^2) (\omega_-^2 - \omega_{ci}^2)} [\omega \omega_{ci}^2 (\omega + \omega_-) k_x k_{-z}] \right] \quad (22)$$

and

$$\beta_z = \frac{e E_{0z} \omega_{pe}^2}{2m_e k \delta_i \omega^2 \omega_0 \omega_- (\omega^2 - \omega_{ce}^2) (\omega_-^2 - \omega_{ce}^2)} \times \left[(\omega_-^2 - \omega_{ce}^2) [\omega^2 k_x^2 + (\omega^2 - \omega_{ce}^2) k_z k_{-z}] - \frac{m_e^2}{m_i^2} \frac{\delta_i}{\delta_e} \frac{(\omega^2 - \omega_{ce}^2) (\omega_-^2 - \omega_{ce}^2)}{(\omega^2 - \omega_{ci}^2) (\omega_-^2 - \omega_{ci}^2)} \{ [\omega^2 k_x^2 + (\omega^2 - \omega_{ci}^2) k_z k_{-z}] (\omega^2 - \omega_{ci}^2) \} \right] \quad (23)$$

For this decay channel the dispersion relation can be obtained⁹ with the help of Eqs. (15) and (19) and is given by

$$\epsilon \|\vec{D}\| = \mu \mu_- E_{0z} E_{0z}^* \quad (24)$$

Here,

$$\mu \mu_- = \frac{P_1 Q_1 + P_2 Q_2 + P_3 Q_3}{E_{0z} E_{0z}^*}, \quad (25)$$

and the values of $P_1, P_2, P_3,$ and $Q_1, Q_2, Q_3,$ are given as follows:

$$P_1 = \frac{e E_{0z}^* \omega_- \omega_{pe}^2 k_z \delta_e}{2 m_e c^2 k (\omega^2 - \omega_{ce}^2) (\omega_-^2 - \omega_{ce}^2) \omega \omega_0} \times [\omega (\omega \omega_- + \omega_{ce}^2) k_{-x} + \omega_{ce}^2 (\omega + \omega_-) k_{0x}], \quad (26)$$

$$P_2 = \frac{ie E_{0z}^* \omega_{pe}^2 \omega_{ce} \omega_- \delta_e k_z}{2 m_e c^2 k \omega \omega_0 (\omega^2 - \omega_{ce}^2) (\omega_-^2 - \omega_{ce}^2)} \times [\omega (\omega + \omega_-) k_x - (\omega^2 - \omega_{ce}^2) k_{0x}], \quad (27)$$

$$P_3 = \frac{e E_{0z}^* \omega_{pe}^2 \delta_e}{2 m_e c^2 k \omega \omega_0 (\omega^2 - \omega_{ce}^2)} \times \left[\frac{(\omega + \omega_-) (\omega^2 - \omega_{ce}^2)}{\omega} k_z^2 + \omega \omega_- k_x^2 \right], \quad (28)$$

$$Q_1 = (\beta_x D_{zz} - \beta_z D_{xz}) D_{yy}, \quad (29)$$

$$Q_2 = (\beta_x D_{zz} - \beta_z D_{xz}) D_{yx}, \quad (30)$$

$$Q_3 = (-\beta_x D_{xz} + \beta_z D_{xx}) D_{yy} + \beta_z D_{xy}^2. \quad (31)$$

III. APPLICATION AND DISCUSSION

For the application point of view, we first consider the parametric decay of a perpendicularly propagating ordinary em wave into a high-frequency UHW and a low-frequency mixed mode KAW. A finite low-frequency ponderomotive force component along the direction of the external magnetic field and the electrostatic part of the low-frequency mixed-mode KAW leads to the dominant coupling. The ponderomotive force component provides a low-frequency electron motion, thereby exciting a KAW at the expense of the pump energy whereas the nonlinear current at the scattered wave frequency is driven by the interaction of the low-frequency KAW and the pump wave.

From the phase-matching conditions and the dispersion relations of the pump and decay waves, we find that in a low- β ($1 > \beta > m_e/m_i$) plasma this coupling is possible in the following parameter space: $0 < (\omega_0^2 - \omega_{pe}^2) < 2\omega_{ce}^2$. As an illustration, we point out the relevance of the present investigation to tokamak heating experiments using ordinary wave for heating. Taking the tokamak plasma parameters to be $N_0 = 2.43 \times 10^{12} \text{ cm}^{-3}$, $B_0 = 5 \text{ kG}$, $T_e \simeq 2 \text{ keV}$, we find $f_{ce} \simeq f_{pe} \simeq 14 \text{ GHz}$. For $f_0 \simeq 19.8 \text{ GHz}$ and $f \simeq 7.35 \text{ MHz}$ the homogeneous threshold power comes out to be $\sim 5.1 \text{ mW/cm}^2$, when the damping is assumed to be collisional damping and

$\nu_{ei} \sim 10^5 \text{ rad/sec}$. However, when the plasma inhomogeneity is taken into account, this parametric decay instability may not have time to develop before the perturbation wave packet is carried into the stable region. When the decay waves are the UHW and mixed-mode KAW the convective threshold is given by¹⁰

$$\frac{E_{0z}^2}{16\pi N_0 T_e} \gtrsim \frac{8}{\beta L} \frac{m_e}{m_i} \frac{\omega_0^2}{\omega^2} \frac{k_{-x}}{k_x^2}.$$

For the above-mentioned tokamak parameters and $k_{-x} L \sim 10$, we find the convective threshold power to be 20 kW/cm^2 . In the present day experiments where ordinary waves are being used for heating the plasmas such a high-power flux is easily available. Moreover, the convective threshold at lower plasma density and higher magnetic field (low β) will be higher.

Second, we consider the parametric decay of an ordinary em wave into another high-frequency ordinary wave and a low-frequency electrostatic IBW. The relevance of this decay process has been pointed out in fusion as well as in magnetospheric and ionospheric modification experiments.

From the phase-matching conditions and dispersion relations of the pump, scattered and IBW, it has been seen that this instability can exist in different parameter spaces. The homogeneous threshold power has been calculated for different values of f near the first, second, third, and fourth harmonics of ion cyclotron frequency f_{ci} . It is found that the homogeneous threshold power comes out to be lowest for f near the first harmonic of the ion cyclotron frequency. For tokamak parameters $N_0 \simeq 1.41 \times 10^{13} \text{ cm}^{-3}$, $B_0 \simeq 14.75 \text{ kG}$, $T_e \simeq 1 \text{ keV}$, $\nu_{ei} \sim 10^4 \text{ rad/sec}$, we find $f_{ce} \simeq 41.32 \text{ GHz}$ and $f_{pe} \simeq 33.68 \text{ GHz}$. For $f \simeq 21.37 \text{ MHz}$ and $\omega_{pe}^2 \simeq 0.9\omega_0^2$ the threshold power comes out to be 30 W/cm^2 .

Now we present the relevance of the present parametric scattering process to a recent ionospheric modification experiment,¹¹ where an ordinary em wave was used in the under dense ionosphere (F layer) and the scattered wave spectrum was observed. The complex nature of the spectrum was attributed to the excitation of other natural modes of the magnetized plasmas.

For illustration point of view, we have applied the result of the present investigation to the F -layer region (200 km). For F -layer parameters $N_0 = 10^4 \text{ cm}^{-3}$, $B_0 = 0.45 \text{ G}$, $T_e \sim 2000 \text{ K}$, we find $f_{ce} \simeq 1.26 \text{ MHz}$, $f_{pe} \simeq 0.89 \text{ MHz}$. For $f_0 \simeq 0.91 \text{ MHz}$ and $f \simeq 42.31 \text{ Hz}$, $\nu_{ei} \simeq 50 \text{ rad/sec}$ the homogeneous threshold power for this parametric decay process comes out to be $2.5 \times 10^{-8} \text{ W/cm}^2$ and effective radiating power (ERP) 125 MW. This is much below the value of ERP used in the experiment of Ref. 11. Thus this decay channel may be responsible for some of the features of the spectrum of scattered wave.

Finally, we point out that our results can be quite useful to the understanding of wave phenomena in the magnetosphere where, in the vicinity of polar cusp region ($R \sim 5R_e$) a broadband electrostatic emission extending from a few Hz to about 30–100 Hz with maximum intensity at about 10–50 Hz has been observed.¹² The mechanism for generating this electrostatic emission remains

highly uncertain. Various mechanisms have been proposed¹³ to explain this electrostatic emission. Here, we propose that the generation of this broadband electrostatic emission can be attributed to the parametric decay process viz. SBS of ordinary em waves of which the auroral kilometeric radiation (AKR) is comprised.^{14,15}

As an illustration for magnetospheric plasma parameters near the polar cusp region at 5 earth radii, $N_0 = 10^2 \text{ cm}^{-3}$, $B_0 = 500 \text{ nT}$, $T_e \sim 1.12 \text{ eV}$, $\Gamma \Gamma_- \sim \nu_{ei}^2$, $\nu_{ei} \sim 10^{-3} \text{ rad/sec}$. One finds $f_{ce} \simeq 14 \text{ kHz}$, $f_{pe} \simeq 89.76 \text{ kHz}$. For $f_0 \simeq 97.08 \text{ kHz}$ and $f \sim 7.12 \text{ Hz}$, the threshold power

flux comes out to be $5 \times 10^{-21} \text{ W m}^{-2} \text{ Hz}^{-1}$. As the fields with higher power have already been observed in AKR in the polar cusp region, this decay is possible. Thus the simultaneous occurrence of the broadband electrostatic noise and the presence of ordinary em waves as AKR can be attributed to the parametric process as discussed here.

In conclusion, we have presented several examples of the relevance of the present parametric processes to the available observations. In addition to this, our present theory may also be useful to the understanding of future experimental data.

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