PHYSICAL REVIEW A VOLUME 38, NUMBER 6 SEPTEMBER 15, 1988

Slow-wave electron cyclotron maser

T. H. Kho and A. T. Lin
Department of Physics, University of California at Los Angeles, Los Angeles, California 90024
(Received 25 April 1988)

The basic physics of a slow-wave electron cyclotron maser (ECM) operating in the Cherenkov regime is considered. This device has the advantage over fast-wave ECM's in that it can be operated with direct axial injection of the electron beam, thus allowing for better control over beam quality and a potentially more compact design. The nonlinear evolution and saturation of the instability are studied using computer simulation. It is shown that high efficiency is attainable and, furthermore, that beam momentum spread is better tolerated in the Doppler-shift-dominated regime than is the case for a fast-wave ECM.

I. INTRODUCTION

Research in electron cyclotron masers (ECM) to date has focused mainly on fast-wave variants, such as the gyrotron and the cyclotron autoresonance maser (CARM).[1-4] In the Gyrotron, the wave phase velocity along the magnetostatic field $\mathbf{B}_0$ is much higher than the speed of light in vacuum, $v_p=\omega/k\gg c$, while in a CARM, $v_p\approx c$. Coupling of the electron-beam kinetic energy to the electromagnetic (em) wave in these fast-wave devices depends on the beam having an initial transverse momentum in the background axial magnetic field.

The production of an electron beam with significant transverse as well as axial momentum is difficult to achieve technically if a high degree of uniformity is required. Most processes of imparting a transverse momentum to the electrons, such as injecting the beam through a transverse undulating magnetic field, will induce a spread in axial momentum. Beam momentum spread is particularly damaging to Doppler-shift-dominated devices such as the CARM, where the resonance condition is given by $\omega=kv_\parallel+\Omega$, where $kv_\parallel\gg\Omega$ is the Doppler shift along the axial magnetic field $\mathbf{B}_0$, and $\Omega=eB_0/\gamma mc$ is the relativistic cyclotron frequency. A small spread in beam axial velocity $v_\parallel$ will be greatly amplified in the Doppler term. Beam momentum spread must be limited to less than a few percent for the CARM to be operationally viable.

It is clearly advantageous to have an ECM which will operate with direct axial beam injection; axial beams (i.e., with axial momentum only) can easily be produced with very little momentum spread. Using single-particle theory, previous authors have shown that direct injection is possible in a slow-wave ECM operating in the Cherenkov regime, i.e., where $v_\parallel>v_p$.[5,6] This has been confirmed experimentally.[7,8] The characteristics of this slow-wave ECM are distinguishable from that of a conventional Cherenkov maser[9]: The instability in the former depends on the axial magnetic field, and the wave vector $\mathbf{k}$ of the em wave is independent of the Cherenkov angle $\theta=\cos^{-1}(v_p/v_\parallel)$. Also, it will be shown here that no axial density bunching of the beam is involved.

An added attraction of this slow-wave ECM is the potential for large frequency upshift with moderate magnetic field and beam energy. The resonance frequency is given by

$$\omega=kv_\parallel-\Omega \tag{1}$$

or

$$\omega=\frac{\Omega}{(\beta_\parallel/\beta_p-1)}\ , \tag{1a}$$

showing that substantial frequency upshift, $\omega\gg\Omega$, can be achieved with $\beta_\parallel\approx\beta_p$, where $\beta_\parallel=v_\parallel/c$ and $\beta_p=v_p/c$. Since $\beta_p<1$, it is not necessary to have a very energetic electron beam to have $\beta_\parallel\approx\beta_p$.

In this paper we examine the basic physics of a slow-wave ECM operating in the Cherenkov regime. The single-particle theory is reviewed and computer simulation is used to investigate the instability in a simple one-dimensional (1D) model. Slow-wave ECM's which operate outside the Cherenkov regime, like all fast-wave ECM's, require an initial transverse beam momentum to couple to the em wave;[10,11] these are not considered here. In Sec. II we review the single-particle theory of em wave-electron interaction in a uniform axial magnetic field, showing why only if $v_\parallel>v_p$ is wave amplification possible with an axial electron beam. The single-particle efficiency due to electron deceleration by a plane monochromatic wave is considered. Section III contains the linear theory and computer simulation of the slow-wave cyclotron instability. The dependence of the efficiency on beam energy, magnetic field, and wave phase velocity is presented. Sensitivity of the slow-wave ECM to beam momentum spread is investigated. Our findings are summarized in Sec. IV.

II. ELECTRON-em WAVE INTERACTION AND SINGLE-PARTICLE EFFICIENCY

Consider a transverse em wave with wave vector $\mathbf{k}$ directed along a uniform axial magnetic field $B_0\hat{\mathbf{z}}$. The motion of an electron in the field of the wave is governed by

$$\frac{d\mathbf{p}}{dt}=-e\mathbf{E}_\perp-e\frac{\mathbf{v}}{c}\times(\mathbf{B}_\perp+\mathbf{B}_0) , \quad (2a)$$

$$\frac{dU}{dt}=-e\mathbf{v}\cdot\mathbf{E}_\perp , \quad (2b)$$

where $\mathbf{p}=\gamma\mathbf{v}$ is electron momentum, $\mathbf{E}_\perp$ and $\mathbf{B}_\perp$ the em wave electric and magnetic fields, respectively, $U=\gamma mc^2$ the electron energy, and e the charge on a positron.

The salient features of electron dynamics can be revealed, without resorting to a full solution of the equations of motion, by considering the constants of motion. Using $\mathbf{B}_\perp=(c/\omega)\mathbf{k}\times\mathbf{E}_\perp$, and assuming constant phase velocity, we obtain from Eqs. (1) and (2),

$$\frac{dU}{dt}=v_p\frac{dp_\parallel}{dt} ,$$

whence

$$\gamma(1-\beta_p\beta_\parallel)=C , \quad (3)$$

where $\beta_p=v_p/c$, $\beta_\parallel=v_\parallel/c$, and C is a constant of motion. Equation (3) defines a characteristic in γ-$\beta_\parallel$ phase space that an electron with given initial energy and momentum will be confined to in interacting with the em wave. This characteristic does not depend on $\mathbf{B}_0$ or the wave amplitude, however, the particular segment of it accessible to an electron with given initial conditions will depend in part on these quantities.

Equation (3) can be rewritten as

$$\beta_\perp^2+\beta_\parallel^2+(1-\beta_p\beta_\parallel)^2/C^2=1 , \quad (4)$$

where $\beta_\perp^2=1-\gamma^{-2}-\beta_\parallel^2$ is the normalized transverse velocity. Equation (4) shows that for a given β_p, there are in general two roots of $\beta_\parallel$ at which $\beta_\perp=0$, i.e., where an axial electron beam can couple to the em wave. We shall denote these roots as β_1 and β_2, with $\beta_2\geq\beta_1$.

Figures 1 and 2 show generic examples of the solutions of Eqs. (3) and (4) for the cases $\beta_p>1$ and $\beta_p<1$, respectively. For $\beta_p>1$, γ has a singularity at $\beta_\parallel=\beta_p^{-1}$, where the transverse magnetic field of the em wave vanishes in the rest frame of the beam. If an axial beam has $\beta_\parallel=\beta_p^{-1}$, it can only gain transverse kinetic energy from the wave. Both β_1 and β_2 coincide with local minima in γ. Interaction of an axial beam (with $\beta_\parallel=\beta_1$ or β_2) with a fast em wave can only result in conversion of wave energy to beam kinetic energy. Wave amplification can only occur if the beam starts with a finite $\beta_\perp$, as in a gyrotron.

For $\beta_p<1$, β_1 and β_2 satisfy $\beta_1\leq\beta_p\leq\beta_2$. Degenerate roots occur at $\beta_1=\beta_2=\beta_p$. Physically, this means that if the initial velocity of an axial beam exactly matches the wave phase velocity, no coupling occurs between electron and wave. This is because in the reference frame of the beam, the transverse electric field vanishes so that an electron observe only a magnetostatic field. Since the electron is stationary in this reference frame, no forces act on it.

However, if the axial beam has initial velocity $\beta_\parallel>\beta_p$, Fig. 2(b) shows that it can interact with the wave by losing kinetic energy to it. This can be understood directly From Eq. (2): Transverse acceleration of an axial beam by the wave is given by

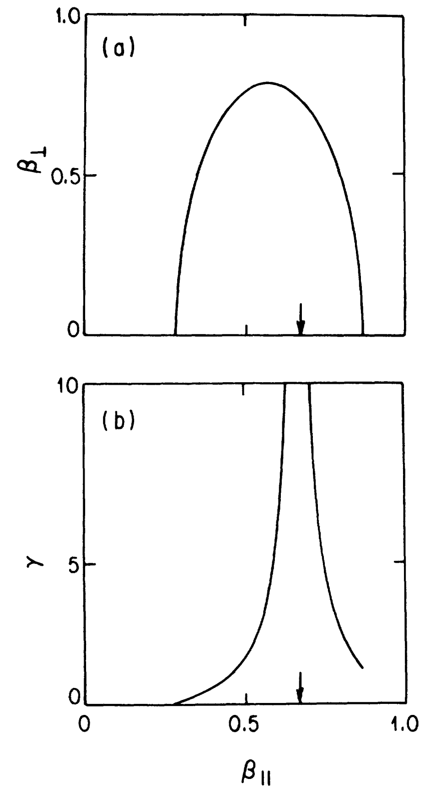

FIG. 1. Single-particle characteristic for fast wave $\beta_p>1$: (a) $\beta_\perp$ vs $\beta_\parallel$, (b) γ vs $\beta_\parallel$. Arrow indicates where $\beta_\parallel=\beta_p^{-1}$.

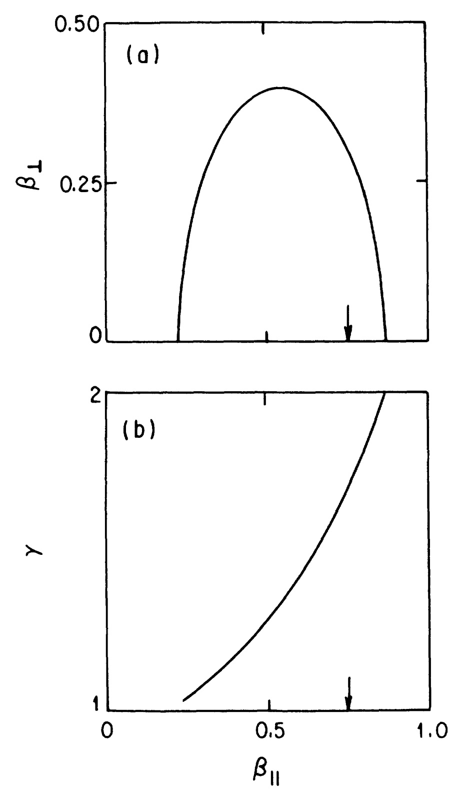

FIG. 2. Single-particle characteristic for slow wave $\beta_p<1$: (a) $\beta_\perp$ vs $\beta_\parallel$, (b) γ vs $\beta_\parallel$. Arrow indicates where $\beta_\parallel=\beta_p$.

$$\frac{d\mathbf{p}_\perp}{dt} = -e\mathbf{E}_\perp - e\frac{\mathbf{v}_\parallel}{c}\times\mathbf{B}_\perp = -e\mathbf{E}_\perp + e\frac{\mathbf{v}_\parallel}{v_p}\mathbf{E}_\perp ,$$

showing that if $v_\parallel > v_p$, transverse acceleration of the beam is dominated by $\mathbf{v}_\parallel \times \mathbf{B}_\perp$, so that $\mathbf{v}_\perp$ is parallel to $\mathbf{E}_\perp$ and hence the beam loses energy to the wave. If $v_\parallel < v_p$, $\mathbf{v}_\perp$, transverse is dominated by the electric field which does work on the electron.

The em wave can originate either from an external source or from noise through self-excitation of the beam. In the former case, wave amplification is possible in principle with or without the axial magnetic field. Self-excitation or exponential gain, however, is possible only if $B_0 \neq 0$, as will be shown in Sec. III.

Regardless of the origin of the em wave, a useful indication of the maximum efficiency achievable in practice in this slow wave interaction is given by the single-particle efficiency $\eta_{\rm SP}$ defined by

$$\eta_{\rm SP} = (\gamma_0 - \gamma_f)/(\gamma_0 - 1) , \tag{5}$$

where γ_0 and γ_f are the initial and final electron Lorentz factors, respectively; γ_f is the minimum value physically permissible for given initial conditions. It can be seen from Fig. 2 that if an electron could be decelerated from β_2 to β_1, a single-particle efficiency of 100% could be achieved by choosing suitable parameters to make $\beta_1 = 0$ (and hence, $\gamma_f = 1$). However, for the purpose of wave amplification, the limiting velocity to which an electron may be decelerated is in fact β_p.

This is most easily seen for the case $B_0 = 0$ (corresponding to an amplifier). Let $p_\perp = \gamma v_\perp$ and $A_\perp$ be the vector potential of the em wave. Since transverse conjugate momentum is conserved in this case:

$$p_\perp(t) - \frac{e}{c}A_\perp(t) = \psi , \tag{6}$$

where ψ is a constant of motion. Equation (6) immediately shows that it is not possible to decelerate an electron from β_2 to β_1 with a final radiation field $A_\perp(t_f) > A_\perp(0)$. Also, from Eq. (4),

$$p_\perp \frac{dp_\perp}{d\beta_{z_\parallel}} = \gamma^2 c^2(\beta_p - \beta_\parallel)/(1 - \beta_p\beta_\parallel) , \tag{7}$$

showing that $dp_\perp/d\beta_\parallel = 0$, i.e., $p_\perp$ is at maximum, when $\beta_\parallel = \beta_p$. Since $A_\perp(t) \propto p_\perp(t)$, maximum amplification of the wave occurs at $\beta_\parallel = \beta_p$. This constraint was not considered by previous authors who pointed out the possibility of 100% efficiency as $B_0 \to 0$.[5,6]

When $B_0 \neq 0$, ψ is no longer constant so the above analysis does not apply. However, consider a Lorentz transformation to the rest frame of the wave.[12] Then as mentioned above, the transverse electric field disappears and the electron observes a helical transverse magnetostatic field (for a circularly polarized wave) and an axial guide field. The equilibrium motion of an electron in this field has been studied previously[13] in the context of a free-electron laser and is given by

$$v'_\perp/v'_\parallel = (B'_\perp/B_0)/(1 - k'v'_\parallel/\Omega') , \tag{8}$$

$$v'^2_\perp + v'^2_\parallel = \text{const} ,$$

where the prime indicates evaluation in the wave's rest frame. As $v_\parallel \to v_p$, $v'_\parallel \to 0$, so

$$v'_\perp/v'_\parallel \simeq B'_\perp/B_0 . \tag{9}$$

Equation (9) shows that $v'_\parallel \to 0$ asymptotically as $B'_\perp \to \infty$; it is not possible to reverse the direction of the electron in this magnetostatic field. Hence, even when $B_0 \neq 0$, v_p must be the limiting value to which the electron can be decelerated by the wave.

Taking the limiting electron velocity to be v_p, the single-particle efficiency, using Eqs. (3) and (5), is given by

$$\eta^*_{\rm SP} = \frac{\gamma_0\beta_p^2(\alpha - 1)}{(\gamma_0 - 1)(1 - \beta_p^2)} , \tag{10}$$

where $\alpha = \beta_{\parallel 0}/\beta_p$, $\beta_{\parallel 0}$ being the initial velocity. For given γ_0, the maximum efficiency is

$$\eta_{\rm SP} = 0.5 , \tag{11}$$

which is achieved when the phase velocity is given by

$$\beta_p = (1 - \gamma_0^{-1})/\beta_{\parallel 0} .$$

III. THEORY AND COMPUTER SIMULATION OF A SLOW-WAVE CYCLOTRON INSTABILITY

Retardation of the em wave in Cherenkov devices may be achieved by using a dielectric-lined or dielectric-filled waveguide.[14] The practical implementation of such devices is a subject of active research. Here we confine ourselves to the basic physics of em wave-electron interaction and simplify the analysis by assuming a 1D model of an axial electron beam directed along a uniform magnetic field $B_0\hat{\mathbf{z}}$ in an arbitrary dielectric medium. We also restrict the analysis to plane em waves with vector $\mathbf{K}$ parallel to $\mathbf{B}_0$ only. Since Cherenkov radiation is at an angle to the axial beam momentum, the latter restriction excludes Cherenkov instability from this model.

Evolution of the em field is given by Maxwells equations:

$$\nabla\cdot\mathbf{B} = 0 , \tag{12a}$$

$$c\nabla\times\mathbf{E} = -\partial\mathbf{B}/\partial t , \tag{12b}$$

$$c\nabla\times\mathbf{B} = 4\pi\mathbf{J} + \partial\mathbf{E}/\partial t . \tag{12c}$$

To model the retardation of em waves by an arbitrary dielectric medium, Eq. (12c) is rewritten as

$$\nabla\times\mathbf{B} = \epsilon_r\partial\mathbf{D}/\partial t \tag{13}$$

where $\mathbf{D}$ is the electric displacement of the beam plasma in the uniform magnetic field and $\epsilon_r > 1$ is an assumed constant which determines the degree to which the dielectric slows em waves.

Derivation of the dispersion relation for a cold axial electron beam in the magnetostatic field is straight forward (see, for e.g., Ref. 15). In the 1D model, space charge and em waves decouple. The dispersion relation for the latter is given by

$$(\omega - kv_\parallel + \Omega)(\omega^2 - k^2c_r^2 - \omega_p^2/\gamma) = -\Omega\omega_p^2/\gamma , \tag{14}$$

where $c_r = c/\sqrt{\epsilon_r}$. Only the left polarized waves are shown in Eq. (14) as the right polarized waves are always stable and so are of no interest here. It is seen from Eq. (14) that if $\Omega = 0$, the different branches of the em wave are also always stable. For finite Ω, solution of Eq. (14) reveals unstable wave growth at the intersection of the slow cyclotron mode $\omega \simeq kv_\parallel - \Omega$ and $\omega \simeq kc_r$ (Fig. 3). The cyclotron resonance $\omega = kv_\parallel - \Omega$ results from the fact that although the equilibrium axial beam is impervious to $\mathbf{B}_0$, any transverse perturbation of the beam will impart cyclotron motion to the electrons. The growth rate at $\omega_0 = kv_\parallel - \Omega = kc_r$ may be derived analytically. Let $\omega = \omega_0 + \Delta\omega$, where $\Delta\omega \ll \omega_0$. Then from Eq. (14),

$$\Delta\omega^2 = -(\omega_p^2/2\gamma_0\omega_0)(\Omega - \Delta\omega) . \quad (15)$$

Assuming $\Omega \gg \Delta\omega$,

$$\Delta\omega^2 = -(\omega_p^2/2\gamma_0)(\Omega/\omega_0) = -(\omega_p^2/2\gamma_0)(\alpha - 1) .$$

Hence, instability requires $\alpha > 1$, which is consistent with the single electron theory discussed above. The growth rate is given by

$$\mathrm{Im}[\Delta\omega] = \omega_p[(\alpha - 1)/2\gamma_0]^{1/2} . \quad (15a)$$

A $1\frac{2}{2}$-dimensional, em particle code[16] was used to simulate the slow-wave cyclotron instability. As in the linear theory above, the dielectric medium was modeled using Eq. (13). An arbitrary number of k modes (parallel to $\mathbf{B}_0$) of the em wave can be included in the simulation. The beam electrons were initialized with the equilibrium energy and axial momentum and the unstable em waves were then allowed to grow from noise.

Figure 3(b) shows the growthrate obtained from theory and a multiple k-mode simulation for $\Omega/\omega_p = 2.0$, and $\beta_r = c_r/c = 0.75$. The unstable waves in the simulation correspond to the slow cyclotron wave with growth rates in agreement with linear theory. The efficiency is about 5%. To optimize the wave-electron interaction, it is desirable to avoid nonlinear mode coupling by keeping only a single k mode of the em wave. This can be done in practice with an appropriate resonator. Only one unstable k mode of the em wave was kept in the computer simulation for the results presented below.

Figure 4 shows the evolution of an initially axial electron beam in the slow-wave cyclotron instability. Transverse acceleration of the axial beam by the em wave results in a helical beam with the same periodicity and helicity as the wave. There is no axial velocity modulation and consequently no density bunching of the beam. This is because the axial ponderomotive force is constant along the beam. Let the transverse magnetic field be

$$\mathbf{B} = B_\perp(t)[\hat{\mathbf{x}}\cos(kz - \omega t) + \hat{\mathbf{y}}\sin(kz - \omega t)]$$

and the transverse beam velocity driven by the em wave be

$$\mathbf{v}_\perp = v_\perp(t)\{\hat{\mathbf{x}}\cos[kz + \phi(t)] + \hat{\mathbf{y}}\sin[kz + \phi(t)]\} ,$$

where $\phi(t)$ is an arbitrary phase with respect to the wave. Hence, $\mathbf{F}_p$, the ponderomotive force given by

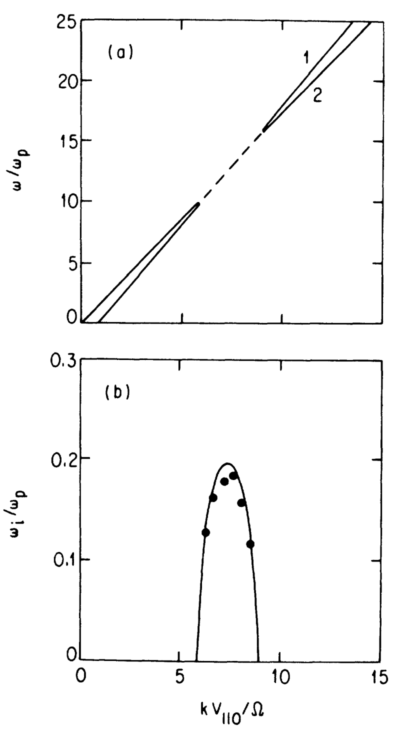

FIG. 3. (a) Dispersion relation for slow-wave cyclotron instability with $\gamma_0 = 2.0$, $\Omega = 2.0\omega_p$, and $c_r/c = 0.75$. Lines 1 and 2 correspond to $\omega = kv_\parallel - \Omega$ and $\omega = kc_r$, respectively. The unstable region is indicated by the dashed line. (b) Theoretical (line) and computational (dots) growth rates of the instability in (a).

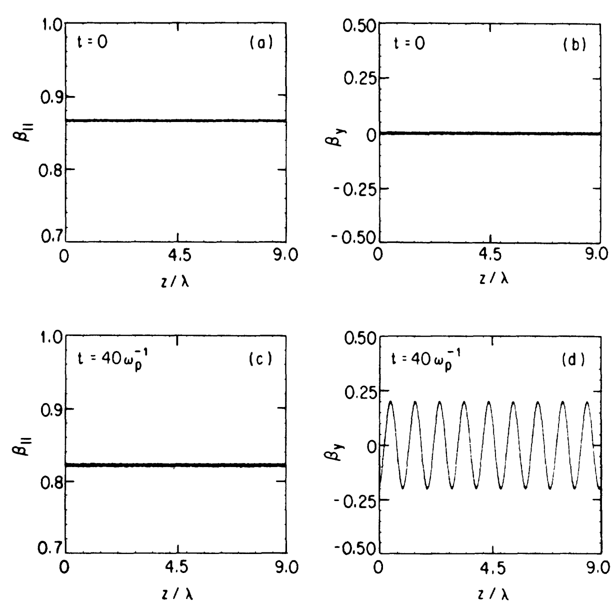

FIG. 4. Phase plots of electron velocity along the beam (in units of the radiation wavelength λ) at time $t = 0$ [(a) and (b)] and just before the cyclotron instability saturated [(c) and (d)].

$$\mathbf{F}_p \propto \mathbf{v}_\perp \times \mathbf{B}_\perp \propto v_\perp B_\perp \sin[\omega t + \phi(t)]\hat{\mathbf{z}} ,$$

is independent of z. The absence of axial density bunching is in marked contrast to a conventional Cherenkov maser.

Also, since the beam is gyro coherent and its phase relative to the wave is independent of z, the beam is always bunched in θ-phase space where θ is defined by $\cos\theta = \mathbf{v}_\perp \cdot \mathbf{E}_\perp/(v_\perp E_\perp)$. This simplifies the analysis of beam to radiation energy conversion efficiency as the whole beam can be treated as a single particle. In Sec. II it was argued that the phase velocity of the wave places an upper limit to the maximum energy that can be extracted from the beam. Another saturation mechanism here is phase detuning between the beam and wave. The efficiency η_d due to the latter effect may be estimated as follows. Assuming the resonance condition [Eq. (1)] is initially satisfied, the change in phase between electron and wave is given by

$$\Delta\theta(t) = \int_0^t \Delta\omega(t)dt , \tag{16}$$

where from Eqs. (1) and (3),

$$\Delta\omega = k\,\Delta v_\parallel + \Omega(\Delta\gamma/\gamma) = \frac{\Omega(1-\beta_p^2)}{\beta_p^2(\alpha-1)} \frac{\Delta\gamma}{\gamma_0} . \tag{17}$$

Since from conservation of energy

$$\Delta\gamma(t) \propto e^{2\omega_i t} , \tag{18}$$

where ω_i is the growth rate of the wave, Eq. (16) may be expressed as

$$\Delta\theta(t) \simeq \frac{1}{2} \frac{\Omega(1-\beta_p^2)}{\beta_p^2(\alpha-1)} \frac{\Delta\gamma}{\gamma_0} \frac{1}{\omega_i} . \tag{19}$$

As the beam gains transverse momentum only from the action of the wave, [from $(\mathbf{v}_\parallel \times \mathbf{B}_\perp)$ and $\mathbf{E}_\perp$], $\mathbf{v}_\perp$ must initially be parallel to $\mathbf{E}_\perp$, and the maximum change in γ occurs when $|\Delta\theta| = \pi/2$, the transition between the deceleration and acceleration phases of the wave. Hence, the efficiency due to wave-electron detuning is given [using Eq. (15) for ω_i] approximately by

$$\eta_d = \left[\frac{\gamma_0}{2}\right]^{1/2} \frac{\omega_p}{\Omega} \frac{\pi\beta_p^2(\alpha-1)^{3/2}}{(\gamma_0-1)(1-\beta_p^2)} . \tag{20}$$

The actual efficiency η is determined by

$$\eta = \min[\eta_{\mathrm{SP}}^*, \eta_d] . \tag{21}$$

From Eqs. (10) and (20),

$$\eta_d/\eta_{\mathrm{SP}}^* = \pi\frac{\omega_p}{\Omega}\left[\frac{\alpha-1}{2\gamma_0}\right]^{1/2} . \tag{22}$$

For most practical applications B_0 must function to collimate the beam so that $(\omega_p/\Omega) < 1$ will prevail. Also, for large frequency upshift $(\alpha-1) \ll 1$. If $\alpha > 2$, the output frequency will be less than Ω [Eq. (1a)], and will be of little practical interest. Hence, in the regime of most interest the saturation mechanism will be due to phase detuning between electron and wave.

Figure 5 shows the efficiency η obtained from computer simulation for the case $\gamma_0 = 2.0$, $\Omega/\omega_p = 2.0$ as a function of the wave phase velocity $\beta_p/\beta_{\parallel 0}$. Also show in the figure are the theoretical values for η_{SP}^* and η_d. The computational results agree with the theoretical values for η_d. The efficiency increases with α, but at the expense of frequency upshift.

The efficiencies in Fig. 5 were for a cold electron beam. It was pointed out earlier that operation in the Doppler-shift-dominated regime $(\omega \approx kv_\parallel)$ is particularly vulnerable to beam momentum spread because of the amplifying effect of the Doppler term. The effect of finite beam momentum spread on the efficiency is shown in Fig. 6 for a Doppler-shift-dominated slow-wave ECM with $\gamma_0 = 2.0$, $\Omega = 2.0\omega_p$, and $c_r/c = 0.75$ $(\omega \simeq 1.7\gamma_0^3\Omega)$. The cold beam efficiency is about 14%. With a 5% momentum spread, the efficiency drops to 3%. Shown for comparison in Fig. 6 are results for a CARM with the same Doppler shift $kv_\parallel$ as the slow-wave ECM, but with $\gamma_0 = 2.8$, $\beta_\perp = 0.35$, and $\Omega = 2.4\omega_p$ $(\omega \simeq \gamma_0^2\Omega)$. It has a cold beam efficiency of 11% which drops rapidly to 1.5% with just a 2% momentum spread. It appears that for the same Doppler shift, the slow-wave ECM is more tolerant of beam momentum spread than the fast-wave CARM.

This difference is probably due to the different bunching processes in the two devices. In the slow-wave ECM, the em wave tends to decelerate (or accelerate) the electrons in the beam coherently as a bunch in θ space; beam momentum spread diffuses the bunch, but as long as an electron has initial axial velocity $v_\parallel > v_p$ it will be decelerated by the wave. In the CARM, electrons are initially approximately gyrotropic in θ space even for a cold beam. The action of the em wave will bunch the electrons[3,4] but only if they remain resonant with the wave; beam momentum spread reduces the number of resonant electrons. The fraction of electron trapped in the decelerating phase of the wave thus depends more critically on beam momentum spread.

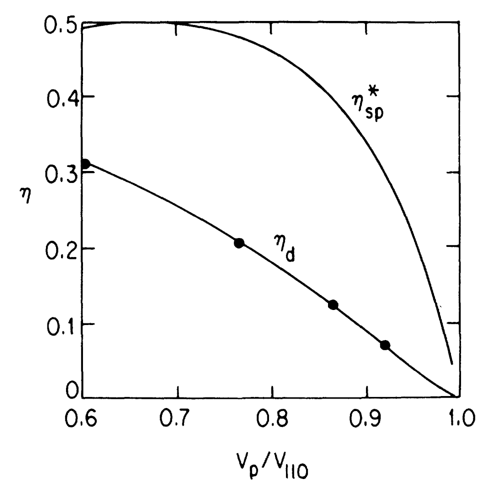

FIG. 5. Efficiency as a function of $v_p/v_{\parallel 0}$ for $\gamma_0 = 2.0$ and $\Omega = 2.0$. Simulation results are represented by dots. The theoretical values due to phase detuning, η_d, and depletion of single-particle energy, η_{SP}^*, are shown as continuous lines.

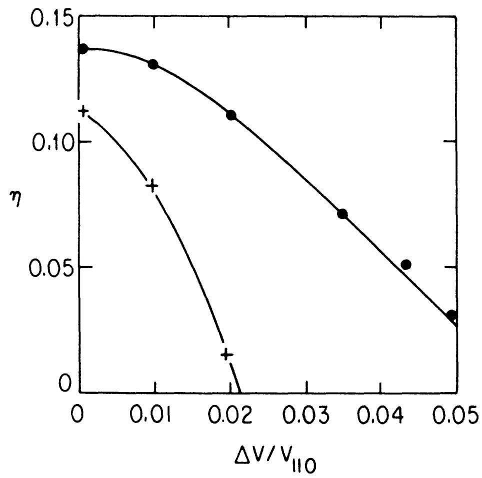

FIG. 6. Effect of a thermal spread on the efficiency of a slow-wave ECM (dots) with $\gamma_0=2.0$, $\Omega=2.0\omega_p$, $c_r/c=0.75$ and a CARM (crosses) with $\gamma_0=2.8$, $\Omega=2.4\omega_p$. The wave vector k and axial velocity $v_{\parallel 0}$ are the same in both cases, with $kv_{\parallel 0}=15.3\omega_p$. $\Delta v=\Delta v_{\parallel}=\Delta v_{\perp}$ is the standard deviation of a Gaussian velocity distribution about the mean values $v_{\parallel 0}$ and $v_{\perp 0}$.

IV. SUMMARY

A slow-wave electron cyclotron maser can be operated with direct axial injection of an electron beam if the beam velocity is greater than the phase wave of the em wave. The instability depends on the axial magnetic field and the output frequency is at the slow-wave cyclotron resonance $\omega=kv_{\parallel}-\Omega$. There is no axial density bunching. However, in θ-phase space, the beam electrons are bunched throughout the interaction with the em wave. Analytic expressions for the growth rate and efficiency were presented.

A primary motivation for direct axial injection is that high quality beams, which are crucial to operation in the Doppler-shift-dominated regime, are more easily attainable. Furthermore, it was shown here that for the same Doppler shift, the slow-wave ECM is more tolerant of beam momentum spread than a fast-wave CARM.

The Cherenkov maser instability was not considered in the simple model used in this work. In a realistic device, mode competition between Cherenkov and slow-wave cyclotron instabilities will be an important issue and will be considered in a future investigation. We note that the characteristics of the slow-wave ECM here are distinguishable from those of a conventional Cherenkov maser: The growth rate and output frequency depend on the magnetic field, there is no axial density bunching, and the wave vector **k** is parallel to the beam. The latter characteristic may allow the slow-wave cyclotron instability to be isolated in a practical device if all nonaxial **k** modes can be damped.

ACKNOWLEDGMENTS

This research was supported by the Air Force Office of Scientific Research (USAF) under Grant No. AFOSR-88-0027, the National Science Foundation under Contract No. NSF-ECS-86-03644, and the San Diego Supercomputer Center.

[1]J. L. Hirshfield and V. L. Granatstein, IEEE Trans. Microwave Theory Tech. **MIT-25**, 522 (1977).

[2]R. S. Symons and H. R. Jory, in *Advances in Electronics and Electron Physics,* edited by L. Morton and C. Morton (Academic, New York, 1981), Vol. 55, p. 1.

[3]V. L. Bratman, N. S. Ginzburg, G. S. Nusinovich, M. I. Petelin, and P. S. Strelkov, Int. J. Electron. **53**, 555 (1982).

[4]A. T. Lin, Int. J. Electron. **57**, 1097 (1984).

[5]N. S. Ginzburg, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **22**, 470 (1979); [Sov. Radiophys. Quantum Electron. **22**, 323 (1979)].

[6]J. L. Vomvoridis, IEEE Trans. Nucl. Sci. **NS-20**, 3124 (1983).

[7]S. Yu, Galuzo, V. I. Kanavets, A. I. Slepkov, and V. A. Pletyushkin, Zh. Tekh. Fiz. **52**, 1681 (1982) [Sov. Phys.—Tech. Phys. **27**, 1030 (1982)].

[8]A. N. Didenko, A. R. Borisov, G. P. Fomenko, A. S. Shlapakovskii, and Yu. G. Shtein, Pis'ma Zh. Tekh. Fiz. **9**, 1331 (1983) [Sov. Tech. Phys. Lett. **9**, 572 (1983)].

[9]T. C. Marshall, *Free-Electron Lasers* (MacMillan, New York, 1985), p. 119.

[10]H. Guo, L. Chen, H. Keren, J. L. Hirshfield, S. Y. Park, and K. R. Chu, Phys. Rev. Lett. **49**, 730 (1982).

[11]A. T. Lin, W. W. Chang, and K. R. Chu, Int. J. Infra. Millimeter Waves **5**, 427 (1984).

[12]J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975), p. 552.

[13]L. Friedland, Phys. Fluids **23**, 2376 (1980).

[14]J. E. Walsh, in *Advances in Electronics and Electron Physics,* edited by L. Marton (Academic, New York, 1982), Vol. 58, p. 271.

[15]Nicholas A. Krall and Alvin W. Trivelpiece, *Principles of Plasma Physics* (McGraw-Hill, New York, 1973), p. 178.

[16]A. T. Lin, J. M. Dawson, and H. Okuda, Phys. Fluids **17**, 1995 (1974).