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Coherent reflection as superradiation from the boundary of a resonant medium

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The reflection of short optical pulses from the boundary of a resonant medium is investigated theoretically by the coupled system of the Maxwell and optical Bloch equations. In order to describe the reflected wave, the slowly-varying-envelope approximation in space is not exploited. The coherent reflection is shown to be in connection with Dicke superradiance. It takes place if the relaxation time of the polarization is longer than the superradiation time of the atoms in an optically thin boundary layer of the medium. That is why much of the relevant information can be obtained already in the thin-medium limit, which is considered separately. The conditions of strong reflection from an extended medium are similar to the ones found for a thin sample. The transition into the incoherent and stationary regime is discussed.

I. INTRODUCTION

Reflection and refraction of a monochromatic plane wave on the boundary of a linear medium are described by Fresnel formulas,[1] which are valid only in the stationary regime, when all the transient relaxation processes in the medium have completed their course. In this work we investigate the case when the medium is resonant, nonlinear, and the incoming wave is a short pulse, generally shorter than the relaxation times of the atoms in the medium.

It is well known from the Fresnel formulas that if a medium absorbs at a frequency then its reflection coefficient will be large too.[1] So the following question arises: Why not take into account the reflected wave in the usual theory of resonant interaction[2] when the dominant process is absorption? As expected, it is the nonlinearity of the resonant process that modifies the results of classical optics, and allows one to neglect the reflected wave. The more careful analysis presented here will show, however, that resonant reflection can be strong even in those cases when the excitation is supposed to create significant inversion in the medium.

We will show that if the lifetime of the macroscopic polarization is long enough, and neither the incident field nor the relaxation processes reduce it, then the field originating from this polarization of the boundary gives rise to a strong reflected wave. This coherent response is in close connection with the effect usually known as superradiation,[3–8] which is the collective emission of an appropriately excited ensemble of "two-level" atoms. When initially all the atoms are in the upper inverted state it is also termed as superfluorescence.[8] We emphasize here the collective behavior and the coherent preparation, and not the initial conditions; therefore, we retain the original name. As it is known,[3–8] superradiation takes place only if its characteristic time T_R is shorter than the relaxation time of the polarization T_2, otherwise the phase memory of the dipoles will be destroyed. The condition $T_2 > T_R$ will be seen to be substantial for coherent reflection too.

The problem of nonstationary reflection has been studied so far in only a few papers. The discussion of Eilbeck[9] is restricted to the linear case by using a frequency-dependent index of refraction in the Fresnel formulas. The work of Rupasov and Yudson[10] also uses the phenomenological boundary conditions of electrodynamics and deduces the equation describing the transmission and reflection of an optically thin layer in the absence of relaxation. These authors treat the case of the extended medium too, but, as it will be shown below, the application of the slowly-varying-envelope approximation in space (SVEAS) used by them cannot give rise to the reflected wave, and the macroscopic boundary conditions yield only the well-known incoherent (Fresnel) reflection of the extended medium.

We shall proceed in a more consistent way. The resulting transmitted wave will be regarded as the superposition of the incident wave and of the secondary wave, emitted by atoms that are excited by the incoming wave. They radiate, of course, in both directions and the backward scattered wave gives rise to reflection. We shall confine ourselves to the case of normal incidence. We note that in a more recent work[11] by Vlasov *et al.* the general integro-differential equations of the boundary value problem have been presented, but the complicated system remained unsolved.

The backward wave within the medium has been considered by Crisp[12] for the case of a 2π secanthyperbolic pulse, regarding the polarization of the medium as a given function of space and time calculated from the slowly-varying-envelope approximation (SVEA) theory of self-induced transparency.[13,14] In the present work we shall follow the evolution of the polarization and the field together, thus solve a time-dependent scattering problem. The role of the boundary will be explicitly taken into account.

II. EQUATIONS OF THE MODEL

Our model is a medium consisting of two-level atoms,[2] on which a given linearly polarized electromagnetic plane wave is falling at the plane boundary $x=0$. We look for the reflected wave at $x=0$ and for the transmitted wave at $x=L$, specifying the other end of the medium. We use the semiclassical rotating-wave approximation (RWA), together with the slowly-varying-envelope approximation in time (SVEAT), but an important point is that the same (slowly-varying-envelope) approximation in space (SVEAS) will not be exploited.

The propagation of a linearly polarized plane wave through the medium which is characterized by a polarization $\mathcal{P}(x,t)$ is described by

$$\frac{\partial^2 \mathcal{E}}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2 \mathcal{E}}{\partial t^2}=\frac{4\pi}{c^2}\frac{\partial^2 \mathcal{P}}{\partial t^2} . \tag{2.1}$$

We shall use the explicit retarded solution of Eq (2.1),

$$\mathcal{E}(x,t)=\mathcal{E}_i(x,t)-\frac{2\pi}{c}\int_0^L \frac{\partial \mathcal{P}}{\partial t}(x',\, t-|x-x'|/c)dx' , \tag{2.2}$$

where

$$\mathcal{E}_i(x,t)=\tfrac{1}{2}E_i(x,t)\exp[i(\omega t-kx)]+\text{c.c.}$$

is a solution of the homogeneous equation corresponding to (2.1), and it is identified with the incoming plane wave. We look for solutions of the form

$$\begin{aligned}\mathcal{E}(x,t)&=\tfrac{1}{2}E(x,t)e^{i\omega t}+\text{c.c.} ,\\ \mathcal{P}(x,t)&=\tfrac{1}{2}P(x,t)e^{i\omega t}+\text{c.c.} ,\end{aligned} \tag{2.3}$$

where $E(x,t)$ and $P(x,t)$ are slowly varying in time compared with $\exp(i\omega t)$. Applying the RWA and SVEAT, we have[15]

$$E(x,t)=E_i(x,t)-\frac{2i\pi\omega}{c}\int_0^L P(x',\, t-|x-x'|/c)\times e^{-i(\omega/c)|x-x'|}dx' . \tag{2.4}$$

The important point is that E and P are still rapidly varying functions of x. It is clear now that the backward wave comes from that part of the integral where $x<x'$, and especially at $x=0$ this yields the reflected wave. The integral equation has the advantage that the boundary conditions need not be specified separately. The reflected wave and the transmitted wave at $x=L$ are determined in the following way:

$$\begin{aligned}E_r&=E(0,t)-E_i(0,t)\\ &=-\frac{2i\pi\omega}{c}\int_0^L P(x',\, t-x'/c)e^{-ikx'}dx' ,\end{aligned} \tag{2.5}$$

$$E_t=E(L,t)=\left[E_i(L,t)-\frac{2i\pi\omega}{c}\int_0^L P(x',\, t-(L-x')/c)\times e^{ikx'}dx'\right]e^{-ikL} . \tag{2.6}$$

The dynamics of the polarization in the two-level atom model will be determined by the optical Bloch equations in the RWA (Ref. 2),

$$\frac{\partial R}{\partial t}=\left[i\Delta-\frac{1}{T_2'}\right]R+i\frac{p}{\hbar}EW , \tag{2.7}$$

$$\frac{\partial W}{\partial t}=\frac{ip}{2\hbar}(E^*R-ER^*) , \tag{2.8}$$

where W is the population difference between the levels, $R/2$ is the slowly varying part of the off-diagonal element of the atomic density matrix $R=U+iV$, p is the transition dipole moment, T_2' is the relaxation time of the polarization, and Δ is the difference between the field frequency and the atomic transition. R and P are connected by the relation $P=pN\langle R\rangle$, where the angular brackets denote summation over the inhomogeneous line and N is the density of the active atoms. The relaxation of the inversion is neglected here (in the case of activated crystals, e.g., $T_2'\ll T_1$). To determine the transmitted and reflected waves we have to solve Eqs. (2.7) and (2.8) simultaneously with (2.4).

In the linear approximation, when W is set equal to -1, and in the case of rapid relaxation, when T_2' is much less than the duration of the excitation, the stationary value of $P=pN\langle R\rangle$ will be proportional to E, giving the usual expression for the susceptibility and a corresponding index of refraction,

$$\chi=\frac{p^2N}{\hbar}\left\langle\frac{\Delta-i/T_2'}{\Delta^2+1/(T_2')^2}\right\rangle,\quad n=(1+4\pi\chi)^{1/2} . \tag{2.9}$$

Putting $P=\chi E$ in (2.4) we get a single integral equation for E, the solution of which yields the Fresnel formulas.[16]

III. REFLECTION FROM AN OPTICALLY THIN MEDIUM

In this section we consider the boundary value problem for an optically thin medium, i.e., when $L\ll\lambda$, which allows a significant simplification. As we shall see in Sec. IV, much of the results of this limiting case will remain valid for the extended medium too.

In the case of a thin sample in Eq. (2.4) we may put $\exp(ik|x-x'|)\approx 1$, and instead of integrating we can take the spatial average of the polarization. With these approximations from Eqs. (2.7) and (2.8) and (2.4) we obtain

$$\frac{\partial R}{\partial t}=\left[i\Delta-\frac{1}{T_2'}\right]R+\left[i\frac{p}{\hbar}E_i\frac{1}{T_R}\langle R\rangle\right]W , \tag{3.1}$$

$$\frac{\partial W}{\partial t}=\frac{ip}{2\hbar}(E_i^*R-E_iR^*)-\frac{1}{2T_R}(\langle R^*\rangle R+\langle R\rangle R^*) , \tag{3.2}$$

where

$$T_R=\frac{\hbar c}{2\pi NL\omega p^2} \tag{3.3}$$

is the superradiation time[2–8] of the sample.

This modified set of the optical Bloch equations has been obtained in the works of Hopf *et al.*[17] and Ben Aryeh *et al.*,[18] where the stationary solution of these equations has been investigated. It has been pointed out by these authors that the imaginary part of the correction represented by the last term in Eq. (3.1) produces a renormalization of the resonance detuning Δ, and this gives rise to optical bistability in the stationary regime. It has been noted in those references that the real component gives rise to superradiance, which is the mechanism discussed in the present paper.

When studying superradiation one does not consider the terms containing E_i in (3.1) and (3.2), while in the usual treatment of the interaction of coherent external radiation with the resonant medium, the terms containing $1/T_R$ are neglected. To obtain the reflected wave one must take both into account.

The solution of these ordinary differential equations yield the reflected and transmitted amplitudes in the following way: In Eqs. (2.5) and (2.6) we make the same approximations that have led us to (3.1) and (3.2) and then obtain

$$E_r = -i\frac{\hbar}{pT_R}\langle R\rangle , \tag{3.4}$$

$$E_t = E_i - i\frac{\hbar}{pT_R}\langle R\rangle . \tag{3.5}$$

Equations (3.4) and (3.5) express the physical fact that the atoms of the medium radiate in both directions equally. The reflected wave is identical with this secondary field emitted in the backward direction, while the transmitted wave is the superposition of this self-field and the incident field.

A. Linear limit

We suppose now that the excitation is weak, so that W remains close to -1 during the whole process. In this case (3.1) and (3.2) reduce to a single complex equation for each Δ,

$$\frac{\partial R}{\partial t} = \left[i\Delta - \frac{1}{T_2'} \right] R - i\frac{p}{\hbar}E_i - \frac{1}{T_R}\langle R\rangle . \tag{3.6}$$

This linear equation can be solved, e.g., by Laplace transforming both sides and then summing over the inhomogeneous line.[19,20] If the latter is approximated by a Lorentz curve[2] of linewidth $2/T_2^*$, then the calculation can be performed simply, and we have

$$\langle R\rangle = -i\frac{p}{\hbar}\int_0^t E_i(t')\exp\left[-\left[\frac{1}{T_2'} + \frac{1}{T_2} + \frac{1}{T_R} \right] \times (t-t') \right] dt' . \tag{3.7}$$

The reflected and the transmitted waves are determined now by (3.4) and (3.5), respectively.

If a step pulse of amplitude E_0 is switched on at $t=0$, then for the time dependence of the reflected wave we obtain from (3.4) and (3.7)

$$E_r = \frac{E_0}{T_R}\frac{\exp\left[-\left[\frac{1}{T_R} + \frac{1}{T_2} \right] t \right] - 1}{\left[\frac{1}{T_R} + \frac{1}{T_2} \right]} , \tag{3.8}$$

where the notation $T_2^{-1} = T_2'^{-1} + T_2^{*-1}$ has been introduced.

The stationary value of the reflection coefficient for the intensity

$$\mathcal{R} = (1 + T_R/T_2)^{-2} \tag{3.9}$$

is achieved during the time $(T_2^{-1} + T_R^{-1})^{-1}$. Until t is less than this value, the medium has no time to answer the excitation, and the reflection coefficient remains smaller than $\mathcal{R}$.

To compare (3.9) with the appropriate Fresnel formula, we note that in the stationary case the index of refraction can be connected with T_2/T_R. From Eqs. (2.9) and (3.3) we have

$$(n^2 - 1)kL = -2i\frac{T_2}{T_R} . \tag{3.10}$$

Using (3.10), the expansion of the Fresnel reflection coefficient for a thin layer ($nkL \ll 1$) is in agreement with (3.9). If $T_2 > T_R$, which in the stationary case corresponds to a large imaginary index of refraction, the reflection coefficient will be large in the transient regime as well. It also follows from (3.7) that after the incoming wave has been switched off, the reflected amplitude disappears with a time delay $(T_2^{-1} + T_R^{-1})^{-1}$.

B. Nonlinear thin medium

In this case we shall first investigate the ideal problem of exact resonance, without inhomogeneous broadening and without polarization damping, $T_2' = \infty$. This allows one to obtain analytical results, too, and then the effects of relaxation terms can also be better understood. With these approximations Eqs. (3.1) and (3.2) can be written into the form

$$\frac{\partial R}{\partial t} = \left[i\frac{p}{\hbar}E_i + \frac{1}{T_R}R \right] W , \tag{3.11}$$

$$\frac{\partial W}{\partial t} = \frac{ip}{2\hbar}(E_i^* R - E_i R^*) - \frac{1}{T_R}|R|^2 , \tag{3.12}$$

and from Eqs. (3.4) and (3.5) the conservation law of energy can be deduced,

$$\frac{c}{4\pi}|E_i|^2 = \frac{c}{4\pi}(|E_t|^2 + |E_r|^2) + NL\hbar\omega\frac{\partial W}{\partial t} . \tag{3.13}$$

We shall solve Eqs. (3.11) and (3.12) assuming that initially all atoms are in their ground state, $W=-1$, and the polarization is absent, $R=0$. It can be easily seen that if E_i is real, then R will remain purely imaginary during the evolution of the system, $R = iV$. As the length of the Bloch vector $|R|^2 + W^2$ is a conserved quantity, it is straightforward to introduce the Bloch angle[2] with $V = -\sin\theta$ and $W = -\cos\theta$. Now (3.11) and (3.12) can be recast into the single equation[10,21]

$$\frac{d\theta}{dt}=\frac{p}{\hbar}E_i-\frac{1}{T_R}\sin\theta \ . \tag{3.14}$$

According to (3.4) and (3.5), the reflected and the transmitted waves can be obtained now in the following way:

$$E_r=\frac{\hbar}{pT_R}V=-\frac{\hbar}{pT_R}\sin\theta \ , \tag{3.15}$$

$$E_t=E_i+\frac{\hbar}{pT_R}V=\frac{\hbar}{p}\frac{d\theta}{dt} \ . \tag{3.16}$$

By omitting the term $(1/T_R)\sin\theta$ in (3.14), we would have the usual equation[2] describing the optical Rabi oscillations. On the other hand, in the most simplified model of superradiation one has Eq. (3.14) with $E_i=0$. The term $(1/T_R)\sin\theta$ takes into account the self-field of the material, and in the absence of excitation this gives rise to the superradiation of the thin layer.[14,22] As we can see, the same term is responsible for the reflected wave, and in this sense we may identify the coherent reflection with superradiation.

From the form of Eq. (3.14) one can see that if $(p/\hbar)E_i<1/T_R$, then θ has a stationary value where $\sin\theta=(p/\hbar)E_iT_R$. If now the duration of the incident pulse is longer than T_R, then the system approximates this stationary value and after $T>T_R$, $\dot\theta\approx 0$, and $E_r\approx -E_i$. The secondary field is in opposite phase compared with the incident wave, and that is why they cancel each other in the forward direction, while a strong reflected wave is generated.

We note that in the case of Eq. (3.14) the area of the incident pulse, defined as

$$A=\frac{p}{\hbar}\int_{-\infty}^{\infty}E_i dt \ , \tag{3.17}$$

is not in a one-to-one correspondence with the final value of θ. The latter is always $\theta(\infty)=0(\text{mod}2\pi)$, as can be seen from (3.14), taking into account that $E_i(\infty)=0$.

The solution of (3.14) for a constant amplitude E_i has been obtained by Rupasov and Yudson.[10] In actual experiments E_i itself depends on time, therefore we have solved Eq. (3.14) for incoming pulses of the form

$$E_i=E_0\text{sech}(t/T) \ . \tag{3.18}$$

The area of this pulse is $A=(p/\hbar)E_0T\pi$.

We begin the analysis by observing that exciting pulses of the form (3.18), with

$$\frac{p}{\hbar}E_0=\frac{1}{T_R}+\frac{1}{T} \ , \tag{3.19}$$

$$A=\left[\frac{T}{T_R}+1\right]\pi \ , \tag{3.20}$$

bring the system exactly in the upper inverted state. Equation (3.14) now has a simple analytic solution,

$$\theta=2\arctan\exp(t/T) \ , \tag{3.21}$$

and the reflected and transmitted pulses have the form

$$\frac{p}{\hbar}E_r=-\frac{1}{T_R}\text{sech}(t/T), \quad \frac{p}{\hbar}E_t=\frac{1}{T}\text{sech}(t/T) \ . \tag{3.22}$$

The reflected intensity is thus proportional to N^2, which is a characteristic feature of superradiation.

The relative strength of the reflected and transmitted waves compared with the incident wave are determined by the ratios of T and T_R. If $T\gg T_R$ then the incident pulse is strongly reflected, while if $T\ll T_R$ it is transmitted without reflection. The inverted state generated by the incident pulse is not stable, of course. After the first response, determined by Eqs. (3.22), a π pulse of superfluorescence

$$E=\frac{p}{\hbar T_R}\text{sech}[(t-t_D)/T_R]$$

will be radiated in both directions with a delay time[3–8] t_D.

The case considered above is a limit between two classes of solutions. To achieve complete inversion with a pulse of duration T, its area A must be larger than $(T/T_R+1)\pi$, or equally $(p/\hbar)E_0>1/T_R+1/T$, while if $A<(T/T_R+1)\pi$, then the inverted state cannot be reached.

The statements above can be demonstrated by the special, but nontrivial analytic solution of Eq. (3.18) when E_i is a 2π secanthyperbolic pulse. This is a frequently investigated and also a practically realizable pulse shape in self-induced transparency experiments.[13,14] So if in (3.18) we put $(p/\hbar)E_0=2/T$, then Eq. (3.14) has the solution

$$\theta=2\arctan\frac{2}{\left[\frac{T}{T_R}-1\right]\exp(t/T_R)+\left[\frac{T}{T_R}+1\right]\exp(-t/T_R)} \ . \tag{3.23}$$

The reflected and transmitted amplitudes in this case are

$$\frac{p}{\hbar}E_r=2\frac{T}{T_R}\frac{\frac{1}{T}\sinh\left[\frac{t}{T}\right]-\frac{1}{T_R}\cosh\left[\frac{t}{T}\right]}{1+\left[\frac{T}{T_R}\cosh\left[\frac{t}{T}\right]-\sinh\left[\frac{t}{T}\right]\right]^2} \ ,$$

$$\frac{p}{\hbar}E_t=2\frac{\frac{1}{T}\cosh\left[\frac{t}{T}\right]-\frac{1}{T_R}\sinh\left[\frac{t}{T}\right]}{1+\left[\frac{T}{T_R}\cosh\left[\frac{t}{T}\right]-\sinh\left[\frac{t}{T}\right]\right]^2} \ , \tag{3.24}$$

respectively. The formulas above allow to enlighten the very different behavior of the reflected and transmitted waves depending on the amplitude and accordingly on the duration of the excitation. Omitting the analytic discussion of the different limiting cases here we refer only to Figs. 1(a) and 2(a). We note that if $A>2\pi$ and $T/T_R<A/\pi-1$, then in both the strong transmitted and weaker reflected waves Rabi oscillations appear.

Let us turn now to the solution of Eqs. (3.1) and (3.2) when the relaxation of the polarization is taken into account, $T_2'<\infty$. For sake of simplicity we confine ourselves to the case when $\Delta=0$ for all the atoms. Quantitative results can be obtained here only numerically, but

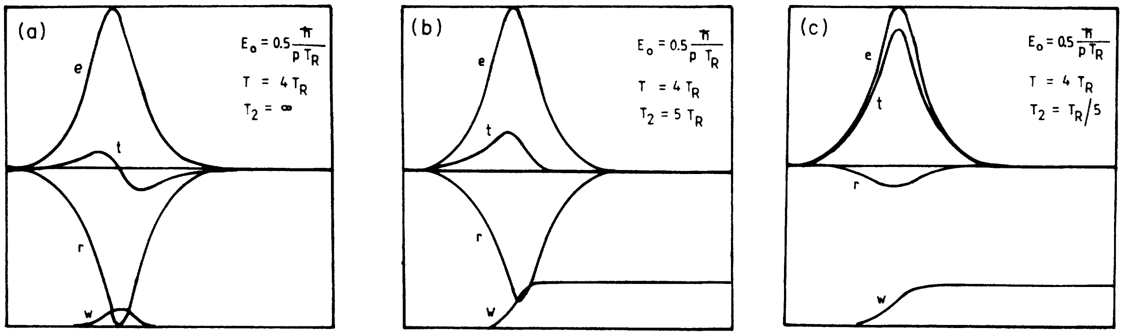

FIG. 1. Time dependence of the transmitted (t) and reflected (r) amplitudes relative to the excitation (e), in the thin-medium limit. The time and amplitude scales are fixed by the exciting pulse (e), having the form of $E_i = 0.5(\hbar/pT_R)\mathrm{sech}(t/4T_R)$. Its area is $A = 2\pi$. The relaxation time T_2' is ∞, $5T_R$, and $0.2T_R$ for (a), (b), and (c), respectively. The vertical scale for the inversion (W) goes between -1 and 1.

the form of Eq. (3.1) shows that R, which gives rise to the reflected wave, will be damped now, and therefore in this case reflection will be less than before. This effect, of course, will be remarkable only if the reflection is strong in the absence of relaxation. See Fig. 1. As $1/T_2'$ grows, the reflected wave diminishes and transmission gets closer to the exciting pulse.

When the reflected wave is small in the undamped case, i.e., when the polarization is already ceased by the external field, then the introduction of the term $1/T_2'$ may cause only a little effect, as can be seen comparing Figs. 2(a), 2(b), and 2(c). The reflected and transmitted waves are essentially the same as without damping. The main difference is that the inversion does not reach $+1$, and later does not fall back to -1, the atomic system dissipates the energy of the excitation. On a longer time scale W must go back to -1 due to a term containing[2] $1/T_1$, which was neglected here.

IV. EXTENDED MEDIUM

In this case we must solve Eqs. (2.4), (2.7), (2.8), and the reflected and transmitted waves can be determined by (2.5) and (2.6), respectively. This is possible only numerically.

Here we have one more parameter that can be chosen freely; it is the length of the sample L. The usual phenomenological treatment of reflection in ordinary optics disguises the physical origin of the reflected wave; nevertheless, it is well known that the latter arises as the backscattering of the dipoles in a boundary layer of the medium. The same will be valid in the case of resonant interaction; one expects that the depth of the material to be taken into account must be of the order of the wavelength.

To see this explicitly, let us substitute Eq. (2.4) into (2.7) and (2.8), and express the length of the medium in units of $1/k = \lambda/(2\pi)$,

$$\frac{\partial R}{\partial t} = \left[i\Delta - \frac{1}{T_2'} \right] R + \left[i\frac{p}{\hbar} E_i e^{-ikx} + \frac{1}{T_\lambda} \int_0^{kL} \langle R(y,t) \rangle e^{-i|kx-y|} dy \right] W \,, \tag{4.1}$$

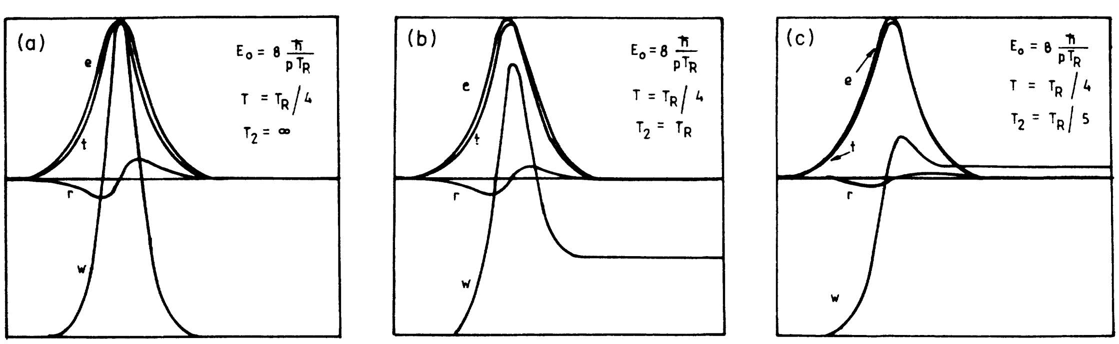

FIG. 2. Same as Fig. 1 with $E_i = 8(\hbar/pT_R)\mathrm{sech}(4t/T_R)$, $A = 2\pi$. The relaxation time T_2' is ∞, T_R, and $0.2T_R$ for (a), (b), and (c), respectively.

$$\frac{\partial W}{\partial t}=\left[-\frac{ip}{2\hbar}E_i e^{-ikx} -\frac{1}{2T_\lambda}\int_0^{kL}\langle R(y,t)\rangle e^{-i|kx-y|}dy\right]R^*+\text{c.c.} \tag{4.2}$$

The solution of the coupled integro-differential equations (4.1) and (4.2) yield the reflected and transmitted waves by

$$E_r=\frac{\hbar}{p}\frac{1}{T_\lambda}\int_0^{kL}\langle R(y,t)\rangle e^{-iy}dy \ , \tag{4.3}$$

$$E_t=\left[E_i+\frac{\hbar}{p}\frac{1}{T_\lambda}\int_0^{kL}\langle R(y,t)\rangle e^{iy}dy\right]e^{ikL} \ . \tag{4.4}$$

Here T_λ is the superradiation time of a layer of thickness $\lambda/(2\pi)$

$$T_\lambda=2\pi\frac{L}{\lambda}T_R=\frac{\hbar}{2\pi p^2 N} \ . \tag{4.5}$$

The first terms in the braces in (4.1) and (4.2) represent the external driving field, while the second ones mean the self-field, originating from the dipoles of the material, and this gives rise to the reflected wave at $x=0$. It can be seen that the first term, i.e., the incident wave creates a polarization proportional to $R_s(x,t)\exp(i\omega t-kx)$, where $R_s(x,t)$ is slowly varying in time and space as well. Now if we put this R into the second term, we see that at $x=0$ the integrand contains the expression $\exp(-2ikx')$, and integration over subsequent intervals of length of $\lambda/4$ give a net result approximately 0. Physically this means that the backscattered waves from the different parts of the bulk of the medium suffer destructive interference. This is why one can apply the SVEAS if one is not interested in the reflected wave. But as our aim is to determine reflection we must not use this approximation. The consideration above suggests also that to obtain just the reflected wave, it is enough to integrate until $L\sim\lambda$ because the bulk of the medium will not contribute to the reflection. Our results below justify this assumption. We note that as R is slowly varying in time, the retardation of its time argument can be neglected if $L\sim\lambda$.

The reasoning above that the length scale to be used must be comparable with the wavelength implies also the relevant time parameter of the investigated effect. The time constant determining the order of that part of the derivative, which is responsible for the reflected wave is the superradiation time of a layer of thickness $\lambda/(2\pi)$.

We note that in the semiclassical picture the terms containing the integral in (4.1) and (4.2) give rise to superradiation,[5,6] when the relaxation time $T_2=\infty$. Equation (4.3) shows that the reflected wave consists of purely this term even in the presence of the external excitation. These facts show that it is reasonable to regard coherent reflection from an extended medium as superradiation originating from the boundary layer of the medium. The numerical calculations below show that the depth of this layer is not larger than $\lambda/2$. We expect therefore that the results for the ideally thin medium, obtained in Sec. III, essentially will retain their validity for the extended medium too. The reflected intensity will not depend on the length of the sample beyond $\lambda/2$.

Our calculations yield the transmitted wave as well. In contrast with reflection, it will depend of course on L even if we have $L\gg\lambda$. To determine the transmitted wave, however, it is not recommended to choose a length scale comparable with the wavelength, unless one is interested in the transmission of a thin layer. Our considerations above and Eqs. (4.1) and (4.2) show that if $L\gg\lambda$, then it is enough to use the traditional theoretical treatment[2] with the SVEA. It must be noted, however, that the initial pulse of SVEA theory is assumed to be already within the medium. Therefore the identification of the initial pulse with the external excitation is allowed only if reflection is weak.

We have investigated the problem first without relaxation, $T_2=\infty$. The conditions of strong reflection for the extended medium are similar to those of the thin medium case: The amplitude of the exciting pulse must satisfy $(p/\hbar)E_i<1/T_\lambda$, otherwise in Eqs. (4.1) and (4.2) the first term will dominate, and the polarization wave will go into the forward direction, leading to a weak reflection. The other condition is that the duration of the incoming pulse T must be greater or at least comparable with T_λ.

Figure 3 shows the results for a $\pi/2$ secanthyperbolic pulse of amplitude $E_0=0.5(\hbar/pT_\lambda)$ and duration $T=T_\lambda$, for $L=0.5\lambda$, λ, and 2.8λ. The reflected waves are relatively strong and nearly equal in all cases, only the transmitted intensity is decreasing. This proves that the reflected wave really originates in the boundary layer of the medium, not deeper than $\lambda/2$.

For weak but short excitations the character of the results can be explained as in the infinitely thin case. When the external pulse is shorter than the superradiation time T_λ, but it is not strong enough to bring back the atoms in their ground state, then after the rapid excitation, which is a coherent preparation, there remains an inversion and a polarization in the medium. This leads to superradiation in both directions [Fig. 4(a)]. The forward wave shows ringing, while the backward wave is a wide flat pulse, because it is the field of the boundary layer only.

When $(p/\hbar)E_i>1/T_\lambda$ and $T<T_\lambda$, then the reflected wave will be weak. This happens in the case, which is usually dealt with when coherent propagation effects, e.g., self-induced transparency, is investigated. To compare the results following from (2.4), (2.7), and (2.8), with those obtained ignoring the role of the boundary, we have considered the behavior of a 2π secanthyperbolic pulse with duration less than T_λ (Fig. 5). The reflected wave is relatively less, if the amplitude of the excitation is greater. The transmitted pulse is a single strong peak similar to the excitation. Already for layers of thickness about λ one can observe the delay of the transmitted pulse, a characteristic feature of coherent interaction, which is longer for a wider pulse. As the area of the propagating pulse within the medium is somewhat less than 2π, because of reflection, therefore according to the area theorem[13] its area must grow until 2π. This leads to the broadening of the transmitted pulse. The opposite effect, narrowing, has been obtained in the case of a 2.25π

pulse.

We have investigated the influence of relaxation on reflection and transmission. In Eqs. (2.7) and (2.8) we have taken into account the term R/T_2'.

We have noted already that in the linear case, when $W \simeq -1$, and for stationary excitation $T \gg T_2$, our equations yield the Fresnel formulas for a finite layer.[18] From Eqs. (2.9) and (4.5) the index of refraction can be expressed as

$$n = \left[1 - 2i\frac{T_2}{T_\lambda}\right]^{1/2} . \tag{4.6}$$

This fact enabled us to control the results of the numerical calculations. In the case $T \gg T_2$ the transmitted and reflected pulses have the same form as the incident one.

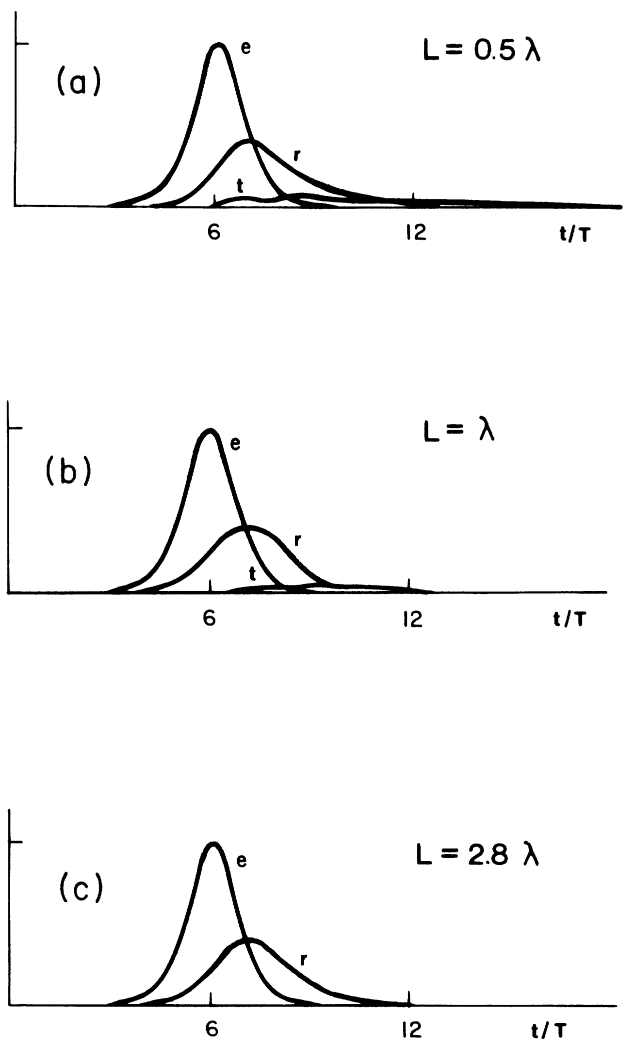

FIG. 3. Time dependence of the transmitted (t) and reflected (r) intensities relative to the intensity of the exciting wave (e). The later is a 0.5π secanthyperbolic pulse, $E_i = 0.5(\hbar/pT_\lambda)\mathrm{sech}[(t-t_0)/T]$, $T = T_\lambda$, $t_0 = 6T$, $T_2' = \infty$. Increasing the length of the medium has no essential effect on the reflection.

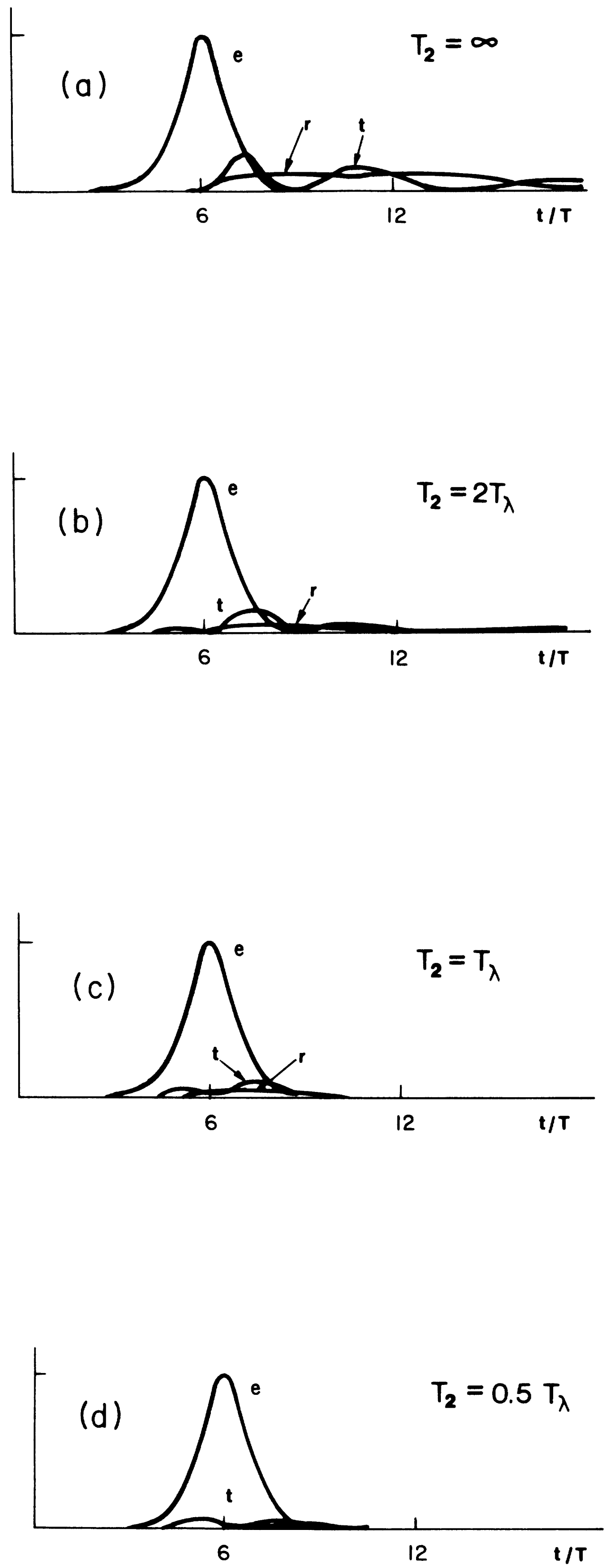

FIG. 4. Same as Fig. 3 with $E_i = 2(\hbar/pT_\lambda)\mathrm{sech}[(t-t_0)/T]$, $T = 0.25T_\lambda$, $A = 0.5\pi$, $L = \lambda$, for different $T_2' - s$. The secondary waves show a superradiant character which is suppressed by decreasing T_2'.

Increasing the length of the system, the reflection coefficient, defined as the reflected intensity divided by the incident intensity, has shown the known oscillatory decreasing behavior.[1] For $T=8T_2$ and $T_2=0.25T_\lambda$ the difference between the analytical and numerical results was less than 10^{-3}.

Now let us turn to the nonstationary case. The inclusion of the relaxation leads to the attenuation of the reflected wave and reduces the pulse delay, both being signs of the vanishing coherent behavior. In accordance with the results of Sec. III, this effect on the reflection is determined by the ratio of T_λ and T_2. Since the coherent reflection can be regarded as superradiation from the boundary layer, it will be reduced if we decrease T_2 and cannot be significant if $T_2 < T_\lambda$ [Figs. 3(b) and 6]. On the other hand, if the direct response of the medium is dominating, e.g., in the case of a short 2π pulse with duration less than T_λ, when reflection is small otherwise, then the inclusion of relaxation influences markedly only the transmitted wave. The effect is determined by the ratio of the duration of the excitation and the relaxation time. The damping of transmission is a bulk effect and therefore in a medium of thickness of the order of λ it will not be very strong (Fig. 5). In this case the absence of coherent interaction is shown by the decreased pulse delay and the reduction of the broadening. In the system remains an approximately homogeneously distributed excitation that will be dissipated by longitudinal relaxation.

V. CONCLUSIONS

In the case of coherent interaction similar to the incoherent case, the reflected wave originates in the boundary layer of the medium, which is less than a wavelength. Therefore the character of the results for the ideally thin

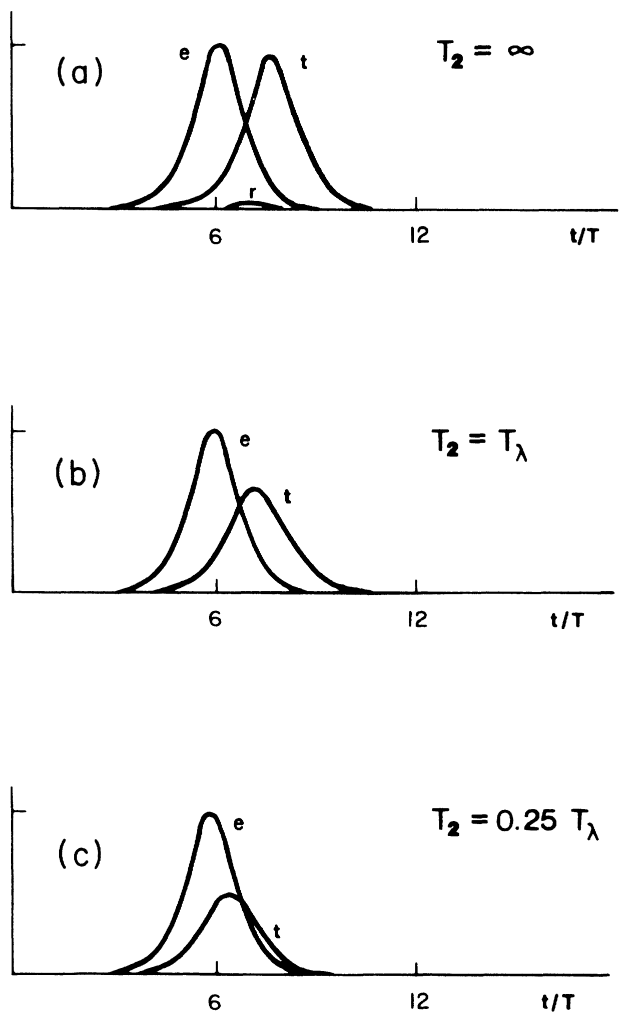

FIG. 5. Reflected (r) and transmitted (t) intensities relative to the excitation (e), for a 2π secanthyperbolic pulse, $E_t = 8(\hbar/pT_\lambda)\mathrm{sech}[(t-t_0)/T]$, $T=0.25T_\lambda$, $t_0=6T$, $L=\lambda$, for different $T_2'-s$.

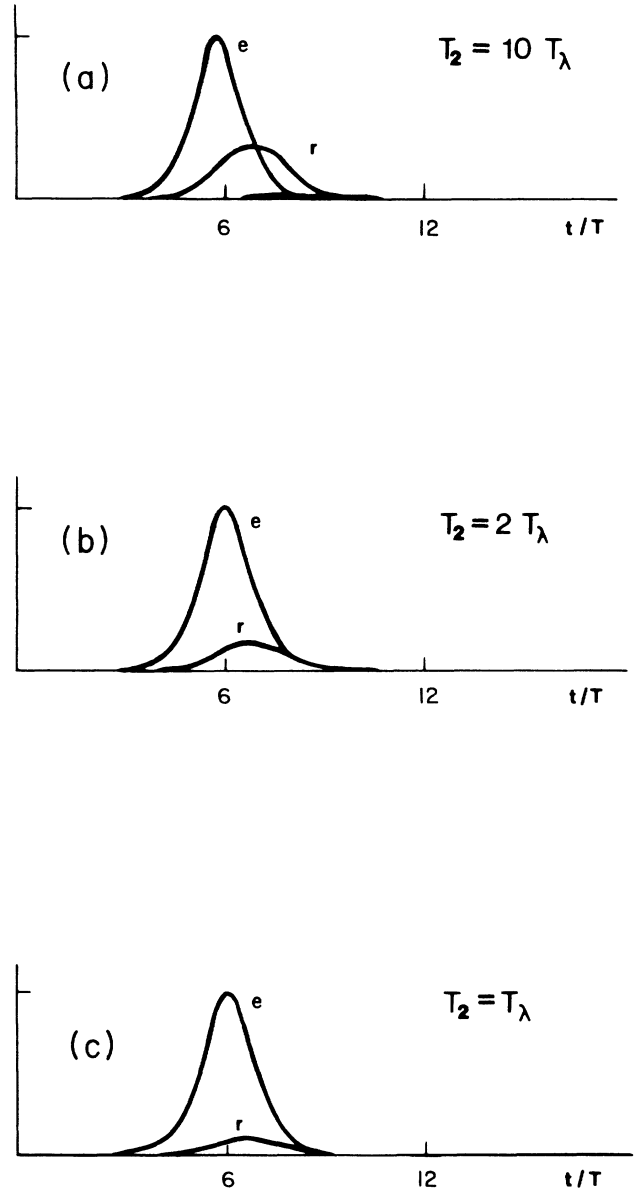

FIG. 6. Effect of relaxation on the pulse shown in Fig. 3(b); $L=\lambda$.

medium presented in Sec. III are relevant for the extended medium too.

To observe a strong coherent reflection, the Rabi frequency of the incident field must be comparable or smaller than the inverse of the superradiation time T_λ of the medium of thickness $\lambda/(2\pi)$. Another condition is that the duration of the incident pulse must be shorter than the relaxation time of the macroscopic polarization, but longer than T_λ. Otherwise either the strong external field will force the atoms to radiate forward, or the macroscopoic dipole moment of the system that creates the coherent reflection will be destroyed. In the other coherent case $T_2 > T_\lambda > T$, the reflection is small, and the transmitted pulse shows the characteristics of self-induced transparency. In the linear incoherent case with $T > T_2 > T_\lambda$ the reflection will be large too, but this can be interpreted also by a great imaginary index of refraction, arising from the transition between the resonant levels.

In an activated crystal with $N = 10^{18}$ cm^{-3} and $p = 3\times 10^{-18}$ cgs units we obtain $T_\lambda = 10^{-11}$ s. If for T_2 a value longer than this can be achieved, then the conditions of observing coherent reflection are realizable.

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