Wave cybernetics: A simple model of wave-controlled nonlinear and nonlocal cooperative phenomena

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A simple theoretical description of nonlinear and nonlocal cooperative phenomena is presented in which the global control mechanism of the whole system is given by the tuned-wave propagation. It provides us with an interesting universal scheme of systematization in physical and biological systems called wave cybernetics, and may be understood as a model realizing Bohm's idea of implicate order in natural philosophy.

One often encounters the problem of the global control mechanism in nonlinear and nonlocal cooperative phenomena, e.g., in the cell membrane process of biological rhythm generation, in chemical oscillation, in the critical dynamics of spin systems, and in the quantum mechanics of many-body systems. To describe those nonlinear and nonlocal cooperative phenomena, many mathematical models with nonlinear differential equations, nonlinear stochastic differential equations, and variational principles have been adopted. However, in view of the nonlocal (i.e., global) control mechanism in the cooperative phenomena, a new mathematical treatment seems to be needed in which the physical nonlinear and nonlocal control mechanism is explicitly presented.

In the present paper, I have developed such a mathematical description of nonlinear and nonlocal cooperative phenomena in which certain wave propagation guides the local variables of the whole system globally. The wave is called a pilot wave and plays an important role in nonlocal cooperative phenomena. My approach provides us also with a physically realistic model of Nelson's stochastic mechanics: In the last decade, stochastic mechanics has been investigated extensively from both physical and mathematical points of view.[1–7] It became then a wide belief that Nelson's stochastic mechanics itself has little in common with quantum mechanics. Rather, it is understood as a mathematical framework for describing statistical phenomena in large physical systems.[3,5–7]

To simplify the analysis, I have restricted myself to a spatially extended physical system. Such a spatial extension may be described by a one-, two-, or three-dimensional manifold M, and location in the system can be represented by coordinates $x=(x^1,x^2,x^3)$. Suppose that the system is made up of infinitely many elements spreading over the manifold M. It is then convenient to choose a distribution density of the elements as a local variable of the system. Let $\rho(x,t)$ be the distribution density of the elements located in x at time t. As time passes, the local variable $\rho(x,t)$ evolves due to dynamical interaction between the elements. Thus the local variable $\rho(x,t)$ is in its entirety a representative of the dynamical state of the system. In this sense, ρ is called a state variable.

I have assumed furthermore the existence of another local degree of freedom in the system. Namely, in each location x of the manifold M we have another local variable manifesting a nonlinear oscillation. The oscillation pattern (i.e., rhythm) of this local variable, say $\Theta(x,t)$, is the key concept in the global control mechanism of cooperative phenomena in question. The local variable $\Theta(x,t)$ in its entirety represents a global implicate order of the system. In this sense, Θ is called a hidden variable. As only the oscillation pattern of the local variable $\Theta(x,t)$ is relevant, I assume $\Theta(x,t)$ to be a complex number with unit absolute value, and write $\Theta(x,t) = \exp[iS(x,t)]$. The local variable $S(x,t)$ is the phase of local oscillation of the hidden variable Θ. As time passes, the hidden variable Θ manifests nonlinear oscillation in each location, and it can be considered as a wave spreading over the manifold M. I call it a pilot wave of the system. Now, I will show that an interesting global control mechanism of nonlocal and nonlinear cooperative phenomena can be incorporated in the present system with state variable ρ and hidden variable Θ.

The hidden variable (i.e., pilot wave) Θ of the system is tuned over the whole manifold M and guides the state variable ρ in each location to evolve by the phase difference. Let v be a vector field on the manifold M given by the gradient,

$$v(x,t)=\nabla S(x,t) \; . \tag{1}$$

It describes the orientation and magnitude of phase difference of the pilot wave Θ between neighboring locations. The elements of the system in each location x flow along the phase difference (1). In other words, the state variable ρ evolves by means of the equation of continuity

$$\frac{\partial \rho}{\partial t}+\operatorname{div}(v\rho)=0 \; . \tag{2}$$

The state variable ρ of the system thus suffers from the guidance of the hidden variable Θ by a local physical law (2). It is the hidden variable Θ of the system which manifests nonlocality.

As long as the physical environment of the system remains stationary, I may be allowed to assume the conservation of energy of the system between state and hidden variables. The total energy of the system is

$$E=\int\left(\tfrac{1}{2}\mid v\mid^2+\tfrac{1}{2}\mid u\mid^2+V\right)\rho\, dx\ , \tag{3}$$

where V denotes a given external potential energy, u is the osmotic velocity of the state variable ρ given by

$$u=\tfrac{1}{2}\nabla \ln\rho\ , \tag{4}$$

and dx is the invariant volume element of the manifold M. Notice that the energy $\mid u\mid^2/2$ is an intrinsic potential energy of the state variable ρ. The condition of energy conservation $dE/dt=0$ yields that

$$\frac{\partial S}{\partial t}+\tfrac{1}{2}\mid v\mid^2+V-\tfrac{1}{2}\mid u\mid^2-\tfrac{1}{2}\operatorname{div}u$$

is a function of t alone.[2] Then, without loss of generality, the hidden variable Θ can be thought to satisfy

$$\frac{\partial S}{\partial t}+\tfrac{1}{2}\mid v\mid^2+V-\tfrac{1}{2}\mid u\mid^2-\tfrac{1}{2}\operatorname{div}u=0\ . \tag{5}$$

For a nonstationary environment given by time-dependent V, the energy-conservation law may be replaced by a least-action principle with respect to the action functional

$$I=\int\int\left(\tfrac{1}{2}\mid v\mid^2-\tfrac{1}{2}\mid u\mid^2-v\right)\rho\, dx\, dt\ . \tag{6}$$

The variational condition $\delta I=0$ yields the same equation as (5).[8] Finally, I see that the system is governed by the energy-conservation law or the least-action principle if the hidden variable Θ is subject to Eq. (5). The pilot wave Θ of the system is tuned globally by Eq. (5) over the whole manifold M, and the state variable ρ of the system evolves due to this globally tuned pilot wave.

For given external potential V, fundamental nonlinear equations (2) and (5) determine the time evolution of both state and hidden variables ρ and Θ. It is convenient to introduce a single variable

$$\psi=\sqrt{\rho}\,\Theta\ . \tag{7}$$

It is a complex order parameter of the system which may be used to illustrate the implicate order of the system. I call it an order variable of the system. Equations (2) and (5) yield a fundamental equation for the time evolution of order variable ψ,

$$i\frac{\partial\psi}{\partial t}=-\tfrac{1}{2}\Delta\psi+V\psi\ , \tag{8}$$

where Δ denotes the Laplacian of the manifold M. This is a Schrödinger-like linear wave equation. I have found the following scheme of global control mechanism in nonlocal and nonlinear cooperative phenomena.

(1) Oscillating local variables (i.e., hidden variables) of the system are turned over the whole manifold and contain the global information of the implicate order of the system. Distribution of the tuned oscillations over the manifold manifests a pilot wave which guides each local variable.

(2) Each local variable (i.e., state variable) evolves due to the local interaction with the globally tuned pilot wave. Resulting time evolution of the state variable manifests nonlocality arising from the global information on the implicate order of the system contained in the hidden variable.

(3) Nonlinear and nonlocal cooperative phenomena in this system with state and hidden variables are described by the order variable subject to the Schrödinger-like wave equation. The time evolution of the state variable is obtained as the absolute square of the order variable.

In many practical problems of cooperative phenomena, the present global control mechanism may arise. Among them are reaction processes of the amoebalike biological cell against the external stimulus and perceptual processes of the dendritic membrane of neurons. In the former case of the amoeba reaction process, the state variable ρ is the mass density of cytoplasm and the hidden variable Θ is the membrane oscillation. Indeed, recent experimental results claim the validity of the present global control mechanism of nonlocal and nonlinear cooperative phenomena. Cytoplasm in each location is controlled to flow in the direction of phase gradient.[9,10] In the latter case of neuronal activity, the state variable ρ is the charge distribution density of ions and the hidden variable Θ is the analog membrane potential below the threshold.[11]

The present global control mechanism thus seems to play the fundamental role especially in various perceptual processes of biological systems. Nonlocal and nonlinear cooperative phenomena described by the mechanism are key concepts of biological perception. The physical system with state and hidden variables proposed in this paper may provide us with a simple mathematical model of biological cybernetics. As the tuned wave of local oscillations plays the essential role there, it will be called wave cybernetics. Details of wave cybernetics will be reported in a forthcoming paper.[12] The idea of cooperative phenomena controlled globally by the wave tuned over the whole system may be understood in the light of Bohm's natural philosophy—the universe (i.e., the whole system) is filled up with a tuned wave from which each element in the universe knows how it should evolve.[13]

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