Effect of spin on the electron-ion scattering and the heating rate of a plasma in very intense laser light

Shahid Rashid*

Physics Department, Kalyani University, Nadia, West Bengal, India

(Received 26 September 1986; revised manuscript received 24 September 1987)

An exact Volkov state solution of the minimally coupled Dirac equation is used to calculate the transition rate dR of an electron scattering via a stationary ion in the presence of a very intense laser field. A consistent picture of the scattering is presented in which the electrons' initial and final states are quasifree states. Accordingly, a modified transition rate dR^* and a modified Maxwell-Boltzmann distribution are developed. They are used to calculate the heating rate W of the isotropic part of a quasifree neutral plasma in the presence of very intense laser light. In order to simplify the expression for the heating rate W, an important transformation, which changes an infinite sum over Bessel functions into a finite integral, is introduced. It is then shown that the leading term of the heating rate W is similar to the expression of Osborn (with corrections) for intensity $I < 10^{16}$ W/cm^2 and $k_B T < 1$ keV. A new correction factor is defined to show the effect of a very intense laser field when the intensity $I > 10^{16}$ W/cm^2. For $k_B T > 1$ keV, a spin-dependent term of order $k_B T/mc^2$ is also discovered. This represents a new term not previously known. We show that the effect of this term on the heating rate is substantial and it would be possible to measure its effect with present-day laser systems.

I. INTRODUCTION

In the 1960s, the possibility of high-intensity lasers led first to speculation and then to realization[1] of laser-induced thermonuclear reactions within a plasma. When the laser intensity I is increased beyond 10^{12} W/cm^2, a host of nonlinear processes comes into play.[2–9] So far as the optimization of the heating rate is concerned, the most relevant process is the multiphoton absorption by the electrons.[8–19] It is characterized by the parameter $X = E_q/E_{\text{ph}}$, where E_q is the average electron energy in the radiation field and $E_{\text{ph}} = \hbar\omega$ is the energy of one photon. When $X > 1$, multiphoton absorption can take place.

In a recent paper Schlesinger and Wright[12] have reviewed different methods to calculate the heating rate W. They start from a Volkov-like solution[20] to the minimally coupled Klein-Gordon equation. Using the Born approximation, the transition rate dR in momentum space is obtained. Taking nonrelativistic limits, the transition rate dR then reduces to a form easily obtained by using the solution of the minimally coupled Schrödinger equation.[5,9,11] Assuming the electrons in the plasma form a Maxwellian gas, the heating rate of the plasma W is then calculated. Their calculation shows that the quantum-mechanical corrections are small for intensity $I < 10^{16}$ W/cm^2. To bring out the effect of multiphoton absorption on the heating rate W, the final result is presented as a correction factor F defined as the ratio of W and the first term of W, which is the one-photon absorption rate.

Although other researchers have studied this problem using different approaches,[10,21] the correction factor F is plotted as a function of the parameter $z = 2E_q/k_B T$, where $k_B T$ is the mean thermal energy of the electron. Results presented are meager, mostly because F turns out to be a double infinite sum and it is difficult to compute large number of terms. Schlesinger and Wright use analytical methods to compute F term by term, where the sum over l, the number of photons absorbed, is converted into an integral. Again the plot is only for $z < 10$.

Brysk[21] showed that if only the leading term in the series expansion of the modified Bessel function[22–24] is retained in the double infinite sum, then F reduces to the generalized hypergeometric series ${}_2F_2$. But the argument of ${}_2F_2$ was obtained as $z/4$, which is wrong and hence Brysk's result could not be relied on.[12] One advantage of his result is the closed form of F which he obtains.

The main purpose of our paper is twofold. First, we want to investigate the effect of the spin of the electron on the heating rate W by using the Volkov solution of the Dirac equation,[20] which in second-order form differs from the Klein-Gordon equation only by a spin term. Basically, the Volkov solution has two terms, of which one is directly proportional to the amplitude of the laser field and hence important for high field cases. Second, we intend to develop a consistent theory of the transition rate when the intensity I of the laser beam is in the range of 10^{16} to 10^{18} W/cm^2 (only infrared lasers are considered here).

The motivation behind our work has been the lack of data for a large range of values of z and also for high intensities of the laser beam, i.e., in the range of 10^{16} to 10^{18} W/cm^2. Moreover, no computation has been done for a hot plasma (1 keV $< k_B T <$ 100 keV) in high laser field.

In Sec. II the transition rate is derived using the Volkov states for the incoming and outgoing states. Because we are considering very high laser intensity and soft photons (where wavelength $\lambda \simeq 1$ μm), we treat the coherent laser field classically. The justification is that one can neglect radiative corrections.[2,9,25,26] The question of asymptotic decoupling of the electron from the laser field[18] is avoided, the reason being that we assume the electron is inside the laser field in both its initial and final states. The Coulomb (shielded) interaction is treated as a perturbation in the Born approximation. To be consistent, a modified transition rate dR^* is introduced to incorporate the time-averaged effect of the laser field. We show that, under nonrelativistic approximation, all the terms of dR^* are of the same order of magnitude and hence all terms should be considered for further calculations.

In Sec. III we develop a modified Maxwellian distribution for the quasifree electrons whose Volkov states are labeled by the *time-averaged* quasi-4-momentum p^*. The heating rate W for the isotropic part of the plasma is obtained by the standard method.[12] An important transformation developed in the Appendix is utilized to put the heating rate W in a form which allows us to drop terms by direct comparison when the intensity $I < 10^{18}$ W/cm^2. It should be pointed out that although nonrelativistic approximation is used, our treatment tacitly retains relativistic effects of the laser field.

In Sec. IV the final expression for W is obtained as the sum of two major terms. The first term W_1 is a double sum which, under low-intensity conditions, reduces to an expression similar to that obtained by Osborn.[10] Following Brysk we reduce the high-intensity expression to a single integral form which can be easily evaluated by a computer. Results are presented for values of z from 0 to 350. We also present a new correction factor F^* for intensity range $10^{16} < I < 10^{18}$ W/cm^2 and $0 < z < 330$. The first term is dominant for $k_B T < 100$ eV and the above results have been plotted for this range of temperature. The other term of the heating rate represents a new term previously unknown and becomes important in the temperature range 100 eV $< k_B T <$ 200 keV. Investigation shows that the term arises solely due to the spin of the electron. At the end of Sec. IV we present our results showing the effect of the spin on the heating rate.

Section V ends our presentation with some general conclusions and also mentions some calculations which could be attempted in the near future. Throughout the paper we consider wavelengths in the range of 0.1 to 10 μm because the most important and promising laser systems, e.g., CO_2 gas lasers, are in this range.

II. TRANSITION RATE AND NONRELATIVISTIC APPROXIMATION

When a laser beam is incident on a plasma, initially it is mostly the electrons which absorb the laser energy by inverse bremsstrahlung and hence their temperature T_e becomes much higher than the ion's temperature T_i.[27] The electrons are assumed to be scattered by a Coulomb potential. Classically, the total Coulomb cross section for an electron-ion encounter is infinite and to avoid this, different cutoff schemes have been proposed.[28-30] In our analysis, we use Lorentz-Heaviside units[31] and the metric $g^{\mu\nu}=(1,-1,-1,-1)$ together with $\hbar=c=1$.

A. Transition rate

The Coulomb (shielded) vector potential is (for electron $e=-|e|$)

$$A_C^\mu=\frac{-Ze}{4\pi|\mathbf{x}|}e^{-\Omega|\mathbf{x}|}b^\mu , \tag{2.1}$$

where

$$b^\mu=(1,0,0,0) . \tag{2.2}$$

The laser beam is represented as a monochromatic field

$$A^\mu=(0,\mathbf{A})=\epsilon^\mu A_0\cos(k\cdot x) , \tag{2.3}$$

where

$$k^\mu=(k_0,\mathbf{k})=|\mathbf{k}|(n_0,\mathbf{n}) \tag{2.4}$$

and the gauge is $\mathbf{k}\cdot\boldsymbol{\epsilon}=0$. The Volkov state solution for an electron in the radiation field A^μ is given by[20]

$$\Psi_i=\sqrt{m/E_i}\left[1+\frac{e}{2n\cdot p_i}\hat{n}\,\hat{A}\right]u_i e^{-i(p_i\cdot x-S_i)} , \tag{2.5}$$

where u_i is a spinor satisfying the normalization condition

$$u_i^\dagger\cdot u_i=|E_i|/m \tag{2.6}$$

and

$$S_i=(2n\cdot p_i)^{-1}\int_{-\infty}^{+n\cdot x}[2e(p_i\cdot A)-(eA)^2]dy . \tag{2.7}$$

The subscript i indicates the incident electron and the caret notation implies a dot product between the 4-vector and the γ matrices.[31] Hence e.g., $\hat{n}=n\cdot\gamma$. The expression (2.5) is an exact solution to the minimally coupled Dirac equation.

The transition matrix element for a quasifree electron scattering off a Coulomb potential is given by (Ref. 31, Chap. 7)

$$S_{fi}=-ie\int d^4x\,\overline{\Psi}_f\cdot A_C\cdot\Psi_i , \tag{2.8}$$

where f indicates final state.

Doing a sum over the final spin states and an average over the initial spin states for $|S_{fi}|^2$, we can reduce it to a trace given by

$$\frac{1}{2}\sum_{\pm s_f,s_i}|S_{fi}|^2=\frac{(me)}{2}\int\int\left[d^4x\,d^4x'\frac{1}{E_f}\frac{1}{E_i}\right]e^{iK}$$
$$\times\mathrm{Tr}\left[\left[\frac{\hat{p}_i+m}{2m}\right]\left[1+\frac{e}{2n\cdot p_i}\hat{A}'\hat{n}\right]\hat{A}'_C\left[1+\frac{e}{2n\cdot p_f}\hat{n}\,\hat{A}'\right]\right.$$

$$\times \left[\frac{p_f+m}{2m}\right]\left[1+\frac{e}{2n\cdot p_f}\hat{A}\,\hat{n}\right]\hat{A}_C\left[1+\frac{e}{2n\cdot p_i}\hat{n}\,\hat{A}\right]\Bigg] , \tag{2.9}$$

where

$$K=(p_f-p_i)\cdot(x-x')+(S_f-S_i)-(S_f'-S_i') \tag{2.10}$$

and the prime indicates x' in the argument. After performing the S integrals of (2.10) as given by (2.9), the double exponentials of e^{iK} can be changed into single exponentials by means of the expansion[22]

$$\exp[Z_0(t-t^{-1})]=\sum_{p=-\infty}^{+\infty} t^p J_p(2Z_0) , \tag{2.11}$$

where J_p is the Bessel function.

Performing the integrals and the trace of (2.10) using the well-known trace theorems,[31] we get

$$\begin{aligned}\tfrac{1}{2}\sum_{s_f,s_i}|S_{fi}|^2=\frac{(8\pi^2e^2)^2}{2E_iE_f}\sum_{r_1,r_2,r_3,r_4=-\infty}^{+\infty} & J_{r_1}(e\mu_1)J_{r_2}(e\mu_2)J_{r_3}(e\mu_1)J_{r_4}(e\mu_2)\\ &\times[\Delta_1^0\Delta_3^0P_{1,1}+e\Delta_3^0(\Delta_1^{+1}+\Delta_1^{-1})(P_{1,3}+P_{1,24})\\ &+e\Delta_1^0(\Delta_3^{+1}+\Delta_3^{-1})(P_{3,1}+P_{24,1})\\ &+e^2\Delta_3^0(2\Delta_1^0+\Delta_1^{+2}+\Delta_1^{-2})P_{1,5}+e^2\Delta_1^0(2\Delta_3^0+\Delta_3^{+2}+\Delta_3^{-2})P_{5,1}\\ &+e^2(\Delta_1^{+1}+\Delta_1^{-1})(\Delta_3^{+1}+\Delta_3^{-1})(P_{3,3}+P_{3,24}+P_{24,3}+P_{24,24})\\ &+e^3(\Delta_3^{+1}+\Delta_3^{-1})(2\Delta_1^0+\Delta_1^{+2}+\Delta_1^{-2})(P_{3,5}+P_{24,5})\\ &+e^3(\Delta_1^{+1}+\Delta_1^{-1})(2\Delta_3^0+\Delta_3^{+2}+\Delta_3^{-2})(P_{5,3}+P_{5,24})\\ &+e^4(2\Delta_1^0+\Delta_1^{+2}+\Delta_1^{-2})(2\Delta_3^0+\Delta_3^{+2}+\Delta_3^{-2})P_{24,24}] ,\end{aligned} \tag{2.12}$$

where

$$\begin{aligned}&\mu_1=A_0[(\epsilon\cdot p_f)/n\cdot p_f-(\epsilon\cdot p_i)/n\cdot p_i]\\ &\mu_2=[A_0^2/(8k_0)](1/n\cdot p_f-1/n\cdot p_i)=\mu_0/(2k_0)\end{aligned} \tag{2.13}$$

and

$$\Delta_{1,3}^s=\frac{\delta(E_f-E_i+e^2\mu_0n_0+(r_{1,3}+2r_{2,4}+s)k_0)}{|\mathbf{p}_f-\mathbf{p}_i+e^2\mu_0\mathbf{n}+(r_{1,3}+2r_{2,4}+s)\mathbf{k}|^2+\Omega^2} . \tag{2.14}$$

Moreover,

$$\begin{aligned}&P_{i,j}=A_iB_jP'_{i,j},\quad P_{ij,k}=(A_i+A_j)B_kP'_{ij,k},\quad P_{i,jk}=A_i(B_j+B_k)P'_{i,jk} ,\\ &P_{ij,kl}=(A_i+A_j)(B_k+B_l)P'_{ij,kl} ,\end{aligned} \tag{2.15}$$

where

$$\begin{aligned}&A_1=-Z/(4\pi),\quad A_2=-A_0A_1/(4n\cdot p_i)=A_3/2,\quad A_4=A_0A_1/(2n\cdot p_f) ,\\ &A_5=-A_2A_4/A_1,\quad B_1=A_1,\quad B_2=-A_4/2,\quad B_3=A_4,\quad B_4=-A_2,\quad B_5=A_5\end{aligned} \tag{2.16}$$

and

$$\begin{aligned}&P'_{1,1}=2E_iE_f-p_i\cdot p_f+m^2,\quad P'_{1,24}=(p_i\cdot n)(p_f\cdot\epsilon)-(p_i\cdot\epsilon)(p_f\cdot n)+2E_f(p_i\cdot\epsilon) ,\\ &P'_{1,3}=E_i(p_f\cdot\epsilon)+(p_i\cdot\epsilon)E_f,\quad P'_{1,5}=E_i(p_f\cdot n)+E_f(p_i\cdot n)-p_i\cdot p_f+m^2 ,\\ &P'_{24,24}=2(p_i\cdot n)(p_f\cdot n)-2E_f(p_i\cdot n)-2E_i(p_f\cdot n)+2p_i\cdot p_f+4(p_i\cdot\epsilon)(p_f\cdot\epsilon)-2m^2 ,\\ &P'_{24,3}=E_i(p_f\cdot n)-E_f(p_i\cdot n)+p_i\cdot p_f+2(p_f\cdot\epsilon)(p_i\cdot\epsilon)-m^2,\quad P'_{24,5}=2(p_f\cdot n)(p_i\cdot\epsilon) ,\\ &P'_{3,3}=2(p_i\cdot\epsilon)(p_f\cdot\epsilon)+p_i\cdot p_f-m^2,\quad P'_{3,5}=(p_i\cdot\epsilon)(p_f\cdot n)+(p_i\cdot n)(p_f\cdot\epsilon) .\end{aligned} \tag{2.17}$$

The rest is obtained by the following correspondence:

$$P_{i,j}=P_{j,i}(i\leftrightarrow f)\ , \tag{2.18}$$

e.g.,

$$P_{5,24}=P_{24,5}(i\leftrightarrow f)=2(p_i\cdot n)(p_f\cdot\epsilon)\ .$$

The transition rate is given by

$$dR=\frac{1}{T}\left[\tfrac{1}{2}\sum|S_{fi}|^2\right]\frac{d^3p_f}{(2\pi)^3}\ , \tag{2.19}$$

where T is a large time. By transformations of the sums in (2.12) over r_1, r_2, r_3, and r_4 and using

$$\delta^2(E_f-E_i+e^2\mu_0 n_0+l_1k_0)=(T/2\pi)\delta(E_f-E_i+e^2\mu_0 n_0+l_1k_0) \tag{2.20}$$

we get the transition rate as

$$dR=(2e^2)\sum_{l_1=-\infty}^{+\infty}\frac{\delta(E_f-E_i+e^2hc\mu_0 n_0+l_1\hbar\omega)}{(|p_f-p_i+e^2\mu_0 n+l_1k|^2+\Omega^2)^2}\frac{d^3p_f}{E_iE_f}\sum_{l_2,l_4=-\infty}^{+\infty}J_{-l_2}(e\mu_2)J_{-l_4}(e\mu_2)J(l_1,l_2,l_4)\ , \tag{2.21}$$

where

$$\begin{aligned}J(l_1,l_2,l_4)=[&e^2J_{12}^0J_{14}^0P_{1,1}+e^3(J_{12}^{+1}+J_{12}^{-1})J_{14}^0(P_{1,3}+P_{1,24})+e^3J_{12}^0(J_{14}^{+1}+J_{14}^{-1})(P_{3,1}+P_{24,1})\\&+e^4J_{12}^0(2J_{14}^0+J_{14}^{+2}+J_{14}^{-2})(P_{1,5}+P_{5,1})+e^4(J_{12}^{+1}+J_{12}^{-1})(J_{14}^{+1}+J_{14}^{-1})\\&\qquad\times(P_{24,24}+P_{3,24}+P_{24,3}+P_{3,3})\\&+e^5(J_{12}^{+1}+J_{12}^{-1})(2J_{14}^0+J_{14}^{+2}+J_{14}^{-2})(P_{5,24}+P_{5,3})\\&+e^5(2J_{12}^0+J_{12}^{+2}+J_{12}^{-2})(J_{14}^{+1}+J_{14}^{-1})(P_{24,5}+P_{3,5})+e^6(2J_{12}^0+J_{12}^{+2}+J_{12}^{-2})\\&\qquad\times(2J_{14}^0+J_{14}^{+2}+J_{14}^{-2})P_{5,5}]\ .\end{aligned} \tag{2.22}$$

In (2.22) the J factors are defined as follows:

$$J_{12}^s=J_{l_1+2l_2+s}(e\mu_1)\ ,\qquad J_{14}^s=J_{l_1+2l_4+s}(e\mu_1)\ . \tag{2.23}$$

Hence the transition rate with absorption of l photons is given by dR^l $(l>0)$ and is the same as the term on the right-hand side of (2.21) for which $l_1=-l$. We now discuss the concept of quasifree states for the electrons.

B. Quasifree states

The unsolved problem of decoupling the electron in the Volkov state from the radiation field has been considered elsewhere.[18,32,33] The justification of applying Volkov states in our analysis is that it can be assumed that the electron does not leave the laser beam at all. As a first approximation, this should be valid if (a) the pulse of the laser beam is long enough and (b) the width of the laser beam covers the whole scattering region. In reality, before the electron gets inside the pulse, it is acted on by a nonlinear temporal change of the beam intensity.[34]

It was pointed out by Fried *et al.*[18] that the Volkov solution cannot be normalized and hence Eq. (4) of Ref. 2 and Eq. (40.10) of Ref. 20 appear to be in error. In our analysis, we introduce the time-averaged method. Hence, for the α-matrices velocity operators [Ref. 31, Eqs. (1.22) and (3.29)], we use

$$\langle\boldsymbol{\alpha}\rangle_t=\langle\Psi_p^\dagger\boldsymbol{\alpha}\Psi_p\rangle_t\ , \tag{2.24}$$

where the subscript t means averaging over one time period of the plane wave and Ψ_p is the Volkov solution (2.5). One can easily show that

$$\langle\Psi_p^\dagger\alpha^i\Psi_p\rangle_t=(1/E)\left[p^i+\frac{e^2A_0^2}{4k\cdot p}k^i\right]=(p^*)^i/E\ . \tag{2.25}$$

We have defined the quasi-3-momentum $\mathbf{p}^*$ in (2.25) and the spinor used here is normalized as in (2.6).[35] Doing the averaging over the unit operator, we get

$$\langle\mathbb{1}\rangle_t=\langle\Psi_p^\dagger\mathbb{1}\Psi_p\rangle_t=1+\frac{e^2A_0^2}{4k\cdot p}\frac{k^0}{p^0}=(p^*)^0/p^0\ , \tag{2.26}$$

where a quasi-energy $(p^*)^0$ has been defined. Combining (2.25) and (2.26) we get the quasi-4-momentum p^* given by

$$(p^*)^\mu=p^\mu+\frac{e^2A_0^2}{4k\cdot p}k^\mu\ . \tag{2.27}$$

Also,

$$(p^*)^2=m^2+\tfrac{1}{2}(eA_0)^2=(m^*)^2\ , \tag{2.28}$$

where we have defined an effective mass m^* given by

$$m^*=m[1+\tfrac{1}{2}(eA_0/m)^2]^{1/2}\ . \tag{2.29}$$

We can define the Kibble parameter as

$$\varepsilon_R=[E_q/(mc^2)]^{1/2}=eA_0/(2m)\ , \tag{2.30}$$

which is relevant whenever the quiver energy of the electron becomes comparable or greater than the rest mass energy of the electron. Hence we see from (2.27) and (2.29) that the radiation field could give rise to relativistic effects to a nonrelativistic electron.

If we consider the electrons to constitute a quasifree gas, where the momentum can be expressed by (2.27), then the final states in d^3p_f of (2.19) should be changed to $d^3p_f^*$ so that the quasifree-to-quasifree transition can be written as (for l photon absorption)

$$(dR^*)^l=(2e^2)\frac{\delta(E_f^*-E_i^*-l\hbar\omega)}{(|\mathbf{p}_f^*-\mathbf{p}_i^*-l\hbar\mathbf{k}|^2+\Omega^2)^2}\frac{d^3p_f^*}{E_iE_f}\sum_{l_2,l_4=-\infty}^{+\infty}J_{-l_2}(e^2\mu_2)J_{-l_4}(e^2\mu_2)J(-l,l_2,l_4)\ , \tag{2.31}$$

where $J(l_1=-l,l_2,l_4)$ is defined in (2.22).

C. Nonrelativistic approximation

For nonrelativistic electron velocities the μ coefficients of (2.13) become

$$\begin{aligned} e\mu_1 &\sim \frac{eA_0}{\omega}\frac{\epsilon\cdot(\mathbf{p}_i^*-\mathbf{p}_f^*)}{mc}\ ,\\ e^2\mu_2 &\sim (eA_0/m)^2(8\hbar k)^{-1}\mathbf{n}\cdot(\mathbf{p}_f^*-\mathbf{p}_i^*)\ . \end{aligned} \tag{2.32}$$

Taking nonrelativistic approximations for the J coefficient of (2.22) and also using the relation

$$J_{n+1}(x)+J_{n-1}(x)=(2n/x)J_n(x) \tag{2.33}$$

for the Bessel functions, we get

$$\begin{aligned} J(l_1,l_2,l_4)\simeq \left[\frac{eZ}{4\pi}\right]^2 \Bigg[& 2m^2J_{12}^0J_{14}^0+\frac{eA_0}{2}\frac{2(l_1+2l_2)}{e\mu_1}J_{12}^0J_{14}^0[-2\epsilon\cdot(\mathbf{p}_i^*+\mathbf{p}_f^*)]\\ &+\left[\frac{eA_0}{2}\right]^2\frac{4(l_1+2l_2)(l_1+2l_4)}{(e\mu_1)^2}J_{12}^0J_{14}^0\left[1+\frac{\mathbf{n}\cdot(\mathbf{p}_i^*-\mathbf{p}_f^*)}{2mc}\right]\\ &+\left[\frac{4}{e\mu_1}\right]\left[\frac{eA_0}{2}\right]^2J_{12}^0\left[\frac{2(l_1+2l_2)^2}{e\mu_1}J_{14}^0+J_{14}^{+1}-J_{14}^{-1}\right]\\ &+\left[\frac{2}{e\mu_1}\right]^2\left[\frac{eA_0}{2}\right]^3(l_1+2l_2)J_{12}^0\left[\frac{2}{e\mu_1}(l_1+2l_4)^2J_{14}^0+J_{14}^{+1}-J_{14}^{-1}\right]\\ &\times\left[\frac{-\epsilon\cdot(\mathbf{p}_i^*+\mathbf{p}_f^*)}{m^2}\right]+\left[\frac{eA_0}{2}\right]^4\left[\frac{2}{e\mu_1}\right]^2\left[\frac{2(l_1+2l_2)^2}{e\mu_1}J_{12}^0+J_{12}^{+1}-J_{12}^{-1}\right]\\ &\times\left[\frac{2(l_1+2l_4)}{e\mu_1}J_{14}^0+J_{14}^{+1}-J_{14}^{-1}\right]\frac{1}{2m^2}\Bigg]\ , \end{aligned} \tag{2.34}$$

where J_{12}^s and J_{14}^s are defined by (2.23) but with μ_1 and μ_2 now given by approximations (2.32). Using (2.32) one can readily check that each term of $J(l_1,l_2,l_4)$, as given in (2.34), are of the same order in (eA_0) and hence further analysis should consider all terms of (2.34) simultaneously.

III. HEATING RATE OF A QUASIFREE PLASMA

In this section we introduce modifications in the Juttner distribution[36] for a relativistic electron gas due to the presence of the laser radiation field.

A. Modified Maxwellian distribution

If the electrons in the plasma are in states specified by the quasi-4-momentum $(p^*)^\mu$ as defined in (2.27) and form a gas, then the Maxwellian distribution function is given by

$$f(E^*)=C^*\exp[-E^*/(k_BT)]\ , \tag{3.1}$$

where C^* is the normalization constant obtained from the requirement that

$$\int\int\int f(\mathbf{p}^*)d^3\mathbf{p}^* = 1 \ . \tag{3.2}$$

Hence,

$$C^* = \left[4\pi (m^*)^2 (k_B T) K_2 \left[\frac{m^*}{k_B T} \right] \right]^{-1} , \tag{3.3}$$

where K_2 is the modified Bessel function of the second order (Ref. 24).

The modified Juttner distribution is obtained from (3.1) as

$$f(\mathbf{p}^*) = C^* \exp \left[\frac{-m^* c^2}{k_B T} \left[1 + \frac{|\mathbf{p}^*|^2}{(m^* c)^2} \right]^{1/2} \right] \tag{3.4}$$

and the nonrelativistic approximation of (3.4) gives us the *modified Maxwellian distribution*

$$[f(\mathbf{p}^*)]_{\rm NR} = (2\pi m^* k_B T)^{-3/2} \exp[(-\mathbf{p}^*)^2/(2m^* k_B T)] \ . \tag{3.5}$$

It reduces to the usual Maxwellian distribution for low intensity of the laser field, i.e., when ε_R of (2.30) is $\ll 1$. When $\varepsilon_R < 1$, the intensity is in the range of 10^{16} to 10^{18} W/cm^2. We call this the high-intensity range (HIR).

It has been shown earlier that the electron distribution becomes non-Maxwellian even for moderate intensities.[37,38] Jones and Lee[39] have also shown that if we deal only with the isotropic part of the distribution then the self-similar solution is a Maxwellian. In our case we only deal with the isotropic part of the electron distribution so that the classical heating rate expression can be used. The anisotropic part will be dealt with in a later paper.

B. Heating rate of a nonrelativistic plasma in the high-intensity range

The heating rate W of the isotropic electrons in the plasma is given by[12]

$$W = N_e N_i \sum_{l=1}^{+\infty} (l\hbar\omega) \int d^3 p_i^* \times \int (dR^*_{\rm NR})^l \times \{ [f(\mathbf{p}_i^*)]_{\rm NR} - [f(\mathbf{p}_f^*)]_{\rm NR} \} , \tag{3.6}$$

where $N_{e\,(i)}$ is the density of the electrons (ions) in the plasma and $(dR^*_{\rm NR})^l$ is given by (2.31) but with $J(-l,l_2,l_4)$ defined by (2.34) and the μ coefficients by (2.32). Performing some of the integrations by the standard models[11,12] and noting that in HIR $1 \ll [2m^*/(lk)]^{1/2}$, we get $W_{\rm HIR}$ as (letting $\Omega \to 0$)

$$\begin{aligned} W_{\rm HIR} = {} & \frac{4m^*}{2\pi} \frac{N_e N_i}{(2\pi m^* k_B T)^{1/2}} \left[\frac{e^2 Z}{4\pi m} \right]^2 \\ & \times \sum_{l=1}^{+\infty} (l\hbar\omega) \sinh \left[\frac{l\hbar\omega}{2k_B T} \right] \\ & \times \int_0^{2\pi} d\phi \int_{-1}^{+1} dx \int_0^{+\infty} dQ \frac{1}{Q^3} \exp \left[\frac{-\delta_1^*}{Q^2} - \gamma^* Q^2 \right] \\ & \times \left\{ 4\pi m^2 (S_1')^2 - \left[\frac{eA_0}{2} \right] \frac{4\pi m^* l\hbar\omega x}{Q} \left[2S_1' S_2' + \left[\frac{eA_0}{2m} \right]^2 S_2' S_3' \right] \right. \\ & \left. + 2\pi \left[\frac{eA_0}{2} \right]^2 [(S_2')^2 + 2S_1' S_3'] + \left[\frac{eA_0}{2} \right]^4 \frac{\pi}{m^2} (S_3')^2 \right\} , \end{aligned} \tag{3.7}$$

where $x = \cos\theta$, $\mathbf{Q} = (\mathbf{p}_i^* - \mathbf{p}_f^*)$, θ is the angle between ϵ and $\mathbf{Q}$, $\delta_1^* = m^*(lk)^2/(2k_B T)$, $\gamma^* = (8m^* k_B T)^{-1}$, and the S_i''s are defined by

$$\begin{aligned} S_1' &= \sum_{l_2=-\infty}^{+\infty} J_{l_2}(A) J_{l+2l_2}(B) , \\ S_2' &= \sum_{l_2=-\infty}^{+\infty} J_{l_2}(A) [J_{l+2l_2+1}(B) + J_{l+2l_2-1}(B)] , \\ S_3' &= \sum_{l_2=-\infty}^{+\infty} J_{l_2}(A) [2J_{l+2l_2}(B) + J_{l+2l_2+2}(B) + J_{l+2l_2-2}(B)] , \end{aligned} \tag{3.8}$$

where

$$A=-\alpha Q(1-x^2)^{1/2}\cos\phi\ ,$$
$$B=\beta Q x\ , \tag{3.9}$$
$$\alpha-(8\hbar k)^{-1}(eA_0/m)^2,\quad \beta=eA_0(\hbar k m c^2)^{-1}\ .$$

Using the transformation developed in the Appendix, we change (3.8) to

$$S_1'=S_l,\quad S_2'=(S_{l+1}+S_{l-1}),\quad S_3'=(2S_l+S_{l+2}+S_{l-2})\ , \tag{3.10}$$

where

$$S_p=\frac{(-1)^p}{\pi}\int_0^{\pi/2}d\underline{\theta}[\cos(\alpha_0\sin 2\underline{\theta}-p\underline{\theta}-\beta_0\sin\underline{\theta})+(-1)^p\cos(\alpha_0\sin 2\underline{\theta}-p\underline{\theta}+\beta_0\sin\underline{\theta})] \tag{3.11}$$

and

$$\alpha_0=\alpha Q(1-x^2)^{1/2}\cos\phi,\quad \beta_0=\beta Q x\ . \tag{3.12}$$

It is now possible to ignore the α_0 factor compared to β_0 because in the HIR $\alpha<\beta$ for the intensity $I\leq 10^{18}$ W/cm^2 and also $\alpha \ll \beta$ for $I\ll 10^{18}$ W/cm^2. Under such approximations

$$S_p\simeq\frac{(-1)^p}{\pi}\int_0^{\pi/2}d\underline{\theta}[\cos(\beta_0\sin\underline{\theta}+p\underline{\theta})+(-1)^p\cos(\beta_0\sin\underline{\theta}-p\underline{\theta})]\ . \tag{3.13}$$

Also, as $(l\hbar k)$ is very small, we can ignore the second term of (3.7). Moreover, because of the factor $(eA_0)^4/m^2$, we can ignore the last term of (3.7). One can then show that

$$W_{\rm HIR}=W_0^*\sum_{l=0}^{+\infty}(l\hbar\omega)\sinh\left[\frac{l\hbar\omega}{2k_BT}\right]\times\sum_{k=0}^{+\infty}\frac{(-1)^k}{(2k+1)!}\frac{4\beta^{2k}}{\pi}(\delta_l^*/\gamma^*)^{(k-1)/2}K_{k-1}(2(\delta_l^*\gamma^*)^{1/2})$$
$$\times\int_0^{\pi/2}d\theta\int_0^{\pi/2}d\theta'[I^{l,l}+(\varepsilon_R^2/2)(I^{l+1,l+1}+4I^{l,l}+2I^{l,l+2}+I^{l-1,l-1}+2I^{l+1,l-1}+2I^{l,l-2})]\ , \tag{3.14}$$

where

$$W_0^*=\frac{8m^*}{(2\pi m^* k_BT)^{1/2}}\left[\frac{e^2Z}{4\pi}\right]^2, \tag{3.15}$$
$$\varepsilon_R=|e|A_0/(2mc^2)\ , \tag{3.16}$$

and

$$I^{l',l''}=\cos(l'\theta+l''\theta')[\sin\theta+\sin\theta')^{2k}+(-1)^{l'}(\sin\theta-\sin\theta')^{2k}]$$
$$+\cos(l'\theta-l''\theta')[(\sin\theta-\sin\theta')^{2k}+(-1)^{l'}(\sin\theta+\sin\theta')^{2k}]\ . \tag{3.17}$$

To check the validity of the many approximations we have introduced in our analysis, we derive the heating rate W for the low-intensity region and compare with expressions for W obtained by other authors.[10,12,21] This is done in Sec. IV.

IV. NUMERICAL RESULTS

We can do the θ integrations and then comparison shows that the second term in the square bracket of (3.14) contributes through its $l=1$ term significantly. Retaining only this term for it, we can reduce $W_{\rm HIR}$ to the form

$$W_{\rm HIR}=W_1+W_0^*(\hbar\omega)\sinh\left[\frac{\hbar\omega}{2k_BT}\right]\beta^2\left[\frac{k_BT}{2m^*c^2}\right], \tag{4.1}$$

where

$$W_1=8\pi W_0^*\sum_{l=1}^{+\infty}(l\hbar\omega)\sinh\left[\frac{l\hbar\omega}{2k_BT}\right]\times\sum_{k=l}^{+\infty}\frac{(-1)^k}{k!}\frac{(2m^*l\hbar\omega)^{k-l}}{(2k+1)}(\beta/2)^{2k}\times\frac{(2k-1)!}{(k-1)!(k-l)!(k+l)!}\times K_{k-1}\left[\frac{l\hbar\omega}{2k_BT}\right]. \tag{4.2}$$

We define a new correction factor $F(z^*)$ as $W_{\rm HIR}$, given by (4.1), divided by the first term of W_1, given by

(4.2). Following the treatment given by Brysk,[21,40] one can easily show that

$$F'(z^*)=F(z^*)+3k_BT\{2m^*c^2\ln[2k_BT/(\hbar\omega)]\}^{-1} . \quad (4.3)$$

In (4.3) the second term is a completely new term and

$$F(z^*)=\tfrac{3}{4}(z^*)^{-3/2}\int_0^{z^*} dz^*\sqrt{z^*}\exp(-z^*/2) \times[I_0(z^*/2)-I_1(z^*/2)] , \quad (4.4)$$

where I_0 and I_1 are the modified Bessel functions.[22] We consider three cases.

Case 1: $\varepsilon_R \ll 1$, $k_BT \le 1$ keV. Under such approximations $m^*=m$ and $z^*=z$ and the second term of (4.3) becomes negligible. Hence $F'(z^*)=F'(z)$ is given by

$$F(z)=\tfrac{3}{4}z^{-3/2}\int_0^{z}dz\sqrt{z}\exp(-z/2) \times[I_0(z/2)-I_1(z/2)] , \quad (4.5)$$

which is similar to the correction factor obtained by Brysk[21] and hence our approximations introduced in our analysis are correct. But our z is four times larger than Brysk's x because Brysk's result is in error.[12] Figures 1 and 2 are plots of $F(z)$ versus z where z is ranging from 0 to 320 obtained by a computer (IBM 3033). The computer handled the singularities successfully for $z<330$, and the result is presented in Table I.

In Fig. 1 we compare our results with that of Schlesinger and Wright.[12] One must note that our z is the same

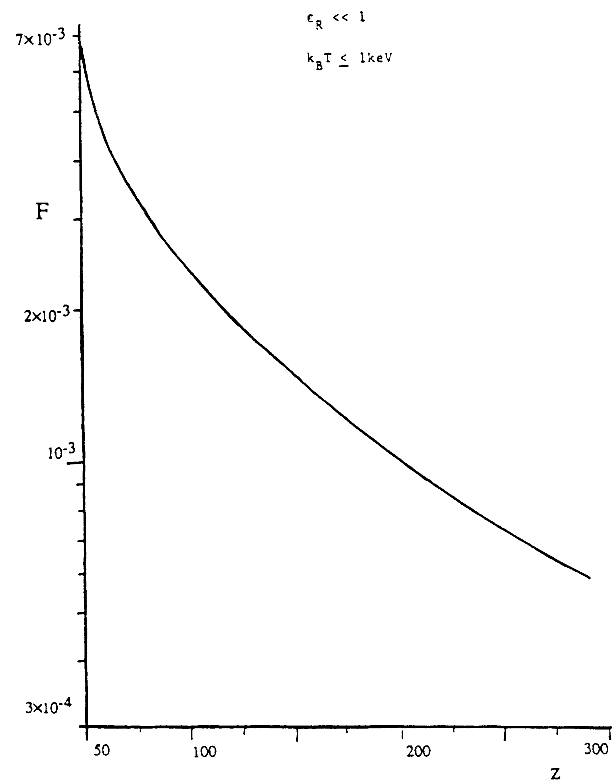

FIG. 2. Plot of correction factor F for z between 50 and 350.

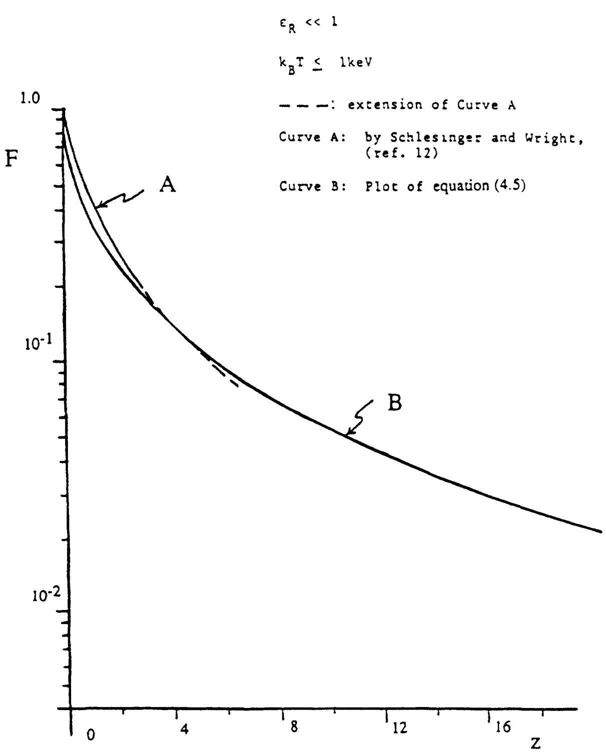

FIG. 1. Plot of correction factor F for z between 0 and 20.

TABLE I. Correction factor F for different values of z. Dash denotes where the computer failed to handle the singularities.

z	F
0	1
1	0.33
4	0.13
9	0.54×10^{-1}
16	0.27×10^{-1}
25	0.15×10^{-1}
36	0.96×10^{-2}
49	0.65×10^{-2}
64	0.45×10^{-2}
81	0.33×10^{-2}
100	0.25×10^{-2}
121	0.20×10^{-2}
144	0.16×10^{-2}
169	0.13×10^{-2}
196	0.10×10^{-2}
225	0.85×10^{-3}
256	0.71×10^{-3}
289	0.60×10^{-3}
324	0.52×10^{-3}
361	–
400	–

as their $\nu/(2T)$ and hence their curve is for the range $0 \le z \le 4$. Our curve is close to their estimate, the nominal difference between the two curves being around 0.05.

Case 2: $k_B T \le 1$ keV, $0.1 \le \varepsilon_R \le 1$. The latter condition implies that the intensity I lies in the range $10^{16}-10^{18}$ W/cm^2. In that case, we can ignore the second term of (4.3) so that $F'(z^*) \sim F(z^*) = F^*(z)$. We have defined a new correction factor $F^*(z)$ given by [see (4.4)]

$$F^*(z) = \tfrac{3}{4} z^{-3/2} \int_0^{m^* z/m} dz \sqrt{z} \exp\left[\frac{-m^* z}{2m}\right] \times \left[I_0\left[\frac{m^* z}{2m}\right] - I_1\left[\frac{m^* z}{2m}\right]\right] . \tag{4.6}$$

Figure 3 shows a plot of F^* versus z for different values of the parameter ε_R [see (3.16)]. The argument z ranges from 0 to 20 and we note that for $\varepsilon_R \le 0.1$ the curve for F^* is practically the same as F in Fig. 1. This is also true for Fig. 4 compared to Fig. 2, where the former is a plot of $F^*(z)$ for z lying in the range 50–300. We note that for a fixed value of z, F^* decreases as the intensity I is increased.

Case 3: $\varepsilon_R \ll 1$, $\hbar\omega = 1$ eV, $k_B T > 100$ eV, i.e., we consider a hot but nonrelativistic plasma. Then $F'(z^*)$ of (4.3) reduces to

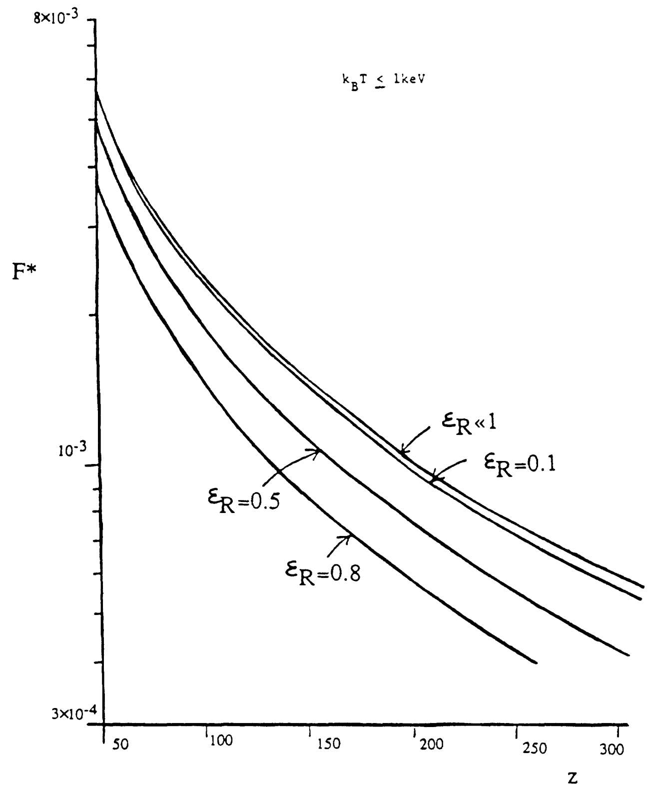

FIG. 4. Plot of F^* for z between 50 and 350 and ε_R between 0.1 and 0.8.

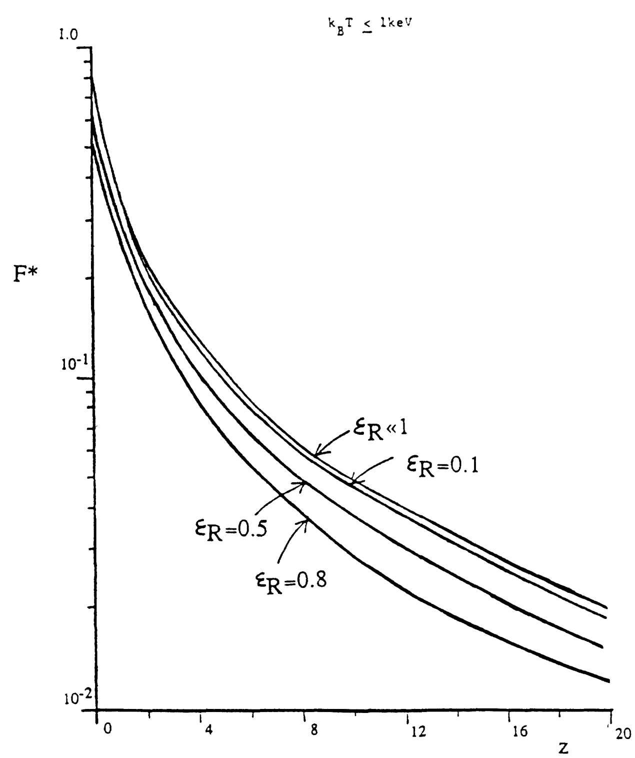

FIG. 3. Plot of correction factor F^* for z between 0 and 25 and ε_R between 0.1 and 0.8.

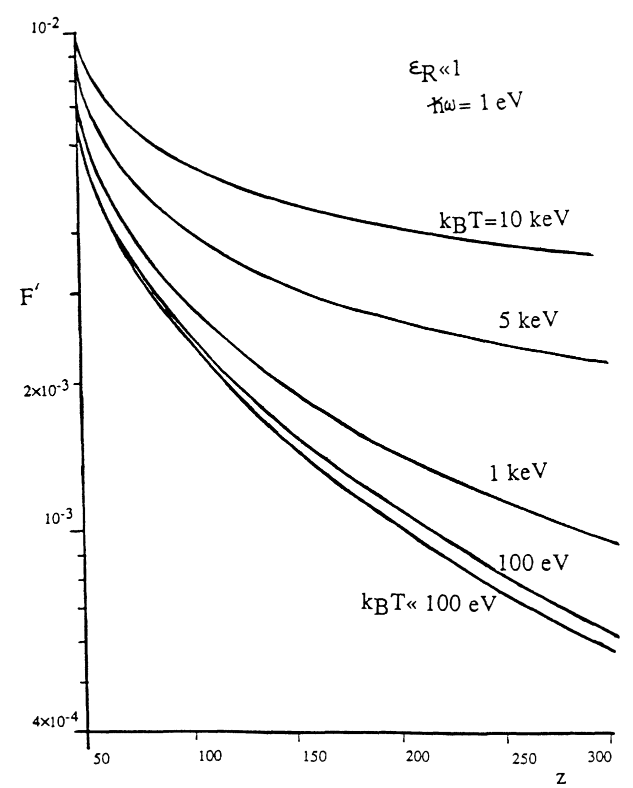

FIG. 5. Plot of correction factor F' for $50 \le z \le 350$ and 100 eV $\le k_B T \le 10$ keV.

$$F'(z)=F(z)+\frac{3k_BT}{2mc^2\ln[2k_BT/(\hbar\omega)]}\;, \tag{4.7}$$

where $F(z)$ is given by (4.5).

Figure 5 shows a plot of F' versus z for k_BT values ranging from 100 eV to 10 keV. Although higher values of k_BT (up to $\sim$200 keV) can be plotted, Fig. 5 is sufficient to bring out the fact that F' increases substantially with increase of the mean thermal energy k_BT of the plasma. This makes it easier to observe the effect of the second term in (4.7) experimentally. Moreover, it does not need very high powered lasers to observe the effect.

V. CONCLUSIONS

In our analysis, the source of the second term in (4.7) can be traced back to the third term of (2.34) or to the fourth term of (2.22). From this term we get the coefficient of [see (2.23)]

$$[J_{l_1+2l_2+1}(e\mu_1)+J_{l_1+2l_2-1}(e\mu_1)]\times[J_{l_1+2l_4+1}(e\mu_1)+J_{l_1+2l_4-1}(e\mu_1)] \tag{5.1}$$

as given by [see (2.17) and (2.18)]

$$1+\tfrac{1}{2}(n\cdot p_i/n\cdot p_f-n\cdot p_f/n\cdot p_i)+(E_f-E_i)(1/n\cdot p_i-1/n\cdot p_f)+(p_i\cdot p_f+2\epsilon\cdot p_i\epsilon\cdot p_f-m^2)/[(n\cdot p_i)(n\cdot p_f)]\;. \tag{5.2}$$

Our second term in (4.7) arises from the term unity in (5.2). The coefficient of (5.1), given in Schlesinger and Wright's treatment of the problem using the Volkov solution of minimally coupled Klein-Gordon equation, is obtained as [see (2.19) of Ref. 12]

$$(\epsilon\cdot\mathbf{p}_i/n\cdot p_i-\epsilon\cdot\mathbf{p}_f/n\cdot p_f)^2\;, \tag{5.3}$$

which is negligible under nonrelativistic approximation. This shows conclusively that the second term of (4.7) is specifically due to the interaction of the electron's spin with the electromagnetic field of the laser beam. Because of this term, we discover a new contribution to the heating rate of a nonrelativistic plasma which is comparably high. Hence, a future project to analyze the physics of the mechanism involved would be important.

ACKNOWLEDGMENTS

We would like to acknowledge the help given by Professor H. Grotch in the project and also the moral support which he extended so graciously. We take this opportunity to thank the Department of Physics, Pennsylvania State University for the use of their facilities.

APPENDIX

The sum in (3.8) is of the type

$$M_l(A,B)=\sum_{l_2=-\infty}^{+\infty}J_{l_2}(A)J_{2l_2-l}(B)\;. \tag{A1}$$

Two cases arise.

Case 1: l is even, i.e.,

$$l=2l'\quad(l'=1,2,\ldots,\infty)\;. \tag{A2}$$

Hence

$$M_{2l'}(A,B)=(2/\pi)\int_0^{\pi/2}d\theta\left[\cos(A\sin2\theta)\sum_{l_2=-\infty}^{+\infty}\cos(2l_2\theta)J_{2(l_2-l')}(B)+\sin(A\sin2\theta)\sum_{l_2=-\infty}^{+\infty}\sin(2l_2\theta)J_{2(l_2-l')}(B)\right]\;, \tag{A3}$$

where 9.1.21 of Ref. 22 has been used. Changing l_2 sums by $l_2-l'=n$ and noting

$$\sum_{n=-\infty}^{+\infty}\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}[2(l'+n)\theta]J_{2n}(B)=\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}(2l'\theta)\cos(B\sin\theta)\;, \tag{A4}$$

we get [using (A2)]

$$M_l(A,B)=(1/\pi)\int_0^{\pi/2}d\theta\{\cos(A\sin2\theta-l\theta-B\sin\theta+\cos(A\sin2\theta-l\theta+B\sin\theta)\}\;. \tag{A5}$$

Case 2: l is odd, i.e.,

$$l=(2l'-1)\quad(l'=1,2,\ldots,\infty)\;. \tag{A6}$$

Again using the transformation $l_2-l'=n$, we get

$$M_{2l'-1}(A,B)=(2/\pi)\int_0^{\pi/2}d\theta\left[\cos(A\sin2\theta)\sum_{n=-\infty}^{+\infty}\cos[2(l'+n)\theta]J_{2n+1}(B)+\sin(A\sin2\theta)\sum_{n=-\infty}^{+\infty}\sin[2(l'+n)\theta]J_{2n+1}(B)\right]\;. \tag{A7}$$

Noting

$$\sum_{n=-\infty}^{+\infty} \begin{Bmatrix} \cos \\ \sin \end{Bmatrix} [2(l'+n)\theta] J_{2n+1}(B) = \begin{Bmatrix} -\sin \\ \cos \end{Bmatrix} [(2l'-1)\theta] \sin(B \sin\theta) \quad \text{(A8)}$$

and using (A6), we obtain

$$M_l(A,B) = (1/\pi) \int_0^{\pi/2} d\theta [\cos(A \sin 2\theta - l\theta - B \sin\theta) - \cos(A \sin 2\theta - l\theta + B \sin\theta)] \ . \quad \text{(A9)}$$

Combining (A5) and (A6), we finally get for general l

$$\sum_{l_2=-\infty}^{+\infty} J_{l_2}(A) J_{2l_2-l}(B) = (1/\pi) \int_0^{\pi/2} d\theta [\cos(A \sin 2\theta - l\theta - B \sin\theta) + (-1)^l \cos(A \sin 2\theta - l\theta + B \sin\theta] \ . \quad \text{(A10)}$$

---

*Present address: Rutgers–State University of New Jersey, R.P.O. 9888, CN 5063, New Brunswick, NJ 08903.

[1]Laser Focus **5**, 14 (1969).

[2]L. S. Brown and T. W. B. Kibble, Phys. Rev. **133**, A705 (1964).

[3]O. von Roos, Phys. Rev. **119**, 1174 (1960).

[4]S. Rand, Phys. Rev. **136**, B231 (1964).

[5]H. Brehme, Phys. Rev. C **3**, 837 (1971).

[6]F. Ehlotsky, Opt. Commun. **13**, 1 (1975).

[7]I. Goldman, Phys. Lett. **8**, 103 (1964).

[8]G. J. Pert, J. Phys. A **5**, 506 (1972).

[9]N. M. Kroll and K. M. Watson, Phys. Rev. A **8**, 804 (1972).

[10]B. K. Osborn, Phys. Rev. A **5**, 1660 (1972).

[11]Y. Shima and H. Yaton, Phys. Rev. A **12**, 2106 (1975).

[12]L. Schlesinger and J. Wright, Phys. Rev. A **20**, 1934 (1979).

[13]L. Schlesinger and J. Wright, Phys. Rev. A **22**, 909 (1980).

[14]M. H. Mittleman, Phys. Rev. A **21**, 79 (1980).

[15]A. L. Nikisov, V. I. Ritus, Zh. Eksp. Teor. Fiz. **46**, 776 (1963) [Sov. Phys.—JETP **19**, 529 (1964)].

[16]J. Bos *et al.*, J. Phys. A **12**, 5 (1979).

[17]A. Weingartshofer *et al.*, Phys. Rev. Lett. **39**, 269 (1977); Phys. Rev. A **19**, 2371 (1979).

[18]Z. Fried, A. Baker, and D. Korff, Phys. Rev. **151**, 1040 (1966).

[19]T. P. Hughes, *Plasma and Laser Light* (Wiley, New York, 1975).

[20]V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii, *Relativistic Quantum Theory* (Pergamon, New York, 1971), Pt. I.

[21]H. Brysk, J. Phys. A **8**, 1260 (1975).

[22]*Handbook of Mathematical Functions,* edited by M. Abramowitz and I. A. Stegun (Dover, New York, 1970).

[23]I. S. Gradsteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products* (Academic, New York, 1980).

[24]G. N. Watson, *Theory of Bessel Functions* (Cambridge University Press, Cambridge, England, 1952).

[25]R. J. Glauber, Phys. Rev. **130**, 2529 (1963).

[26]R. J. Glauber, Phys. Rev. **131**, 2766 (1963).

[27]L. Spitzer, Jr., *Physics of Fully Ionized Gases,* 2nd ed. (Interscience, New York, 1962).

[28]S. Chapman, Mon. Not. R. Astron. Soc. **82**, 294 (1922).

[29]R. S. Cohen, L. Spitzer, and P. McR. Roulty, Phys. Rev. **80**, 230 (1950).

[30]E. Everhart, Phys. Rev. **99**, 1287 (1955).

[31]J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).

[32]J. Kupersztych, Phys. Rev. Lett. **42**, 498 (1979).

[33]H. Grotch, E. Kazes, and D. A. Owen (unpublished).

[34]H. Kruger and C. Jung, Phys. Rev. A **17**, 1706 (1978).

[35]J. J. Sakurai, *Advanced Quantum Mechanic* (Addison-Wesley, Reading, MA, 1967).

[36]Jüttner, Ann. Phys. **36**, 861 (1911); **34**, 856 (1911).

[37]A. B. Langdon, Phys. Rev. Lett. **44**, 575 (1980).

[38]J. Dawson and C. Oberman, Phys. Fluids **5**, 517 (1962).

[39]R. D. Jones and K. Lee, Phys. Fluids **25**, 2307 (1982).

[40]W. Feller, *An Introduction to Probability Theory and Its Applications,* 3rd ed. (Wiley, New York, 1957), Vol. 1.