Effects of multiplicative white noise on laser light fluctuations

M. R. Young and Surendra Singh
Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701

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The steady-state statistical properties of the single-mode laser with multiplicative white-noise loss fluctuations are investigated. Analytic expressions for the intensity distribution and its moments are derived and it is shown that the multiplicative noise significantly modifies the laser characteristics. These predictions are then tested by photoelectric measurements of a He:Ne laser with fluctuating loss. Good agreement between theory and experiment is obtained.

I. INTRODUCTION

Instabilities and phase transitions in nonequilibrium systems have been the subject of great interest in many branches of natural sciences.[1] In the neighborhood of an instability such systems are extremely sensitive to the presence of noise. There are two types of noise present in nonequilibrium systems.[2] The so-called additive noise arises due to the microscopic processes by which the system evolves and is independent of the macroscopic state of the system. Under the influence of additive noise a system executes a random walk in the state space leading to a distribution of the values of the state variables. This type of noise, however, leaves the local stability properties of the system unchanged. In particular, the critical points and the extrema of the probability distribution of the state variables coincide with the solutions of the deterministic system and no new instabilities which are not expected from a deterministic description arise. This type of noise has been studied extensively.

Multiplicative noise arises due to the randomness of the environment to which the system is coupled. This noise enters the system dynamics via its coupling to state variables. Although the importance of multiplicative noise in the context of electronic oscillators has been known for many years,[3,4] it is only recently that its importance for nonequilibrium systems has been realized. The effects of multiplicative noise are far less intuitive than those produced by additive noise. Thus, for example, multiplicative noise can change the local stability properties of the deterministic solutions. As a result the critical points as well as the extrema of the probability distribution of the state variables may differ from those of the deterministic description and new instabilities which are unexpected from the deterministic description may appear.[2-5] Since the multiplicative noise is state dependent it may not be negligible even for large systems. This is in contrast to additive noise which scales inversely as the system size and is usually important only in the neighborhood of an instability.

In quantum optics both types of noise have been studied in lasers where sources of both noise are present. The additive noise in the laser arises due to quantum-mechanical spontaneous-emission fluctuations and the multiplicative noise arises due to the fluctuations of the gain or the loss. Two laser systems that have been studied rather extensively are the single-mode He:Ne laser[6-9] and the single-mode dye laser.[10-13] The single-mode laser threshold instability in the He:Ne laser is perhaps the best studied nonequilibrium phase transition.[14] It has the phenomenology of a second-order phase transition and various experimental studies of this instability seem to indicate that, at least in the region of threshold, only the additive noise is important.[6-9] In the case of the dye laser, on the other hand, the experiments[10-13] suggest that the behavior of the dye laser is dominated by multiplicative noise[15] which arises due to pump fluctuations and the turbulence in the dye jet. It is not clear what the nature of the underlying instability is. Earlier theoretical investigations[16] had suggested a first-order-phase-transition-type instability. This has, however, never been observed. Furthermore, there are indications that the fluctuations of the dye flow are chaotic.[12] It remains to be investigated if the effects of chaotic fluctuations are any different from those of the random fluctuations. It is therefore clear that the fluctuations of the dye laser and the nature of the underlying instability as well as the role of multiplicative noise in lasers can be better understood by studying a system where the fundamental instability is well characterized and where multiplicative noise can be introduced in a controlled fashion. Also, since the additive spontaneous-emission noise is always present in lasers, it would be of interest to see what new features appear when additive and multiplicative noise strengths are similar.

In this paper[17] we report on the investigations of the effects of multiplicative noise on the single-mode laser threshold transition. Only the multiplicative white noise arising from laser loss fluctuations is considered here. In Sec. II we present an outline of the theoretical model which describe the effects of multiplicative noise and summarize the steady-state fluctuation properties predicted by the model. Section III describes the experimental setup and the method used to introduce multiplicative noise. A number of corrections to data and the procedure to determine certain key parameters are described in some detail. Experimental results and the principal conclusions of the paper are presented in Secs. IV and V, respectively.

II. EQUATION OF MOTION

We consider a single-mode electromagnetic field of frequency ω interacting with a set of two level atoms inside a laser cavity. If the laser electric field $\mathcal{E}(\mathbf{r},t)$ at position $\mathbf{r}$ at time t is written as

$$\mathcal{E}(\mathbf{r},t)=u(\mathbf{r})E(t)e^{-i\omega t}+\text{c.c.} \ , \tag{1}$$

where $u(\mathbf{r})$ is the cavity-mode function and $E(t)$ is a slowly varying complex field amplitude, then on resonance when the field frequency ω coincides with the atomic transition frequency, $E(t)$ obeys the following equation of motion:[18]

$$\dot{E}(t)=(A-\mathcal{C}-B\,|\,E\,|^2)E(t)+\xi(t) \ . \tag{2}$$

Here A, $\mathcal{C}$, and B are the gain, loss, and self-saturation coefficients which are real on resonance and $\xi(t)$ is a complex random process representing additive quantum noise due to spontaneous emission. The properties of $\xi(t)$ can be determined by utilizing a fully quantum-mechanical treatment of the laser. However, it has been shown that near threshold $\xi(t)$ may be taken to be a δ-function-correlated (white noise), Gaussian random process of zero mean. Accordingly the noise properties of $\xi(t)$ are determined by

$$\langle \xi(t)\rangle=0=\langle \xi^*(t)\rangle \ , \tag{3a}$$

$$\langle \xi^*(t_1)\xi(t_2)\rangle=4S\delta(t_1-t_2) \ , \tag{3b}$$

where S represents the strength of the random process $\xi(t)$. If the laser gain or the losses are fluctuating then the coefficients A, $\mathcal{C}$, and B have a noise component. Since these coefficients occur multiplied by the field amplitude, the corresponding noise appears as multiplicative noise in the equation of motion. In this paper we consider the laser loss to have a fluctuating component by writing

$$\mathcal{C}=C+\eta(t) \ , \tag{4}$$

where C is the mean loss coefficient and $\eta(t)$ represents loss fluctuations. We take $\eta(t)$ to be a δ-correlated (white-noise) complex Gaussian random process with mean zero and strength D

$$\langle \eta^*(t_1)\eta(t_2)\rangle=4D\delta(t_1-t_2) \ . \tag{5}$$

The noise process $\eta(t)$ is assumed to be statistically independent of $\xi(t)$. The coefficient C is real. The choice of a complex or real $\eta(t)$ depends on the manner in which loss fluctuations are introduced. If loss fluctuations change the phase of the field, then $\eta(t)$ must be taken to be a complex process. Our experiments correspond to a complex noise process. The validity of the white-noise assumption [Eq. (5)] will be examined later.

In terms of new dimensionless variables

$$E'=\left[\frac{B}{S}\right]^{1/4}E \ , \tag{6a}$$

$$t'=(BS)^{1/2}t \ , \tag{6b}$$

$$a=(A-C)/\sqrt{BS} \ , \tag{6c}$$

$$\xi'=S^{-3/4}B^{-1/4}\xi \ , \tag{6d}$$

$$\eta'=\frac{D}{\sqrt{SB}}\eta\equiv Q\eta \ , \tag{6e}$$

we can rewrite Eq. (2) with the help of Eq. (4) as

$$\dot{E}=E(a-|\,E\,|^2)+\eta E+\xi \ , \tag{7}$$

where we have dropped the primes on the new variables with the understanding that in the rest of the paper we will be dealing with the new scaled dimensionless variables. The noise sources in Eq. (7) are statistically independent with

$$\langle \eta\rangle=0=\langle \xi\rangle \ , \tag{8a}$$

$$\langle \xi^*(t_1)\xi(t_2)\rangle=4\delta(t_1-t_2) \ , \tag{8b}$$

$$\langle \eta^*(t_1)\eta(t_2)\rangle=4Q\delta(t_1-t_2) \ . \tag{8c}$$

The parameter a is the so-called pump parameter which is negative below and positive above the conventional threshold of laser oscillations which occurs at $a=0$ corresponding to $A=C$. The term conventional will be used to refer to the situation when no multiplicative noise is present. The pump parameter a is sometimes also expressed in terms of the mean number of photons n_0 at threshold in the conventional single-mode laser as a $[(A-C)/C]\sqrt{\pi n_0}$. The number[9] $n_0=C/\sqrt{BS\pi}$ is of order 10^4.

The stochastic equation of motion (7) for the field amplitude is to be interpreted in the Stratonovich sense.[19] This is because the multiplicative noise process η, which represents loss fluctuations introduced into the system, is a Gaussian process with negligibly small (compared to the correlation time of the laser) correlation time. Using this interpretation Eq. (7) leads to the following Fokker-Planck equation for the probability density $p(E,t)$ of the field to be characterized by the complex amplitude E at time t,

$$\dot{p}(E,t)=\left[-\frac{\partial}{\partial E}E(a-|\,E\,|^2)p(E,t)+2\frac{\partial^2}{\partial E\partial E^*}(1+Q\,|\,E\,|^2)p(E,t)\right]+\text{c.c.} \tag{9}$$

The steady-state solution of this equation depends only on the field intensity $I=|\,E\,|^2$ and has been found independently by a number of authors.[17,20,21] From this solution we find the steady-state intensity probability density $P_s(I)$ to be

$$P_s(I)=\text{const}\times(1+QI)^{\nu-1}\exp\left[-\frac{I}{2Q}\right] , \tag{10}$$

where $\nu=a/2Q+1/2Q^2$. The effects of multiplicative noise on the steady-state fluctuation properties of the laser can now be discussed in terms of Eq. (10) and its moments. We note that in the limit $Q\to 0$ we recover the conventional intensity distribution

$$\mathcal{P}_s(I)=\text{const}\times\exp(\tfrac{1}{2}aI-\tfrac{1}{4}I^2) \tag{11}$$

for the conventional laser.

The steady-state intensity probability density $P_s(I)$ has only a single peak and the peak position shifts to smaller values as Q increases. The position of the peak in the probability determines the most probable light intensity I_m. From Eq. (10) we find

$$I_m = \begin{cases} 0, & a < 2Q \\ a - 2Q, & a > 2Q \end{cases} . \tag{12}$$

This means that laser oscillations cannot build up for $a < 2Q$. In the language of oscillators then we may call $a = 2Q$ the threshold of oscillation. In the conventional laser with additive noise only, the threshold of oscillation is reached at $a = 0$. Thus we see that the effect of multiplicative noise is to shift the threshold of oscillation from $a = 0$ to $a = 2Q$. This threshold shift due to multiplicative noise has also been noted in several other contexts. Figure 1 illustrates this shift of threshold of oscillation for $Q = 1$. It is seen that the most probable value of the light intensity does not begin to move away from zero to nonzero values until $a = 2$. By treating I_m as an order parameter it is easily shown that the single-mode laser with multiplicative white noise undergoes a phase transition with the phenomenology of a second-order phase transition. This shift of laser threshold was observed experimentally in Ref. 17.

Analytic expressions for the moments of light intensity can be derived from Eq. (10). For the mean $\langle I \rangle$, the normalized variance κ_2, and the normalized skewness κ_3, we obtain

$$\langle I \rangle = 2Q \left[\frac{a}{2Q} + \frac{e^{(-1/2)Q^2}}{(2Q^2)^\nu \Gamma\left[\nu, \frac{1}{2Q^2}\right]} \right] , \tag{13}$$

$$\kappa_2 = \frac{\langle (I - \langle I \rangle)^2 \rangle}{\langle I \rangle^2} = \frac{a+2Q}{\langle I \rangle} + \frac{2}{\langle I \rangle^2} - 1 , \tag{14}$$

$$\begin{aligned} \kappa_3 &= \frac{\langle (I - \langle I \rangle)^3 \rangle}{\langle I \rangle^3} \\ &= \frac{1}{Q^2 \langle I \rangle^3}(2a^2Q + 2a + 4aQ^2 + 8Q^3) \\ &\quad + \frac{1}{Q^2 \langle I \rangle^2}(a^2Q^2 + 6aQ^3 - 2aQ + 2Q^2 + 8Q^4 - 2) \\ &\quad - 3\kappa_2 - 1 , \end{aligned} \tag{15}$$

where the incomplete gamma function $\Gamma(\nu, z)$ is given by

$$\Gamma(\nu, z) = \int_z^\infty dx\, e^{-x} x^{\nu - 1} . \tag{16}$$

The behavior of the mean light intensity as a function of the pump parameter is illustrated in Fig. 2 for several different values of the multiplicative strength. It is seen that the multiplicative noise tends to increase the mean light intensity for a given value of the pump parameter. This increase, despite the fact that the peak of the distribution shifts to lower intensity values, is possible because the distribution develops a long tail with increasing noise strength Q. This long tail is also responsible for the enchanced intensity fluctuations seen in Fig. (3). For the conventional laser ($Q = 0$) the relative intensity fluctuations decrease monotonically from a value of 1 corresponding to a thermal state for large negative values of the pump parameter to a value of zero corresponding to a coherent state for large positive values of the pump parameter. For nonzero values of Q the relative intensity fluctuations may reach a maximum in excess of unity before decreasing as the pump parameter a is varied from negative values to positive values. The maximum occurs approximately at $a = -(2Q + 1/2Q)$.

In the conventional laser it is a good approximation to neglect the saturation term in Eq. (2) for large negative values of a. For the laser with multiplicative noise this approximation leads to the steady-state intensity distribution,

$$P_L(I) = \text{const} \times (1 + QI)^{-|a|/2Q - 1} , \tag{17}$$

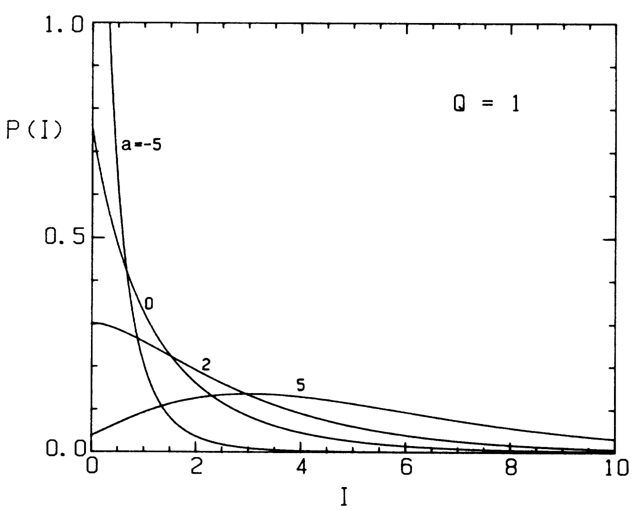

FIG. 1. Forms of the steady-state intensity probability density $P_s(I)$ for a fixed value Q of the multiplicative noise strength and several different values of the pump parameter a.

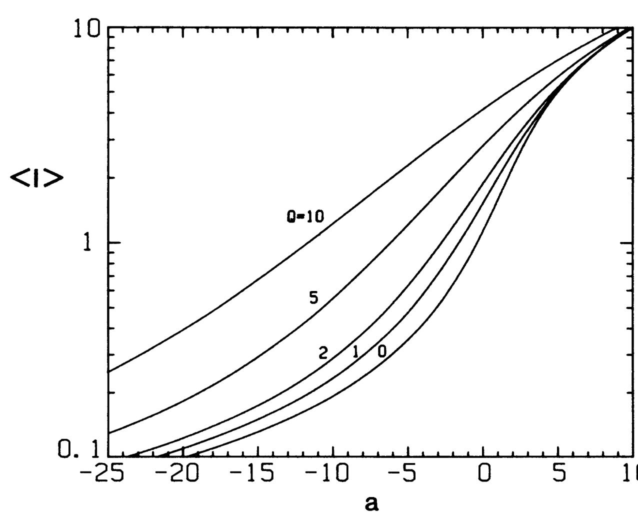

FIG. 2. Variation of the mean light intensity $\langle I \rangle$ with pump parameter a for several different values of Q.

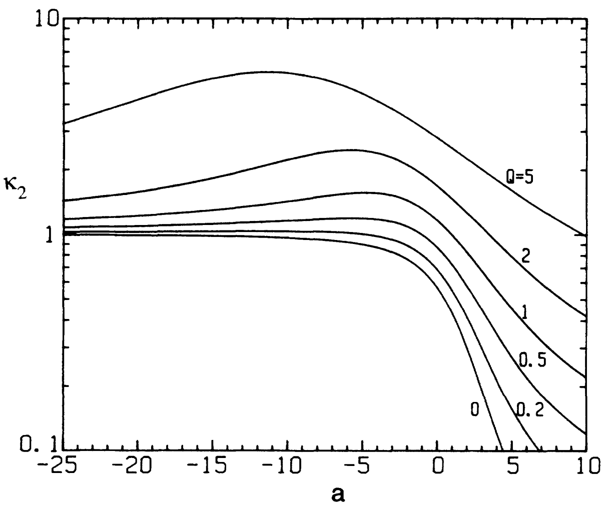

FIG. 3. Variation of the normalized variance κ_2 of the light intensity with a for several different values of Q.

not all of whose moments exist. Our experimental results are in disagreement with the predictions of Eq. (17) indicating that for the laser with multiplicative noise the nonlinear term is important even for large negative values of a.

Multiplicative noise also modifies the near-threshold and above-threshold behavior of the laser. At the threshold of oscillation the value of κ_2 and $\langle I \rangle$ depends on Q as opposed to the conventional laser where they have values $\kappa_2=0.57$ and $\langle I \rangle = 1.13$ independent of the strength of the additive noise. In the present case we have for large values of Q,

$$\langle I \rangle_{\text{th}} = 2Q + \frac{1}{2Q} , \tag{18}$$

$$\kappa_{2,\text{th}} = 1 - \frac{1}{2Q^2} . \tag{19}$$

As the pump parameter a is increased beyond $a = 2Q$ the relative intensity fluctuations die out and we obtain the following expressions for the mean $\langle I \rangle$ and the relative variance κ_2,

$$\langle I \rangle \to a - 2Q , \tag{20}$$

$$\kappa_2 \to \frac{2}{\langle I \rangle^2} + \frac{2Q}{\langle I \rangle} . \tag{21}$$

The intensity distribution approaches a Gaussian with mean and variance given by Eqs. (20) and (21). Its explicit form is

$$P_s(I) \to \text{const} \times \exp \left[- \frac{(I - \langle I \rangle)^2}{4(1 + Q\langle I \rangle)} \right] . \tag{22}$$

Thus the laser eventually reaches a coherent state albeit at a slower rate compared to the conventional laser where $\kappa_2 \to 2/\langle I \rangle^2$. These new features which result due to the presence of the multiplicative noise were tested in the photon-counting measurements of a He:Ne laser where a controlled amount of multiplicative noise was introduced.

III. EXPERIMENTAL PROCEDURE

The experimental setup used in these experiments[17] consisted of a 20-cm-long standing-wave He:Ne laser operating at $\lambda = 633$ nm near threshold. Large longitudinal-mode spacing (750 MHz) compared with 1500 MHz atomic linewidth and an intracavity aperture to discriminate against off-axis modes ensured a single-longitudinal- and single-transverse-mode operation at all power levels available from the laser. The laser was operated with the field frequency tuned to the center of atomic line with the help of a mirror mounted on a piezoelectric transducer. A photomultiplier tube monitoring a part of the output light intensity, an electronic amplifier, and a movable knife edge formed the part of a feedback loop which could control the operating point of the laser at any desired level. The whole setup was enclosed in a temperature-controlled housing which was placed on top of a heavy vibration-isolated platform. Once thermal equilibrium had been reached the laser was naturally quite stable. Its frequency drifted by no more than 5 MHz over a period of 10–15 minutes—and the electronic feedback loop could hold the laser light intensity constant at any desired level to better than 1% for the same duration in the entire threshold regime. The feedback unit only responded to slow changes occurring on the time scale of several seconds.

In order to introduce multiplicative noise an acousto-optic modulator (AOM) was inserted into the laser cavity. The input to the driver (not shown) for the modulator was derived from a noise voltage source. The noise voltage had a Gaussian amplitude distribution with flat spectrum in the 10 Hz to 10 MHz frequency range. The maximum root-mean-square (rms) voltage was 20 mV with a peak to rms ratio of 5. The output could be attenuated from 0–40 dB in steps of 1 dB. The transfer characteristics of the driver-modulator combinations were determined by passing a narrow laser beam (0.6 mm diam) through the modulator outside the cavity and analyzing the spectrum of the photocurrent produced by the first-order beam when the input to the driver was the noise voltage. The photocurrent spectrum again was found to be flat from 10 Hz to 10 MHz within 2 dB. Since the intensity of the first-order beam is proportional to the loss suffered by the zeroth-order beam, the photocurrent spectrum reflected the spectrum of the loss fluctuations. The relation between the rms value of the modulator loss and the noise voltage was found to be linear except at the highest voltages in the neighborhood of $V_{\text{rms}} = 20$ mV. This means that when the modulator is placed inside the laser cavity and driven by the noise voltage, the loss fluctuations introduced by the AOM may be modeled by a Gaussian random process with a flat spectrum up to frequencies of 10 MHZ. Also, since the 10^{-7} sec correlation time of the loss fluctuations corresponding to the 10 MHz bandwidth is negligible compared to the correlation time of the laser, we may also treat loss fluctuations as a white-noise process. The laser is then modeled by Eq. (7). The strength Q of the multiplicative noise can be varied by changing the rms value of the input noise voltage.

To perform the experiment, noise voltage after proper attenuation is applied to the AOM and the laser is stabilized by the feedback loop at some operating level characterized by its mean light intensity. The main beam of light coming out of the laser cavity after passing through an interference filter and a set of calibrated filters, if needed, falls on a fast-counting photomultiplier tube (PMT). The photoelectric pulses appearing at the output of the PMT are amplified and passed through a nonupdating discriminator which produces standard negative-logic nuclear instrumentation module (NIM) pulses of 20 ns duration. These pulses are fed to an electronic gate which is opened for a counting time $T=3$ μs. The pulses appearing from the gate are counted by a scaler. The number of pulses n registered by the scaler during the counting interval T is transferred to a computer memory in such a way that the contents of the memory location n are incremented by 1. After the transfer the scaler is cleared. This process is repeated 10^5 times and a histogram is built up. Successive counting cycles are separated by at least 250 μs in order to ensure independent counting samples. At the end of the measurement the number N_n stored at the memory location n becomes a measure of $p(n)\equiv p(n,T)$, the probability of detecting n photoelectric pulses during the counting interval T. Since the counting interval $T=3$ μs is short compared with the correlation time of the laser (40–200 μs) we can write the relation between $p(n)$ and the steady-state intensity probability density in the form[22]

$$p(n)=\int_0^\infty \frac{(\alpha IT)^n}{n!}e^{-\alpha IT}P_s(I)dI \ , \tag{23}$$

when the electronic deadtime can be ignored. In Eq. (23) α is the quantum efficiency of detection. Using this relation we can extract the moments of the light intensity I from the measured moments of n. In particular, the mean, the normalized variance κ_2, and the normalized skewness κ_3 are given by

$$\langle I\rangle=\alpha T\langle n\rangle \ , \tag{24}$$

$$\kappa_2=\langle n(n-1)\rangle/\langle n\rangle^2-1 \ , \tag{25}$$

$$\kappa_3=\langle n(n-1)(n-2)\rangle/\langle n\rangle^3-3\langle n(n-1)\rangle/\langle n\rangle^2+2 \ . \tag{26}$$

In practice, several corrections had to be applied to the measured moments of n before information about the moments of the light intensity could be extracted. The measured moments were corrected for the finite deadtime of the counting electronics. The counting deadtime was determined by the discriminator which had the longest deadtime (30 ns). These corrections have been discussed by many authors.[8,23] If δ denotes the ratio of the deadtime to the counting time T, then Eq. (23) is modified to read

$$p(n)=\int_0^\infty \left[\frac{\gamma(n-1,\beta_{n-1})}{(n-1)!}-\frac{\gamma(n,\beta_n)}{n!}\right]P_s(I)dI \ , \tag{27}$$

where the incomplete γ function $\gamma(n,\beta_n)$ is defined by

$$\gamma(n,\beta_n)=\int_0^{\beta_n} x^n e^{-x}dx \ , \tag{28}$$

with $\beta_n=\alpha TI(1-n\delta)$. Corrections to Eqs. (24)–(26) due to this modification are usually expressed as a power series in δ. The convergence of this power series is worst for light fields with large intensity fluctuations. The convergence of the series for the kth moment requires $C_k\langle n\rangle\delta\ll 1$, where the constant C_k depends on the statistics of the light field. For a thermal light field $C_k=k$. For the experiments reported here C_k can be several times greater than k depending on the strength of the multiplicative noise. This condition puts severe limitations on the counting rate usable in the experiment. With the deadtime ratio $\delta=0.01$ (deadtime 30 ns and the counting time $T=3$ μs) the mean $\langle n\rangle$ was kept around 0.6 by using a few calibrated neutral density filters. Even under these conditions the inclusion of terms up to third order given in Ref. 8 was not sufficient. Fourth-order terms were calculated[21] and included in all corrections applied in order to obtain a satisfactory convergence.

The data were also corrected for the effects of background light mainly from the laser gas discharge.[7,8] An interference filter, a polarizer, and two small apertures were used in front of the counting PMT to reduce the background light from the laser. The background count rate was never more than 8–10% at the lowest working point and usually much less at higher working points. The background light is statistically independent of the laser light and has a bandwidth large compared with the inverse of the counting time so that its photocount statistics may be taken to be Poissonian. If β denotes the ratio of the background count rate to the total count rate, the mean, the normalized variance, and the normalized skewness for the laser light alone are obtained by multiplying the corresponding quantities for the total light field by $(1-\beta)$, $(1-\beta)^{-2}$, and $(1-\beta)^{-3}$, respectively. Corrections due to the finite counting time T were not considered in these measurements.[8]

Measurements were made for several different operating points of the laser for a fixed value of the multiplicative noise strength determined by the rms value of the noise voltage. The measurements were repeated after changing the noise voltage driving the AOM. In all measurements the laser was operated on resonance.

IV. EXPERIMENTAL RESULTS

The corrected data are compared with the theoretical predictions of Sec. III in Figs. 4 and 5. The experimental values of κ_2 versus $\langle I\rangle$ are shown in Fig. 4, which is a reproduction of Fig. 2 of Ref. 17 with some new data in it. The scale constant αT [Eq. (24)] was chosen to give best fit with the $Q=0$ (conventional laser) curve. This was done by plotting κ_2 against $\log_{10}\langle I\rangle$ for the $Q=0$ case. Since κ_2 (and κ_3) is independent of αT, a variation of αT corresponds to a translation along the $\log_{10}\langle I\rangle$ axis. The value of αT determined in this way was not adjusted any further. The parameter Q was estimated from a curve of κ_2 versus Q for a fixed value of $\langle I\rangle$. The values of Q determined in this way were in agreement with the AOM loss versus noise voltage measurements. Once αT and Q are known, the pump parameter a is determined from a knowledge of $\langle I\rangle$. We found it con-

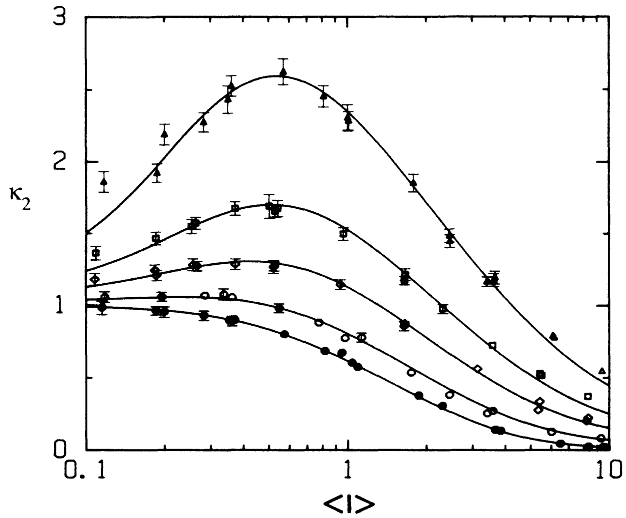

FIG. 4. The measured values of the normalized variance κ_2 as a function of the mean light intensity $\langle I \rangle$ for (bottom to top) $Q=0$, 0.25, 0.66, 1.15, 2.13. Solid curves are theoretical predictions based on Eqs. (13) and (14).

venient to plot κ_2 and κ_3 against $\langle I \rangle$ for in the experiment it was the mean light intensity that characterized the operating point of the laser. Superimposed upon the experimental points are the theoretical curves derived from Eqs. (14) and (15). It will be seen that there is good agreement between theory and experiment. The normalized variance κ_2 for the conventional laser ($Q=0$) is subthermal remaining always below unity. As the operating point of the laser is raised, κ_2 monotonically decreases to zero. For the laser with multiplicative noise ($Q>0$) the relative variance exhibits a peak before decreasing to zero with increasing light intensity. In the vicinity of these peaks the laser can exhibit superthermal fluctuations far in excess of unity. A similar peak in κ_2 was observed by

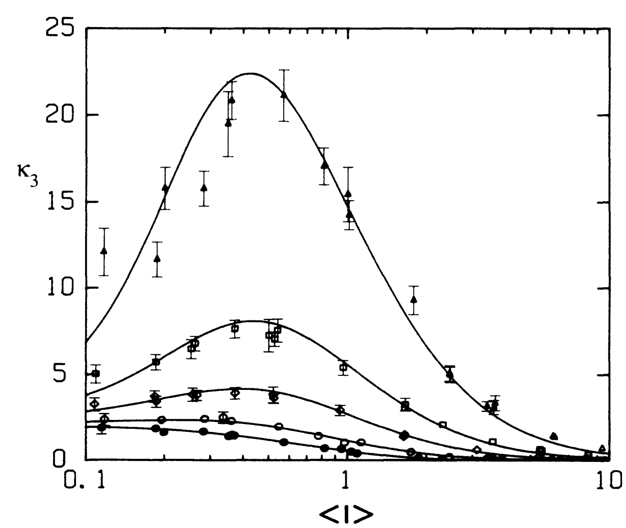

FIG. 5. The measured values of the normalized skewness κ_3 as a function of the mean light intensity $\langle I \rangle$ for (bottom to top) $Q=0.0$, 0.25, 0.66, 1.15, 2.13. Solid curves are theoretical predictions based on Eqs. (13) and (15).

Lett, Short, and Mandel[10] in the dye laser with large colored multiplicative noise.

Enhanced intensity fluctuations are also evident in Fig. 5, where normalized skewness κ_3 is plotted against the mean light intensity. The theoretical predictions as derived from Eq. (15) are indicated by the continuous curves. Once again there is good agreement between theory and experiment. It will be seen that κ_3 for nonzero values of Q exhibits a pronounced peak before decreasing to zero with increasing light intensity. For a thermal light field $\kappa_3=2$, but for the laser with multiplicative noise κ_3 can be much greater than this value. Higher-order moments exhibit similar superthermal intensity fluctuations. The magnitude of the departures from the thermal statistics increases as the strength of the multiplicative noise is increased. Although these peaks occur for negative values of the pump parameter, they can be accounted for only if the nonlinear term in Eq. (2) is included. They also require the inclusion of the additive term no matter how strong the multiplicative noise is.

The excess fluctuations are a reflection of the fact that the multiplicative noise produces highly skewed intensity probability densities. This is clearly reflected in the measured histograms for the probability $p(n)$ of detecting n photoelectric pulses in a counting interval $T=3$ which were presented in Ref. 17. For all the histograms $\langle I \rangle$ was kept constant, but the noise parameter Q was varied. Although $p(n)$ reflects $P_s(I)$ only indirectly, it was evident that increasing multiplicative noise strength gives rise to intensity distributions with long tails and large intensity fluctuations. These measurements also illustrated the shift of the peak of the distribution to smaller values of I with increasing Q (cf. Fig. 1). It is also clear that the threshold of laser oscillation as characterized by the appearance of a peak in $P_s(I)$ at a nonzero value of I shifts to higher values of a as Q is increased. Equation (27) is not easily inverted to yield $P_s(I)$ from the measured $p(n)$.[24] We therefore chose to compare the measured forms of $p(n)$ with those derived from Eqs. (10) and (27). The values of the parameters δ and αT for the computations of $p(n)$ were chosen to be 0.01 and 1.71, respectively.

V. CONCLUSIONS

The overall good agreement between theory and experiment confirms the validity of the model used to describe the effects of multiplicative white noise on laser light fluctuations. In the experiments reported here, multiplicative noise strengths less than or comparable to that of the additive spontaneous emission noise were used in order to study the changes that laser light fluctuations undergo when external noise is introduced into the laser. At higher multiplicative noise strengths, no qualitative changes in the fluctuation properties of the laser are expected to occur. Our measurements demonstrate that the nonlinearity of light-matter interaction is important even for negative values of the laser pump parameter when multiplicative noise is present.

Some of the features observed here have also been seen in the dye laser.[10] It is, however, not possible without also studying the effects of multiplicative colored noise in a He:Ne laser to make a definite statement about the nature of the instability in the dye laser. Further experiments with a controlled amount of colored noise are needed. Finally, since gain variations due to plasma current and power supply fluctuations also contribute to multiplicative noise, one may ask how much is their contribution. If we assume that these fluctuations can also be represented by a Gaussian white-noise process, then their strength must produce Q values which are indistinguishable from $Q=0$ within the experimental uncertainties. From Fig. 4 we estimate that plasma tube current and power supply fluctuations produce $Q<0.05$ in the laser used in these experiments. The assumption of a white-noise process for these gain fluctuations is questionable. More realistic estimates based on a colored noise model would produce a smaller upper limit on Q values. These estimates are probably too optimistic for most commercial ion or even He:Ne lasers under normal operating conditions.

Lasers are but one example of nonequilibrium systems which by their very nature are susceptible to external fluctuations in their environment. We have demonstrated that these external fluctuations can alter the behavior of nonequilibrium system drastically and that a correct description of such a system must take into account the internal noise, the external noise, and the nonlinearity of its response.

ACKNOWLEDGMENTS

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