Modulational polarization instability of light in a nonlinear birefringent dispersive medium

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The interplay between linear birefringence, nonlinear polarization changes, and chromatic dispersion may lead to phase-matched parametric four-photon mixing. As a result, new modulational polarization instabilities are predicted in single-mode fibers: Instability of an intense light beam may occur even when the wave is coupled to a spatially stable eigenmode, and propagates in the normal-dispersion regime.

A uniform wave train propagating in a nonlinear dispersive medium may be unstable with respect to weak modulations. Such a modulational instability (MI) is found in the nonlinear Schrödinger (NLS) equation, describing, for example, a Bose-gas condensate,[1] self-focusing[2] and self-phase modulation[3] of light, or deep-water gravity waves.[4]

The development of MI-generated[5] or-induced[6] sidebands from an initially quasimonochromatic light field has been recently observed in optical fibers. Propagation in a fiber obeys the one-dimensional scalar NLS equation when one neglects both diffraction and changes in the state of polarization. As a consequence of this description, MI is predicted only in the regime of anomalous dispersion.[3] It has been known, however, since the early studies by Bespalov and Talanov[2] and Berkhoer and Zakharov,[7] that an incoherent (i.e., purely intensity-dependent) coupling between two NLS equations leads to extension of the instability domain. Incoherent interaction and extended MI occur in a variety of different physical contexts: for example, in the propagation of two transverse electromagnetic waves in a nonlinear dielectric[2,7,8] or a plasma,[9] or of Langmuir and transverse or sonic waves.[10] In particular, MI is possible also in the normal dispersion regime, when two light fields with different polarization[7] or frequency[8] interact in an isotropic medium.

So far a different situation that is of considerable interest in nonlinear optics has not been discussed, namely the coherent coupling between two polarizations in a weakly anisotropic and weakly nonlinear dispersive dielectric; take, for example, an optical fiber. In a fiber, a coherent (i.e., sensitive to the input relative phase between the waves) coupling occurs between the two circularly polarized components of the field, as induced by the anisotropy of the linear dielectric tensor. Recent studies have revealed the intriguing dynamical behavior (involving bifurcations, instabilities, and chaos) of the steady-state spatial evolution of the polarization in nonlinear anisotropic dielectrics.[11] The aim of this paper is to show how chromatic dispersion affects the spatiotemporal stability of both spatially stable and unstable solutions, thus establishing an interesting link between modulational and spatial polarization instabilities. The present results are of applicative relevance when propagating short optical pulses in fibers,[6,12] and are indicative of a basic limit set by dispersion (and not inertia of the nonlinearity) to the ultimate speed of operation of all-optical fiber switches and couplers.[13] A light pulse propagating in a glass fiber obeys the coherently coupled NLS equations

$$\begin{aligned}-i(\partial A_+/\partial z+v_g^{-1}\partial A_+/\partial t)+\tfrac{1}{2}\alpha\partial^2 A_+/\partial t^2 &= \beta A_+ +\kappa A_- +\tfrac{2}{3}R(|A_+|^2+2|A_-|^2)A_+ ,\\ -i(\partial A_-/\partial z+v_g^{-1}\partial A_-/\partial t)+\tfrac{1}{2}\alpha\partial^2 A_-/\partial t^2 &= \beta A_- +\kappa A_+ +\tfrac{2}{3}R(|A_-|^2+2|A_+|^2)A_- ,\end{aligned} \tag{1}$$

where $A_\pm$ are the complex amplitudes of two counterrotating circularly polarized modes, obtainable from the linearly polarized nearly degenerate eigenmode amplitudes as $A_\pm=(1/\sqrt{2})(A_x \pm iA_y)$. In Eq. (1) we posed $\beta\equiv(\beta_x+\beta_y)/2$ and $\kappa\equiv(\beta_x-\beta_y)/2$, where β_x and β_y are the linear modes propagation constants ($\beta_x>\beta_y$). The group-velocity v_g and its dispersion $\alpha=dv_g^{-1}/d\omega$ can be taken identical for both modes,[14] which ensures mode overlap at least over the distances of interest. The nonlinearity (assuming isotropic electronic distortion mechanism) coefficient R $(\mathrm{W}^{-1}\mathrm{m}^{-1})=n_2k_0/A_{\text{eff}}$, where n_2 specifies the intensity-dependent index ($n=n_0+n_2I$), k_0 is the vacuum wave number, and A_{eff} is the common effective mode area.

For a generic input polarization state, the steady-state solution of Eqs. (1) is expressible in terms of Jacobian elliptic functions.[11] Investigation of the modulational stability of spatially periodic steady solutions would require a relatively involved numerical computation of Floquet exponents. However, we show below that a physically transparent insight in the MI's of Eqs. (1) can be obtained in a simple manner from the stability analysis of the spatial eigenmodes by means of standard techniques. We consider first a linearly polarized input field, aligned with the fast axis of the fiber. The continuous wave solution of Eqs. (1) is

$$A_x=0, \quad A_y=P^{1/2}\exp[i(RP+\beta_y)z] , \tag{2}$$

where P is the input power. We write a weak modulation in terms of circular modes as

$$A_\pm=\pm[i(P/2)^{1/2}+a_\pm]\exp[i(RP+\beta_y)z] , \tag{3}$$

where the sidebands take the form

$$a_\pm=c_{1\pm}\exp(i\eta)+c_{2\pm}\exp(-i\eta)$$

with real $c_{j\pm}$ $(j=1,2)$ and $\eta\equiv Kz-\Omega t$. By inserting the expression (3) in Eqs. (1) and retaining only linear terms, one easily obtains for the perturbation a_+

$$-i(\partial a_+/\partial z+v_g^{-1}\partial a_+/\partial t)+\tfrac{1}{2}\alpha\partial^2 a_+/\partial t^2 =\kappa(a_+-a_-)+\tfrac{1}{3}RP[(a_+-a_+^*)+2(a_--a_-^*)] . \tag{4}$$

The linearized equation for a_- is obtainable from Eq. (4) upon interchanging $+$ with $-$. Note that the presence of κ introduces a coupling proportional to both the in-phase and out-of-phase quadratures of the sideband modes $a_\pm$. The corresponding set of real equations for the quantities $u_\pm=c_{1\pm}+c_{2\pm}$ and $v_\pm=c_{1\pm}-c_{2\pm}$ leads to the dispersion relation

$$[(K-\Omega/v_g)^2-s_1][(K-\Omega/v_g)^2-s_2]=0 , \tag{5}$$

where

$$s_1=\alpha\Omega^2/2(\alpha\Omega^2/2+2RP)$$

and

$$s_2=(\alpha\Omega^2/2+2\kappa-\tfrac{2}{3}RP)(\alpha\Omega^2/2+2\kappa) .$$

Modulational instability occurs at those modulation frequencies Ω such that $K(\Omega)$ takes on complex values, indicating that a perturbation would grow exponentially with the fiber length. From Eq. (5) we obtain that in the normal dispersion regime $(\alpha>0)$ instability occurs when $s_2<0$, i.e., for

$$\Omega<\Omega_{c1}\equiv[\gamma(p-1)]^{1/2} \quad \text{with } p\geq 1 , \tag{6}$$

where $\gamma\equiv4\kappa/|\alpha|$, and $p\equiv P/P_c$ is the ratio of the input power to the critical bifurcation power[11] $P_c=3\kappa/R$. The exponential growth rate of the modulating field power is given by the gain

$$g_1(\Omega)\equiv2|\mathrm{Im}[K(\Omega)]|=|\alpha|[(\gamma+\Omega^2)(\Omega_{c1}^2-\Omega^2)]^{1/2} . \tag{7}$$

This gain is shown in Fig. 1 for increasing values of p, considering a carrier wavelength of $\lambda=0.53$ μm, and a nonpolarization-preserving fiber characterized by a linear beat length $L_b=\pi/\kappa=2$ m and a dispersion $\alpha=60$ psec^2/km. In this case, taking the fused silica value $n_2=3.2\times10^{-16}$ cm^2/W and $A_{\rm eff}=1\times10^{-7}$ cm^2, the condition $p=1$ would correspond to an input power $P=124$ W. Maximum gain is attained for vanishing modulation frequency $\Omega=\Omega_{m1}=0$ whenever $1\leq p\leq2$, otherwise Ω_{m1} satisfies the condition

$$\alpha\Omega_{m1}^2=\tfrac{2}{3}RP-4\kappa . \tag{8}$$

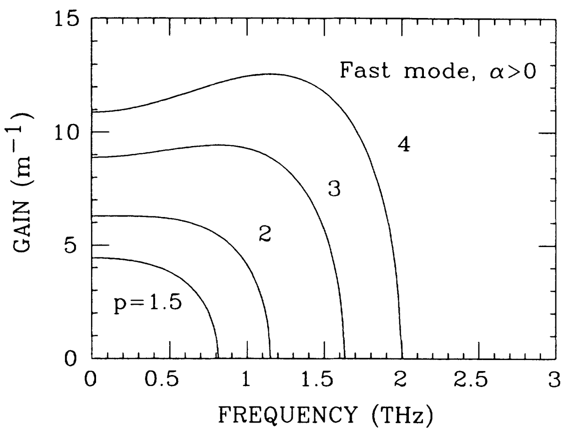

FIG. 1. Gain of the modulational polarization instability for a wave at $\lambda=0.53$ μm, linearly polarized along the fast axis of a birefringent fiber with $L_b=2$ m and $\alpha=60$ psec^2/km, for increasing values of normalized power p.

The associated peak gain is $g_{m1}=4\kappa(p-1)^{1/2}$ for $1\leq p\leq2$, while $g_{m1}=\frac{2}{3}RP$ for $p>2$. Equation (8) turns out to be the phase-matching condition between pump and orthogonally polarized (i.e., in the slow mode) sidebands,[15] corrected for the presence of pump power-dependent contributions to the wave vectors (optical Kerr effect). Unlike MI occurring with incoherently coupled NLS equations,[8] nonzero and even peak gain is possible for vanishing modulation frequency. In fact, for slow modulations the present modulational polarization instability (MPI) reduces to the continuous-wave polarization instability.[11] For sufficiently long pulses, the spatial instability effect has potential for power or phase-controlled switching of light beams in glass-fiber couplers.[13] With this in mind, the results reported in Fig. 1 indicate that when rapidly varying waveforms are involved, the decay of parametric gain imposes a finite bandwidth to such all-optical switches. In the anomalous dispersion regime $(\alpha<0)$, both s_1 and s_2 can be negative. Therefore, two distinct complex roots are possible for K. The condition $s_1<0$ implies the following constraints for the modulation frequency:

$$(\gamma)^{1/2}=\Omega_{c2}>\Omega>\Omega_{c3}=[\gamma(1-p)]^{1/2} \quad \text{if } p<1, \text{ otherwise } \Omega_{c2}>\Omega>0 . \tag{9}$$

The gain is

$$g_2(\Omega)=|\alpha|[(\Omega^2-\Omega_{c2}^2)(\Omega_{c3}^2-\Omega^2)]^{1/2} . \tag{10}$$

As can be seen from Fig. 2, now the condition of maximum growth is obtained for $\Omega=\Omega_{m2}=0$ when $p\geq2$; for $0<p<2$, Ω_{m2} again satisfies Eq. (8). The gain values reported in Fig. 2 refer to the case $\lambda=1.55$ μm, $L_b=2$ m, and $\alpha=-17.8$ psec^2/km. Furthermore, imposing $s_1<0$ yields

$$\Omega<\Omega_{c4}=(3\gamma p)^{1/2}=(4RP/|\alpha|)^{1/2} . \tag{11}$$

This is the conventional (scalar) MI condition,[1-6] whose gain is

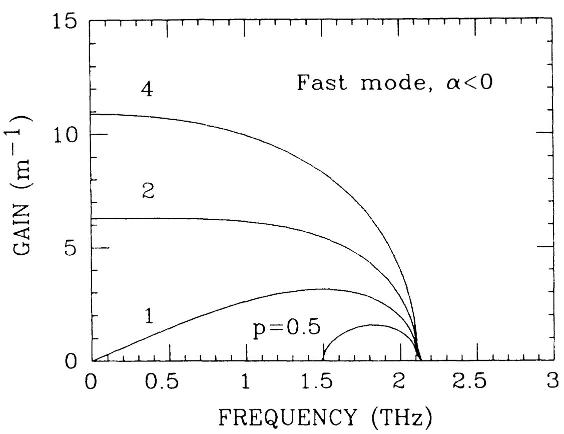

FIG. 2. Same as in Fig. 1, with $\lambda=1.55\ \mu$m, and $\alpha=-17.8$ psec^2/km.

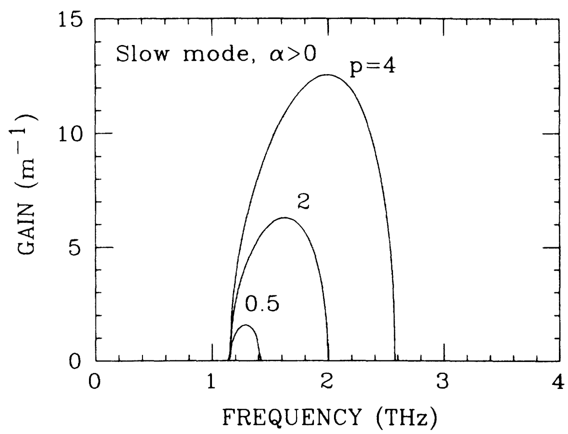

FIG. 3. Same as in Fig. 1, but when the input polarization is along the slow axis.

$$g_3(\Omega)=|\alpha|\,\Omega(\Omega_{c4}^2-\Omega^2)^{1/2} . \tag{12}$$

Maximum growth rate $g_{m3}=2RP$ is attained at $\Omega=\Omega_{m3}=\Omega_{c4}/\sqrt{2}$. This frequency shift yields peak four-photon parametric gain for the sidebands, with identical linear polarization along the fast pump mode.

Proceeding along similar lines, we study the MI of the solution of Eqs. (1) (slow mode)

$$A_x=P^{1/2}\exp[i(RP+\beta_x)z], \quad A_y=0 , \tag{13}$$

known to be spatially stable in the steady state.[11] From linearized equations similar to Eq. (4), one obtains a dispersion relation of the form of Eq. (5), where s_1 is unchanged and κ is replaced by $-\kappa$ in the expression for s_2.

For carrier wavelengths longer than the zero-dispersion value of $\lambda\cong1.3\ \mu$m, condition $s_2\geq0$ is always satisfied; conversely one has $s_1<0$ when $\Omega<\Omega_{c4}$ [see Eq. (11)]. Furthermore, the gain is the same as in Eq. (12). Therefore only scalar MI occurs, i.e., the growing modulation has the same linear polarization of the pump.

A novel behavior is found for $\alpha>0$. In this case, $s_1>0$ always, whereas $s_2<0$ holds if

$$(\gamma)^{1/2}=\Omega_{c2}<\Omega<\Omega_{c5}=[\gamma(1+p)]^{1/2} . \tag{14}$$

The associated gain (see also Fig. 3) is

$$g_4(\Omega)=|\alpha|\,[(\Omega_{c2}^2-\Omega^2)(\Omega^2-\Omega_{c5}^2)]^{1/2} , \tag{15}$$

whose peak value occurs for Ω satisfying Eq. (8), with κ replaced by $-\kappa$.

We have shown that MI may occur, once that the polarization changes along the fiber are accounted for, even in the case of an input field linearly polarized and oriented along the slow fiber axis and in the regime of normal dispersion. This is an unexpected result, since this situation is stable with respect to polarization perturbations when chromatic dispersion is neglected, and is also stable with respect to the growth of modulations when the state of polarization is not included in the description. Moreover, it is remarkable that, unlike the fast mode (requiring $p>1$), when $\alpha>0$ MPI occurs for the slow mode as soon as $p>0$. A comparison between Figs. 1 and 3 clearly shows that the interplay among birefringence, nonlinearity, and dispersion substantially affects the instabilities that one would predict when considering these ingredients separately. In fact, dispersion has a stabilizing effect over rapid modulations of the spatially unstable fast mode, and conversely destabilizes the slow mode as the sideband offset falls within a certain frequency range. On the other hand, a basic difference with respect to incoherent MI (Refs. 7–10) is that the linear anisotropy is dramatically enhanced at high powers (thus forcing a nonlinear nonreciprocity into the medium), so that the present MPI is strongly sensitive to the initial relative phase between the waves (or input polarization state). Note that the cross-intensity coupling terms appearing in Eq. (1) are not essential for the existence of coherent MI: for example, in a directional coupler[13] the same instability is present while the cross-intensity coupling is zero.

Finally, we consider a limiting situation of physical relevance. When a fiber is so highly birefringent ($\kappa\gg1$ m^{-1}), or the input power P is so low, that the condition $p\ll1$ is fulfilled,[16] then from Eqs. (6) and (14) one finds that with normal dispersion both eigenmodes are modulationally stable. Conversely, if $\alpha<0$, just scalar MI occurs when Eq. (11) is satisfied. As can be easily shown, no modulational instability may occur for $\alpha>0$, even for an arbitrary input polarization state.

The relatively large gain values (see Fig. 3) should allow for the experimental observation of MPI using a few meters long fiber, before the onset of Raman scattering. For example, using 100-ps pulses (in order to suppress Brillouin scattering) from a Q-switched mode-locked pump source emitting at $\lambda=0.53\ \mu$m, coupled into the slow mode of a fiber with $L_b=2$ m, polarization noise-generated sidebands with a frequency shift of 2 THz are expected in the orthogonal polarization as the peak power P is increased to about 200 W.

In conclusion, when one correctly includes the changes in the state of polarization in the description of wave propagation in a nonlinear dispersive and anisotropic medium, entirely new modulational instabilities may occur. As a possible important practical application of these effects, consider that although MI does not necessarily imply the existence of stable solitary waves, the instability suggests that, with the proper choice of fiber parameters and initial modulation frequency, linear birefringence could induce the generation of trains of compressed solitonlike pulses also in the visible.[17] Furthermore, it has been recently shown that scalar MI has potential for generating squeezed light.[18] Therefore, a quantized version of Eqs. (1) would offer interesting new possibilities of sideband squeezing.

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