Comment on "Obtainment of thermal noise from a pure quantum state"

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Yurke and Potasek [Phys. Rev. A **36**, 3464 (1987)] have shown that pure states of correlated pairs of quantum systems can lead to mixed-state behavior if only one of the systems is observed. We note that such behavior is well known and describe how the mixed-state properties of these states have led to their application in thermodynamical problems via the thermofield formalism. The correlations have been found to lead to manifestly nonclassical behavior (such as squeezing) if both systems are observed.

In a recent paper Yurke and Potasek[1] described the remarkable thermal properties of the individual modes in a two-mode squeezed state. This phenomenon is of interest because the mixed-state behavior of the single mode occurs even though the complete two-mode system is in a pure state. We note here that this property has been discussed at length by others.[2–5] It has led to the use of these states (and their fermionic counterparts) in finite-temperature quantum field theory via thermofield dynamics.[2] Thermofield states have been applied to a range of thermodynamic problems in quantum optics,[3] in particular problems involving the amplification and attenuation of light.[5] They have also been used to describe the production of particles by a blackhole through the Hawking mechanism.[6]

The motivation behind the thermofield formalism is to exploit the analogy between the ensemble averages of statistical mechanics and the vacuum expectation values of quantum field theory. The statistical average of an observable A is conventionally given by

$$\langle A \rangle = Z^{-1}(\beta)\mathrm{Tr}[A \exp(-\beta H)] , \tag{1}$$

where H is the Hamiltonian, $Z(\beta)$ is the partition function, and β is the inverse temperature ($\beta = 1/k_B T$). The thermofield representation of this ensemble average is written as a pure-state expectation value

$$\langle A \rangle = \langle 0(\beta) | A | 0(\beta) \rangle , \tag{2}$$

where $|0(\beta)\rangle$ is the temperature-dependent vacuum state. Equating expressions (1) and (2) gives

$$\langle 0(\beta) | A | 0(\beta) \rangle = Z^{-1}(\beta) \sum_n \langle n | A | n \rangle \exp(-\beta E_n) , \tag{3}$$

where $|n\rangle$ are the eigenstates of the Hamiltonian with eigenenergies E_n. If we expand the temperature-dependent vacuum state in terms of these eigenstates,

$$|0(\beta)\rangle = \sum_n |n\rangle f_n(\beta) , \tag{4}$$

then we find that Eq. (3) can only be satisfied if the expansion parameters $f_n(\beta)$ are vectors satisfying the orthogonality relation

$$f_n^*(\beta) f_m(\beta) = Z^{-1}(\beta) \exp(-\beta E_n) \delta_{nm} . \tag{5}$$

The thermofield vacuum is a vector in an enlarged space spanned by $|n\rangle$ and $f_n(\beta)$. Such a representation may be realized by inventing an additional fictitious dynamical system identical to that under consideration. Quantities associated with this fictitious system are conventionally denoted by a tilde. The additional system is therefore characterized by a Hamiltonian $\tilde{H}$ and orthonormal eigenvectors $|\tilde{n}\rangle$ with eigenenergies E_n. The thermofield vacuum state may be expressed in terms of this double basis

$$|0(\beta)\rangle = Z^{-1/2}(\beta) \sum_n \exp(-\beta E_n/2) |n, \tilde{n}\rangle . \tag{6}$$

If the thermal system is a field-mode then the eigenstates $|n\rangle$ are photon number states. In this case, the thermofield vacuum state is formally identical to the two-mode squeezed states discussed by Yurke and Potasek[1] and others.[3,4,7] We have provided a detailed discussion of the link between two-mode squeezed states and the thermofield formalism elsewhere.[3] Note that the thermofield vacuum is a superposition of only those states with equal numbers of photons in the real and fictitious modes. It is this property that is responsible for destroying off-diagonal matrix elements in the thermofield vacuum expectation value. The thermofield formalism allows us to use pure-state wave functions, in place of density matrices, to represent thermal states. The price paid for this ability is that we have to contend with a doubling of the number of dynamical variables—a fictitious one for each real one.

If the fictitious mode is actually real, in a genuine two-mode system, then the thermofield vacuum state becomes a two-mode squeezed state. Such states have been predicted in a number of nonlinear optical processes and particularly in parametric amplifiers,[3] where single-mode thermal properties are well known.[3,4,8] The mechanism that is responsible for producing this thermal effect is the correlation between the two modes induced by the parametric amplifier (or similar device). These correlations are neglected when expectation values are evaluated for single-mode properties and the properties of the second mode are not investigated. That is, the single-mode properties are described by a thermal density matrix that is obtained by tracing the full two-mode density matrix over the unobserved mode.[1,3,4,8,9]

When the properties of both modes of a two-mode squeezed state are observed then the nonclassical correlations between the modes become apparent. These correlations are directly responsible for the strong intensity-locking between the modes as well as the *normal-mode* squeezing that is the definitive feature of two-mode squeezed states.[9] It is intriguing to note that the thermal behavior of individual modes in a two-mode squeezed state, as well as their nonclassical intensity-locking and squeezing properties have a common origin in the build up of correlations between the two modes. The mechanism for thermalization inherent in these states is indeed intrinsically quantum mechanical.[1]

The novel properties of the two-mode squeezed states are by no means unique. It has been shown that correlated pairs of atoms or spins exhibit similar thermal and nonclassical properties.[10] Moreover, the thermofield formalism can be extended to provide a pure-state representation of arbitrary mixed states of any quantum system.[11] It remains to see how useful the thermofield formalism will be in practical calculations in quantum optics.

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