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Period-doubling bifurcation in an electronic circuit with a fast-recovery diode and square-wave input

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An inductance-resistance diode circuit with square-wave input shows period-doubling and chaotic behaviors similar to a circuit with sinusoidal-wave input. These near-resonance phenomena using a fast-recovery diode agree well with our theoretical model that includes neither diffusion capacitance nor reverse recovery time but only includes the varactor characteristics of the junction capacitance. Computer solutions and experiments show only period-doubling $2T$ bifurcations which agree very well with each other.

The phenomena of period doubling and chaotic behavior in an inductance-resistance-capacitance circuit with a varactor diode as a nonlinear capacitor were first reported by Linsay.[1] The system can easily be set up in the laboratory and many features of the chaotic behavior have been studied such as crisis,[2] Feigenbaum's universality,[3] noisy effects,[4] intermittency,[5] and Hopf bifurcations.[6] Effects from a system with a slow reverse recovery-time diode (an ordinary rectifier diode) have been analyzed by Hunt *et al.* on the basis of the effect of the reverse recovery time,[7] and by Brorson *et al.* based on the total nonlinearity of the junction and its diffusion capacitance.[8] In our experiments, however, a system with a fast-recovery diode was also found to generate period-doubling $2T$. Thus the analysis of some other dominant physical process and more precise interpretation of these nonlinear physical phenomena are important to obtain a clear understanding of the physical conditions present in this system. The purpose of this paper is to present another realistic physical property of a nonlinear diode resonator having fast-recovery time. This model provides another exact description of the bifurcation, but in this case it depends only on the varactor characteristic of the junction capacitance.

In previous work with ordinary rectifier diodes, the period doubling and chaotic behavior were attributed to the effects of the diffusion capacitance of the diode or its reverse recovery time. Since an ordinary rectifier diode has a high diffusion capacitance when conducting, the discharge time of the diffusion capacitance must be taken into account in order to understand the chaotic behavior. In the case of fast-recovery diodes, however, the discharge time is fast and hence the diffusion capacitance can be considered to have no effect on the discharge time and only the nonlinearity of the junction capacitance must be considered for producing bifurcations. We emphasize that in order to determine the effect of the varactor characteristics of the junction capacitance we must neglect the diffusion capacitance and the reverse recovery time and this condition is achieved by using a fast-reverse recovery-time diode (FE6D; General Instruments). The experiment is carried out using a FE6D diode ($C_0 \approx 266$ pF and $\gamma \approx 0.41$), a 199-Ω resistor, a 55-mH inductor (dc resistance of 51 Ω), and a function generator (Tektronix FG503, $\tau = 7$ μsec, $V_1 = 7$ V, and $V_2 = -5$ V). See Figs. 1 and 2.

In our model, the following assumptions are made. (1) The diode has a finite forward bias voltage Φ and the voltage remains at Φ during the time the diode is conducting. (2) When the diode is off, it has a voltage-dependent capacitance such that $C(V) = C_0/(1+V/\Phi)^{\gamma}$ until the forward voltage is Φ. (3) When the induced current in the inductor reaches 0, the capacitance is charged to maximum. Then the charge stored in the diode dissipates through the inductor and resistor until $Q(V) = Q(-\Phi)$. When the charge stored in the diode dissipates, energy is transferred to the inductor to produce induced current. (4) When the stored charge in the diode reaches $Q(-\Phi)$, the diode is no longer a capacitor any more, and the diode only as a voltage source with a forward bias voltage of Φ. (5) The conduction condition continues until the forward current becomes 0. Then the diode acts as a capacitor, since we neglected any diffusion capacitance, with a starting charge of $Q(V) = Q(-\Phi)$, a charge caused by the forward bias voltage.

Using the above assumptions we can divide the phenomenon into four steps as shown in Fig. 2(a). The

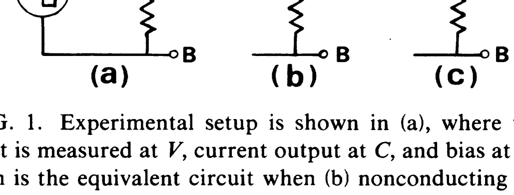

FIG. 1. Experimental setup is shown in (a), where voltage output is measured at V, current output at C, and bias at B; also shown is the equivalent circuit when (b) nonconducting and (c) conducting.

initial currents at each step are $I_m^0, I_m^1, I_m^2, I_m^3, I_{m+1}^0, \ldots$. We want to show the recurrence relations for the steps $I_m^0 \to I_{m+1}^0$ via $I_m^0 \to I_m^1 \to I_m^2 \to I_m^3, \to I_{m+1}^0$, $I_m^1 \to I_{m+1}^1 \cdots$, by using the following equation:

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + V(Q) = \begin{cases} V_1, & [2n\tau \le t \le (2n+1)\tau] \\ V_2, & [(2n+1)\tau \le t \le (2n+2)\tau] \end{cases} \quad (1)$$

where the external signal applied is a square-wave pulse with V_1 and V_2, the time interval is τ, and n is an integer.

Step (1). I_m^0 is the initial current when the diode becomes conducting with charge $Q(V) = Q(-\Phi)$. Then the current is determined by the initially existing induced current under the external voltage V_2 since the diode does not play any significant role. This situation is maintained until the external voltage switches to V_1 and the current at this instant is I_m^1, which is the initial current of the next step. Then the current relation between I_m^0 and I_m^1 can be obtained by setting $V(Q) = -\Phi$ and by using Eq. (1) to give

$$I_m^1 = \left[I_m^0 - \frac{V_2 + \Phi}{R} \right] \exp\left[-\frac{R}{L}(\tau - \tau_{m-1}^1) \right] + \frac{V_2 + \Phi}{R} , \quad (2)$$

where τ is the pulse width of V_2 and τ_{m-1}^1 is the time interval from the beginning of V_2 to I_m^0. During this time the charge stored in the diode is still $Q(-\Phi)$, the diffusion capacitance is 0, and the forward bias remains at Φ. Step (2). I_m^1 begins to increase under the external voltage V_1 until the current is 0. When the current is 0, the charge in the diode is $Q(-\Phi)$. The current at this time is $I_m^2 = 0$, which is the initial current of the next step. The current relation between I_m^1 and I_m^2 can be obtained by using Eq. (1) to give

$$I_m^2 = \left[I_m^1 - \frac{V_1 + \Phi}{R} \right] \exp\left[-\frac{R}{L}\tau_m^0 \right] + \frac{V_1 + \Phi}{R} = 0 , \quad (3a)$$

with

$$\tau_m^0 = \frac{L}{R} \ln \left[1 - \frac{R I_m^1}{V_1 + \Phi} \right] , \quad (3b)$$

where τ_m^0 is the delay time due to the presence of the inductor and is the interval between when the external voltage V_1 is applied and when the current becomes 0. The other properties of the diode are the same as in step (1). In step (3) the diode now acts as a capacitor (a voltage-dependent junction capacitance) and we get the current shape by using Eq. (1) with external voltage V_1 and the voltage applied to the diode $V(Q)$, which varies from $-\Phi$, the forward bias voltage. The current increases the external voltage switches to V_2 and the current becomes I_m^3, which is the initial current of the next step. In step (4) the physical condition is the same as in step (3) except that the external voltage is V_2. The current decreases until $V(Q)$ becomes $-\Phi$. The time interval for this step is τ_m^1 and the current at the end of this step is I_{m+1}^0. The cycle is then repeated.

The relations can be visualized using the return map

$$\begin{aligned} &\tau_m^0 \to \tau_{m-1}^0, \quad \tau_m^1 \to \tau_{m+1}^1 , \\ &I_m^0 \to I_{m+1}^0, \quad I_m^1 \to I_{m+1}^1 , \\ &V_m|_{\max} \to V_{m+1}|_{\max}, \ldots . \end{aligned} \quad (4)$$

Our theoretical results simulated by computer are compared with the experimental results by giving real values to the elements and by setting $\tau_{m-1}^1 = 5$ μsec and

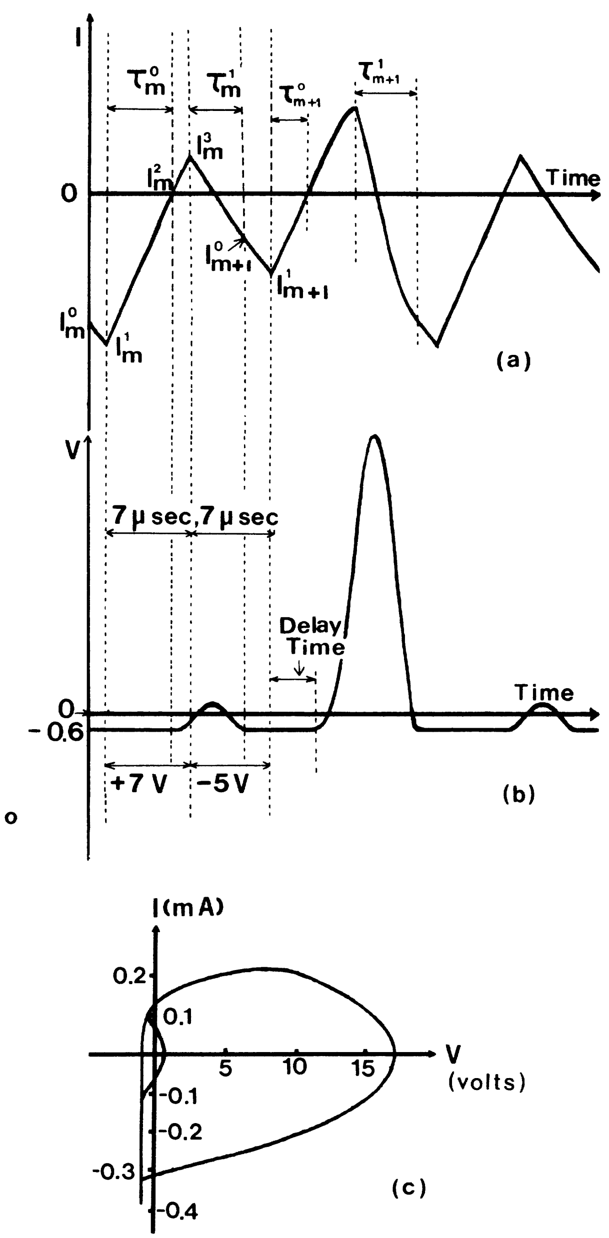

FIG. 2. Theoretical results for period-doubling $2T$ bifurcation: (a) voltage shape, (b) current shape, (c) phase diagram of current vs voltage (significant values are given).

$I_m^0=-0.01$ A as the initial conditions. Steps (1) and (2) are solved by using the derived Eqs. (2) and (3), and steps (3) and (4) are solved numerically by using the four-step Runge-Kutta method. The theoretical results are shown in Fig. 2, and the experimental results in Fig. 3. They agree with one another with regard to voltage shapes, current shapes, the phase diagram of voltage, and the current, to within the approximations for C_0 and γ of the diode and the measurement error of the voltage across the resistor. The error range for γ is 0.002 and that of C_0 is 10 pF; the voltage shown in Fig. 3(a) is the voltage applied to the resistor and diode, while the voltage in Fig. 2(a) is the voltage applied to the diode.

The predicted current and voltage shapes show that the delay time is due only to the dissipating time of the inductor energy, and not due to either the diffusion capacitance or the reverse recovery time.

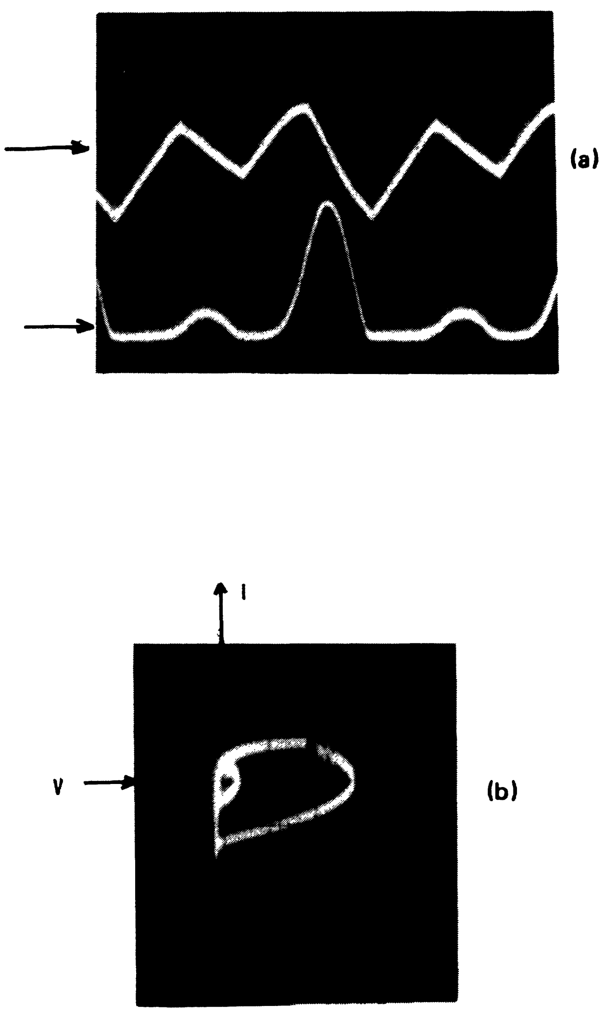

FIG. 3. Experimental results of bifurcation when using a square-wave input with +7 V, −5 V, and 7 μsec of pulse width. (a) The upper shape is the current shape and the lower shape is the voltage shape (5 μsec/div, 0.25 mA/div, and 5 V/div). (b) Phase diagram of current vs voltage (arrows show reference lines).

Using Eqs. (3a) and (3b) we can evaluate the current shapes if we only have the initial condition for τ_m^0 since $I_m^2=0$. The recurrence relations can be expressed as follows: (1) when the initial condition for τ_m^0 is given, the initial current in step (4) can be obtained; (2) this current gives the discharge time for the junction capacitance τ_m^1 and the initial current for step (1), I_{m+1}^0; (3) using τ_m^1 and I_{m+1}^0, the initial value of I_m^1 can be obtained; (4) using Eqs. (3a) and (3b) the next delay time τ_{m+1}^0 can be obtained. Thus the recurrence relation can be expressed for $\tau_m^0\to\tau_{m+1}^0$, a recurrence relation similar to that of Hunt *et al.*, though our relation expressing the delay time is different from the reverse recovery time for a diode as used by Hunt *et al.*[7]

If we calculate steps (3) and (4) without consideration of the nonlinearity of the capacitor by letting $C(V)=C_0$, the map does not bifurcate. Furthermore, when the applied voltage is increased, the circuit does not bifurcate any more, and it now has a stable output. For example, when $\tau=10$ μsec, $V_2=-5$ V, and $V_1=7$ V, the circuit bifurcates, but when $V_1=20$ V the circuit does not bifurcate. These experimental results are consistent with our calculations. Both the experimental results and our theoretical calculations show only period-doubling $2T$ or stable output. We attribute these results to the voltage-dependent capacitance of the unction capacitance. In the case when V_1 is +7 V, the nonlinearity of the junction capacitance is large enough to cause bifurcation, while in the case when V_1 is +20 V the nonlinearity of the junction capacitance is not large enough to cause bifurcation. At other input pulse widths near the circuit resonance frequencies, the theoretical calculations and experimental results agree in all aspects.

In order to show the importance of the nonlinearity of the junction capacitance, we have varied the value of γ only and have obtained very interesting phenomena ($\tau=7$ μsec, $V_1=+7$ V, and $V_2=-5$ V). (1) When $1/(1-\gamma)=10$, the equations produce period-doubling $2T$; (2) when $1/(1-\gamma)=5$, the equations produce chaos; (3) when $1/(1-\gamma)=2.5$, the equations produce period-doubling $4T$; (4) when $1/(1-\gamma)=2$, the equations produce period-doubling $2T$; (5) when $1/(1-\gamma)=\frac{10}{9}$, the equations produce a stable output; (6) when $1/(1-\gamma)=0.25$, the equations produce period-doubling $2T$. These results show the importance of the nonlinear capacitance producing bifurcations.

Thus the resultant relations that produce bifurcations can be expressed as follows:

$$I_m^i\to I_{m+1}^i=F(I_m^i;V_1,V_2),\quad i=0,1,3$$

$$V_m\,|_{\max}\to V_{m+1}\,|_{\max}=G(V_m\,|_{\max};V_1,V_2)\,, \qquad (5)$$

$$\tau_m^i\to\tau_{m+1}^i=H(\tau_m^i;V_1,V_2),\quad i=0,1+.$$

These calculations show how the nonlinearity of the capacitance produces bifurcation in the case of fast recovery diodes.

In conclusion, a nonlinear junction capacitance coupled with inductance produces bifurcation where the diffusion capacitance and the reverse recovery time are insignificant and therefore have not been considered. In addition, the calculated current shape, voltage shape, and phase diagram agree well with the experimental results.

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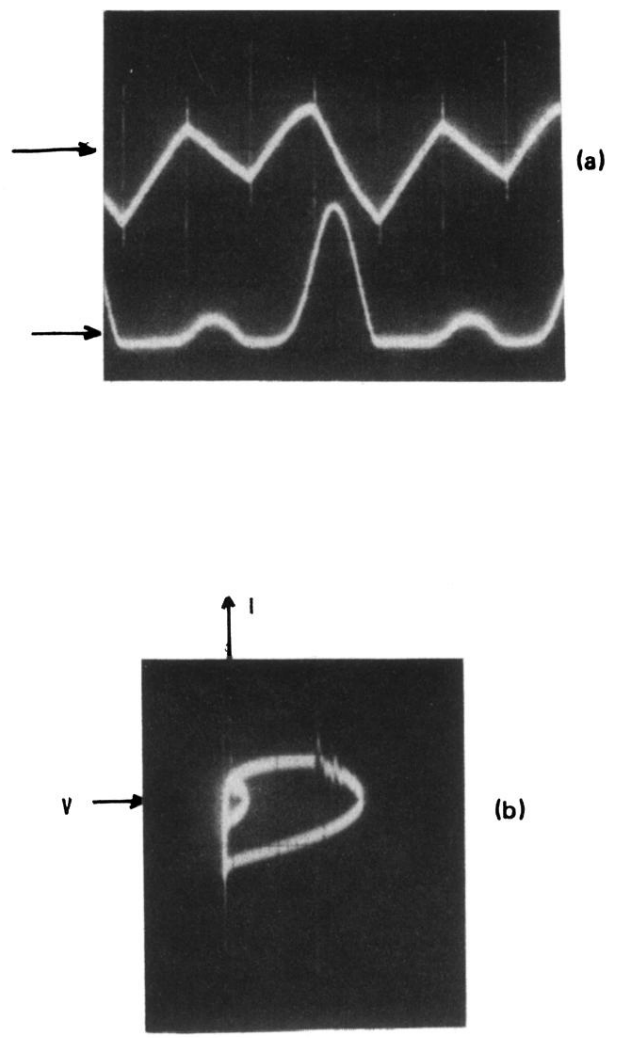

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