Collision-limited lifetimes of atom traps

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We carry out a simple calculation of the rate at which atoms confined within an atom trap are kicked out of the trap by elastic collisions with background gas molecules. The calculated rates are in good agreement with the observed rates and are remarkably insensitive to the trap depth.

Several techniques for the trapping of neutral atoms have been demonstrated recently using cold sodium atoms. Trap lifetimes ranging between 0.5 and 100 sec have been achieved using both magnetic[1,2] and optical[3-5] traps. These traps are very shallow, their depths ranging between 5 and 500 mK, and only very cold atoms can be confined within them. In all of the above-referenced work, it was surmised that the observed trap lifetimes were determined by elastic collisions between the cold trapped atoms and the much hotter residual gas molecules in the vacuum system. It was reasoned that even very weak collisions of this type transfer enough kinetic energy to the trapped atoms to eject them from the shallow traps. The purpose of the paper is to quantify that reasoning by carrying out a simple calculation of trap lifetimes as determined by elastic collisions. The results are in good agreement with the observed values. Additionally, they demonstrate that trap lifetimes are quite insensitive to trap depth and even to the collision parameters.

Consider an atom of mass m at rest at the bottom of a shallow atom trap having a potential well depth W_t. Let a residual gas molecule having mass M and speed S collide with the trapped atom. Since the collisions of interest here will be seen to occur at relatively large internuclear distances, it is reasonably safe to assume that the interaction potential between the two particles is adequately described using $V(r)=C_n/r^n$, where r is the internuclear distance. We calculate the speed s imparted to the trapped atom during the collision by using the impulse approximation.[6] In this approximation the impulse given the trapped atom is calculated in first order by assuming that the motion of each particle is unaffected by the collision. Later it well be shown that the impulse approximation is valid for typical trap depths of interest. We find

$$s=nC_n a_n/mSb^n ,$$

where b is the impact parameter for the collision and

$$a_n=\int_{-\infty}^{\infty} dx(1+x^2)^{-(n+2)/2} .$$

For future reference, $a_5=\frac{16}{15}$ and $a_6=5\pi/16$. The atom will escape from the trap if $s>v_0$, where $mv_0^2/2=W_t$. Thus, for a given S, the initially trapped atom is knocked out of the trap for impact parameters less than or equal to b_m, where $b_m=(nC_n a_n/mv_0 S)^{1/n}$.

Any residual gas molecule of speed S which hits the surface (from outside) of a sphere of radius b_m centered on the trapped atom will cause the trapped atom to be ejected from the trap. The rate at which residual molecules impinge on this surface is $R(S)=(SdN_S/4)(4\pi b_m^2)$, where dN_S is the density of residual molecules having speeds between S and $S+dS$. The total rate at which trapped atoms are ejected from the trap is found by integrating over all S. Assuming that the residual gas molecules are described by a Maxwellian velocity distribution with temperature T, the result of this integration is

$$R_n=2\pi^{1/2}N\left[\frac{2kT}{M}\right]^{(n-2)/2n}\left[\frac{nC_n a_n}{mv_0}\right]^{2/n} \times\Gamma((2n-1)/n) , \quad (1)$$

where N is the density of the residual molecules and $\Gamma(x)$ is the gamma function. Of course, $N=p/kT$, where p is the pressure of the residual gas molecules. The $1/e$ lifetime, τ_n, of atoms in the trap is $1/R_n$.

Consider conditions typical of the experiments of Refs. 1–5. All of those experiments demonstrated the trapping of the sodium atom. The residual gas molecules found in a typical ultrahigh vacuum chamber fabricated primarily of stainless steel, as used in those works, are largely composed of N_2, CO, H_2, and CH_4. The interaction potentials for N_2 and CO with sodium have been calculated[7,8] and appear to be reasonably well approximated using $n=5$ for the internuclear separations of interest. The parameter C_5 depends on magnitude of the N-N and C-O separations and also upon the direction of approach to Na relative to that separation. Taking this into consideration, approximate average values for C_5 are 6×10^{-51} and 1.6×10^{-50} erg cm^5 for Na-N_2 and Na-CO, respectively.[7,8]

Before proceeding further, we examine the validity of our initial assumptions using parameters appropriate for Na-N_2 collisions. For a N_2 residual gas temperature of 300 K, typical for the experiments, $R(S)$ is maximized for $S=1.14(2kT/M)^{0.5}=4.8\times10^4$ cm/sec; the vast majority of scatterings which result in ejection of the atom occur for values of S between half this amount and twice this amount. Considering a very deep trap having $W_t=1$ K, we find an upper bound on v_0 of 2.7×10^3 cm/sec. From these numbers we find that $b_m>0.5$ nm; internuclear distances this large justify treating the collisions with long-range potentials. The impulse approximation is valid as long as $2V(b_m)/MS^2 \ll 1$. Using the above numbers we find that this ratio is less than 3×10^{-2}, justifying the use of the impulse approximation.

With the knowledge that our approximations are valid, the trap lifetime for a $1/r^5$ potential is

$$\tau_5 = 5.8\times 10^{-19}\frac{T^{0.7}M^{0.3}}{p}\left[\frac{mW_t}{C_5^2}\right]^{0.2} \text{ sec} ,$$

where p is the pressure in Torr and W_t is the trap depth in degrees K. This equation displays the remarkable insensitivity of τ to the trap depth W_t, as was noted experimentally in Ref. 5. Inserting the constants for sodium and N_2, we find $\tau = 2.5\times 10^{-8}(W_t)^{0.2}/p$. Complete data for the lifetime as a function of the chamber pressure for a "magnetic molasses" trap having an equivalent trap depth of 500 mK is given in Ref. 5. It was found that $\tau = (2.0\times 10^{-8}\text{ sec})/p$. For $W_t = 500$ mK, our calculation yields $\tau = (2.2\times 10^{-8}\text{ sec})/p$, in what must be regarded as fortuitously good agreement with this experiment.

Reasonable agreement with the lifetimes reported for other types of traps is also obtained, although the experimental parameters are not as well defined. A trap lifetime is 0.8 sec is reported in Ref. 1 for an 18-mK-deep magnetic trap at a pressure of about 10^{-8} Torr; we calculate $\tau = 1.1$ sec for these parameters. Reference 3 reports a lifetime of about 1 sec for a 6-mK-deep dipole-force optical trap at a pressure of about 4×10^{-9} Torr; for those parameters we find $\tau = 2.2$ sec. A comparison with the lifetimes measured in Ref. 2 cannot be made since values for T and p are not reported for that cryogenic apparatus. The calculated values for τ_5 are reduced to about 70% of the above if the residual gas is taken to be CO instead of N_2.

It should be noted that the calculated lifetimes are also rather insensitive to the exact type of collision involved. For example, the interaction potential for a Na-rare-gas collision is weakly attractive and proportional to $1/r^6$. For Na-Ar, C_6 is about 2×10^{-58} erg cm^6.[9] Evaluating Eq. (1) as before, we find $\tau = (3.4\times 10^{-8}\text{ sec})/p$. These lifetimes are only about 50% longer than those calculated for $1/r^5$ collisions with N_2.

The simple calculation presented here is intended as a lowest-order treatment of the problem. Many complications have been ignored. For instance, actual interaction potentials are not as simple as those used here. Furthermore, in an optical trap a significant fraction of the atoms is in an excited state and the interaction potential for such an atom is different than that for a ground-state atom. Nonetheless, this simple calculation does demonstrate that elastic collisions between trapped Na atoms and residual gas molecules do quantitatively explain the observed lifetimes of atom traps. It also indicates that the lifetime of an atom trap is remarkably insensitive to the trap depth. Finally, the calculation makes it clear that increases in trap lifetimes will be most readily obtained by improving the vacuum in which a trap is located. However, since a good vacuum of 2×10^{-10} Torr roughly corresponds to a lifetime of only 100 sec, large increases in trap lifetimes will require truly excellent vacuums.

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