# Buildup of x-ray laser gain by fluctuations in channeled relativistic beam systems

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A theory for the buildup of stimulated x-ray radiation from spontaneous emission in relativistic channeling beam systems is presented. Explicit expressions for the startup operator for amplification is obtained from a first-principles quantum Hamiltonian including channel fluctuations (phonons or plasmons). Treating the radiation field as classical, an analytic solution for the evolution of the radiation intensity is derived and threshold conditions are obtained. The evolution toward stimulated emission shows a slow buildup followed by a rapid transition. Time scales of these two stages are analyzed.

## I. INTRODUCTION

A relativistic beam of channeled particles may occupy bound-energy eigenstates in the transverse direction. The channeling system should be free of imperfections (impurities or dislocations) to reduce dechanneling of the occupied bound states of the beam. Spontaneous dipolar transitions between these discrete eigenstates give rise to narrow-width, highly polarized, and intense x-ray radiation which is strongly forward-peaked.[1] Channeling radiation is obtained from relativistic electrons or positrons channeled in crystals[1–3] or in other hollow structures.[4] In such systems the radiation from the discrete transverse energy states is Doppler upshifted towards the x-ray region. This implies that for nondegenerate channeling systems the stimulated emission from different pairs of discrete levels can exhibit coherence in the Glauber sense.[5] Previous treatments considered the channeling gain of stimulated radiation only in the steady-state limit.[5,6] Furthermore, these works considered the amplification of an externally applied coherent source of x-ray radiation. Such coherent sources are not yet available, and it is highly desirable to consider the time-dependent growth of coherent stimulated radiation from incoherent spontaneous emission. In this paper I present a temporal and spatial theory for the buildup of stimulated radiation from spontaneous emission via propagation and interaction with channel fluctuations. Using a quantum density-matrix theory it is shown that fluctuations in the channels (phonons or plasmons) may evoke a buildup of stimulated emission. I will obtain an explicit expression for the startup mechanism, characteristic time scales, and conditions for reaching the stimulated-emission stage.

The theoretical framework presented here starts from a coupled Hamiltonian of relativistic channeling particles, channel fluctuations, and radiation field, where the radiation field is treated as classical.[5,7,8] An explicit expression for the temporal and spatial evolution of the radiation is obtained using the Heisenberg equation of motion, which has the Maxwell-Bloch form with modifications resulting from the relativistic motion of the channeling particles.[5,8] The present treatment of the radiation is spatially one dimensional and therefore ignores features of transverse gradients associated with the radiation and the channeling beam. This effect can be incorporated in an approximate way by means of a filling factor.[9] But at the front of the beam a three-dimensional theory is required for the radiation field and this effect is ignored in the present treatment. We assume that the average photon density is large, $\bar{N}_R \gg 1$, except for the very initial stage of the startup where $\bar{N}_R \sim 1$ and a quantum-field theory of the radiation should be used.[10]

In this paper I have neglected radiation losses. It is possible to make use of the Borman anomalous transmission so that x-ray losses can be small.[11–13] Another possibility is to consider the concept of distributed-feedback x-ray laser, by Bragg reflections in the forward direction from the periodic atomic structure in the crystal.[14] This mirrorlike structure may generate standing waves and reduce the losses on the atomic sites.[15]

The problem of stimulated channeling radiation buildup is similar to the problems of superfluorescence in a medium composed of inverted two-level atoms[16] and nonlinear stimulated Raman scattering.[17,18] But whereas Refs. 16–18 include the fluctuations phenomenologically using a Langevin operator and damping constants, we identify the startup mechanism from first principles. Furthermore, in channeling radiation systems the relativistic motion of the particle beam is of special importance and leads to additional effects.

A classical theory of the startup in a free-electron-laser (FEL) oscillator was considered in the small-signal regime for a one-dimensional system.[9] The fluctuation source in the FEL was taken as contributing to the startup because of the fact that the electrons of the beam are discrete and initially uncorrelated. This startup mechanism is ignored in the present treatment. In this work we use a quantum theory, and the startup includes the beam interaction with the channel fluctuations.

The theory presented in this work may be of special interest in other systems of x-ray lasers, such as laser-produced hot plasmas where x-ray lasing was obtained experimentally.[19]

In Sec. II we derive the startup mechanism. An analytic solution for the evolution of the radiation intensity is presented in Sec. III.

 

## II. A STARTUP MECHANISM FOR STIMULATED CHANNELING RADIATION

Let us consider a system of a particle beam interacting with channel fluctuations such as lattice vibrations (phonons) or electronic charge fluctuations (plasmons). The particles interact also with the radiation field of frequency $\omega$. The Hamiltonian of the particle beam may be written as $H = H_0 + H_I$. Here $H_0$ describes the free Hamiltonian of the channeling particles in the ensemble-average channeling potential,[8]

$$H_0 = \sum_{l,\mathbf{q}} \hbar[\omega_l + \omega(\mathbf{q})] a_l^\dagger(\mathbf{q},t) a_l(\mathbf{q},t) \ , \tag{1}$$

and $H_I$ is the interaction Hamiltonian given by[8,20]

$$H_I = \sum_{l,l'} \sum_{\mathbf{q},\mathbf{k}} \hbar \Omega_{l'l}(\mathbf{k},t) a_{l'}^\dagger(\mathbf{q}+\mathbf{k},t) a_l(\mathbf{q},t) \ , \tag{2}$$

where $a_l^\dagger(\mathbf{q},t)$ $[a_l(\mathbf{q},t)]$ is the creation (annihilation) operator of the bound transverse state $l,\mathbf{q}$ is the particle momentum mainly in the channeling direction $z$. The momentum $\mathbf{q}$ is situated in the $y$-$z$ plane for planar channels and in the $z$ direction for axial channels. $\omega_l$ and $\omega(\mathbf{q})$ are the directional particle energies. The interaction field $\Omega_{l'l}(\mathbf{k},t) = \Omega_{l'l}^F(\mathbf{k},t) + \Omega_{l'l}^E(\mathbf{k},t)$ represents the scattering of particles from channeling state $l$ to $l'$ with momentum transfer $\mathbf{k}$. $\Omega_{l'l}^F(\mathbf{k},t)$ is the channel fluctuation field,[8,20] and $\Omega_{l'l}^E(\mathbf{k},t)$ is the interaction of the positive-frequency component of the electric field,

$$\mathbf{E}(z,t) = \mathcal{E}(z,t)\exp[i\omega(t - z/c)] \ ,$$

where $\mathcal{E}(z,t)$ is the slowly varying part of the electric field.[5,10] In Eq. (2) we consider the system in the laboratory frame. It is possible to use the beam frame and the Lorentz transformations to transform the results to the laboratory frame. But in the beam frame the channel fluctuation field in Eq. (2) should be written in a relativistic moving frame, and thus the moving frame does not simplify the treatment.

We define the spatial Fourier transform of the density matrix by

$$n_{ij}(\mathbf{q},t) = \sum_{\mathbf{p}} a_i^\dagger(\mathbf{p}-\mathbf{q},t) a_j(\mathbf{p},t) \ .$$

Using the Heisenberg equation of motion

$$i\hbar \frac{\partial}{\partial t} n_{ij}(\mathbf{q},t) = [n_{ij}(\mathbf{q},t), H]$$

and the anticommutation relations

$$[a_i(\mathbf{p}',t), a_j^\dagger(\mathbf{p},t)] = \delta_{i,j}\delta_{\mathbf{p}',\mathbf{p}} \ ,$$

a kinetic equation for the particle beam in the slowly varying representation for the radiation field is obtained,

$$\frac{\partial}{\partial t} n_{ij}(\mathbf{q},t) = i(\Delta_{ij} - q_z v) n_{ij}(\mathbf{q},t) + F_{ij}(\mathbf{q},t) + E_{ij}(\mathbf{q},t) \ , \tag{3}$$

where the detuning is $\Delta_{ij} = \omega_{ij} - \omega(1 - v/c)$, $\omega_{ij} = \omega_i - \omega_j$, $\hbar[\omega(\mathbf{p}) - \omega(\mathbf{p}-\mathbf{q})] \simeq q_z v$, $1 - v/c \simeq 1/2\gamma^2$, $\gamma$ is the relativistic factor for particle with velocity $v$, and

$$F_{ij}(\mathbf{q},t) = i \sum_{\mathbf{k},l} [\Omega_{li}^F(\mathbf{q}-\mathbf{k},t) n_{lj}(\mathbf{k},t) - n_{il}(\mathbf{k},t)\Omega_{jl}^F(\mathbf{q}-\mathbf{k},t)] \ , \tag{4}$$

$$E_{ij}(\mathbf{q},t) = i \sum_{\mathbf{k},l} [\Omega_{li}^E(\mathbf{q}-\mathbf{k},t) n_{lj}(\mathbf{k},t) - n_{il}(\mathbf{k},t)\Omega_{jl}^E(\mathbf{q}-\mathbf{k},t)] \ . \tag{5}$$

Inserting Eqs. (4) and (5) into Eq. (3) and integrating over $t$, an integral equation for $n_{ij}(\mathbf{q},t)$ is obtained,

$$n_{ij}(\mathbf{q},t) = n_{ij}(\mathbf{q},0) e^{i(\Delta_{ij} - q_z v)t} + i \int_0^t dt' e^{i(\Delta_{ij} - q_z v)(t-t')} \sum_{\mathbf{k},l} [\Omega_{li}(\mathbf{q}-\mathbf{k},t') n_{lj}(\mathbf{k},t') - n_{il}(\mathbf{k},t')\Omega_{jl}(\mathbf{q}-\mathbf{k},t')] \ . \tag{6}$$

Upon iterating by inserting Eq. (6) into the right-hand side (rhs) of $F_{ij}(\mathbf{q},t)$ in Eq. (4), using the relation $[\Omega_{ij}^F(-\mathbf{k},t)]^\dagger = \Omega_{ji}^F(\mathbf{k},t)$, and averaging over fast oscillations in $t$ for $i \neq j$, two terms are obtained.[8] One is the startup term depending on the initial value of the density matrix $n_{ij}(\mathbf{q},0)$, and the second is the decay term of the density matrix. Explicitly $F_{ij}(\mathbf{q},t)$ obeys the following equation:

$$F_{ij}(\mathbf{q},t) = \delta F_{ij}(\mathbf{q},t) - \Gamma_{ij} n_{ij}(\mathbf{q},t) \ , \tag{7}$$

where the effect of the radiation field on the fluctuations is ignored. Here $\delta F_{ij}(\mathbf{q},t)$ is the startup operator given by

$$\delta F_{ij}(\mathbf{q},t) = i \sum_{\mathbf{k},l} e^{-ik_z vt} [\Omega_{li}^F(\mathbf{q}-\mathbf{k},t) n_{lj}(\mathbf{k},0) e^{i\Delta_{lj}t} - n_{il}(\mathbf{k},0) e^{i\Delta_{il}t} \Omega_{jl}^F(\mathbf{q}-\mathbf{k},t)] \ . \tag{8}$$

The ensemble average of $\delta F_{ij}$ is $\langle \delta F_{ij} \rangle = 0$, but the ensemble-average correlation is $\langle \delta F_{ij}^\dagger \delta F_{ij} \rangle \neq 0$, and $\delta F_{ij}$ is responsible for the buildup of the radiation field. In the usual treatment one writes an ensemble-average equation for the density matrix, and the startup term vanishes. The decay rate $\Gamma_{ij}$ of the off-diagonal density matrix is given by[7,8]

$$\Gamma_{ij} = -R_l \left[ \sum_{\mathbf{k},l} [G_{li}(\mathbf{k}) + G_{lj}^\dagger(\mathbf{k})] \right] \ , \tag{9a}$$

where $R_l$ is the real part and $G_{ij}(\mathbf{k})$ is

$$G_{ij}(\mathbf{k})=\left[\int_0^t d\tau\langle\Omega_{ij}^F(\mathbf{k},t)[\Omega_{ij}^F(\mathbf{k},t-\tau)]^\dagger\rangle \times\exp[i(\Delta_{ij}+k_z v)\tau-\eta_{\mathbf{k}}\tau]\right]_t . \quad (9b)$$

In Eq. (9b) the subscript $t$ denotes a time average over oscillating terms and $\eta_{\mathbf{k}}$ is a small convergence factor. The $\mathrm{Im}(G_{ij})$ contribution to Eq. (9a) and Eq. (7) can be added to the detuning $\Delta_{ij}$ in Eq. (3), but this contribution is small and is ignored in the present treatment. By introducing the ensemble average $\langle\Omega^F(\Omega^F)^\dagger\rangle$ here in Eq. (9b) I have also ignored higher-order correlations between fluctuations. The average $\langle\Omega^F(\Omega^F)^\dagger\rangle$ is proportional to the occupation number of the fluctuations (phonons or plasmons) and to their zero-point motion.

## III. INTENSITY OF CHANNELING RADIATION

In the following part we obtain the buildup of the radiation field while propagating in the channels. For simplicity, we assume that the particle beam has a population inversion $W$ in a two-level system and that these bound states remain undepleted (low-signal-gain condition). Population inversion between channeling bound states can be obtained by directing a cold-particle beam at a tilt angle of the order of milliradian relative to the channel direction.[21] This initial transverse energy of the particle beam increases the probability of being captured in higher-energy states. The population inversion can be partially maintained by using a set of progressive layers of thickness shorter than the occupation length of the considered bound states,[13] as in superlattice crystals,[22,23] where the beam particles are recaptured while entering progressive layers. Using the local time variable $\tau=t-z/c$ and Fourier-transforming Eqs. (3), (5), and (7), we get the coupled propagation equations for the particle beam and electric field,[5]

$$(1-v/c)\frac{\partial}{\partial\tau}n_{21}(z,\tau)+v\frac{\partial}{\partial z}n_{21}(z,\tau) =(i\Delta_{21}-\Gamma)n_{21}(z,\tau)+i\kappa\epsilon(z,\tau)+\delta F_{21}(z,\tau) , \quad (10)$$

$$\frac{\partial}{\partial z}\epsilon(z,\tau)=-i\kappa_2 n_{21}(z,\tau) , \quad (11)$$

where in Eq. (5) a dipolar interaction was assumed,[5]

$$\Omega_{l'l}^E(\mathbf{k},t)=-(1-v/c)d_{ll'}\epsilon(\mathbf{k},t)/\hbar ,$$

$$\kappa=(1-v/c)d_{12}W/\hbar ,$$

$$\kappa_2=2\pi N\omega d_{21}(1-v/c)/c ,$$

$\Gamma\equiv\Gamma_{21}$, and $N$ is the average density of particles in the beam. The treatment of the electric field in Eq. (11) is one dimensional and does not include diffraction effects. Dividing Eq. (10) by $(1-v/c)$ we find that by the transformation $(z,t)\to(z,\tau)$ the frequencies of the system $\omega_s$ are transformed to $\omega_s/(1-v/c)$.

I will use the Laplace-transform technique to solve Eqs. (10) and (11). Defining $\tilde\epsilon(s,\tau)$ and $\tilde n_{21}(s,\tau)$ as the Laplace transform of $\epsilon(z,\tau)$ and $n_{21}(z,\tau)$, using the Laplace transform of Eqs. (10) and (11) and eliminating $\tilde\epsilon(s,\tau)$, we obtain for the resonance case ($\Delta_{21}=0$) (Ref. 24)

$$\tilde\epsilon(s,\tau)=-i\frac{\kappa_2}{s}e^{\beta(s)\tau}\tilde n_{21}(s,0)-i\kappa_2(1-v/c)^{-1}\int_0^\tau d\tau' e^{\beta(s)(\tau-\tau')}\left[\frac{v}{s}n_{21}(0,\tau')+\frac{1}{s}\delta\tilde F_{21}(s,\tau')\right] , \quad (12)$$

where $\beta(s)=(-\Gamma_1-v_1 s+\kappa_1\kappa_2 s^{-1})$, $v_1=v/(1-v/c)$, $\Gamma_1=\Gamma/(1-v/c)$, and $\kappa_1=\kappa/(1-v/c)$. In Eq. (12) I assumed that no external radiation enters into the medium so that $\epsilon(0,\tau)=0$. The inverse Laplace transform of Eq. (12) is[25]

$$\begin{aligned}\epsilon(z,\tau)=&-i\kappa_2 e^{-\Gamma_1\tau}\int_0^z dz' n_{21}(z-z',0)I_0(\mathfrak{z}(z',\tau))\Theta_0(z'-v_1\tau)\\ &-i\kappa_2(1-v/c)^{-1}\int_0^\tau d\tau' e^{-\Gamma_1\tau'}\Big[v n_{21}(0,\tau-\tau')I_0(\mathfrak{z}(z,\tau'))\Theta_0(z-v_1\tau')\\ &+\int_0^z dz'\delta F_{21}(z-z',\tau-\tau')I_0(\mathfrak{z}(z',\tau'))\Theta_0(z'-v_1\tau')\Big]\end{aligned} \quad (13)$$

where $\mathfrak{z}(z,\tau)=[4(z-v_1\tau)\kappa_1\kappa_2\tau]^{1/2}$, $\Theta_0(x)$ is the Heaviside function, and $I_n(x)$ are the modified Bessel functions.

The radiation intensity $I_R(z,\tau)$ is obtained from the ensemble average $I_R(z,\tau)=(fc/2\pi\hbar\omega)\langle\epsilon^\dagger(z,\tau)\epsilon(z,\tau)\rangle$, where $f$ is a filling factor[9] incorporating, in an approximate way, transverse propagation aspects. Using the fact that at $\tau=0$ the density matrix is uncorrelated so that $\langle n_{21}^\dagger(z,0)n_{21}(z',0)\rangle=\delta(z-z')/[N(1-v/c)]$, and ignoring highly oscillating terms, we obtain from Eq. (13)

$$\begin{aligned}I_R(z,\tau)=A_s\Big[&e^{-2\Gamma_1\tau}[(z-v_1\tau)\Theta_0(z-v_1\tau)Q(z,\tau)]\\ &+\int_0^\tau d\tau' e^{-2\Gamma_1\tau'}\Theta_0(z-v_1\tau')[2\Gamma_1(z-v_1\tau')Q(z,\tau')+v_1 I_0^2(\mathfrak{z}(z,\tau'))\Big] ,\end{aligned} \quad (14)$$

where $A_s=fc\kappa_2^2/[(1-v/c)2\pi\hbar\omega N]$, $Q(z,\tau)=I_0^2(\mathfrak{z}(z,\tau))-I_1^2(\mathfrak{z}(z,\tau))$, and $\mathfrak{z}(z,\tau)=[2g_0(z-v_1\tau)\Gamma_1\tau]^{1/2}$. In deriving Eq. (14) I used the relation[26]

$$\int^z dy\, yI_0^2(y)=z^2[I_0^2(z)-I_1^2(z)]/2$$

and the fact that at $z=0$ the correlation

$\langle n_{21}^{\dagger}(0,\tau)n_{21}(0,\tau')\rangle=\delta(\tau-\tau')/(Nv)$ is small. The factor $g_0=2\kappa_1\kappa_2/\Gamma_1$ is the steady-state gain coefficient in the short coherence length limit for an externally incident electric field on the medium.[5,6] Equation (14) includes the details of the buildup of the forward stimulated radiation intensity from spontaneous emission.

I will now consider some illustrative limits of the buildup of the x-ray stimulated gain. For short times $\Gamma_1\tau\rightarrow 0$, only the first term in Eq. (14) contributes. In this limit $I_R(z,\tau)=A_s z$ (since for $x\ll 1$, $I_0(x)\simeq 1$ and $I_1(x)\simeq x/2$). This is the spontaneous emission due to population inversion.[6,17] In the limit $\Gamma_1\tau\rightarrow\infty$ only the second term of Eq. (14) contributes to the buildup of the steady-state stimulated radiation.

In the high-gain limit $gz\gg 1$, where $g$ is the gain coefficient, the integrand

$$u(z,\tau')=(z-v_1\tau')Q(z,\tau')\exp(-2\Gamma_1\tau')$$

of the second term in Eq. (14) is sharply peaked in time and can be expanded around the peak using the relation, for $x\gg 1$ (Ref. 27),

$$I_n(x)\simeq(2\pi x)^{-1/2}[1-(4n^2-1)/8x]\exp(x)\ .$$

The result is

$$u(z,\tau')\propto\exp(gz)\exp\{-[(\tau'-\tau_M)/\Delta\tau_M]^2\}\ .$$

Here $\tau_M$ is the buildup time and $\Delta\tau_M$ is the time interval for the rapidly increased gain towards its stimulated steady state. Using this expansion for $u(z,\tau')$ in Eq. (14) we obtain for the case of short coherence lengths ($g_0 l\ll 1$, $l=v/\Gamma$) that the following hold true: $\tau_M=g_0 z/2\Gamma_1$, $g=g_0$, and $\Delta\tau_M=(g_0 z/2)^{1/2}/2\Gamma_1$. Thus we find that $\Delta\tau_M/\tau_M=1/(2g_0 z)^{1/2}\ll 1$. The steady-state radiation intensity is $I_R(z,\infty)=B(z)\exp(gz)$, where the intensity factor is

$$B(z)=(A_s/2g_0)/(2\pi g_0 z)^{1/2}\ .$$

This intensity factor is directly proportional to the level fluctuation $\Gamma^{3/2}$ and is inversely proportional to $z^{1/2}$. The case $g_0 l\ll 1$ is of special interest in electron or positron beams channeling in crystals where the coherence length may be small.[1-3,5]

For the case of long coherence lengths ($g_0 l\gg 1$, $l<z$) we get the following: $\tau_M=z/2v_1$, $g=(2g_0/l)^{1/2}$, $\Delta\tau_M=(z/g)^{1/2}/2v_1$, and $\Delta\tau_M/\tau_M=1/(gz)^{1/2}\ll 1$. For this case the intensity factor

$$B(z)=(A_s/2g_0)/(\pi gz)^{1/2}$$

increases with level fluctuations as $\Gamma$ and has also a $z^{-1/2}$ spatial dependence. The steady-state gain coefficient $g$ obtained here due to the interaction with channel fluctuations is the same as that obtained from amplification of an external signal.[5] The case of large coherence lengths $g_0 l\gg 1$ is of importance in hollow structures such as zeolite crystals[4] containing hollow channels with a diameter larger than in regular crystals.

## IV. SUMMARY

I have demonstrated that in channeling radiation for the high-gain region ($gz\gg 1$) the evolution towards stimulated emission is comprised of two stages which depend on the level of the channel fluctuations. The first one is a relatively slow buildup of gain up to time $\tau_M$. The second stage is a rapid transition to the steady-state stimulated emission in a short interval of time $\Delta\tau_M$ such that $\Delta\tau_M\ll\tau_M$. In the low-gain region ($gz\ll 1$) Eq. (14) gives the result that the steady-state emission ($\Gamma_1\tau\rightarrow\infty$) is of the order of the spontaneous emission $A_s z$.

Recent estimates suggest that to get into the high-gain regime may require beam currents of the order of Ma/cm$^2$ for particle energies near 10 MeV;[5,6,13] this requirement is in the limit of the present capabilities of high-current technology. It is possible to increase amplification length by using a set of progressive layers of thickness shorter than the dechanneling length and recapture the beam particle in bound states while entering progressive layers.[13,23] A more promising possibility of entering the high-gain regime is by tuning the channeling radiation to be Bragg reflected in the forward direction from the periodic atomic structure. This mirrorlike structure is based on the concept of the distributed-feedback laser and is presented elsewhere.[15]

A detailed buildup theory of stimulated radiation including transverse effects is in preparation. Additional study on the refinement of the very initial stage of the startup (when the average density of photons $\bar{N}_R\sim 1$) using a quantum-field theory for the radiation is in progress.

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