Nonclassical photon states generated by stimulated Compton scattering

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(Received 4 January 1988; revised manuscript received 21 March 1988)

The stimulated Compton scattering of photons between two oppositely propagating coherent lights by a relativistic electron moving along the same line is studied analytically. The system is described in a moving frame so that the electron becomes nonrelativistic and the frequencies of the two lights coincide. Except for the replacement of the static wiggler by a propagating light, our system is essentially the same as a free-electron laser working in the Compton regime and described in the usual Bambini-Renieri frame. The Q representation of two-mode coherent states is adopted to describe the radiation fields, and the electron states are expressed in terms of plane waves. A perturbative solution, with the quantum recoil of the electron as the perturbation parameter, of the time evolution of the system is carried out far beyond the first order. It demonstrates that two nonclassical effects, i.e., squeezing and photon antibunching, occur as results of the scattering. It also confirms once more the fact that quantum recoils play the exclusive role in generating such nonclassical photon states. A very important improvement in the present treatment is that the electron part of the density matrix is traced out before the calculation of the squeezing effect. As a result, it is found that there is no squeezing if the initial state is a vacuum. This is, perhaps, a significant new discovery. Many-electron effects are ignored in the present study.

I. INTRODUCTION

As the experimental techniques become ever more sophisticated, it is now possible to investigate very-fine-scale phenomena in the nonlinear interaction of light with matter in the laboratories. Two very exciting examples are photon antibunching and squeezed states, related, respectively, to the particle and wave aspects of light. They are intrinsically quantum-mechanical phenomena; and radiation fields with such unusual characteristics are called nonclassical photon states. In a sense, the well-known coherent states can be considered as the borderline between classical and nonclassical states. Therefore the coherent states can serve as the natural reference in our discussion of nonclassical photon states.

It is well known that the photon statistics of a coherent light has the Poisson distribution, which implies that the detection of a photon does not change the probability for the detection of the next photon, one way or the other. For a chaotic classical light, this probability is always enhanced; hence it is described as photon bunching. That the same probability can be reduced in the resonance fluorescence from a single two-level atom driven by a laser light was first predicted theoretically by Kimble and Mandel[1] and by Carmichael and Walls[2] in 1976; it was observed experimentally by Kimble *et al.* in 1977.[3] Since the latter is the opposite of the former, it is logically called photon antibunching.[4] It should be pointed out that the photon statistics of a light with photon bunching must have a probability distribution broader than the Poisson distribution, while that corresponding to photon antibuncing must be narrower than the Poisson distribution. Therefore the alternative terms for photon bunching and antibunching are super-Poissonian and sub-Poissonian, respectively. Since, in the Poisson distribution, the variance equals the mean value exactly, the existence of photon antibunching can be determined by the criterion

$$\langle n^2\rangle - \langle n\rangle^2 < \langle n\rangle \ , \tag{1.1}$$

which means that the variance of the photon numbers is less than the mean of the photon numbers.

It is also well known that coherent states as well as vacuum states are minimum-uncertainty states; that is, the uncertainties in the two quadrature components of a radiation field in one of these states are equal and their product assumes the minimum value allowed by the Heisenberg principle. It has been speculated since 1971 (Refs. 5 and 6) that the uncertainty in one component can be reduced below the symmetrical lower limit of a vacuum state, at the expense of increased uncertainty of the other component, without violating the Heisenberg principle. Such highly unusual states of radiation are called squeezed states and have been generated in laboratories very recently.[7,8] An excellent review on this subject was published very recently by Loudon and Knight.[9]

To give squeezed states more precise definition, let us express the cavity electric field operator in terms of quadrature operators as follows:

$$\hat{E}(z,t) = \tfrac{1}{2}\mathcal{E}(z)[a\exp(-i\omega t) + a^\dagger \exp(i\omega t)]$$

$$= \mathcal{E}(z)[\hat{X}\cos(\omega t) + \hat{Y}\sin(\omega t)] \ , \tag{1.2}$$

where a and $a^\dagger$ are, respectively, the annihilation and creation operators of photons in the radiation field, and

$$\hat{X} \equiv (a+a^\dagger)/2, \quad \hat{Y} \equiv (a-a^\dagger)/2i \tag{1.3}$$

are the two quadrature operators satisfying the commutation relation

$$[\hat{X},\hat{Y}]=i/2\ , \qquad (1.4)$$

with their variances satisfying the uncertainty relation

$$\langle(\Delta X)^2\rangle\langle(\Delta Y)^2\rangle\geq\tfrac{1}{16}\ . \qquad (1.5)$$

Then the criterion for the existence of squeezing is either

$$\langle(\Delta X)^2\rangle<\tfrac{1}{4}\ \text{ or }\ \langle(\Delta Y)^2\rangle<\tfrac{1}{4}\ . \qquad (1.6)$$

The fact that photon antibunching and squeezing are intrinsically quantum-mechanical phenomena can best be seen in terms of the Gluaber-Sudarshan P representation of coherent states[10,11] corresponding to the normal ordering of operators. Arranged in the normal ordering of operators, Eqs. (1.1) and (1.6) become

$$\langle:(\Delta n)^2:\rangle<0\ , \qquad (1.7)$$

$$\langle:(\Delta X)^2:\rangle<0\ \text{ or }\ \langle:(\Delta Y)^2:\rangle<0\ . \qquad (1.8)$$

The inequalities (1.7) and (1.8) can be true only if the "probability distribution" in the P representation assumes negative values somewhere; this would make no sense at all in the classical world. Unfortunately, the probability distributions in the P representation are usually very much singular. On the other hand, the probability distributions in the Q representation, corresponding to the antinormal ordering of operators, are positive definite and, hence, easier to handle; so we will use the Q representation exclusively in this article.

Many nonlinear radiation-matter interactions have been studied as possible ways of generating nonclassical photon states, as reviewed by Refs. 4 and 9. In this paper, we will show analytically that nonclassical photon states can be generated by the stimulated Compton scattering of two oppositely propagating coherent lights by a relativistic electron moving along the same line. A similar problem was studied by Peřinová *et al.*[12] They treated the electron as a two-level system and their solutions have been obtained in the short-time approximation. We treat the electron as a multilevel system with energy varying nonlinearly and our solution is valid for much longer time, especially when the quantum recoil of the electron is small.

Very closely related to our problem are the recent intensive studies of the nonclassical effects in free-electron lasers (FEL's) working in the Compton regime.[13–16] In the usual quantum theory of the FEL,[17] one of the "lights" is actually the static periodic magnetic field of the undulator which becomes a quasi-plane-wave radiation only after a relativistic Lorentz transformation (Weizsäcker-William approximation). If we trace out the counterpropagating light, our solution will reduce to that of the FEL. However, there is a very important point that should be mentioned. In Refs. 13–16, the quantum state of the electron is locked with that of the laser photons and the squeezing effect was calculated from the combined electron-photon states. This practice makes the true meaning of squeezing very doubtful. In our present treatment, we trace out the electron part of the quantum states; and the meaning of squeezing becomes clear and unquestionable.

II. BASIC FORMULATION

We consider a one-dimensional problem of two coherent lights propagating along the z axis in opposite directions. Photons are scattered back and forth between the two lights by a relativistic electron moving in the positive z direction. We will describe this system in a moving frame so that the electron becomes nonrelativistic and the frequencies of the two lights coincide. This is the so-called Bambini-Renieri frame in the terminology of the FEL. The Hamiltonian of the system in this frame can be written as

$$H=\frac{p^2}{2m_e}+\hbar\omega(a_f^\dagger a_f+a_b^\dagger a_b)+\hbar\Lambda(a_f^\dagger a_b e^{-2ikz}+a_b^\dagger a_f e^{2ikz})\ , \qquad (2.1)$$

where p is the operator for the electron momentum, m_e is the electron mass, $a_f^\dagger$ ($a_b^\dagger$) is the creation operator for the forward (backward) propagating field, Λ is the coupling constant, and $\hbar k=\hbar\omega/c$ is the momentum of a photon. This Hamiltonian implies the conservation of photon number and linear momentum.

We assume that the initial quantum state of the complete system can be written as the product of a plane wave of momentum p_0 for the electron and coherent states identified by complex numbers u_0 and v_0 for the backward- and forward-propagating light, respectively; i.e.,

$$|\psi_0\rangle\equiv K_0 e^{ip_0z/\hbar}e^{-|u_0|^2/2-|v_0|^2/2}\times\sum_{m=0}^{\infty}(u_0^m/\sqrt{m!})\sum_{n=0}^{\infty}(v_0^n/\sqrt{n!})|m\rangle_b\,|n\rangle_f\ , \qquad (2.2)$$

where $|m\rangle_b$ ($|n\rangle_f$) is the photon number state of the backward- (forward-) propagating radiation and K_0 is the normalization constant. The quantum state evolved from the initial state $|\psi_0\rangle$ can always be written as

$$|\psi(t)\rangle=K_0\exp(-ip_0^2t/2m_e\hbar+ip_0z/\hbar)\times\sum_l\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}C_{l,m,n}(t)e^{-2ilkz-i(m+n)\omega t}\times|m\rangle_b\,|n\rangle_f\ . \qquad (2.3)$$

Substituting Eq. (2.3) into the Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle=H|\psi(t)\rangle \qquad (2.4)$$

with the Hamiltonian given by Eq. (2.1), we obtain the following partial-difference-differential equation:

$$i\frac{d}{dt}C_{l,m,n}(t)=(-2l\Delta+l^2E)C_{l,m,n}(t)+\Lambda\sqrt{m(n+1)}C_{l+1,m-1,n+1}(t)+\Lambda\sqrt{(m+1)n}\,C_{l-1,m+1,n-1}(t)\ , \qquad (2.5)$$

where $C_{l,m,n}(t)$ is the joined probability amplitude that the electron has had l net recoils (a recoil corresponding to the process described by the term $a_f^\dagger a_b e^{-2ikz}$ in the Hamiltonian is considered as $+1$, while that corresponding to $a_b^\dagger a_f e^{2ikz}$ is considered as -1), and the backward- and the forward-propagating lights have m and n photons, respectively, at time t, with the normalization condition

$$\sum_l \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} |C_{l,m,n}(t)|^2 = 1 , \tag{2.6}$$

and

$$\Delta \equiv kp_0/m_e, \quad E \equiv 2\hbar k^2/m_e \tag{2.7}$$

are two constants related to the initial momentum and to the quantum recoil, respectively, of the electron.

Our task is to find the solution to Eq. (2.5) with the initial condition

$$C_{l,m,n}(0) = \delta_{l,0} e^{-|u_0|^2/2 - |v_0|^2/2} (u_0^m/\sqrt{m!})(v_0^n/\sqrt{n!}) \tag{2.8}$$

as implied by Eq. (2.2). Equation (2.5) is the type of Raman-Nath equation[18] characterized by a term proportional to l^2 which is the stumbling block in the attempt to find exact solutions to this type of equation. Fortunately, a good approximate solution to Eq. (2.5) can be obtained quite easily by using the Q representation of two-mode coherent states.

III. PROBABILITY AMPLITUDE IN Q REPRESENTATION

Suppose the solution of Eq. (2.5) is available; then we can construct the density matrix for the system as follows:

$$\rho(t) = \sum_l \sum_{l'} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{m'=0}^{\infty} \sum_{n'=0}^{\infty} C^*_{l',m',n'}(t) C_{l,m,n}(t) |l\rangle |m\rangle_b |n\rangle_f \langle n'|_f \langle m'|_b \langle l'| , \tag{3.1}$$

where $|l\rangle$ denotes the quantum state of the electron with wave function $K_0 e^{i(p_0/\hbar - 2lk)z}$ satisfying the orthogonal condition

$$\langle l' | l \rangle = K_0^2 \int \exp[2i(l'-l)kz] dz = \delta_{l',l} . \tag{3.2}$$

On the other hand, a two-mode coherent state is defined as

$$|u,v\rangle \equiv \exp\left[-\frac{|u|^2}{2} - \frac{|v|^2}{2}\right] \times \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} (u^m v^n/\sqrt{m!n!}) |m\rangle_b |n\rangle_f , \tag{3.3}$$

where u and v are two complex variables. Following Kano[19] the probability density function in the Q representation can be written as

$$\begin{aligned} Q(z,u,u^*,v,v^*,t) &\equiv \langle u,v | \rho(t) | u,v \rangle \\ &\equiv \exp(-|u|^2 - |v|^2) \\ &\quad \times A^*(z,u^*,v^*,t) A(z,u,v,t) , \end{aligned} \tag{3.4}$$

where

$$A(z,u,v,t) = \sum_l \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} C^*_{l,m,n}(t) e^{-2ilkz} \times u^m v^n/\sqrt{m!n!} \tag{3.5}$$

is in a sense the probability amplitude in the Q representation. It is much easier to calculate in our problem than the probability distribution itself. The normalization condition for the distribution function is

$$K_0^2 \int dz \int \frac{d^2u}{\pi} \int \frac{d^2v}{\pi} Q(z,u,u^*,v,v^*,t) = 1 . \tag{3.6}$$

Using Eq. (2.5) we can obtain a partial differential equation for $A(z,u,v,t)$ as follows:

$$\left[i\frac{\partial}{\partial \tau} - i\frac{\delta}{k}\frac{\partial}{\partial z} + \lambda \left[e^{-2ikz} v \frac{\partial}{\partial u} + e^{2ikz} u \frac{\partial}{\partial v} \right] - \epsilon \frac{1}{4k^2}\frac{\partial^2}{\partial z^2} \right] A(z,u,v,\tau) = 0 , \tag{3.7}$$

where we have introduced the following dimensionless parameters:

$$\delta \equiv \Delta/\Omega, \quad \lambda \equiv \Lambda/\Omega, \quad \epsilon \equiv E/\Omega, \quad \tau \equiv \Omega t , \tag{3.8}$$

with $\Omega \equiv (\Delta^2 + \Lambda^2)^{1/2}$, which implies that $\delta^2 + \lambda^2 = 1$. The initial condition for Eq. (3.7) can be obtained by using Eq. (2.8) as

$$A(z,u,v,0) = \exp(-|u_0|^2/2 - |v_0|^2/2 + u_0^* u + v_0^* v) . \tag{3.9}$$

IV. PERTURBATIVE SOLUTION

Our task is now to find the solution to Eq. (3.7). Without the second-derivative term, the solution can be easily obtained; with it, exact analytical solution is apparently impossible. Therefore, let us consider a perturbative solution, with ϵ as the parameter, of the form

$$A(z,u,v,\tau) = A_0(z,u,v,\tau) + \epsilon A_1(z,u,v,\tau) + \epsilon^2 A_2(z,u,v,\tau) + \cdots \tag{4.1}$$

satisfying the initial condition

$$A_l(z,u,v,0) = \delta_{l,0} A(z,u,v,0) . \tag{4.2}$$

Substitution of Eq. (4.1) into Eq. (3.7) yields

$$\hat{D}_0 A_0(z,u,v,\tau)=0 , \quad (4.3)$$

$$\hat{D}_0 A_{l+1}(z,u,v,\tau)+\hat{D}_1 A_l(z,u,v,\tau)=0 , \quad (4.4)$$

where

$$\hat{D}_0 \equiv i\frac{\partial}{\partial\tau}-i\frac{\delta}{k}\frac{\partial}{\partial z}+\lambda\left[e^{-2ikz}v\frac{\partial}{\partial u}+e^{2ikz}u\frac{\partial}{\partial v}\right] , \quad (4.5)$$

$$\hat{D}_1 \equiv -\frac{1}{4k^2}\frac{\partial^2}{\partial z^2} . \quad (4.6)$$

The solution to Eq. (4.3) can be expressed in the form

$$A_0(z,u,v,\tau)=\exp\left[-\frac{|u_0|^2}{2}-\frac{|v_0|^2}{2}+F_1(z,\tau)u+F_2(z,\tau)v\right] \quad (4.7)$$

with

$$F_1(z,\tau)\equiv f_1(\tau)e^{2ikz}+f_2(\tau) , \quad (4.8a)$$

$$F_2(z,\tau)\equiv f_3(\tau)+f_4(\tau)e^{-2ikz} . \quad (4.8b)$$

Substitution of Eq. (4.7) into Eq. (4.3) leads to the simultaneous equations for the f's as follows:

$$i\frac{d}{d\tau}f_1+2\delta f_1+\lambda f_3=0 , \quad (4.9a)$$

$$i\frac{d}{d\tau}f_2+\lambda f_4=0 , \quad (4.9b)$$

$$i\frac{d}{d\tau}f_3+\lambda f_1=0 , \quad (4.9c)$$

$$i\frac{d}{d\tau}f_4-2\delta f_4+\lambda f_2=0 , \quad (4.9d)$$

with the initial conditions implied by Eq. (3.9) and

$$f_1(0)=0,\ \ f_2(0)=u_0^*,\ \ f_3(0)=v_0^*,\ \ f_4(0)=0 . \quad (4.10)$$

The solution to Eqs. (4.9) can be easily obtained to be

$$f_1(\tau)=iv_0^*\lambda(\sin\tau)e^{i\delta\tau} , \quad (4.11a)$$

$$f_2(\tau)=u_0^*(\cos\tau+i\delta\sin\tau)e^{-i\delta\tau} , \quad (4.11b)$$

$$f_3(\tau)=v_0^*(\cos\tau-i\delta\sin\tau)e^{i\delta\tau} , \quad (4.11c)$$

$$f_4(\tau)=iu_0^*\lambda(\sin\tau)e^{-i\delta\tau} . \quad (4.11d)$$

Let us assume that $|u_0|^2+|v_0|^2\equiv N\gg 1$, where N is the expectation value of the total number of photons in the system. We then also have $|u|^2+|v|^2\approx N$ because the probability that this condition is not satisfied is negligible. Then, as long as $l\ll N$, the higher-order terms can be approximated by an expression of the form

$$A_l(z,u,v,\tau)=A_0(z,u,v,\tau)[G(z,u,v,\tau)]^l/l! , \quad (4.12)$$

where

$$G(z,u,v,\tau)\equiv G_1(z,\tau)u^2+G_2(z,\tau)uv+G_3(z,\tau)v^2 , \quad (4.13)$$

with

$$G_1(z,\tau)\equiv g_1(\tau)e^{4ikz}+g_2(\tau)e^{2ikz}+g_3(\tau) , \quad (4.14a)$$

$$G_2(z,\tau)\equiv g_4(\tau)e^{2ikz}+g_5(\tau)+g_6(\tau)e^{-2ikz} , \quad (4.14b)$$

$$G_3(z,\tau)\equiv g_7(\tau)+g_8(\tau)e^{-2ikz}+g_9(\tau)e^{-4ikz} . \quad (4.14c)$$

As will be verified later, using Eq. (4.12) we have

$$\frac{\partial^2}{\partial z^2}A_l=A_l[4k^2\lambda^2\sin^2\tau(e^{-i(\delta\tau+2kz)}u_0^*v-e^{i(\delta\tau+2kz)}v_0^*u)^2][1+O(l/N)] . \quad (4.15)$$

Substituting Eq. (4.12) into Eq. (4.4) and using Eq. (4.15) under the condition that $l\ll N$, we obtain

$$\hat{D}_0 G(z,u,v,\tau)\approx\lambda^2\sin^2\tau(e^{-i(\delta\tau+2kz)}u_0^*v-e^{i(\delta\tau+2kz)}v_0^*u)^2 , \quad (4.16)$$

which leads to the following simultaneous equations for the $g(\tau)$'s:

$$i\frac{d}{d\tau}g_1+4\delta g_1+\lambda g_4=\lambda^2(v_0^*)^2(\sin^2\tau)e^{2i\delta\tau} , \quad (4.17a)$$

$$i\frac{d}{d\tau}g_2+2\delta g_2+\lambda g_1=0 , \quad (4.17b)$$

$$i\frac{d}{d\tau}g_3+\lambda g_6=0 , \quad (4.17c)$$

$$i\frac{d}{d\tau}g_4+2\delta g_4+2\lambda g_1+2\lambda g_7=0 , \quad (4.17d)$$

$$i\frac{d}{d\tau}g_5+2\lambda g_2+2\lambda g_8=-2\lambda^2u_0^*v_0^*\sin^2\tau , \quad (4.17e)$$

$$i\frac{d}{d\tau}g_6-2\delta g_6+2\lambda g_3+2\lambda g_9=0 , \quad (4.17f)$$

$$i\frac{d}{d\tau}g_7+\lambda g_4=0 , \quad (4.17g)$$

$$i\frac{d}{d\tau}g_8-2\delta g_8+\lambda g_5=0 , \quad (4.17h)$$

$$i\frac{d}{d\tau}g_9-4\delta g_9+\lambda g_6=\lambda^2(u_0^*)^2(\sin^2\tau)e^{-2i\delta\tau} . \quad (4.17i)$$

The solution to Eqs. (4.17), satisfying the initial condition

$$g_1(0)=g_2(0)=\cdots=g_9(0)=0 , \quad (4.18)$$

can be easily obtained as follows:

$$g_1(\tau)=\frac{i}{2}\lambda^2(v_0^*)^2e^{2i\delta\tau}[h''(\tau)+2i\delta h'(\tau)+2\lambda^2h(\tau)] , \quad (4.19a)$$

$$g_2(\tau)=\lambda^3u_0^*v_0^*[h'(\tau)+2i\delta h(\tau)] , \quad (4.19b)$$

$$g_3(\tau)=-i\lambda^4(u_0^*)^2e^{-2i\delta\tau}h(\tau)\ , \quad (4.19c)$$

$$g_4(\tau)=-\lambda^3(v_0^*)^2e^{2i\delta\tau}[h'(\tau)+2i\delta h(\tau)]\ , \quad (4.19d)$$

$$g_5(\tau)=-i\lambda^2(u_0^*)^2v_0^*[h''(\tau)+4\delta^2h(\tau)]\ , \quad (4.19e)$$

$$g_6(\tau)=-\lambda^3(u_0^*)^2e^{-2i\delta\tau}[h'(\tau)-2i\delta h(\tau)]\ , \quad (4.19f)$$

$$g_7(\tau)=-i\lambda^4(v_0^*)^2e^{2i\delta\tau}h(\tau)\ , \quad (4.19g)$$

$$g_8(\tau)=\lambda^3u_0^*v_0^*[h'(\tau)-2i\delta h(\tau)]\ , \quad (4.19h)$$

$$g_9(\tau)=\frac{i}{2}\lambda^2(u_0^*)^2e^{-2i\delta\tau}[h''(\tau)-2i\delta h'(\tau)+2\lambda^2h(\tau)]\ , \quad (4.19i)$$

where

$$h(\tau)\equiv[3\sin(2\tau)-4\tau-2\tau\cos(2\tau)/16\ , \quad (4.20a)$$

$$h'(\tau)\equiv[\cos(2\tau)-1+\tau\sin(2\tau)]/4\ , \quad (4.20b)$$

$$h''(\tau)\equiv[-\sin(2\tau)+2\tau\cos(2\tau)]/4\ . \quad (4.20c)$$

Examining Eqs. (4.19) we observe the existence of the symmetric relation

$$g_i(\tau;u_0^*,v_0^*,\delta)=g_{10-i}(\tau;v_0^*,u_0^*,-\delta)\ , \quad (4.21)$$

where u_0, v_0, and δ are parameters; and examining Eqs. (4.20) we also observe that $h'(\tau)$ and $h''(\tau)$ are the first and the second derivatives, respectively, of $h(\tau)$.

We have now completely determined the expression for $A_l(z,u,v,\tau)$ in the form given in Eq. (4.12); we can use it in Eq. (4.1) to obtain

$$A(z,u,v,\tau)\approx A_0(z,u,v,\tau)\sum_{l=0}[\epsilon G(z,u,v,\tau)]^l/l!+\cdots\ , \quad (4.22)$$

where the upper limit of the summation is still uncertain; if it can be extended to infinity, then the summation will give a very simple exponential function. What we have to worry about is that the expression we have obtained for $A_l(z,u,v,\tau)$ is valid only when $l \ll N$.

The most critical factors in evaluating the accuracy of our perturbative solution are the time-dependent parts of $G(z,u,v,\tau)$; i.e., $h(\tau)$, $h'(\tau)$, and $h''(\tau)$. From Eqs. (4.20) we have, for $\tau \ll 1$,

$$h(\tau)=-4\tau^5/5!+\cdots\ , \quad (4.23a)$$

$$h'(\tau)=-\tau^4/3!+\cdots\ , \quad (4.23b)$$

$$h''(\tau)=-4\tau^3/3!+\cdots\ , \quad (4.23c)$$

and, for $\tau\sim 1$ or $\tau \gg 1$, we have

$$h(\tau)\sim h'(\tau)\sim h''(\tau)\sim\tau\ . \quad (4.23d)$$

On the other hand, from Eqs. (4.19) we can see that all the $q(\tau)$'s are proportional to u_0^2, u_0v_0, or v_0^2; therefore, putting the definitions of Eq. (4.13) and Eqs. (4.14) together, we can see that $G(z,u,v,\tau)$ is of quartic form in u_0, v_0, u, and v. Since we have $|u|^2+|v|^2\sim|u_0|^2+|v_0|^2\equiv N$, the time-independent parts of $G(z,u,v,\tau)$ should be of the order of N^2. Putting all these factors together, we have, for $\tau \ll 1$,

$$G(z,u,v,\tau)\sim\lambda^2N^2\tau^3,\quad \partial G(z,u,v,\tau)/\partial z\sim k\lambda^2N^2\tau^3\ , \quad (4.24a)$$

and, for $\tau\sim 1$ or $\tau\gg 1$, we have

$$G(z,u,v,\tau)\sim N^2\tau,\quad \partial G(z,u,v,\tau)/\partial z\sim kN^2\tau\ . \quad (4.24b)$$

We are now in the position to verify Eq. (4.15). Using Eqs. (4.11) and Eqs. (4.8) in Eq. (4.7) and taking the derivative, we have

$$\frac{\partial A_0}{\partial z}=2k\lambda\sin\tau(e^{-i(\delta\tau+2kz)}u_0^*v-e^{i(\delta\tau+2kz)}v_0^*u)A_0$$
$$\sim k(\sin\tau)NA_0\ . \quad (4.25)$$

From Eq. (4.12) we obtain

$$\frac{\partial}{\partial z}A_l(z,u,v,\tau)=\frac{G^l}{l!}\left[\frac{\partial A_0}{\partial z}+A_0\frac{l}{G}\frac{\partial G}{\partial z}\right]\ . \quad (4.26)$$

Using Eqs. (4.23) and Eqs. (4.24) we can estimate the order of the magnitude of the ratio between the two terms in Eq. (4.26) as

$$\frac{A_0(l/g)\partial G/\partial z}{\partial A_0/\partial z}\sim\begin{cases}\tau^2l/N & \text{for } \tau\ll 1 & (4.27a)\\ l/N & \text{for } \tau\sim 1 & (4.27b)\\ \tau l/N & \text{for } \tau\gg 1\ . & (4.27c)\end{cases}$$

Therefore, for $\tau\ll 1$ or $\tau\sim 1$, we can write

$$\frac{\partial}{\partial z}A_l(z,u,v,\tau)=\frac{G^l}{l!}A_0[2k\lambda\sin\tau(e^{-i(\delta\tau+2kz)}u_0^*v-e^{i(\delta\tau+2kz)}v_0^*u)][1+O(l/N)]\ . \quad (4.28)$$

Repeating a similar procedure once more, we verify Eq. (4.15).

Let us then consider the ratio between two consecutive terms in the perturbation series. From Eq. (4.12) we have

$$\epsilon A_l(z,u,v,\tau)/A_{l-1}(z,u,v,\tau)=\epsilon G(z,u,v,\tau)/l\ . \quad (4.29)$$

From Eqs. (4.24) we have

$$\epsilon G(z,u,v,\tau)\sim\begin{cases}\epsilon\lambda^2N^2\tau^3\sim\hbar(k\Lambda N)^2t^3/m_e & \text{for } \tau\ll 1 & (4.30a)\\ \epsilon N^2 & \text{for } \tau\sim 1 & (4.30b)\\ \epsilon N^2\tau & \text{for } \tau\gg 1\ . & (4.30c)\end{cases}$$

We can now examine the behavior of the summand in Eq.

(4.22) as a function of l under the following three different situations.

(1) $\epsilon G \leq 1$, the summand will decrease monotonically as l increases.

(2) $\epsilon G > 1$, the summand will increase first, reach a peak, and then decrease rapidly as l increases; the peak will occur at $l = l_0$ when two consecutive terms are about equal; i.e., $\epsilon G / l_0 \sim 1$; and we have $l_0 \ll N$.

(3) Same as the second situation except that $l_0 \sim N$.

Under the first two situations, it is safe to extend the summation of Eq. (4.22) to infinity. Then we can have

$$A(z,u,v,\tau) \approx A_0(z,u,v,\tau)\exp[\epsilon G(z,u,v,\tau)] . \quad (4.31)$$

Using Eq. (4.31) in Eq. (3.4) we can obtain the probability distribution function in Q representation. Under the third situation, the exponential expression as an approximation to the perturbation series will break down; this will occur when

$$\epsilon N \tau \equiv ENt \sim 1 . \quad (4.32)$$

V. STATISTICS OF A GAUSSIAN DISTRIBUTION IN PHASE SPACE

The solution obtained in Sec. IV will lead to Gaussian distributions in the phase space of Q representation for both the forward- and the backward-propagating radiation. Therefore it will be convenient to derive the general expressions for the various expectation values of physical quantities relevant to the nonclassical properties of radiation with arbitrary Gaussian distribution in phase space.

Let $w \equiv x + iy$ be the complex variable of the phase space and let $a^\dagger$ (a) be the creation (annihilation) operator for a single-mode radiation. Then, according to the rule of Q representation, any operator expressed in terms of a and $a^\dagger$ in antinormal order can be replaced by a corresponding function of the complex variable as follows:[20]

$$F(a,a^\dagger) \rightarrow F(w,w^*) . \quad (5.1)$$

Using this rule, we can derive the classical expressions corresponding to some important operators as follows.

(1) From the definitions of $\hat{X}$ and $\hat{Y}$ by Eqs. (1.3), we have

$$\hat{X} \rightarrow (w + w^*)/2 = x , \quad (5.2a)$$

$$\hat{Y} \rightarrow (w - w^*)/2i = y , \quad (5.2b)$$

$$\hat{X}^2 \rightarrow (w^2 + 2ww^* + w^{*2} - 1)/4 = x^2 - \tfrac{1}{4} , \quad (5.3a)$$

$$\hat{Y}^2 \rightarrow -(w^2 - 2ww^* + w^{*2} + 1)/4 = y^2 - \tfrac{1}{4} . \quad (5.3b)$$

(2) Let $\hat{n} \equiv a^\dagger a$ be the photon number operator, and we have

$$\hat{n} \rightarrow ww^* - 1 = (x^2 + y^2) - 1 , \quad (5.4)$$

$$(\hat{n})^2 \rightarrow w^2 w^{*2} - 3ww^* + 1$$
$$= (x^2 + y^2)^2 - 3(x^2 + y^2) + 1 . \quad (5.5)$$

An arbitrary Gaussian distribution in phase space can be expressed as

$$P(x,y) \propto \exp[-(ax^2 + 2bxy + cy^2 + dx + ey)] . \quad (5.6)$$

Using the normalized $P(x,y)$ we can evaluate the following expectation values:

$$\langle x \rangle = -(cd - be)/D , \quad (5.7)$$

$$\langle y \rangle = -(ae - bd)/D , \quad (5.8)$$

$$\langle x^2 \rangle = (cd - be)^2/D^2 + c/D , \quad (5.9)$$

$$\langle y^2 \rangle = (ae - bd)^2/D^2 + a/D , \quad (5.10)$$

$$\langle x^4 \rangle = (cd - be)^4/D^4 + 6c(cd - be)^2/D^3 + 3c^2/D^2 , \quad (5.11)$$

$$\langle y^4 \rangle = (ae - bd)^4/D^4 + 6a(ae - bd)^2/D^3 + 3a^2/D^2 , \quad (5.12)$$

$$\langle x^2 y^2 \rangle = ac(ae^2 + cd^2 - 2bde)^2/D^4 + [(6ac + bde)(ae^2 + cd^2) - (ae^2 + cd^2)^2/2 - 12abcde]/D^3$$
$$+ [3ac + 2bde - 5(ae^2 + cd^2)/2 + d^2e^2/4]/D^2 - 1/D , \quad (5.13)$$

where

$$D \equiv 2(ac - b^2) . \quad (5.14)$$

Using Eqs. (5.7)–(5.10) we obtain

$$\langle (\Delta X)^2 \rangle = c/D - \tfrac{1}{4} , \quad (5.15a)$$

$$\langle (\Delta Y)^2 \rangle = a/D - \tfrac{1}{4} . \quad (5.15b)$$

Substitution of Eqs. (5.15) into Eq. (1.6) gives the criterion for the existence of squeezing as either

$$c/D < \tfrac{1}{2} \text{ or } a/D < \tfrac{1}{2} . \quad (5.16)$$

Using Eqs. (5.9) and (5.10) in Eq. (5.4), we obtain

$$\langle n \rangle = [(ae - bd)^2 + (cd - be)^2]/D^2 + (a + c)/D - 1 , \quad (5.17)$$

and using Eqs. (5.9)–(5.13) in Eq. (5.5), we obtain

$$\langle (\Delta n)^2 \rangle = 4(a + c)^2(ae^2 + cd^2 - 2bde)/D^3$$
$$+ [4bde - 4(ae^2 + cd^2) - 2(ad^2 + ce^2)]/D^2$$
$$- 2/D - \langle n \rangle - 1 . \quad (5.18)$$

The existence of photon antibunching is determined by the condition

$$q \equiv [\langle (\Delta n)^2 \rangle - \langle n \rangle]/\langle n \rangle < 0 , \quad (5.19)$$

where q is the parameter introduced by Mandel.[21]

It should be pointed out that the parameters a, b, and c are each of the order of 1, while (d^2+e^2) is of the order of $\langle n \rangle$. Therefore, when $\langle n \rangle \gg 1$, the expressions in Eqs. (5.17) and (5.18) can be simplified as

$$\langle n \rangle \approx [(ae-bd)^2+(cd-be)^2]/D^2 , \tag{5.20}$$

$$\langle (\Delta n)^2 \rangle \approx 4(a+c)^2(ae^2+cd^2-2bde)/D^3 + [4bde-4(ae^2+cd^2)-2(ad^2+ce^2)]/D^2 - \langle n \rangle . \tag{5.21}$$

VI. STATISTICS OF THE FORWARD-PROPAGATING RADIATION

The photon statistics of the forward-propagating radiation mode should be calculated from the marginal probability distribution

$$P_f(x,y) \equiv K_0^2 \int dz \int \frac{d^2u}{2\pi} Q(z,u,u^*,v,v^*,\tau) , \tag{6.1}$$

where we let $v \equiv x+iy$. However, it is not easy to carry out the integration with respect to z. Therefore, assuming that the order of integrations is exchangeable, the integration with respect to z will be the last step in our calculation of various expectation values. Let us define an intermediate distribution function

$$R_f(z,x,y,\tau) \equiv \int \frac{d^2u}{2\pi} Q(z,u,u^*,v,v^*,\tau) , \tag{6.2}$$

and let the intermediate expectation value of an expression $F(x,y)$ be denoted by

$$\langle\langle F(x,y) \rangle\rangle_z \equiv \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy \, F(x,y) R_f(z,x,y,\tau) , \tag{6.3}$$

where the subscript z is to emphasize the fact that this "expectation value" is still a function of z. Then the true expectation value is finally obtained as

$$\langle F(x,y) \rangle \equiv K_0^2 \int \langle\langle F(x,y) \rangle\rangle_z dz . \tag{6.4}$$

Using Eq. (3.4) in Eq. (6.2) we have

$$R_f(z,x,y,\tau) \propto \exp[-(ax^2+2bxy+cy^2+dx+ey)] , \tag{6.5}$$

where a, b, c, d, and e are functions of z and τ as follows:

$$a = 1-\epsilon(G_3^*+G_3) - \frac{\epsilon^2}{H}\{ |G_2|^2+\epsilon[G_1^*(G_2)^2+G_1(G_2^*)^2]\} , \tag{6.6a}$$

$$b = i\epsilon \left[G_3^*-G_3-\frac{\epsilon^2}{H}[G_1^*(G_2)^2-G_1(G_2^*)^2] \right] , \tag{6.6b}$$

$$c = 1+\epsilon(G_3^*+G_3) - \frac{\epsilon^2}{H}\{ |G_2|^2-\epsilon[G_1^*(G_2)^2+G_1(G_2^*)^2]\} , \tag{6.6c}$$

$$d = -\left[F_2^*+F_2+\frac{\epsilon}{H}[F_1^*G_2+F_1G_2^* + 2\epsilon(F_1^*G_1G_2^*+F_1G_1^*G_2)] \right] , \tag{6.6d}$$

$$e = i\left[F_2^*-F_2-\frac{\epsilon}{H}[F_1^*G_2-F_1G_2^* - 2\epsilon(F_1^*G_1G_2^*-F_1G_1^*G_2)] \right] , \tag{6.6e}$$

where the F's and the G's are defined by Eqs. (4.8) and (4.14) and

$$H \equiv 1-4\epsilon^2 |G_1|^2 . \tag{6.7}$$

Because of this last expression in the denominators of Eqs. (6.6), we will have a singularity as $\epsilon |G_1| \to \frac{1}{2}$ or, roughly, as $\epsilon N\tau \to 1$. This is the same condition as that of Eq. (4.32) when the exponential expression as an approximation to the perturbation series breaks down.

Substituting the expressions in Eqs. (6.6) into Eqs. (5.15), (5.20), and (5.21), we can obtain the corresponding intermediate expectation values. However, the existence of exponential functions of z in the denominators of these intermediate expectation values still makes it very difficult to carry out the final integrations with respect to z. So from now on we will keep only the first-order terms in ϵ in all the analytical expressions.

A. Squeezing effect

Using Eqs. (6.6a)–(6.6c) in Eqs. (5.15), we have

$$\langle\langle (\Delta X)^2 \rangle\rangle_z \approx \frac{1}{4}+\frac{\epsilon}{2}(G_3^*+G_3) , \tag{6.8a}$$

$$\langle\langle (\Delta Y)^2 \rangle\rangle_z \approx \frac{1}{4}-\frac{\epsilon}{2}(G_3^*+G_3) . \tag{6.8b}$$

We then have to carry out the integration with respect to z. Using Eqs. (3.2) and (4.14c) we obtain

$$\langle G_3^*+G_3 \rangle \equiv k_0^2 \int (G_3^*+G_3) dz = g_7^*+g_7 = 2N_f\lambda^4 \sin(2\delta\tau+2\phi) h(\tau) , \tag{6.9}$$

where we have used Eq. (4.19g) and let

$$v_0 \equiv \sqrt{N_f} e^{-i\phi} . \tag{6.10}$$

with N_f being the expectation value of the photon number in the forward-propagating radiation initially. Using Eq. (6.9) in Eqs. (6.8) we obtain

$$\langle (\Delta X)^2 \rangle \approx \tfrac{1}{4}+\epsilon\lambda^4 N_f S(\tau) , \tag{6.11a}$$

$$\langle (\Delta Y)^2 \rangle \approx \tfrac{1}{4}-\epsilon\lambda^4 N_f S(\tau) , \tag{6.11b}$$

with

$$S(\tau) \equiv \sin(2\delta\tau+2\phi)[3\sin(2\tau)-4\tau-2\tau\cos(2\tau)]/16 , \tag{6.12}$$

where we have used Eq. (4.20a). It is obvious that $S(\tau)$ plays the key role in determining the squeezing effect; if it is positive then squeezing occurs in the y quadrature; if negative, it occurs in the x quadrature; either way, squeezing always exists as long as both ϵ and N_f are not vanishing. This last point is very interesting because it implies that no squeezing is possible without considering quantum recoils of the electron or if the initial state of the radiation is a vacuum state. It should also be pointed out that the appearance of ϕ in the expression for $S(\tau)$ means that the phase angle of the initial radiation also plays an important role in squeezing, as expected.

From Eqs. (6.11) we can see that the degree of squeezing will increase as the initial photon number N_f increases. But, of course, it will not increase to the point that we have a negative value for either $\langle(\Delta X)^2\rangle$ or $\langle(\Delta Y)^2\rangle$; firstly, this point will not be reached before our exponential expression as an approximation to the perturbation series breaks down when $\epsilon N\tau\sim 1$, since we have $N_f<N$, $\lambda<1$, and $S(\tau)\sim\tau$; secondly, even before this breakdown, we might have to include higher-order terms in ϵ to be accurate enough. Therefore we must keep in mind that expressions in Eqs. (6.11) are reliable only for short-time behavior.

A plot of $S(\tau)$ as a function of τ with $\phi=0$, $\pi/4$, $\pi/2$, $3\pi/4$ with fixed $\delta=1/\sqrt{2}$ is presented in Fig. 1.

B. Photon antibunching

We now substitute Eqs. (6.6) into Eq. (5.20) to obtain

$$\langle\langle n\rangle\rangle_z \approx |F_2|^2+\epsilon\{[F_1F_2G_2^*+2(F_2)^2G_3^*]+\text{c.c.}\}\ , \tag{6.13}$$

$$\langle\langle(\Delta n)^2\rangle\rangle_z \approx |F_2|^2 +\epsilon\{[F_1F_2G_2^*+4(F_2)^2G_3^*]+\text{c.c.}\}\ , \tag{6.14}$$

where c.c. stands for complex conjugate. The next step is to carry out the integration with respect to z.

Using Eqs. (4.8) and (4.14) we have

$$J_0(\tau)\equiv\langle |F_2|^2\rangle\equiv K_0^2\int |F_2|^2dz=|f_3|^2+|f_4|^2=|v_0|^2(\cos^2\tau+\delta^2\sin^2\tau)+|u_0|^2\lambda^2\sin^2\tau\ , \tag{6.15a}$$

$$\begin{aligned}J_1(\tau)\equiv\langle F_1F_2G_2^*\rangle+\text{c.c.}&=[f_1f_3g_4^*+(f_1f_4+f_2f_3)g_5^*+f_2f_4g_6^*]+\text{c.c.}\\&=2\delta\lambda^4(|u_0|^4-|v_0|^4)\sin\tau[(\sin\tau)h'(\tau)+2(\cos\tau)h(\tau)]\ ,\end{aligned} \tag{6.15b}$$

$$\begin{aligned}J_2(\tau)\equiv\langle (F_2)^2G_3^*\rangle+\text{c.c.}&=(f_3)^2g_7^*+2f_3f_4g_8^*+(f_4)^2g_9^*\\&=4\delta\lambda^4|v_0|^4\sin\tau\cos\tau\, h(\tau)-2\delta\lambda^4|u_0|^4(\sin^2\tau)h'(\tau)\\&\quad+4\delta\lambda^4|u_0|^2|v_0|^2\sin\tau[(\sin\tau)h'(\tau)-2(\cos\tau)h(\tau)]\ ,\end{aligned} \tag{6.15c}$$

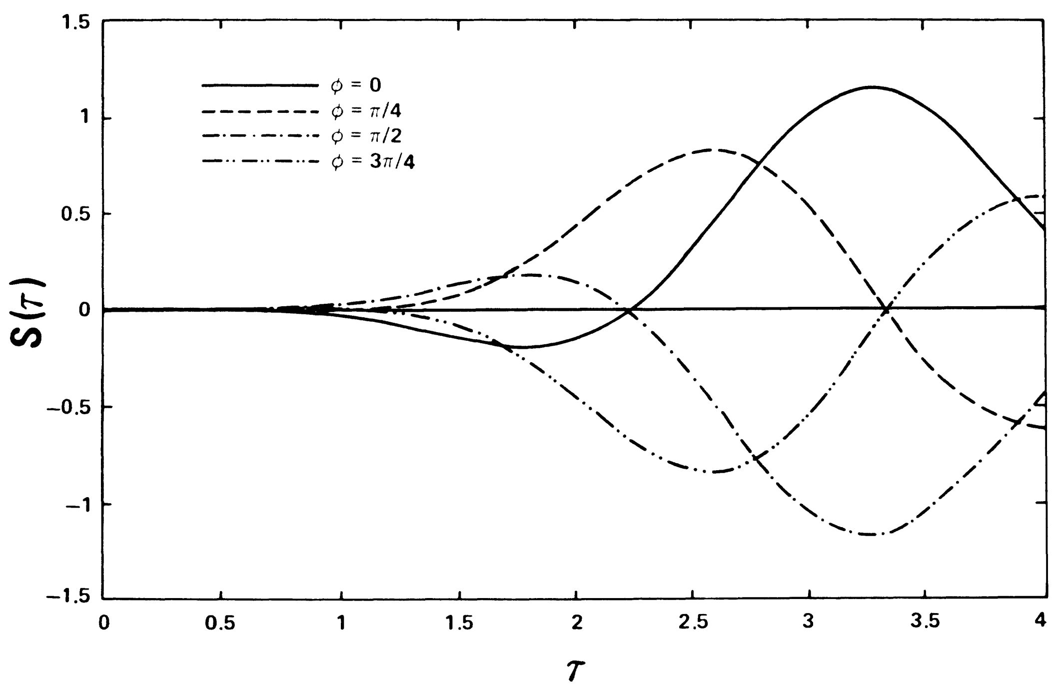

FIG. 1. $S(\tau)$ vs τ. The lines correspond to $\phi=0$, $\pi/4$, $\pi/2$, and $3\pi/4$, while $\delta=1/\sqrt{2}$. $S(\tau)$ is the indicator for squeezing. When it is negative, squeezing occurs in the x quadrature; otherwise, in the y quadrature.

where we have used Eqs. (4.11) and (4.19) to obtain the last expressions in each of Eqs. (6.15).

Substituting Eqs. (6.15) into Eqs. (6.13) and (6.14), we have

$$\langle n \rangle \approx J_0(\tau)+\epsilon[J_1(\tau)+2J_2(\tau)] , \qquad (6.16a)$$

$$\langle (\Delta n)^2 \rangle \approx J_0(\tau)+\epsilon[J_1(\tau)+4J_2(\tau)] . \qquad (6.16b)$$

We now use Eqs. (6.16) in Eq. (5.19) to obtain

$$q \approx 2\epsilon J_2(\tau)/J_0(\tau)=2\epsilon\delta\lambda^4 |u_0|^2 T(\tau) , \qquad (6.17)$$

where

$$T(\tau)\equiv \frac{\rho(\rho-2)\sin(2\tau)h(\tau)+(2\rho-1)(\sin^2\tau)h'(\tau)}{\lambda^2\sin^2\tau+\rho(\cos^2\tau+\delta^2\sin^2\tau)} , \qquad (6.18)$$

with

$$\rho\equiv |v_0|^2/|u_0|^2=N_f/N_b \qquad (6.19)$$

being a parameter which is the ratio of the expectation values of initial photon numbers in the two lights.

From Eq. (6.17) we can see that the condition for the existence of photon antibunching is that δ and $T(\tau)$ must have opposite signs. Since δ can have either sign, depending on whether the initial momentum of the electron p_0 is positive or negative in the moving frame, we conclude that photon antibunching is always possible, one way or the other. We must again keep in mind that Eq. (6.17) is reliable only for short time because we only consider the first-order perturbation. A plot of $T(\tau)$ as a function of τ with $\delta^2=\lambda^2=\frac{1}{2}$ and $\rho=0,0.5,1,2$ is presented in Fig. 2. From Eq. (6.17) we also notice that if ϵ vanishes, q will vanish too; this implies that quantum recoils of the electron are essential for the occurrence of photon antibunching.

VII. SUMMARY

We have analytically studied the stimulated Compton scattering (SCS) in a one-dimensional system consisting of a relativistic electron and two oppositely propagating lights. The two lights are both in coherent states initially and are of quite different frequencies in the laboratory frame. We have adopted a convenient frame, moving along the same direction as the electron, in which the two frequencies become identical due to the Doppler effect. We have considered only a narrow range of the speed of the electron, such that it is nonrelativistic in the moving frame adopted and still has an initial momentum much greater than that of the photon. This is essentially the same as the so-called Bambini-Renieri frame popular in free-electron laser theory.

We have described the time evolution for the exchange of photons between the two lights by SCS in terms of the Q representation of two-mode coherent states. Our main interest has been to see whether the nonclassical phenomena such as squeezing and photon antibunching occur as a result of SCS.

We have obtained a perturbative solution far beyond the first order of the perturbation parameter ϵ which is related to the quantum recoil of the electron. As long as $\epsilon N\tau \ll 1$, we can extend the perturbation series to infinite order to obtain a simple exponential expression for the distribution function in phase space. However, in the last

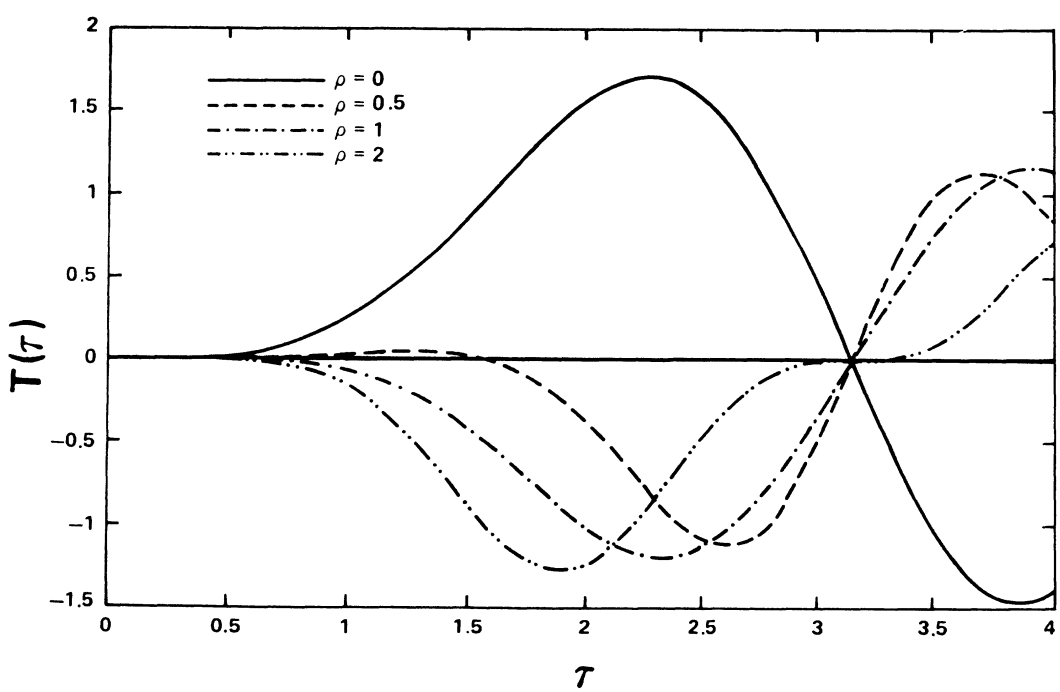

FIG. 2. $T(\tau)$ vs τ. The lines correspond to $\rho=0$, 0.5, 1, and 2, while $\delta=\lambda=1/\sqrt{2}$. $T(\tau)$ is the indicator for photon antibunching. When it is negative, the initial momentum of the electron p_0 must be positive for photon antibunching to exist; otherwise, p_0 must be negative.

stage of our calculations, we have kept terms only up to the first order of the perturbation parameter. Our main conclusions are that, within this limitation, squeezing always exists and photon antibunching is always possible if we are free to pick the right initial electron momentum.

Unlike the various studies of the closely related problem of the FEL in the literature, we have treated the quantum states of the electron as separated from those of the photons. A very important result of such treatment is the revelation that squeezing is not possible if the initial quantum state of the radiation is a vacuum state, contrary to the conclusion of the various studies mentioned above.

We have also reconfirmed that quantum recoils play the exclusive role in generating nonclassical photon states by SCS.

There exists a symmetry between the two lights. Therefore, although we have carried out the detailed calculations for the forward-propagating light only, the corresponding formulas for the other light can always be easily obtained by exchanging u_0 and v_0 and replacing δ by $-\delta$.

We have completely ignored many-electron effects in the present study. In a possible future publication, we will include these effects.

ACKNOWLEDGMENT

This work was supported by the U.S. Navy, Office of Naval Research, under Grant No. N00014-82-K-0521.

[1]H. J. Kimble and L. Mandel, Phys. Rev. A **13**, 2123 (1976).
[2]H. J. Carmichael and D. F. Walls, J. Phys. B **9**, 1199 (1976).
[3]H. J. Kimble, M. Dagenais, and L. Mandel, Phys. Rev. Lett. **39**, 691 (1977).
[4]See, for example, the review article by H. Paul, Rev. Mod. Phys. **54**, 1061 (1982).
[5]D. Stoler, Phys. Rev. D **4**, 1925 (1971).
[6]E. Y. C. Lu, Nuovo Cimento Lett. **2**, 1241 (1971).
[7]R. E. Slusher, L. W. Hollberg, B. Yurke, J. C. Mertz, and J. F. Valley, Phys. Rev. Lett. **55**, 2409 (1985); R. E. Slusher, P. Grangier, A. LaPorta, B. Yurke, and M. J. Potasek, Phys. Rev. Lett. **59**, 2566 (1987).
[8]L. A. Wu, H. J. Kimble, J. L. Hall, and H. Wu, Phys. Rev. Lett. **57**, 2520 (1986).
[9]R. Loudon and P. L. Knight, J. Mod. Opt. **34**, 709 (1987); and the recent collection of articles in J. Opt. Soc. Am. B **4**, No. 10 (1987) (special issue on squeezed states).
[10]R. J. Glauber, Phys. Rev. Lett. **10**, 84 (1963).
[11]E. C. G. Sudarshan, Phys. Rev. Lett. **10**, 277 (1963).
[12]V. Peřinová, J. Peřina, A. Lukš, C. Sibilia, and M. Bertolotti, Opt. Acta **31**, 735 (1984).
[13]W. Becker, M. O. Scully, and M. S. Zubairy, Phys. Rev. Lett. **48**, 475 (1982).
[14]G. Dattoli and M. Richetta, Opt. Commun. **50**, 165 (1984).
[15]C. T. Lee, Phys. Rev. A **31**, 1213 (1985).
[16]M. Orszag and R. Ramírez, J. Opt. Soc. Am. B **3**, 895 (1986).
[17]J. M. Madey, J. Appl. Phys. **42**, 1906 (1971).
[18]C. W. Raman and N. S. Nath, Proc. Indian Acad. Sci. **2**, 405 (1937).
[19]Y. Kano, J. Math. Phys. **6**, 1913 (1965).
[20]See, for example, H. Von Haken, *Light and Matter Ic,* Vol. 25 of *Handbuch der Physik,* edited by S. Flügge (Springer, New York, 1970), p. 62.
[21]L. Mandel, Opt. Lett. **4**, 205 (1979).