Squeezing and dressed-state polarization of driven atoms coupled to a frequency-dependent vacuum reservoir

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We consider the response of an isolated two-level atom to strong-field excitation in the special case where the atom decays via coupling to a frequency-dependent photon-mode density. It is found that atomic squeezing substantially in excess of that found in the case of free-space atoms arises. The enhanced squeezing is related to a polarization of the atom-field dressed states that is also found to occur.

INTRODUCTION

Squeezing[1] of laser-driven atoms and the fluorescent light scattered from them as received the attention of several authors over the last few years. A number of different situations have been investigated. Walls and Zoller[2] drew initial attention to this general problem with their work on the squeezing of atomic and field observables in the case of a single, externally driven, two-level atom in free space. A number of other workers have analyzed closely related single-atom[3–5] and multiple atom[6–8] effects. The subject of atomic squeezing has also arisen in the general context of optical bistability, where situations involving both single-atom[9] and collective atomic[10–14] behavior have been analyzed. Recently, an experimental observation of field squeezing under conditions similar to those analyzed in the context of optical bistability has been reported.[15] It should also be noted that collective atomic squeezing has also been discussed in relation to Rydberg atom masers.[16–18]

In the present paper, we concentrate on the atomic squeezing that occurs when a single atom, whose relaxation is determined through coupling to a frequency-dependent reservoir of photon modes, is exposed to a strong external driving field. It is the presence of the frequency-dependent reservoir that differentiates the present situation from those previously analyzed. Such a frequency-dependent mode density arises physically when, for example, an atom is placed inside a mode-degenerate optical resonator.[19] By controlling the frequency distribution of the reservoir modes, one can modify an atom's relaxational dynamics and, as we point out, generate significantly larger values of atomic squeezing than are possible in the presence of the essentially frequency-independent photon-mode density characteristic of free space.[2] Because mode-degenerate cavities provide one means of producing frequency-dependent photon reservoirs, we discuss this problem in terms of atoms confined within cavities. In using this description, the implicit assumption is made that the cavity is of a type that possesses a frequency-dependent total mode density.

FORMALISM

Consider a single atom. The atom is driven by an applied field, and resides in an open-sided optical cavity that modifies the free-space density of photon modes. As modified, the photon-mode density consists of a spectrally flat background of free-space-like modes plus a spectrally concentrated peak of cavitylike modes. The frequency of maximum mode density is denoted by ω_c. This system[20] can be described by the Hamiltonian

$$H=\frac{\omega_a}{2}\sigma_3+\Omega(e^{i\omega_L t}\sigma+e^{-i\omega_L t}\sigma^\dagger)+\int|k|c_k^\dagger c_k dk+\int|k|b_k^\dagger b_k dk+\int g_c(k)(c_k^\dagger\sigma+\sigma^\dagger c_k)dk+\int g_b(k)(b_k^\dagger\sigma+\sigma^\dagger b_k)dk\ , \quad (1)$$

where the σ's are the usual Pauli matrices describing a two-level atom, ω_a (ω_L) is the frequency of the atomic transition (driving field), and Ω is the driving-field Rabi frequency. The operators c_k and b_k correspond, respectively, to cavity-type and free-space-type photon-modes. The coupling constants $g_b(k)$ and $g_k(k)$ are connected to the corresponding photon-mode densities. Since $|g_b(k)|^2$ is needed only in the neighborhood of ω_a, ω_L, and ω_c, it may be treated as a constant. The coupling $|g_c(k)|^2$ is taken to be a single Lorentzian of full width at half maximum Γ. For further discussion see Ref. 20.

The first step in solving the problem of time evolution of the system, described by Hamiltonian (1), requires the derivation of atomic Bloch equations, i.e., equations describing the evolution of mean values of the atomic observables σ, $\sigma^\dagger$, and σ_3. In doing so, it is convenient to introduce the reservoir response functions

$$\int|g_c(k)|^2\exp[i(|k|-\omega_c)t]dk\simeq\gamma_c\Gamma e^{-\Gamma|t|}\ , \quad (2a)$$

$$\int|g_b(k)|^2\exp[i(|k|-\omega_c)t]dk\simeq\gamma_b\delta(t)\ . \quad (2b)$$

Note that Eq. (2a) expresses the fact that photon modes associated with the cavity resonance have a finite response time Γ^{-1}. It is clear that interaction of the atom with the cavity (background) modes contributes an amount γ_c (γ_b) to the overall undriven spontaneous emission rate $\gamma_{\rm tot}$, if the atoms and the cavity are resonant ($\omega_a = \omega_c$).

Assuming $\Omega, \Gamma \gg \gamma_b, \gamma_c$, the Bloch equations may be derived by eliminating the photon operators through a first-order expansion in γ_b or γ_c (Born approximation). However, if the case $\Omega > \Gamma$ or $\Delta_1 \equiv (\omega_L - \omega_a) > \Gamma$ is to be included, one is not allowed to perform a Markov approximation for the cavity modes. This is due to the fact that for Ω or Δ_1 greater than Γ the atoms change their state, before the cavity field has a change to adjust to it. The resulting Bloch equations must therefore contain characteristic memory terms. The memory extends to times of the order of Γ^{-1} and affects the damping terms as well as dynamical frequency shifts. The explicit (but rather complicated) form of these Bloch equation is published elsewhere.[20] Here we concentrate on qualitative discussion and presentation of some quantitative results.

A similar approach can be used to obtain the equations for all the single-time correlation functions, such as atom-field correlations, etc. Contrary to the Markovian theory (valid in free space), the equations for multitime correlation functions have to be derived separately from the Bloch equations, since the quantum regression theorem (see, for example, Ref. 21) does not hold in our present case.

One should stress that the equations obtained this way are valid only in the sense of a perturbative expansion in γ_ν/Γ and γ_ν/Ω, where $\nu = b, c$. This expansion can be systematically corrected to include the terms of higher orders. The calculations of higher-order terms for single-time correlation functions have in fact been done by us in order to study the regime when γ_c is comparable to Γ or Ω, and will be presented in Ref. 20. For most of the purposes of this communication the lowest-order results are sufficient.

Let us now concentrate on stationary (long-time, $t \to \infty$) atomic-squeezing effects. Such squeezing is characterized by variances of different components of the Bloch vector. The relative squeezing parameters are defined as

$$r_1 = \lim_{t\to\infty} \frac{\Delta^2\sigma_1(t)}{|\langle \sigma_3(t)\rangle|} = \lim_{t\to\infty} \frac{1 - \langle \sigma_1(t)\rangle^2}{|\langle \sigma_3(t)\rangle|} , \tag{3a}$$

$$r_2 = \lim_{t\to\infty} \frac{\Delta^2\sigma_2(t)}{|\langle \sigma_3(t)\rangle|} = \lim_{t\to\infty} \frac{1 - \langle \sigma_2(t)\rangle^2}{|\langle \sigma_3(t)\rangle|} . \tag{3b}$$

The atomic states are squeezed with respect to σ_1 (σ_2) when r_1 (r_2) is smaller than 1. As we see, the parameters r_1 and r_2 can be calculated directly from the stationary solutions of the modified Bloch equations. It is the parameter r_1 which shows squeezing for Δ_1 comparable to Ω in free space[2] and which is expected to do so in the cavity as well.

DISCUSSION

We now qualitatively consider the behavior of the steady-state values of σ_1 and σ_3 in free space and in a cavity in order to obtain some insight into resulting atomic squeezing. In free space and for a fixed, but large Ω, the atomic inversion σ_3 vanishes approximately for $\Delta_1 = 0$ and approaches -1 for $\Delta_1 \gg \Omega$. On the other hand, the atomic coherence component σ_1 vanishes for both $\Delta_1 = 0$ and $\Delta_1 \gg \Omega$, but attains a nonzero maximum somewhere in between. The ratio r_1 achieves its minimum value (i.e., squeezing is maximized) in the region $\Delta_1 \simeq \Omega$, where σ_1 is maximized. The extent of squeezing in free space is determined by the quantitative behavior of σ_1 and σ_3 as a function of Δ_1.

Let us now consider atomic behavior in the presence of a frequency-dependent photon-mode density as described in the preceding. For simplicity, we assume that the density of background modes is negligibly small ($\gamma_b \ll \gamma_c$). Provided that

$$(\Omega^2 + \Delta_1^2)^{1/2} \gg \Gamma ,$$

it is possible to tune the cavity close to particular dressed-state transition frequencies [i.e., ω_L, and $\omega_L \pm (\Omega^2 + \Delta_1^2)^{1/2}$]. By tuning close to the sideband frequencies $[\omega_L \pm (\Omega^2 + \Delta_1^2)^{1/2}]$ one enhances one of the transitions between the dressed states.[22] In this way, one is able to modify the steady-state dressed-state inversion which is proportional to

$$\omega_{\rm dr} = \frac{\Omega}{\Omega'} \langle \sigma_1 \rangle + \frac{\Delta_1}{\Omega'} \langle \sigma_3 \rangle , \tag{4}$$

where $\Omega' = (\Omega^2 + \Delta_1^2)^{1/2}$. In effect, one can polarize the dressed-state populations.[23] Note that this polarization effect (an asymmetry in populations within dressed-state doublets) is induced by a difference between the density of photon modes at $\omega = \omega_L + \Omega'$ and $\omega = \omega_L - \Omega'$. By inducing a dressed-state polarization, one also modifies the values of the steady-state atomic observables $\langle \sigma_i \rangle$ ($i = 1, 2, 3$).

Note also that the polarizing effects discussed in the preceding can be obtained even in the resonant case. Figure 1 shows the population inversion of the dressed states for $\Delta_1 = 0$ and $\Delta_2 = 30\gamma_c$ as a function of Ω (for $\Delta_1 = 0$, the inversion is simply proportional to σ_1). The reduction of relative Bloch vector fluctuations (as represented by $r_1 - 1$) is shown in Fig. 2. Although no squeezing is obtained in the resonant case, the comparison with the free-space results ($\Gamma = 10\,000\gamma_c$) shows dramatic differences. Note that σ_1 does not vanish as it does in the free-space case.

Similar results are presented in Figs. 3 and 4 for the nonresonant case ($\Delta_1 \neq 0$). As we see, dressed-state polarization is again observed. For $\Delta_1 \neq 0$, the dressed-state inversion is no longer simply equal to σ_1, but it becomes ever more nearly so as Ω grows larger than Δ_1. Thus in the $\Omega > \Delta_1$ region, at least, we can see from Fig. 3 that the cavity enhances the value of σ_1. It turns out that the behavior of σ_3 as a function of Ω is only weakly affected

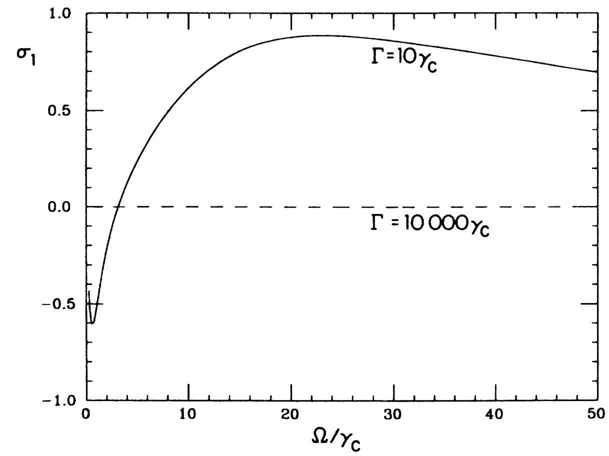

FIG. 1. σ_1 vs Ω/γ_c for $\Delta_1=0$. In free space ($\Gamma=10\,000\gamma_c$), σ_1 is identically zero. In cavity ($\Delta_2=30\gamma_c$, $\Gamma=10\gamma_c$), σ_1 displays a broad maximum that peaks in the vicinity of $\Omega\simeq\Delta_2$. In the resonant case ($\Delta_1=0$), σ_1 corresponds to the dressed-state inversion. $\gamma_b/\gamma_c=0.01$.

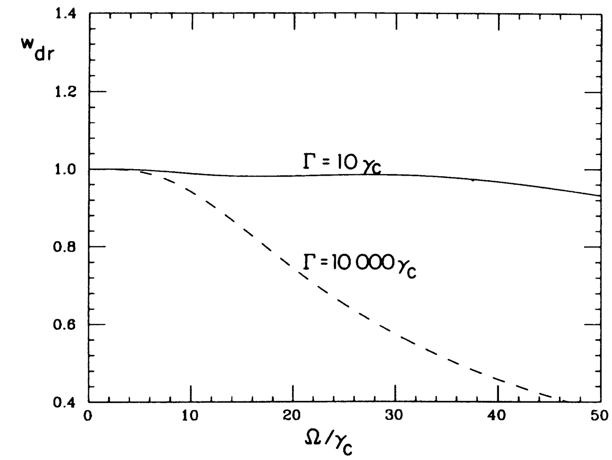

FIG. 3. Dressed-state inversion ω_{dr} [as defined in Eq. (4)] vs Ω/γ_c for $\Delta_1=10\gamma_c$. In free space ω_{dr} is 1 for small Ω (because of large Δ_1) and decays to zero as Ω grows. In the cavity ($\Delta_2=30\gamma_c$, $\Gamma=10\gamma_c$), ω_{dr} stays close to 1 for much larger Ω due to a broad maximum at $\Omega'\simeq\Delta_2$. $\gamma_b/\gamma_c=0.01$.

by the cavity.

Results plotted in Fig. 4 indicate that the cavity increases both the extent of squeezing and the range of Ω values over which it occurs. As mentioned in the preceding, free-space squeezing is optimized when $\Omega\simeq\Delta_1$ and attains a maximum value of about 16%.[2] In the cavity, optimal squeezing occurs for $\Omega'\simeq\Delta_2$ and attains a relative value of 60%. The cavity width $\Gamma=10\gamma_c$, laser-atom detuning $\Delta_1=10\gamma_c$, and the laser-cavity detuning $\Delta_2=30\gamma_c$ were chosen by trial and error to maximize the squeezing.

Unfortunately, the large atomic squeezing effects described in the preceding do not carry over to the light scattered by atoms. In fact, total field variances (associated with both background and cavity field modes) are squeezed by only a few percent. While the field scattered into the background modes is directly related to the instantaneous value of the atomic dipole moment, squeezing of this field appears to be destroyed because of the small fraction of photons actually scattered into the background modes (recall we have restricted our attention to the case of $\gamma_b \ll \gamma_c$). The cavity field, on the other hand, is connected to the atomic dipoles through convolution-type time integrals extending over times of the order of Γ^{-1}. The convolution process evidently degrades the squeezing of the cavity field. If one considers the spectrum of squeezing associated with the cavity field, one

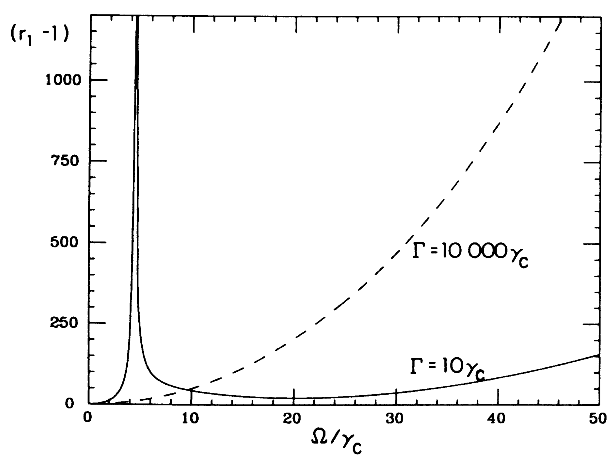

FIG. 2. Relative variance (r_1-1) vs Ω/γ_c for $\Delta_1=0$. In free space r_1-1 grows with Ω. The growth corresponds to the fact that $\sigma_3\to0$ for $\Omega\to\infty$ as $1/\Omega^2$. In the cavity ($\Delta_2=30\gamma_c$, $\Gamma=10\gamma_c$) we find that r_1 is always greater than 1 and is maximum for "small" Ω (due to the fact that σ_3 crosses 0). For large Ω, r_1 displays a broad minimum centered around $\Omega\simeq\Delta_2$. $\gamma_b/\gamma_c=0.01$.

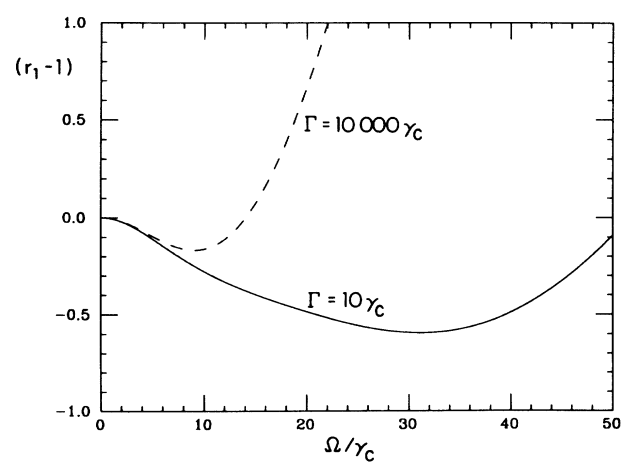

FIG. 4. Relative variance (r_1-1) vs Ω/γ_c for $\Delta_1=10\gamma_c$. In free space the variance has a minimum at $\Omega\simeq\Delta_1$ amounting to 16% of squeezing. In the cavity ($\Delta_2=30\gamma_c$, $\Gamma=10\gamma_c$), the minimum appears at $\Omega'\simeq\Delta_2$ amounting to 60% of squeezing. $\gamma_b/\gamma_c=0.01$.

finds that squeezing occurs mainly for the sideband frequencies, and that the maximum squeezing observed is roughly 15%. Interestingly, however, peak squeezing occurs in a region of parameters different from that which maximizes squeezing in free space.

The most interesting quantum-statistical result of our investigation, however, consists of the large atomic squeezing and dressed-state polarization effects described. From the intuitive physical description of these effects, it is clear that they should not be suppressed by many-body effects. Control of the spectral properties of the vacuum apparently provides one with a powerful means of modifying fundamental light-matter interactions.

Finally, we should mention that we have investigated the optimal conditions for the squeezing and polarizing effects described here, and found that they correspond to moderately broad cavities ($\Gamma > \gamma_b, \gamma_c$), large Rabi frequencies ($\Omega \gg \gamma_{tot}$), cavity frequencies close to one of the transition frequencies between the dressed states of the system, and atom-laser detunings roughly comparable to Ω'.

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