Shot-noise-generated $1/f$ fluctuations in one-dimensional systems

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We study the fluctuations in a one-dimensional conductor, coupled to a current source. The latter produces shot noise at the boundary of the conductor. The conductor is treated as a classical system of mobile negative charges (with a neutralizing fixed positive background). The current source is represented by considering time-dependent boundary conditions for the system. We also assume the existence of dissipation in the system, and describe the interacting particles by a Langevin equation. The problem is then transformed into an equivalent quantum-mechanical problem (in an imaginary time) of charged interacting bosons and is solved in the limit of small density fluctuations. We show how the fluctuations, induced by the source, decay along the wire. We find that whereas the boundary conditions represent uncorrelated shot noise, the power spectrum of the current fluctuations in the conductor has a $1/f$ tail at low frequencies.

I. INTRODUCTION AND RESULTS

The progress made recently in the technology of submicrometer[1] devices has brought forward the issue of quantum fluctuations in small systems. Such fluctuations become important at low temperatures. In addition to the considerable theoretical interest they raise, they also have obvious practical implications. As an apt example let us consider small elements (e.g., tunnel junctions) coupled to an external current source. Recently it has been shown that new interesting effects may be found in such systems.[2,3] It should be emphasized that all the present treatments of these systems are semiphenomenological, and a detailed, microscopic description of a current source does not yet exist. In particular, the fluctuations within the source are usually neglected in the treatments of current-biased systems.

More specifically, if we consider a Josephson junction coupled to a dc current source, the standard approach is to describe such a source by the introduction of a term $-I_{dc}\hat{\Theta}$ in the effective Hamiltonian of the junction. Here I_{dc} is the current bias and $\hat{\Theta}$ is the phase difference between the macroscopic wave function on the left-hand side of the junction and that on the right-hand side of the junction. An alternative approach that was suggested recently[3] consists of representing the bias by a term $-I_{dc}t\hat{P}_{\Theta}$ in a Hamiltonian which contains an explicit time dependence. Here the operator $\hat{P}_{\Theta}$ is the canonical conjugate of $\hat{\Theta}$. In either approach the current source produces a uniform bias on the system (junction) and any fluctuations arising *from the source* are ignored.

In this work we study the time evolution of source-induced fluctuations. This may enable us in principle to evaluate how important such fluctuations are in various physical systems. It is known that due to the discreteness of the charge carriers any current source produces shot noise, whose power spectrum is frequency independent.[4] One may expect that the interactions among the charge carriers will produce correlations in their motion and modify the power spectrum. In order to develop some physical insight into these processes, let us imagine that a current source is coupled to an ideal, one-dimensional conducting lead. Within a semiclassical framework the charge carriers may be viewed as being emitted from the source at random intervals and then move along the ideal lead. Due to the interactions among themselves they tend to rearrange at increasingly uniform intervals, thus minimizing the interaction energy. Obviously, a necessary condition for this to take place is the existence of some dissipative mechanism, which allows the system to get rid of superfluous configurational energy. Under such conditions we may expect the correlations to smooth the shot noise.

In the present work we concentrate on the time evolution of fluctuations which originate at the source. We consider here only a system which consists of a source and a lead, ignoring the effect of any device that may be coupled to that system. We limit ourselves to a classical treatment of such a system. The metallic lead is represented by a classical one-dimensional charged plasma. The source (and the fluctuations induced by the source) are included by imposing the appropriate (time-dependent) boundary conditions on the system.

The two main results we obtain are

(i) Within our picture of dissipative interacting plasma the fluctuation at the source decay and, as we go farther away from the source, the current (and charge-density) profile of the system becomes more uniform. *Thus the fluctuations along the lead are smoothed,* and their effect on a device that may be coupled to the system is weakened.

(ii) The fluctuations at the source (i.e., the boundary conditions) are chosen to be uncorrelated disturbances, to resemble shot noise.[4] *We find that the power spectrum at an arbitrary point in the lead has a $1/\omega$ tail at low frequencies.* The mechanism we find for the noise has some generality and may explain various cases in which $1/\omega$ noise is found.[5]

The outline of this paper is as follows. The model is defined in Sec. II, where the dissipative classical plasma is described in terms of an effective boson Hamiltonian. This Hamiltonian is analyzed and its energy spectrum is found. In Sec. III we discuss boundary conditions that simulate, to some extent, coupling to a realistic current source. The time evolution of the fluctuations is calculated, and we show the existence of $1/\omega$ noise in the system. The generality and applicability of our results are discussed in Sec. IV.

II. FORMULATION OF THE PROBLEM: AN EFFECTIVE BOSON HAMILTONIAN

As was emphasized in the Introduction, we are mainly interested in the evolution of the current fluctuations, induced by the current source, in a metallic lead. Thus, we consider here only the metallic lead, ignoring its coupling to another physical system. Within our classical approach the charge is represented by an ensemble of classical, negatively charged particles which interact via the potential $W(x_1,x_2,\ldots,x_N)$, x_i $(i=1,2,\ldots,N)$ being the coordinate of the ith particle. The source will be represented later by imposing time-dependent boundary conditions on the system. Notice, also, that eventually we shall consider the thermodynamic limit, where the size of the system (i.e., the lead) is large. In order for the system to be electrically neutral we introduce fixed, immobile, uniformly distributed, positive charges. The energy due to the interaction of the positive charges among themselves, as well as the energy due to the interaction of the negative charge carriers with the positive background, are constants of motion and do not contribute to the dynamics of the system. Thus one has to consider only the kinetic energy of the negative charges and the potential energy due to the interactions among themselves. To make our picture more realistic, and to allow for processes that enable the system to get rid of excessive energy and approach configurations with lower energy, we also introduce dissipation. Our starting point is thus to write down a Langevin equation for the system:

$$\dot{x}_i=-\frac{1}{\gamma}\frac{\partial W}{\partial x_i}+\frac{1}{\gamma}\eta_i(t)\ , \tag{2.1}$$

where

$$\langle \eta_i(t)\rangle=0;\langle \eta_i(t)\eta_j(t')\rangle=\gamma^2\sigma\delta_{ij}\delta(t-t') \tag{2.2}$$

and γ is the friction coefficient.

Let us denote the solution of Eq. (2.1) for a particular set of $\{\eta_i\}$ by $\overline{X}(t,\{\eta_i\})$. [$\overline{X}$ is a vector whose components are the solutions for each particle, $x_i(t,\{\eta_i\})$.] Averaging then over all possible realizations of $\{\eta_i\}$ with the corresponding weight $\mathcal{P}(\{\eta_i\})$, we define

$$\tilde{\rho}(X,t)\equiv\overline{\delta(X-\overline{X})}=\int \mathcal{D}\eta\, \delta(X-\overline{X}(t,\{\eta_i\}))\mathcal{P}\{\eta_i\}\ . \tag{2.3}$$

Here X is a vector in the N-dimensional phase space. One can then obtain directly the associated Fokker-Planck equation[6]

$$\frac{\partial\tilde{\rho}}{\partial t}=\sum_i \frac{\sigma}{2}\frac{\partial^2\tilde{\rho}}{\partial x_i^2}+\frac{\partial}{\partial x_i}\left[\frac{1}{\gamma}\frac{\partial W}{\partial x_i}\tilde{\rho}\right]\ . \tag{2.4}$$

We now make the substitution

$$\tilde{\rho}(X,t)=\tilde{\rho}_{\rm eq}^{1/2}\psi(X,t)\ , \tag{2.5}$$

where the stationary equilibrium solution of Eq. (2.4) is given by

$$\tilde{\rho}_{\rm eq}=\exp\left[-\frac{2}{\gamma\sigma}W\right]\ . \tag{2.6}$$

We then obtain an equation for $\psi(X,t)$:

$$-\frac{\partial\psi}{\partial t}=-\frac{\sigma}{2}\sum_i\frac{\partial^2\psi}{\partial x_i^2}+\sum_i\left[\frac{1}{2\gamma^2\sigma}\left[\frac{\partial W}{\partial x_i}\right]^2-\frac{1}{2\gamma}\frac{\partial^2 W}{\partial x_i^2}\right]\psi\ . \tag{2.7}$$

This has the form of a Schrödinger equation in imaginary time, with an effective potential, $\mathcal{V}$, given by

$$\mathcal{V}=\sum_i\left[\frac{1}{2\gamma^2\sigma}\left[\frac{\partial W}{\partial x_i}\right]^2-\frac{1}{2\gamma}\frac{\partial^2 W}{\partial x_1^2}\right]\ . \tag{2.8}$$

The "Hamiltonian" of this "Schrödinger equation" is given by

$$\mathcal{H}=-\frac{\sigma}{2}\sum_i\frac{\partial^2}{\partial x_i^2}+\mathcal{V}\ . \tag{2.9}$$

The associated eigenvalue problem is

$$\mathcal{H}\psi_l=E_l\psi_l\ , \tag{2.10}$$

with $E_0=0$, $\psi_0=\tilde{\rho}_{\rm eq}^{1/2}$. The time evolution of a wave function, $\psi(t)$, is given by

$$\psi(t)=\sum_l\langle\psi_l\,|\,\psi(0)\rangle e^{-E_l t}\,|\,\psi_l\rangle\ , \tag{2.11}$$

where $\psi(0)$ is the initial wave function.

To make further progress we assume that the potential W has the form

$$W=\tfrac{1}{2}\sum_{i,j}v(x_i-x_j)\ , \tag{2.12}$$

where v is a two-particle interaction term. The potential $\mathcal{V}$ then becomes

$$\mathcal{V}=\sum_i\left[\frac{1}{2\gamma^2\sigma}\sum_{j,j'(\neq i)}v'(x_i-x_j)v'(x_i-x_j')-\frac{1}{2\gamma}\sum_{j\,(\neq i)}v''(x_i-x_j)\right]\ . \tag{2.13}$$

To simplify the analysis, we define the particle density in real space:

$$\rho(x)=\sum_{i=1}^{N}\delta(x-x_i)\ . \tag{2.14}$$

We then note that

$$\frac{1}{2\gamma}\sum_{\substack{i,j=1\\(i\neq j)}}^{N}v''(x_i-x_j)=\frac{1}{2\gamma}\sum_{i,j=1}^{N}v''(x_i-x_j)-\frac{1}{2\gamma}Nv''(0)=\frac{1}{2\gamma}\int\int\rho(x)v''(x-x')\rho(x')dx\,dx'-\frac{1}{2\gamma}Nv''(0) \tag{2.15}$$

and

$$\frac{1}{2\gamma^2\sigma}\sum_i\sum_{j,j'(\neq i)}v'(x_i-x_j)v'(x_i-x_j')=\frac{1}{2\gamma^2\sigma}\int dx\int dx'\int dx''\rho(x)\rho(x')\rho(x'')v'(x-x')v'(x-x'')-\frac{1}{\gamma^2\sigma}\int dx\int dx'\rho(x)\rho(x')v'(x-x')v'(0)+\frac{1}{2\gamma^2\sigma}N[v'(0)]^2\ . \tag{2.16}$$

Writing Eq. (2.15) in Fourier space, we have

$$\frac{1}{2\gamma}\sum_{\substack{i,j=1\\(i\neq j)}}v''(x_i-x_j)=-\frac{1}{2\bar{\rho}\gamma}\sum_q q^2v(q)\rho_q\rho_{-q}-\frac{1}{2\gamma}Nv''(0) \tag{2.17}$$

and similarly from Eq. (2.16)

$$\frac{1}{2\gamma^2\sigma}\sum_i\sum_{j,j'(\neq i)}v'(x_i-x_j)v'(x_i-x_{j'})=\frac{1}{2\gamma^2\sigma}\frac{\bar{\rho}^2}{\sqrt{N}}\sum_{q',q''}(-q'\cdot q'')v(q')v(q'')\rho_{q'}\rho_{q''}+\frac{1}{\gamma^2\sigma}\bar{\rho}v'(0)\sum_{q'}(q'\cdot\tilde{q}_0)v(q')\rho_{q'}\rho_{-q'}+\frac{1}{2\gamma^2\sigma}N[v'(0)]^2\ . \tag{2.18}$$

The Fourier transforms are

$$\rho_q=\frac{1}{\sqrt{N}}\int e^{iq\cdot x}\rho(x)dx,\quad \rho(x)=\frac{\bar{\rho}}{\sqrt{N}}\sum_q\rho_q e^{-q\cdot x}\ , \tag{2.19}$$

$$v(q)=\int v(x)e^{iq\cdot x}dx,\quad v(x)=\frac{1}{\Omega}\sum_q v(q)e^{eq\cdot x}\ . \tag{2.20}$$

Here Ω is the volume of the system, $\bar{\rho}$ is the average particle density (we assume unit charge per particle) $\bar{\rho}=N/\Omega$, and $\tilde{q}_0$ is a unit vector in the direction of $\text{grad}v(x)|_{x=0}$. The second term on the right-hand side of Eq. (2.18) is zero. If we consider only small fluctuations from a uniform charge distribution, the dominant contributions from the first term on the right-hand side of Eq. (2.18) will be those with $q'=0$, $q''=0$, or $q'+q''=0$. Keeping only these contributions, Eq. (2.18) becomes, up to a constant,

$$\frac{1}{2\gamma^2\sigma}\bar{\rho}^2\sum q^2v(q)v(-q)\rho_q\rho_{-q}\ . \tag{2.21}$$

The effective potential (2.13) is then given, up to a constant, by

$$\mathcal{V}=\tfrac{1}{2}\bar{\rho}\sum_q q^2v(q)\left[\frac{1}{\gamma}+\frac{v(-q)\bar{\rho}}{\gamma^2\sigma}\right]\rho_q\rho_{-q}\ . \tag{2.22}$$

The original Hamiltonian, Eq. (2.9), is invariant under particle exchange, $x_i\Longleftrightarrow x_j$. This means that it is possible to choose eigenfunctions which have definite parity under exchange of any given pair of particles. The ground state is symmetric [cf. (2.6)]. We assume that the low-lying excitations are also symmetric. The Hamiltonian (2.9) may be regarded, then, as if describing a system of interacting bosons, with the potential $\mathcal{V}$. For our purposes it is convenient to write the boson Hamiltonian in terms of $J(x)$ and $\rho(x)$, the current and density operators.[7] It assumes the form (up to constants)

$$\mathcal{H}=\tfrac{1}{2}\int\Gamma^+(x)\frac{1}{\rho(x)}\Gamma(x)dx+\tfrac{1}{2}\int\int\rho(x)v(x-x')\rho(x')dx\,dx'\ , \tag{2.23}$$

with

$$\Gamma^+(x)=J(x)+\tfrac{1}{2}i\nabla\rho(x)\ . \tag{2.24}$$

We now may write the Hamiltonian in terms of the Fourier transforms ρ_q and

$$\Gamma_q^+=\frac{1}{\sqrt{N}}\int\Gamma^+(x)e^{-iq\cdot x}dx\ .$$

If we apply the assumption of small density fluctuations, we obtain the Hamiltonian[8]

$$\mathcal{H}=\frac{1}{2\bar{\rho}}\sum\Gamma_q^+\Gamma_{-q}+\tfrac{1}{2}\bar{\rho}\sum V(q)\rho_q\rho_{-q}\ , \tag{2.25}$$

which is quadratic in ρ. Identifying the second term on the right-hand side of (2.25) with $\mathcal{V}$ of (2.22) enables us to write

$$V(q)=q^2v(q)\left[\frac{1}{\gamma}+\frac{v(-q)}{\gamma^2\sigma}\bar{\rho}\right]\ . \tag{2.26}$$

Under the assumption of small fluctuations, the Fourier transforms ρ_q and Γ_q^+ can be written as linear combinations of Bose creation and destruction operators. As a result, the Hamiltonian is a bilinear form in Bose operators and can be diagonalized exactly.[8] The excitation energies are given by

$$\omega_q=[\tfrac{1}{4}\sigma^2 q^4+\bar{\rho}\sigma q^2 V(q)]^{1/2} . \tag{2.27}$$

It is interesting to note that the special form we choose for the interaction, Eq. (2.13), gives rise to an excitation spectrum that behaves as q^2 for small q [for small $v(q)$'s that tend to a constant as $q\to 0$], which is different from the usual sound wave spectrum ($\omega_q \sim |q|$) found in interacting Bose systems.[9] The same situation was observed before for a nearest- and next-nearest-neighbor interaction on a lattice.[10] The interaction considered in that reference was the analog of our $\mathcal{V}$. While the classical spectrum obtained was $\sim |q|$, it could be shown that the exact quantum-mechanical spectrum was bounded from above by const$\times q^2$.

In the following discussion we shall regard the ρ_q (rather than the x_i) as the basic degrees of freedom in our problem. Since we are interested in the continuum limit, the number of these degrees of freedom goes to infinity (i.e., $N\to\infty$).

III. DESCRIPTION OF A CURRENT SOURCE AND $1/f$ NOISE

Before we continue with the analysis of the current-current correlations in our system, we recall several relations which are useful for our purposes. The operators ρ_q and J_q may be expressed in terms of a set of operators $\{\alpha_q\}$ which obey the Bose commutation relations. Thus within our approximations (of small density fluctuations)

$$\begin{aligned}\rho_q&=-(Y_q+Z_q)(\alpha_q^\dagger+\alpha_{-q}) ,\\ J_q&=\tfrac{1}{2}q(Y_q-Z_q)(\alpha_q^\dagger-\alpha_{-q}) ,\end{aligned} \tag{3.1}$$

where

$$\begin{aligned}Y_q^2&=\frac{1}{2}\left[1+\left[\frac{\sigma}{2}q^2+\bar{\rho}V(q)\right][\tfrac{1}{4}\sigma^2 q^4+\sigma\bar{\rho}q^2V(q)]^{-1/2}\right] ,\\ Z_q^2&=\frac{1}{2}\left[-1+\left[\frac{\sigma}{2}q^2+\bar{\rho}V(q)\right][\tfrac{1}{4}\sigma^2 q^4+\sigma\bar{\rho}q^2V(q)]^{-1/2}\right]\end{aligned} \tag{3.2}$$

The Hamiltonian [Eq. (2.25)] is diagonalized in the α scheme:

$$\mathcal{H}=\sum_q \omega_q \alpha_q^\dagger \alpha_q . \tag{3.3}$$

Note also that

$$\rho_q\,|\psi_0\rangle=-(Y_q+Z_q)\,|\psi_q\rangle . \tag{3.4}$$

The "wave function" at a given time, when viewed as a function of ρ_q must be real and non-negative. [The reason is that the product of $\tilde{\rho}_{\rm eq}^{1/2}$ (which is Gaussian in ρ_q) is real and positive, and $\tilde{\rho}$ is a probability density.]

We will construct a wave function obeying the above constraint to describe at $t=0$ a density perturbation localized at the origin,[11]

$$\psi(0)=\frac{\exp\left[+\lambda\sum_p c_p\rho_p\right]|\psi_0\rangle}{\langle\psi_0|\exp\left[+\lambda\sum_p c_p\rho_p\right]|\psi_0\rangle} . \tag{3.5}$$

The expectation value of the q component of the density is

$$\langle\rho_q\rangle=\int \rho_q\tilde{\rho}(x,0)=\frac{\langle\psi_0|\rho_q\exp\left[+\lambda\sum_p c_p\rho_p\right]|\psi_0\rangle}{\langle\psi_0|\exp\left[+\lambda\sum_p c_p\rho_p\right]|\psi_0\rangle} . \tag{3.6}$$

Assuming that λ, the parameter describing the strength of the perturbation, is small,

$$\langle\rho_q\rangle=+\lambda\langle\psi_0|c_q\rho_q\rho_{-q}|\psi_0\rangle=\lambda c_q(Y_q+Z_q)^2 . \tag{3.7}$$

It may be easily verified that the time evolution of a weak disturbance at the origin is described by

$$\rho(x,t)=\frac{1}{2\pi}\int dq\rho(q)e^{-\omega_q t}e^{-iqx}dx . \tag{3.8}$$

(For simplicity we have dropped $\langle\ \rangle$ expectation value notation.)

It is important to note that the expectation value of the current, for example, cannot be obtained directly because it involves velocities in addition to coordinates and the knowledge of $\tilde{\rho}$ yields only expectation values for functions of the coordinates. To obtain the current we have to use the continuity equation. Equation (3.8) gives the time-dependent density profile in the frame of reference in which the average current is zero. Transforming to the laboratory frame of reference, we obtain

$$\rho(x,t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}dq\,\rho(q)e^{-\omega_q t}e^{-iq(x-v_d t)} , \tag{3.9}$$

where v_d is the drift velocity in the positive x direction. The continuity equation yields for the current

$$J(x,t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}dq\,\rho(q)\frac{qv_d+i\omega_q}{q}e^{-\omega_q t}e^{-iq(x-v_d t)} . \tag{3.10}$$

What happens now is that instead of one perturbation we have a series of perturbations appearing at the origin at times $\{t_i\}$. We will further assume that each perturbation is removed from the system a time $T=L/v_d$ after it is generated by the source; here L is the length of the system. The last assumption represents a sink at which the current is absorbed. If the number of perturbations present in the system is small, we may still use the linear approximation to obtain

$$J(x,t)=\frac{1}{2\pi}\sum_i \int_{t-t_i<T} dq\, \rho_i(q)\frac{qv_d+i\omega_q}{q}e^{-\omega_q(t-t_i)} \times e^{-iq[x-v_d(t-t_i)]}\Theta(t-t_i) . \quad (3.11)$$

The shot noise in the source is simulated by the fact that the times t_i are uncorrelated apart from having an average difference $[t_{i+1}-t_i]=\tau$, where the square brackets denote averaging with respect to the distribution of $\{t_i\}$. Note also that the magnitude of $\{\rho_i\}$ can be chosen from a certain distribution.

Next, we calculate the Fourier transform of the current-current autocorrelation (which is proportional to the power spectrum of the noise),

$$[J(x,\omega)J(x,-\omega)]=\frac{1}{(2\pi)^2}\sum_{i,j}(e^{i\omega(t_i-t_j)})\left|\int_{-\infty}^{+\infty}dq\,\rho(q)\frac{qv_d+i\omega_q}{q}\frac{e^{-iqx}}{\omega_q+iqv_d+i\omega}\right|^2 . \quad (3.12)$$

All the cross terms in Eq. (3.12) (i.e., $i\neq j$) will average to zero, so that we remain with

$$[J(x,\omega)J(x,-\omega)] =\frac{1}{(2\pi)^2}\frac{T}{\tau}\left|\int_{-\infty}^{+\infty}dq\,\rho(q)\frac{qv_d+i\omega_q}{q} \times\frac{e^{-iqx}}{\omega_q+iqv_d+i\omega}\right|^2 . \quad (3.13)$$

Using the relation $\omega_q=Dq^2$, where

$$D=\left[\tfrac{1}{4}\sigma^2+\bar{\rho}\frac{v(0)\sigma}{\gamma}\left[1+\frac{\bar{\rho}v(0)}{\gamma\sigma}\right]\right]^{1/2} , \quad (3.14)$$

and keeping in mind that $\rho(q)$ is analytic in the complex q plane (because $\rho(x)$ is localized), we obtain for positive x (the lead is in the region $0<x<L$)

$$[J(x,\omega)J(x,-\omega)]=\frac{T}{\tau}\left|\frac{\rho(q_1)(v_d+iDq_1)e^{-iq_1x}}{D(q_1-q_2)}\right|^2 , \quad (3.15)$$

where

$$q_1=\frac{-\frac{iv_d}{D}-i\left[\frac{v_d^2}{D^2}+4i\frac{\omega}{D}\right]^{1/2}}{2} \quad (3.16)$$

and

$$q_2=\frac{-i\frac{v_d}{D}+i\left[\left[\frac{d}{D}\right]^2+4i\frac{\omega}{D}\right]^{1/2}}{2} . \quad (3.17)$$

In the small ω regime ($\omega\ll v_d^2/D$) $q_1=-i(v_d/D)$ and $q_2=0$, so $[J(x,\omega)J(x,-\omega)]$ is independent of ω. For $\omega>(v_d^2/D)$

$$q_1=\left[+\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right]\left[\frac{\omega}{D}\right]^{1/2}$$

and

$$q_2=\left[-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right]\left[\frac{\omega}{D}\right]^{1/2} .$$

We find in the region of small ω, $(\omega/D)^{1/2}x\ll 1$ (but still $\omega>v_d^2/\omega$), that

$$[J(x,\omega)J(x,-\omega)]\alpha\frac{v_d^2}{\omega}\frac{T}{\tau} \quad (3.18)$$

if, for $q\to 0$, $\rho_q\to$const.

Thus we have obtained $1/\omega$ noise[12] that vanishes when v_d vanishes, has a natural lower cutoff at v_d^2/D and is proportional to the size of the system ($T=L/v_d$). The lower cutoff increases as v_d (i.e., the applied voltage) is increased.

IV. DISCUSSION

We have considered here a one-dimensional system of classical charges which interact through a short-range potential. This system is coupled to a current source and a sink which are represented by time-dependent boundary conditions. We have shown that the noise spectrum at a point that is sufficiently close to the source has a $1/\omega$ tail. This result may explain, e.g., the measured noise spectrum of traffic on the roads of Japan.[13] In that case the analog of a current with shot noise may be any disturbance in the traffic. The short-range (self-avoiding) interactions among the cars are probably sufficient to produce the $1/\omega$ noise similarly to the model discussed above. In fact, the range of the interactions is not crucial in the foregoing analysis. If instead of short-range interactions we use bare Coulomb interactions (i.e., $1/r$ potential in a one-dimensional system, embedded in a three-dimensional world), the spectrum will be modified by logarithmic corrections and so will be the noise.

A relevant question at this point is what may actually be measured. In traffic measurement it is possible to measure directly the current flowing through a given point. It has been emphasized, however, by Landauer,[14] that for an electron current the reading of an ampermeter connected to the electrodes of the sample consists of the sum of the contributions of all moving charges in the sample. The measured noise has to be modified by the factor $|(e^{-iq_1L}-1)/q_1|^2$ due to an integration over x [cf. Eq. (3.15)]. Now, remember that our $1/\omega$ result for the noise at a point was a result of the assumption that

$\rho_i(q)$ is constant in the vicinity of the origin. This is the case, e.g., for highway traffic. In the case of a lead coupled to an external 3D source (sink) the situation is different. To guarantee charge neutrality we should consider perturbations $\rho(x)$ which integrate to zero. Thus, we may consider, for example, a disturbance which looks like a derivative of a δ function and contributes an additional factor of q_1. This factor cancels the factor of $1/q_1$ that arises from the integration over x (see the discussion above), and the $1/\omega$ tail is recovered again.

Clearly, there is a possibility of other localized disturbances that integrate to zero (e.g., higher derivatives of the δ function.) Such higher-order disturbances, when present in addition to the simplest (derivatives of δ function) disturbances, will contribute white noise (plus possible higher powers of ω) to the $1/\omega$ tail. The presence of this white noise may affect the observability of the $1/\omega$ tail, because this tail has a lower cutoff at $\omega = v_d^2/D$ (see below) and does not extend down to zero. It may be expected, however, that this effect, which depends on the relative strength of the different disturbances, will not be very important because the simplest disturbances are the easiest to create and are expected, therefore, to dominate.

It should be realized that the whole description of our dissipative system is in a certain sense a low-frequency analysis. Note, for example, that the ions' positions fluctuate in time, and therefore, our description is not expected to hold for frequencies larger than the Debye frequency. Furthermore, the correct description of the system is in terms of a memory kernel accounting for the dissipation and force-force correlation rather than a δ function correlation assumed here. Therefore, our treatment should hold only for frequencies smaller than $1/\Delta t$, where Δt is the width of the memory kernel. Thus, the $1/\omega$ tail may be observed for frequencies that are small compared to Δt^{-1} but larger than v_d^2/D. [For a current density of 10^3 A/m^2 and charge carrier density of 10^{23} cm^{-3}, $v_d \approx 2\times10^{-6}$ cm/sec, and $v_d^2/D \approx 10^{-12}$ Hz (for $D=4$ cm^2/sec). Other numbers (which still imply a lower cutoff frequency) apply to semiconductors.] The lower cutoff can be varied without affecting the upper cutoff, so that our effect can be observed over a finite range. Otherwise (when v_d is large and D is small), white noise is expected. The latter should not be confused with the Nyquist noise, which is an equilibrium noise and may be calculated from the *ground state* of our Hamiltonian. The noise we are treating here is the additional noise generated by disturbing the ground state (equilibrium). The main idea is that there are two sources of noise: thermal and the additional disturbance we generate. The observed correlation is actually the average over the total noise. This implies that even in the absence of the external disturbance the right thing to do is to consider the square of the integrated current and only then average over the equilibrium distribution. What we did here was to average the current over a special nonequilibrium distribution, take the square of it, and only then average over the possible (nonequilibrium) distributions. The two sources for the noise are uncorrelated, and the Nyquist noise may always be added by hand to the noise calculated here.

The physical situation discussed here is very different from the situation in, e.g., vacuum tubes.[15–18] The main difference is the role played by dissipation. In our case dissipation is important, because the electrons lose energy to the ionic system; this is represented by a dissipative term in the Langevin equation. The mechanism by which the electronic system loses energy in a vacuum tube is quite different and takes place mainly at the anode. That is the reason why electrons in a vacuum tube are not expected to obey a Langevin equation. In the former, fluctuations are reduced by a *frequency-independent* factor due to negative feedback effects of the space charge on the current. Here the frequency dependence of the noise is affected by the dissipation.

The mechanism presented in this paper that generates the power-law noise is quite different from other mechanisms that have been suggested previously, including a wide distribution of trapping times and resistance fluctuations.[5] The basic description presented in this article can be applied to higher dimensions; it turns out, however, that the analog of Eq. (3.18) will yield white noise for $d>2$. (In fact, for $d=2$ we will have white noise for $1/r$ potential and logarithmic noise for short-range potentials). In two dimensions the observation of a $1/\omega$ tail is still possible under certain conditions on the correlations of the integrated power.

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